Fractales del Tipo Newton Asociados al Polinomio:
$p(z)=z^9+3z^6+3z^3-1 \quad ,z\in\mathbb{C}$

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Resumen

En esta nota mostramos algunos fractales del tipo Newton asociados al polinomio: $p(z)=z^9+3z^6+3z^3-1 \quad ,z\in\mathbb{C}$.

Introducción: El polinomio $p(z)=z^9+3z^6+3z^3-1$

- Ceros del Polinomio $p(z)$:

$$p(z)=0 \Rightarrow z=z_n \ ,n=1,2,3,4,5,6,7,8,9 \tag{1}$$

- Ceros reales:

$$p(z)=0 \wedge z\in\mathbb{R} \Rightarrow z=z_1=r=\sqrt[3]{\sqrt[3]{2}-1} \tag{2}$$

- Ceros complejos:

$$z_2=r\left(-\frac{1}{2}+\frac{1}{2}i\sqrt{3}\right) \tag{3}$$

$$z_3=r\left(-\frac{1}{2}-\frac{1}{2}i\sqrt{3}\right) \tag{4}$$

$$z_4=\frac{1}{2}\sqrt[3]{u} \tag{5}$$

$$z_5=\frac{1}{2}\sqrt[3]{v} \tag{6}$$

$$z_6=\sqrt[3]{u}\left(-\frac{1}{4}+\frac{1}{4}i\sqrt{3}\right) \tag{7}$$

$$z_7=\sqrt[3]{v}\left(-\frac{1}{4}-\frac{1}{4}i\sqrt{3}\right) \tag{8}$$

$$z_8=\sqrt[3]{u}\left(-\frac{1}{4}-\frac{1}{4}i\sqrt{3}\right) \tag{9}$$

$$z_9 = \sqrt[3]{v}\left(-\frac{1}{4}+\frac{1}{4}i\sqrt{3}\right) \tag{10}$$

donde

$$u = -8-4\sqrt[3]{2}+4i\sqrt{3}\sqrt[3]{2} \quad , \quad v = -8-4\sqrt[3]{2}-4i\sqrt{3}\sqrt[3]{2} \tag{11}$$

El cero real de $p(z)$ es el conocido radical de Ramanujan:

$$r = \sqrt[3]{\sqrt[3]{2}-1} = \sqrt[3]{\frac{1}{9}}-\sqrt[3]{\frac{2}{9}}+\sqrt[3]{\frac{4}{9}} = \left(4\sqrt[3]{\frac{2}{3}}-5\sqrt[3]{\frac{1}{3}}\right)^{1/8} \tag{12}$$

Una fórmula que relaciona r y π es:

$$\pi = 4\sum_{n=1}^{\infty} f_n r^{3n} \tag{13}$$

donde

$$f_n = \left(\frac{3}{2}\right)^n \sum_{k=0}^{[(n-1)/2]} \binom{n-1}{n-2k-1}\frac{(-1)^k}{2k+1} + \sum_{k=0}^{[n/2]} \binom{k}{k-2n} 3^{3k-n} u(k) v(k, n-2k) \tag{14}$$

$$u(n) = \begin{cases} 0 & , n = 0, 2, 4, 6, \ldots \\ \dfrac{(-1)^{(n-1)/2}}{n} & , n = 1, 3, 5, 7, \ldots \end{cases} \tag{15}$$

$$v(n,k) = \begin{cases} 1 & , k \le n \\ 0 & , k > n \end{cases} \tag{16}$$

$$f_n = \left\{\frac{3}{2}, \frac{21}{4}, \frac{13}{4}, 0, -\frac{243}{40}, -\frac{387}{16}, -\frac{3195}{112}, -3, \ldots\right\} \tag{17}$$

Fractales:
Algunos fractales tipo Newton asociados al polinomio $p(z)$

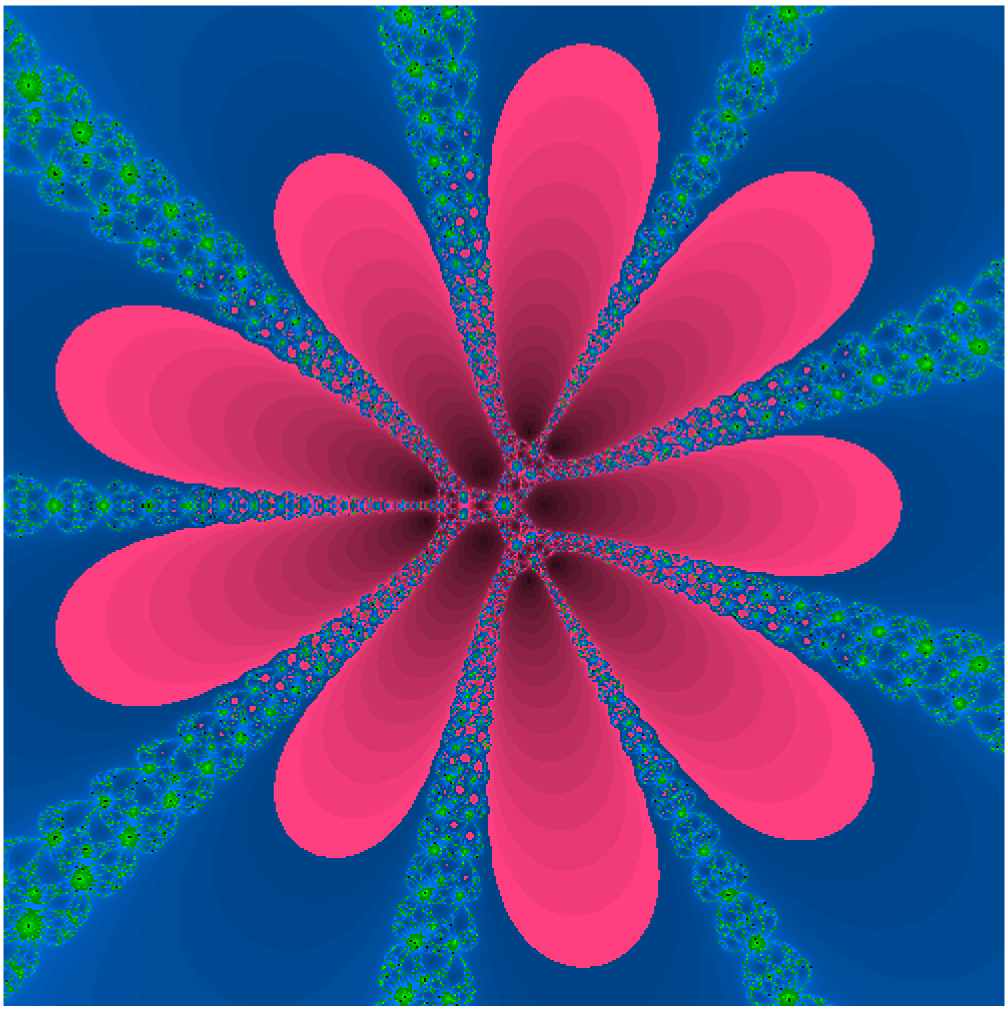

Figura 1. $p(z) \quad , z \in (-8-8i, 8+8i)$

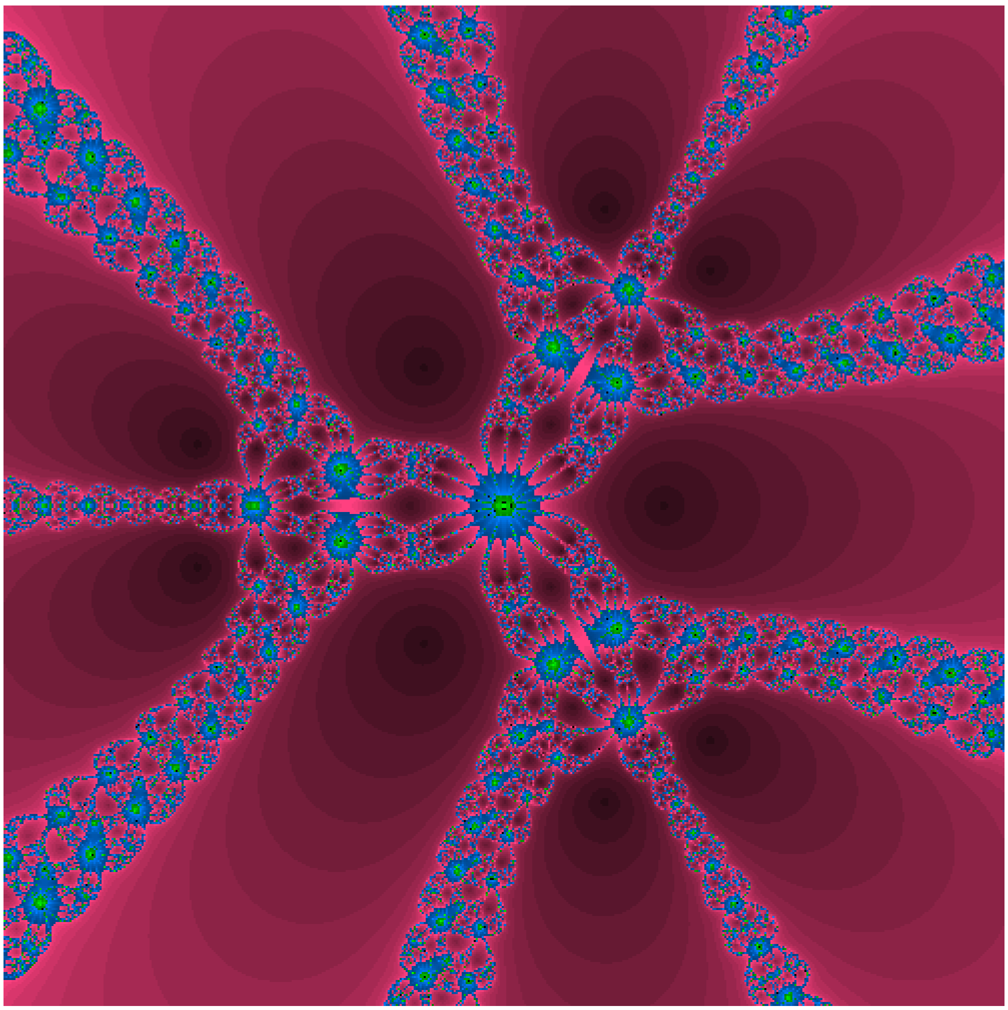

Figura 2. $p(z) \quad , z \in (-2-2i, 2+2i)$

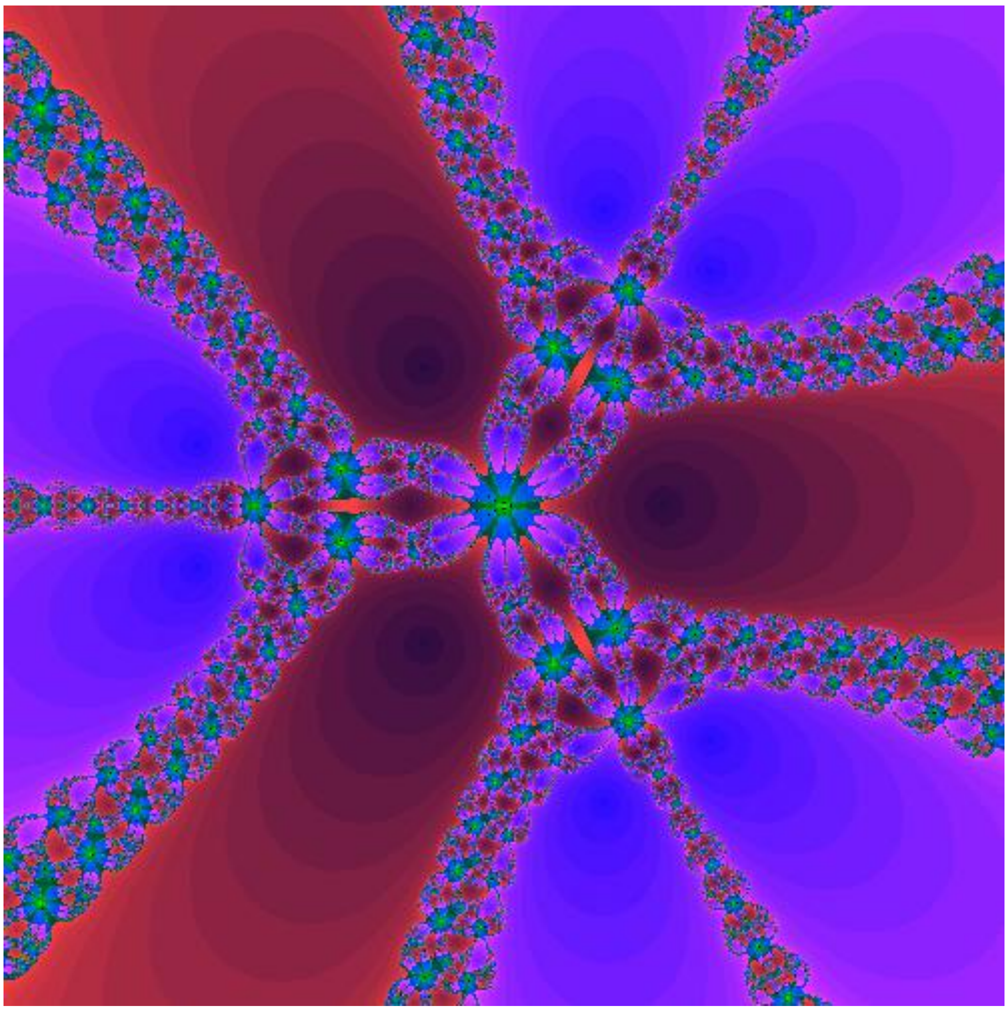

Figura 3. $p(z) \quad , z \in (-2-2i, 2+2i)$

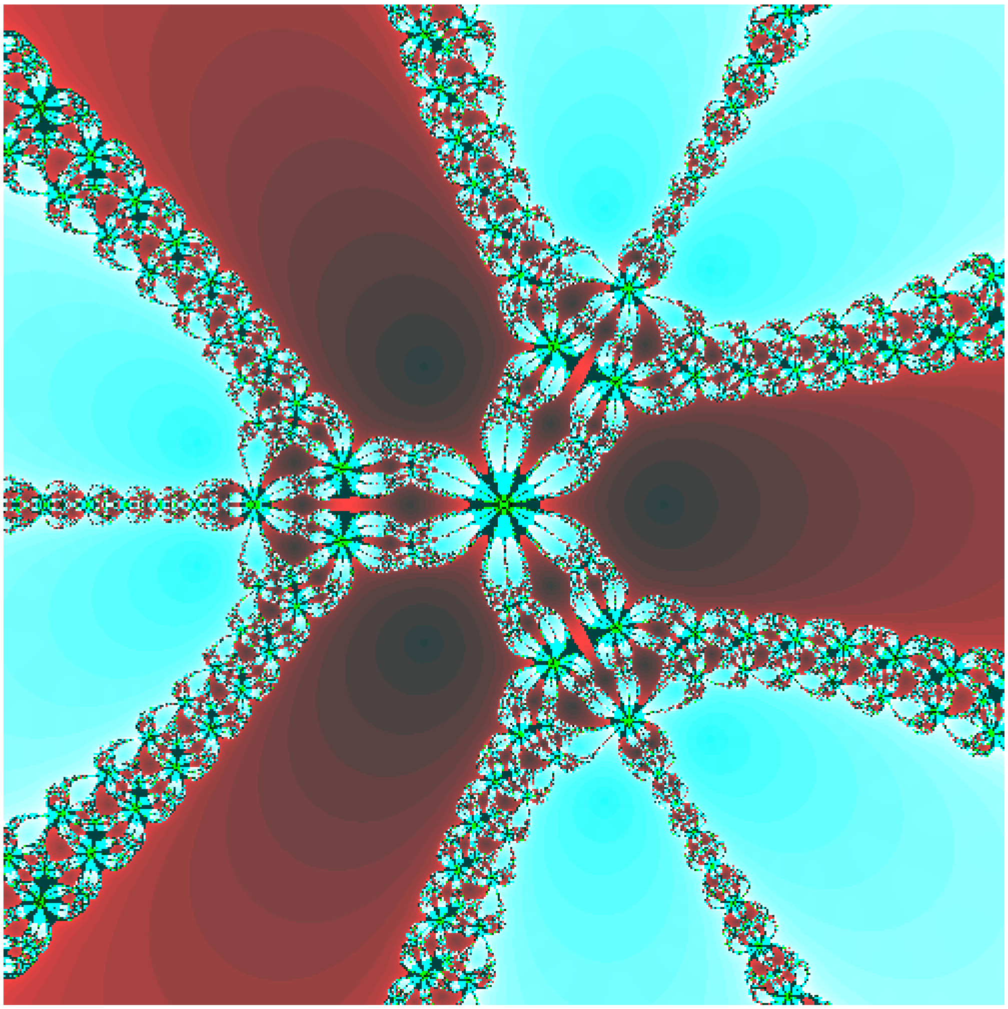

Figura 4. $p(z)\quad ,z \in (-2-2i, 2+2i)$

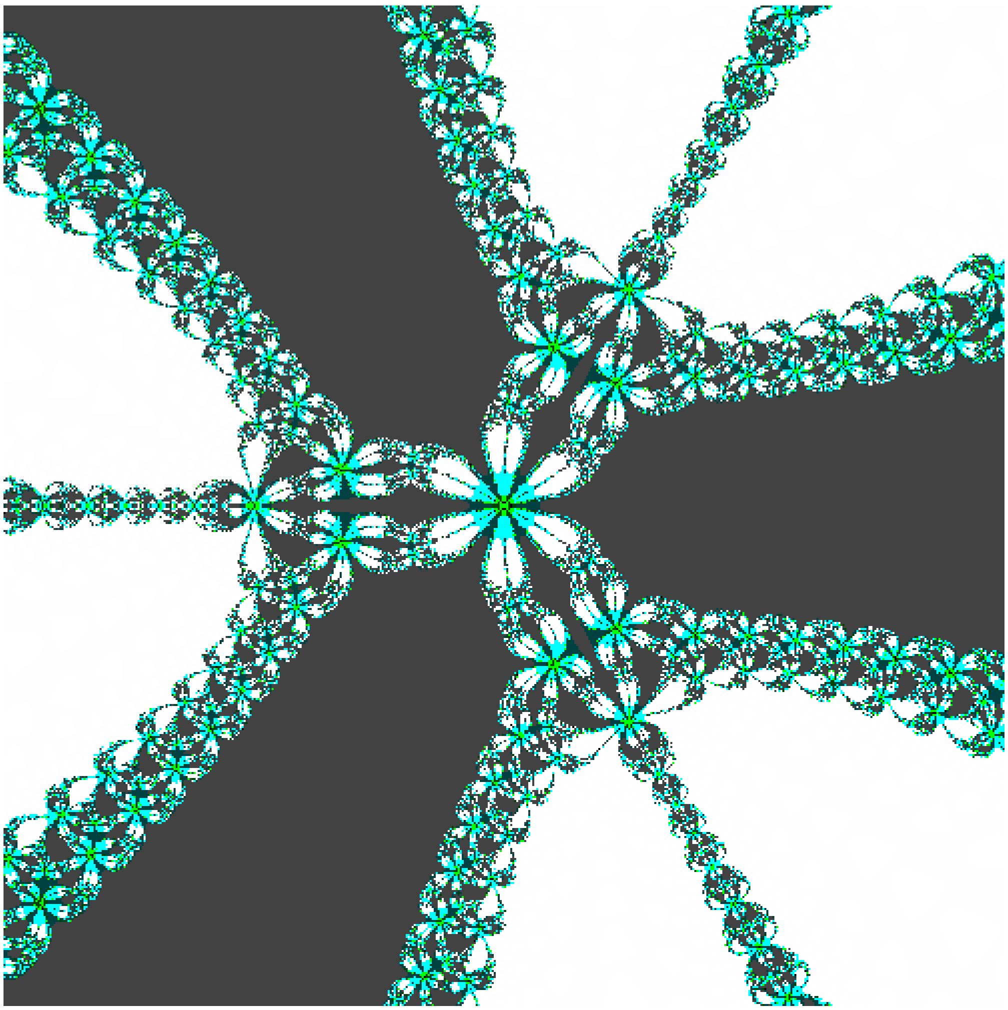

Figura 5. $p(z) \quad , z \in (-2-2i, 2+2i)$

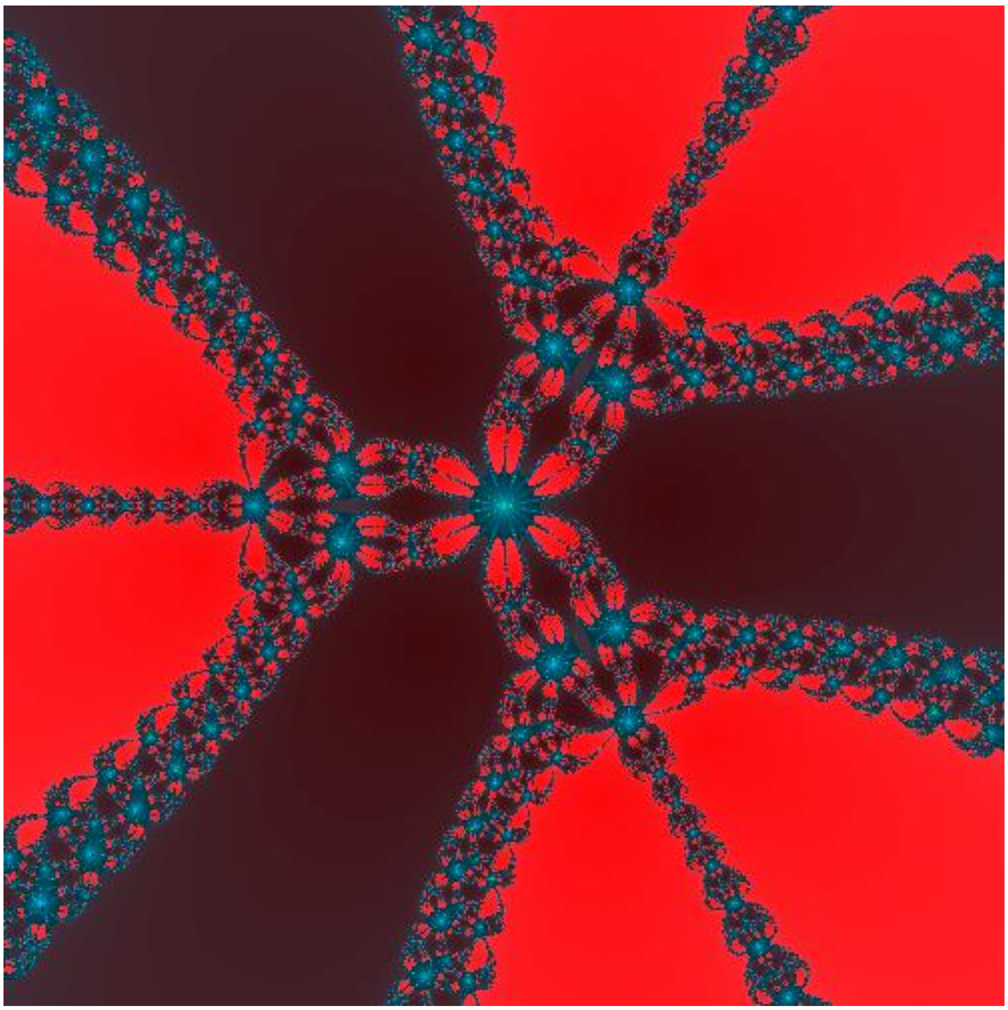

Figura 6. $p(z)$ $,z \in (-2-2i, 2+2i)$

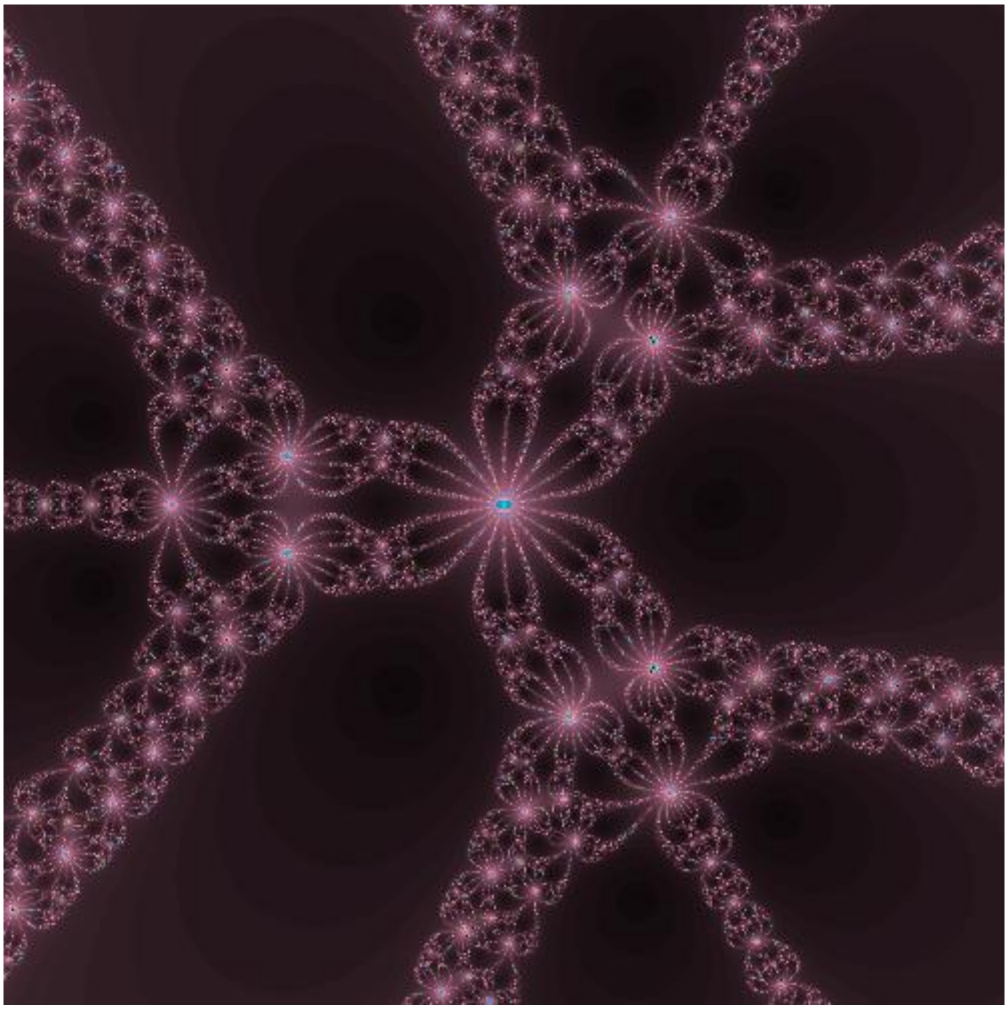

Figura 7. $p(z)$, $z \in (-1.5-1.5i, 1.5+1.5i)$

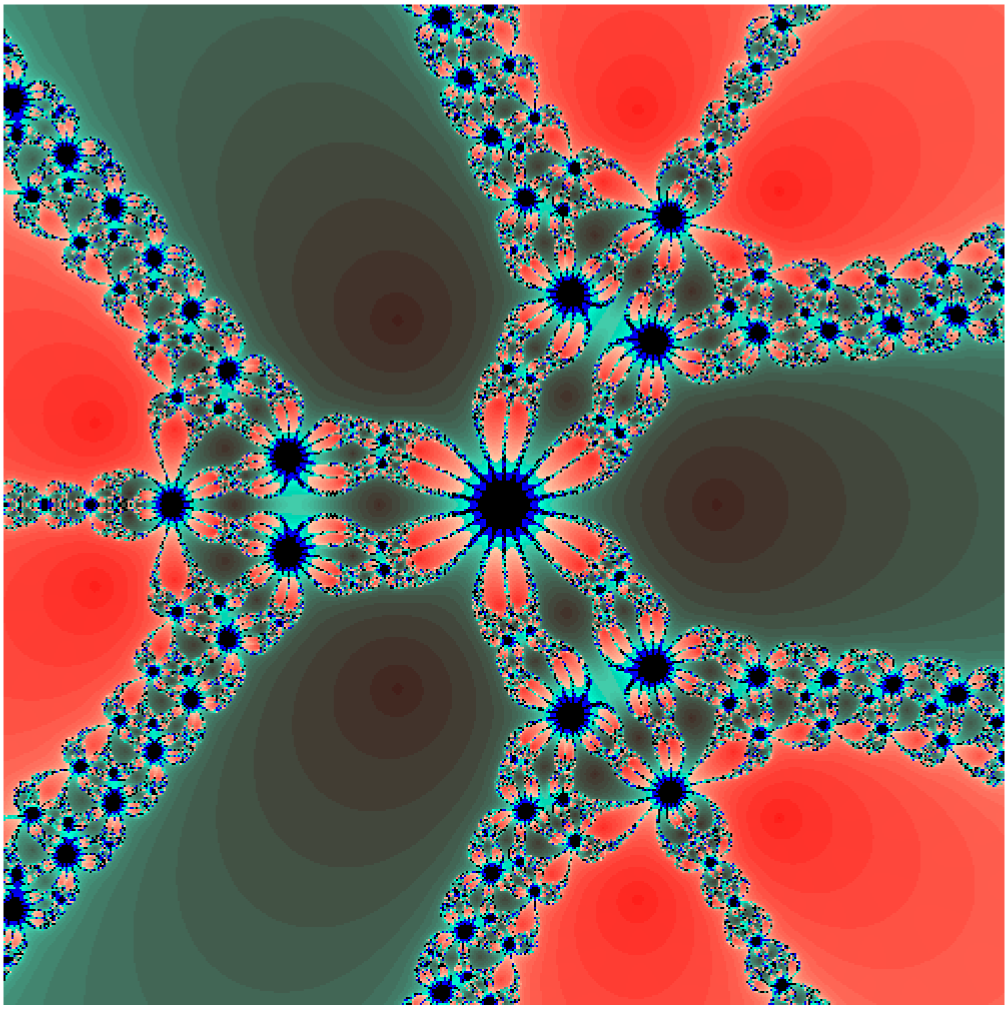

Figura 8. $p(z)$, $z \in (-1.5-1.5i, 1.5+1.5i)$

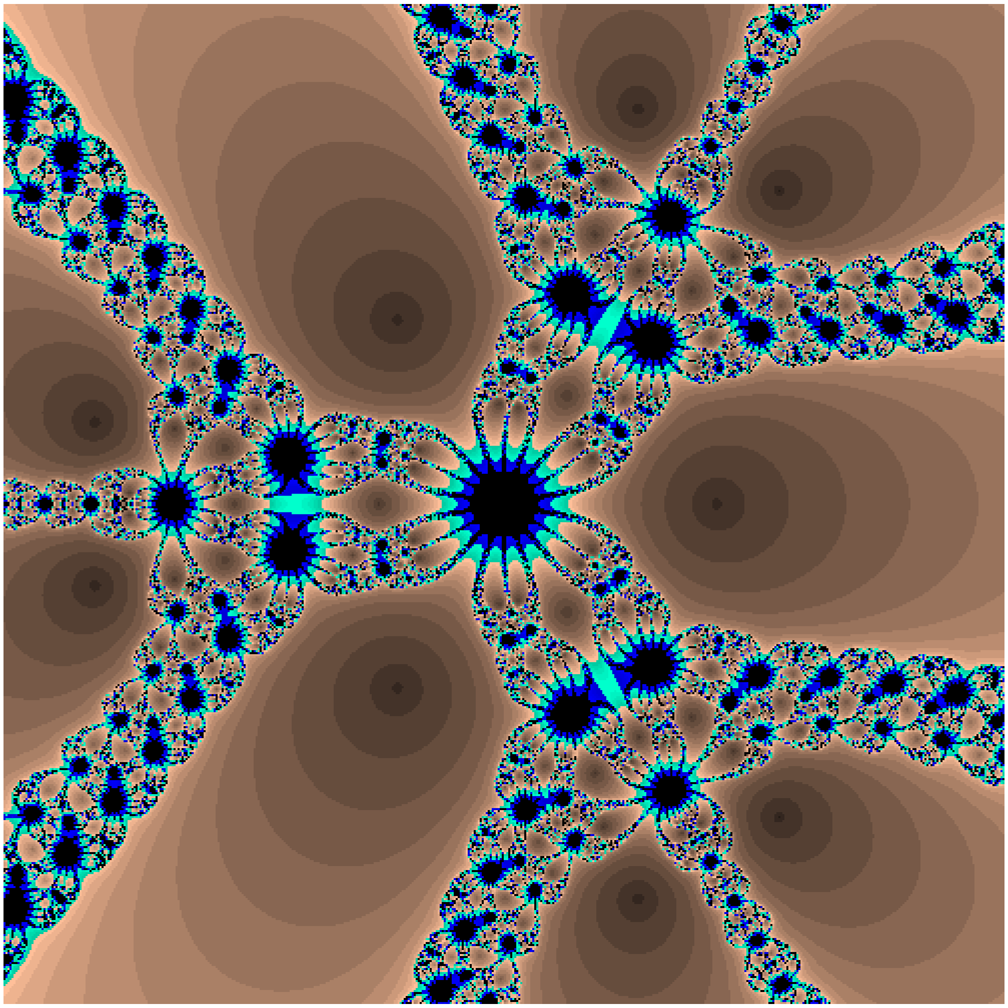

Figura 9. $p(z) \quad , z \in (-1.5-1.5i, 1.5+1.5i)$

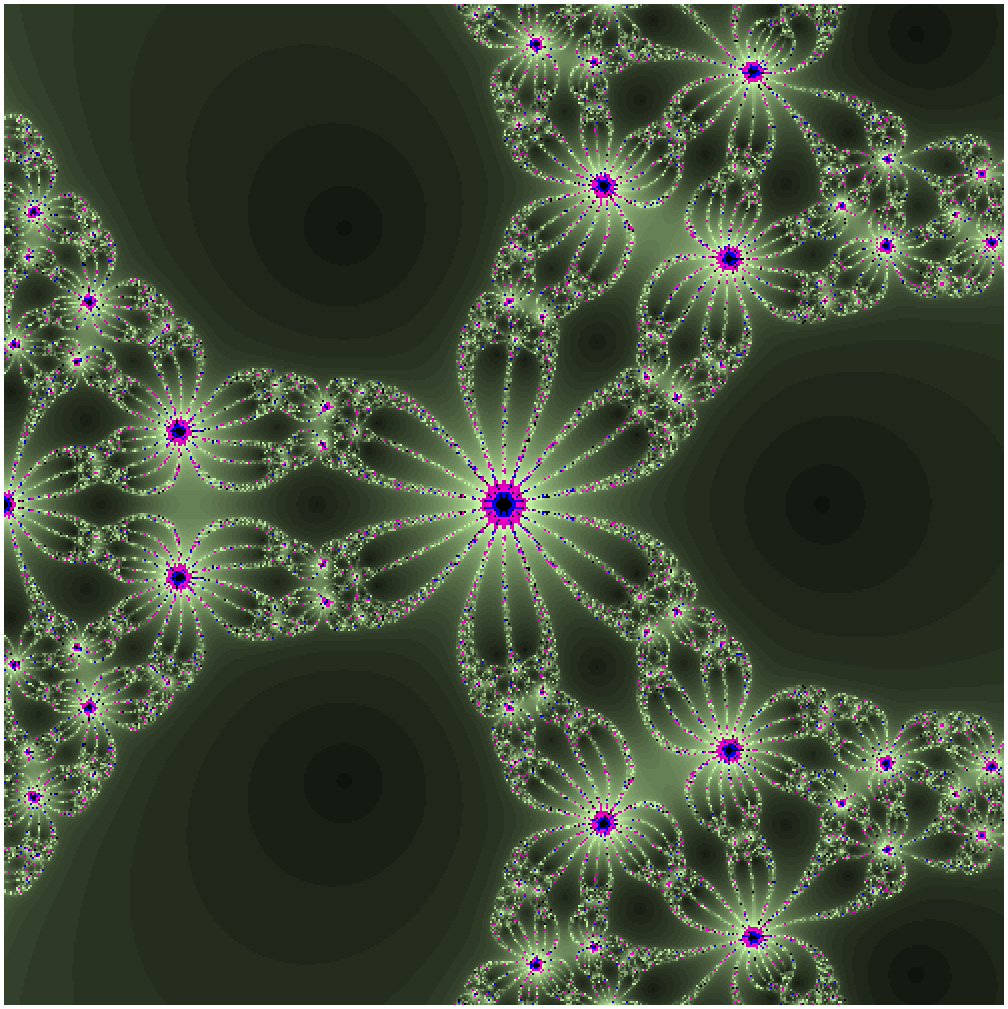

Figura 10. $p(z) \quad , z \in (-1-1i, 1+1i)$

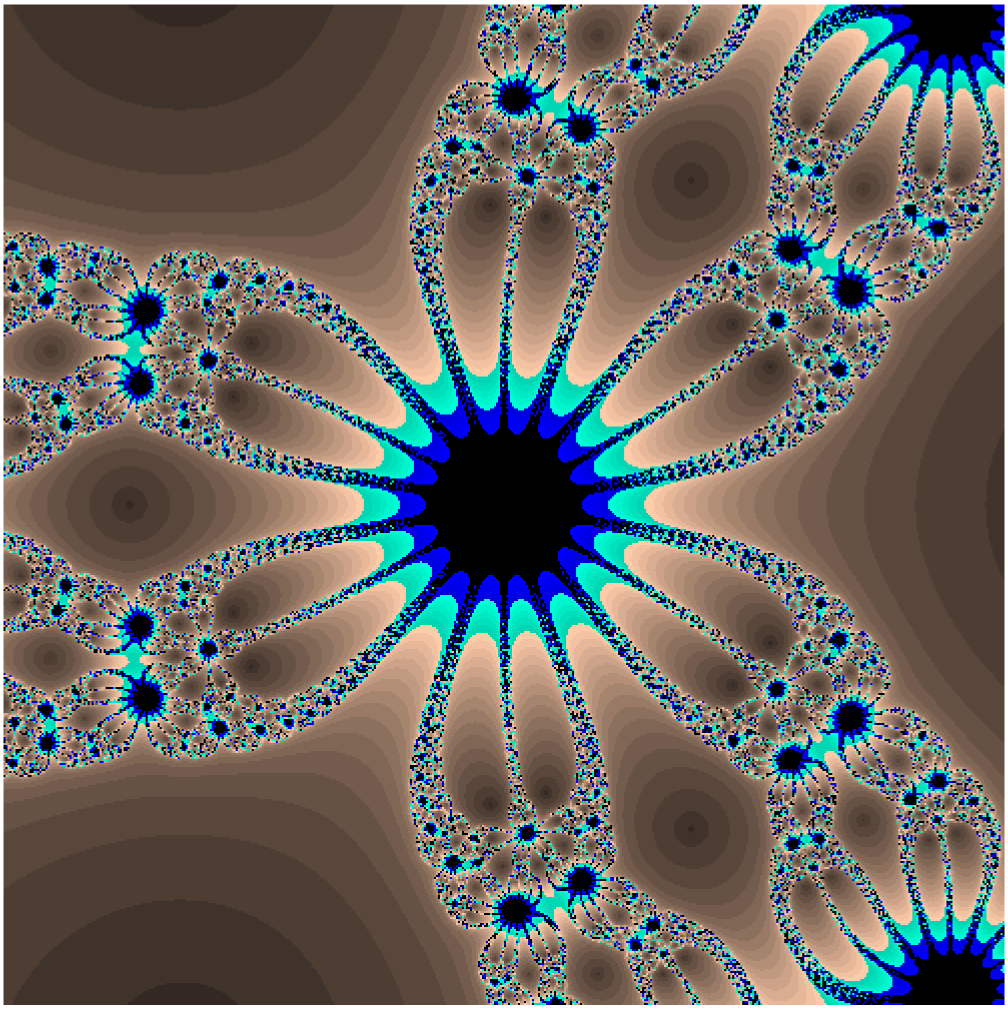

Figura 11. $p(z) \quad , z \in (-0.5-0.5i, 0.5+0.5i)$

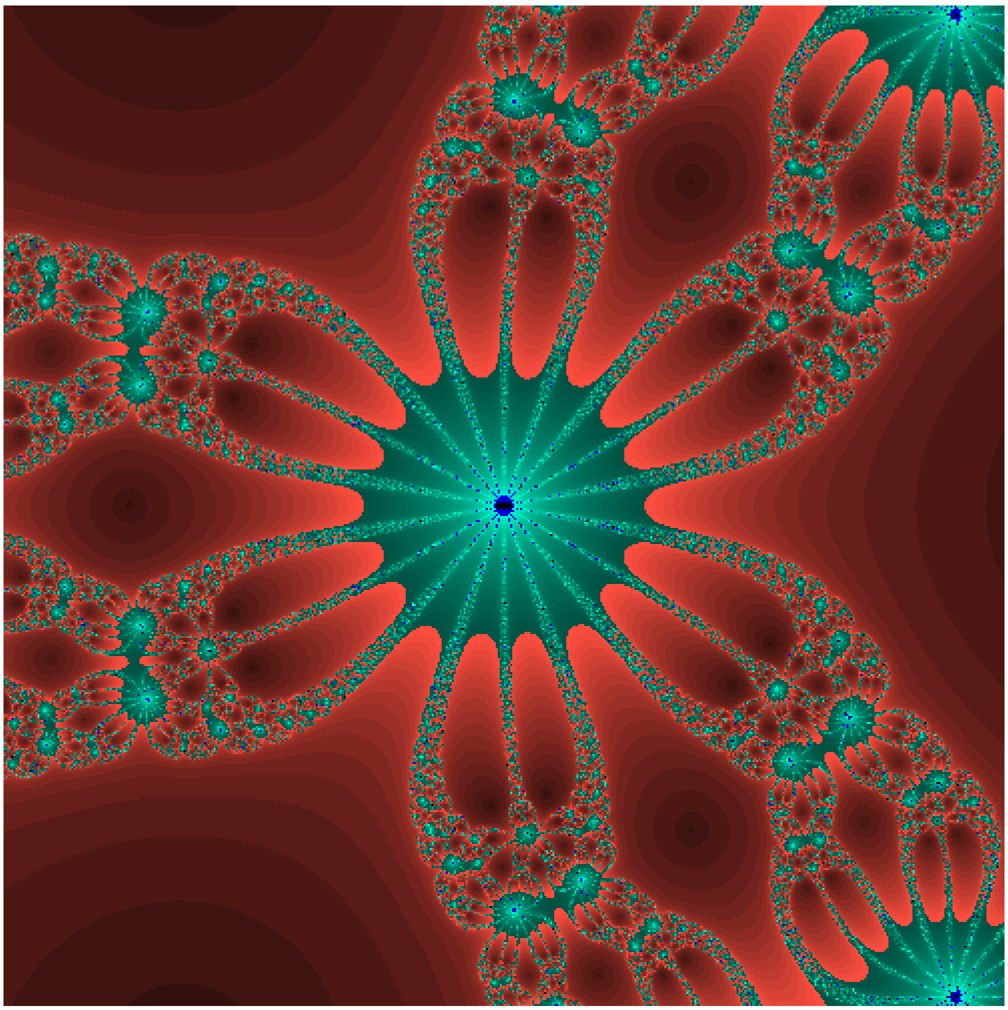

Figura 12. $p(z)$, $z \in (-0.5-0.5i, 0.5+0.5i)$

Referencias

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