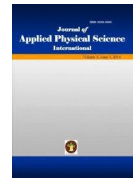
Journal of Applied Physical Science International

10(3): 144-150, 2018
ISSN: 2395-5260 (P), ISSN: 2395-5279 (O)



A DERIVATION OF THE KERR–NEWMAN METRIC USING ELLIPSOID COORDINATE TRANSFORMATION

YU-CHING, CHOU[1,2*]
[1]Health 101 Clinic, 1F., No.97, Guling St., Zhongzheng District, Taipei City 100, Taiwan.
[2]Archilife Research Foundation, 2F.-1, No.3, Ln. 137, Changchun Rd., Zhongshan District, Taipei City 104, Taiwan.

AUTHOR'S CONTRIBUTION

Author YCC designed the study, wrote the protocol, managed the literature searches, the analyses of the study, and wrote the manuscript. The author read and approved the final manuscript.

ARTICLE INFORMATION

Editor(s):
(1) Andrey Trubitsyn, Ryazan State Radio Engineering University, Russia.
(2) Roberto Oscar Aquilano, National University of Rosario (UNR), Argentina.
Reviewers:
(1) W. Obeng-Denteh, Kwame Nkrumah University of Science and Technology, Ghana.
(2) Rongjia Yang, Hebei University, China.
(3) Igor Bulyzhenkov, Lebedev Physics Institute, Russia.
(4) Sie Long Kek, Universiti Tun Hussein Onn Malaysia, Malaysia.
(5) Taxiarchis Papakostas, Technological Educational Institute of Crete (TEI of Crete), Greece.

Received: 15th June 2018
Accepted: 13th September 2018
Published: 16th September 2018

Original Research Article

ABSTRACT

The Kerr–Newman metric describes a special rotating charged mass and is the most general solution for the asymptotically stable "black-hole" solution in the Einstein–Maxwell equations in general relativity. Because these are nonlinear partial differential equations, it is difficult to find an exact analytical solution other than spherical symmetry. This study presented a new derivation of the Kerr–Newman metric which is an extension of the authors' previous research. Using the ellipsoid symmetry of space-time in the Kerr metric, an ellipsoidal coordinate transformation method was performed and the Kerr–Newman metric was more intuitively obtained.

Keywords: Einstein–Maxwell equations; exact solutions; Kerr–Newman black holes.

1. INTRODUCTION

According to the no-hair theorem of black hole, the only physical characteristics of the Einstein–Maxwell equations are three quantities: mass (*M*), electric charge (*Q*), and angular momentum (*J*). In a static case, the angular momentum vanishes and a spherical symmetry, well-known as Schwarzschild metric, is present [1]. The Reissner–Nordstr metric [2,3] depends on whether there is an electric charge. The rotating axisymmetric generalization of the Schwarzschild metric is the Kerr metric [4], whereas the charged rotating generalization of the Reissner–Nordstr metric is the Kerr–Newman metric [5]. These four metrics are often referred to as the "black hole" exact solutions of general relativity.

The Einstein field equations are a set of nonlinear differential equations, where finding an exact analytical solution has proven to be difficult.

**Corresponding author: Email: unclejoe0306@gmail.com;*

Numerous methods for solving the Einstein field equations have been proposed [6,7] (e.g., the Newman–Janis algorithm using complex transformation [8,9], the Newman–Penrose formalism [10,11], and Bäcklund transformations [12,13]). The solution of the rotating Einstein equation has axis symmetry, and the physical derivation can be made from the Ernst equation [14,15]. Despite their great success in dealing with Einstein's equations, these methods are technically complex and expert-oriented.

The Kerr–Newman metric is the most general static/stationary black-hole solution of the Einstein–Maxwell equations. Therefore, it is obviously pertinent to the mathematical framework of general relativity. Traditionally, the general method of the Kerr–Newman solution can be found in *The Mathematical Theory of Black Holes* written by Chandrasekhar [16]. However, the calculation is based on familiarity with spin coefficients and the Newman–Penrose formalism for general relativity, meaning that college students find it too difficult to understand. A concise method for solving the Einstein–Maxwell equations is required.

One study showed that space-time in the Kerr metric has ellipsoid symmetry [17]. Using the oblate coordinate transformation, it is possible to derive the Kerr metric [18]. Another study further rewrote the empty ellipsoidal coordinate into an orthogonal metric form [19]. Our previous research proved that the Kerr metric can be obtained from the ellipsoidal metric ansats in orthogonal form [20]. The aim of this study was to extend our previous research to derive the Kerr–Newman metric by using ellipsoid coordinate transformation. The proposed derivation is similar to that by Schwarzschild, but different from that by Newman and Chandrasekhar.

2. EINSTEIN–MAXWELL EQUATIONS

For all physical quality detailed in this paper, we have adopted $c = G = 1$. Einstein's equation of general relativity is as follows:

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi T_{\alpha\beta} \tag{1}$$

$R_{\alpha\beta}$ is the Ricci tensor, which can be obtained from the Riemann tensor:

$$R_{\alpha\beta} = R^{\mu}_{\alpha\mu\beta} = \partial_\rho \Gamma^{\rho}_{\beta\alpha} - \partial_\beta \Gamma^{\rho}_{\rho\alpha} + \Gamma^{\rho}_{\rho\lambda}\Gamma^{\lambda}_{\beta\alpha} - \Gamma^{\rho}_{\beta\lambda}\Gamma^{\lambda}_{\rho\alpha} \tag{2}$$

$T_{\alpha\beta}$ is the energy-momentum tensor, which in our problem is electromagnetism.

$$T_{\alpha\beta} = \frac{1}{4} g_{\alpha\beta} F_{\mu\nu} F^{\mu\nu} - g_{\beta\nu} F_{\alpha\mu} F^{\nu\mu} \tag{3}$$

Where $F_{\mu\nu}$ is the electromagnetic field strength tensor (note the $T_{\alpha\beta}$ has a zero trace)

$$T = g^{\alpha\beta} T_{\alpha\beta} = \frac{1}{4} g_{\alpha\beta} g^{\alpha\beta} F_{\mu\nu} F^{\mu\nu} - g_{\beta\nu} g^{\alpha\beta} F_{\alpha\mu} F^{\nu\mu} = 0 \tag{4}$$

Because there are four dimensions $g_{\alpha\beta} g^{\alpha\beta} = 4$, Chrostoffel symbols $\Gamma's$ are the connection coefficients obtained through

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \left(\partial_\mu g_{\nu\beta} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu} \right) \tag{5}$$

Einstein's equation can be rewritten in the following form (See Appendix A)

$$R_{\alpha\beta} = 8\pi T_{\alpha\beta} \tag{6}$$

Maxwell's equations are as follows:

$$g^{\mu\nu} \nabla_\mu F_{\nu\alpha} = 0 \tag{7}$$

$$\nabla_{[\mu} F_{\nu\rho]} = 0 \tag{8}$$

Where ∇ is the covariant derivative operator, and the covariant derivative of a rank two tensor $T^{\nu\sigma}$ is defined as

$$\nabla_\mu T^{\nu\sigma} = \partial_\mu T^{\nu\sigma} + \Gamma^{\sigma}_{\mu\lambda} T^{\lambda\nu} + \Gamma^{\nu}_{\mu\lambda} T^{\sigma\lambda} \tag{9}$$

3. SCHWARZSCHILD, REISSNER–NORDSTR, AND KERR SOLUTIONS

The Schwarzschild metric is the first exact solution for the Einstein field equations of general relatively. Although the metric has spherical symmetry, it cannot be used to describe rotation or a charged heavenly body. The Schwarzschild metric is as follows:

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \tag{10}$$

Where, M is the mass of a celestial body.

The Reissner–Nordstr metric is another exact solution for the Einstein field equations, which describe a spherically charged celestial body, as shown in Eq. (11):

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 - \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \tag{11}$$

Where, Q is the charge of a celestial body.

When Q approaches zero, the Reissner–Nordstr metric becomes the Schwarzschild metric. When M

approaches zero, the Reissner–Nordstr metric becomes Minkowski space-time with an electric field, as shown in Eq. (12)

$$ds^2 = \left(1+\frac{Q^2}{r^2}\right)dt^2 - \left(1+\frac{Q^2}{r^2}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \tag{12}$$

The Kerr metric is a generalized form of the Schwarzschild metric and another exact solution for general relativity. It is used to describe a vacuum space-time near a rotational, spherical, symmetrical heavenly body. The Kerr metric in the Boyer–Lindquist coordinate system can be expressed as

$$ds^2 = \left(1-\frac{2Mr}{\rho^2}\right)dt^2 + \frac{4Mra\sin^2\theta}{\rho^2}dtd\phi - \frac{\rho^2}{\Delta_K}dr^2 - \rho^2 d\theta^2 - (r^2+a^2+\frac{2Mra^2\sin^2\theta}{\rho^2})\sin^2\theta d\phi^2 \tag{13}$$

Where, $\rho^2 \equiv r^2 + a^2\cos^2\theta$ and $\Delta_K \equiv r^2 - 2Mr + a^2$ is the delta function of the Kerr metric. a is the spin parameter or specific angular momentum, which is related to the angular momentum J by $a = J/M$.

The Kerr metric has been rewritten in an orthogonal form in some studies [21] as

$$ds^2 = \frac{\Delta_K}{\rho^2}(dt - a\sin^2\theta\, d\phi)^2 - \frac{\rho^2}{\Delta_K}dr^2 - \rho^2 d\theta^2 - \frac{(r^2+a^2)^2\sin^2\theta}{\rho^2}\left(d\phi - \frac{a}{r^2+a^2}dt\right)^2 \tag{14}$$

4. TRANSFORMATION OF THE ELLIPSOID SYMMETRICAL ORTHOGONAL COORDINATE

According to the covariance principle of general relativity covariance, the gravity equation remains unchanged in the coordinate transformation. Thus, Minkowski space-time can be applied as a beginning coordinate:

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 \tag{15}$$

Applying the following ellipsoid to coordinate changes to Eq. (15), with a being the coordinate transformation parameter [22]

$$x \to (r^2+a^2)^{\frac{1}{2}}\sin\theta\cos\phi\,, y \to (r^2+a^2)^{\frac{1}{2}}\sin\theta\sin\phi\,, z \to r\cos\theta\,, t \to t \tag{16}$$

The metric under the new coordinate system becomes

$$ds^2 = dt^2 - \frac{\rho^2}{r^2+a^2}dr^2 - \rho^2 d\theta^2 - (r^2+a^2)\sin^2\theta d\phi^2 \tag{17}$$

Eq. (17) represents an empty space-time of ellipsoid symmetry. If a approaches zero, it morphs into a polar coordinate with spherical symmetry, as shown in Eq. (18):

$$ds^2 = dt^2 - dr^2 - r^2 d\theta^2 - r^2\sin^2\theta d\phi^2 \tag{18}$$

To eliminate the off-diagonal terms ($dtd\phi$) in Kerr metric or Kerr-Newman metric and to let $g_{00}g_{11} = -1$, we can refer to Eq. (14) to define a new coordinate system ($dTd\emptyset$) as follows:

$$dT = dt - a\sin^2\theta\, d\phi \tag{19a}$$

$$d\emptyset = d\phi - \frac{a}{r^2+a^2}dt \tag{19b}$$

After the coordinate transformation, Eq. (17) is transformed into a simplified orthogonal metric, as presented in Eq. (20):

$$ds^2 = \frac{r^2+a^2}{\rho^2}dT^2 - \frac{\rho^2}{r^2+a^2}dr^2 - \rho^2 d\theta^2 - \frac{(r^2+a^2)^2\sin^2\theta}{\rho^2}d\emptyset^2 \tag{20}$$

5. CALCULATING THE RICCI TENSOR

To solve the Kerr–Newman metric, this study started from the ansats of equation Eq. (21) and introduced two new terms $e^{2\nu(r,T)}, e^{2\lambda(r,T)}$:

$$ds^2 = e^{2\nu(r,T)}dT^2 - e^{2\lambda(r,T)}dr^2 - \rho^2 d\theta^2 - \frac{h^2\sin^2\theta}{\rho^2}d\emptyset^2 \tag{21}$$

Where $\rho^2 \equiv r^2 + a^2\cos^2\theta\,, h \equiv r^2 + a^2$

The metric tensor in the matrix form is as follows:

$$g_{\mu\nu} = \begin{pmatrix} e^{2\nu(r,T)} & 0 & 0 & 0 \\ 0 & -e^{2\lambda(r,T)} & 0 & 0 \\ 0 & 0 & -\rho^2 & 0 \\ 0 & 0 & 0 & -\frac{h^2\sin^2\theta}{\rho^2} \end{pmatrix} \tag{22}$$

Chrostoffel symbols and the Ricci tensor can be calculated via Eq. (5) and Eq. (2). In this research, we use the Wolfram Cloud to ensure the correctness of calculations. Totally 14 non-zero Chrostoffel symbols are listed in Eqs. (23)–(32):

$$\Gamma^1_{00} = e^{2(\nu-\lambda)}\,\partial_1\nu \tag{23}$$

$$\Gamma^1_{11} = \partial_1\lambda \tag{24}$$

$$\Gamma^0_{10} = \Gamma^0_{01} = \partial_1\nu \tag{25}$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{r}{\rho^2} \tag{26}$$

$$\Gamma^3_{13} = \Gamma^3_{31} = \frac{2r}{h} - \frac{r}{\rho^2} \tag{27}$$

$$\Gamma^1_{22} = -re^{-2\lambda} \tag{28}$$

$$\Gamma^3_{32} = \Gamma^3_{23} = \cot\theta\left(\frac{h}{\rho^2}\right) \tag{29}$$

$$\Gamma^1_{33} = -re^{-2\lambda}\sin^2\theta\left(\frac{2h}{\rho^2} - \frac{h^2}{\rho^4}\right) \tag{30}$$

$$\Gamma^2_{22} = -\frac{h^2\sin\theta\cos\theta}{\rho^2} \tag{31}$$

$$\Gamma^2_{33} = -\sin\theta\cos\theta\left(\frac{h^3}{\rho^6}\right) \tag{32}$$

After such calculations, the Ricci tensor of all non-zero components can be obtained as follows:

$$R_{00} = [\partial_0^2\lambda + (\partial_0\lambda)^2 - \partial_0\lambda\partial_0\nu] + e^{2(\nu-\lambda)}\left[-\partial_1\nu\,\partial_1\lambda + (\partial_1\nu)^2 + \partial_1^2\nu + \frac{2r}{h}\partial_1\nu\right] \tag{33}$$

$$R_{01} = R_{10} = \frac{2}{r}\partial_0\lambda \tag{34}$$

$$R_{11} = e^{2(\lambda-\nu)}[\partial_0^2\lambda + (\partial_0\lambda)^2 - \partial_0\lambda\partial_0\nu] + \partial_1\nu\,\partial_1\lambda - (\partial_1\nu)^2 - \partial_1^2\nu + \frac{2r}{h}\partial_1\lambda \tag{35}$$

$$R_{22} = e^{-2\lambda}\left(r(\partial_1\lambda - \partial_1\nu) - 1 + \frac{2r^2}{\rho^2} - \frac{2r^2}{h}\right) + \frac{h^2}{\rho^4}\left(\frac{5r^2 - 4\rho^2}{h}\right) \tag{36}$$

$$R_{33} = \sin^2\theta\left(\frac{2h}{\rho^2} - \frac{h^2}{\rho^4}\right)\left[e^{-2\lambda}\left(r(\partial_1\lambda - \partial_1\nu) - \frac{r^2}{\rho^2}\right) + \frac{h^2}{\rho^4}\left(\frac{5r^2-4\rho^2}{h}\right)\left(\frac{2h}{\rho^2} - \frac{h^2}{\rho^4}\right)^{-1}\right] \tag{37}$$

6. COMPONENTS OF THE ELECTROMAGNETIC FIELD STRENGTH TENSOR AND THE STRESS TENSOR

Because of spherical symmetry, the only non-zero components of the electric and magnetic field are the radial components, which should be independent of θ and ϕ. Therefore, the radial component of the electric field has a form of

$$E_r = E_1 = F_{01} = -F_{10} = f(r,T) \tag{38}$$

The other components are zero because there are no currents or magnetic monopoles. In the matrix form, we have

$$F_{\alpha\beta} = \begin{pmatrix} 0 & f(r,T) & 0 & 0 \\ -f(r,T) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{39}$$

The components of the stress-energy tensor can now be computed using Eq. (3). By considering the first term in parenthesis, conducting the summation gives

$$\begin{aligned}\frac{1}{4}g_{\alpha\beta}F_{\mu\nu}F^{\mu\nu} &= \frac{1}{4}g_{\alpha\beta}\left(F_{\mu0}F^{\mu0} + F_{\mu1}F^{\mu1}\right) \\ &= \frac{1}{4}g_{\alpha\beta}(F_{10}F^{10} + F_{01}F^{01}) \\ &= \frac{1}{4}g_{\alpha\beta}(2F_{01}F^{01}) = \frac{1}{2}g_{\alpha\beta}F_{01}F^{01}\end{aligned} \tag{40}$$

For the second term, we get

$$g_{\beta\nu}F_{\alpha\mu}F^{\nu\mu} = g_{\beta\nu}F_{\alpha0}F^{\nu0} + g_{\beta\nu}F_{\alpha1}F^{\nu1} = g_{\beta1}F_{\alpha0}F^{10} + g_{\beta0}F_{\alpha1}F^{01} \tag{41}$$

Thus, we can write Eq. (3) as

$$T_{\alpha\beta} = \frac{1}{2}g_{\alpha\beta}F_{01}F^{01} - g_{\beta1}F_{\alpha0}F^{10} - g_{\beta0}F_{\alpha1}F^{01} \tag{42}$$

The components of the stress-energy tensor can now be easily obtained. We have

$$T_{00} = -\frac{1}{2}g_{00}F_{01}F^{01} = \frac{1}{2}e^{2\nu(r,T)}f(r,T)^2 \tag{43}$$

$$T_{11} = -\frac{1}{2}g_{11}F_{01}F^{01} = -\frac{1}{2}e^{2\lambda(r,T)}f(r,T)^2 \tag{44}$$

$$T_{22} = \frac{1}{2}g_{22}F_{01}F^{01} = \frac{1}{2}\rho^2 f(r,T)^2 \tag{45}$$

$$T_{33} = \frac{1}{2}g_{33}F_{01}F^{01} = \frac{1}{2}\rho^2\left(\frac{h^2}{\rho^4}\right)\sin^2\theta f(r,T)^2 = T_{22}\left(\frac{h^2}{\rho^4}\right)\sin^2\theta \tag{46}$$

$$T_{01} = 0 \tag{47}$$

From, $R_{01} = 8\pi T_{01} = 0$, we get $\frac{2}{r}\partial_0\lambda = 0, \lambda = \lambda(r)$ λ is time dependent.

Using the results and adding them to Eqs. (33) and (35), we obtain

$$e^{-2\lambda(r)}R_{00} + e^{-2\nu(r,T)}R_{11} = \frac{2r}{h}(\partial_1\nu(r,T) + \partial_1\lambda(r)) = 0 \tag{48}$$

Solving this equation yields $\nu(r,T) + \lambda(r) = const.$ Next, the time coordinate is redefined in Eq. (21) by replacing $dT \to e^{const.}dT$, so that $\nu(r,T) = \nu(r) = -\lambda(r)$.

Therefore, $e^{2\nu(r)} = e^{-2\lambda(r)}$ (49)

7. SOLVING THE MAXWELL EQUATIONS

To solve the Maxwell equations for the form of the electromagnetic field strength tensor in Eq. (41), the r component of Eq. (9) gives us

$$\partial_0 F_{01} - \Gamma^{\alpha}_{00} F_{\alpha 1} - \Gamma^{\alpha}_{01} F_{1\alpha} = 0$$

$$\partial_0 F_{01} - \Gamma^{0}_{00} F_{01} - \Gamma^{0}_{01} F_{10} = \partial_0 F_{01} - F_{01}(\Gamma^{0}_{00} + \Gamma^{0}_{01}) = 0 \quad (50)$$

Because $\Gamma^0_{00} = 0$ and $\Gamma^0_{01} = \partial_0 \lambda = 0$ from the above equation, we have $\partial_0 F_{01} = 0$, implying that the time-radial component of the electromagnetic field strength tensor is not time dependent:

$$F_{01} = f(r) \quad (51)$$

To find the explicit form of f, we use the following identity: for any given antisymmetric rank two tensor, $T^{\mu\nu}$, and diagonal metric, the following identity is true

$$\nabla_{\mu} T^{\mu\nu} = \frac{1}{\sqrt{|g|}} \partial_{\mu}(\sqrt{|g|} T^{\mu\nu}) \quad (52)$$

In our metric, we have $\sqrt{|g|} = r^2 \sin\theta$. If we apply the aforementioned identity to Eq. (9), we obtain

$$\nabla_{\mu} F^{\mu\nu} = \frac{1}{r^2 \sin\theta} \partial_{\mu}(r^2 \sin\theta T^{\mu\nu}) = 0 \quad (53)$$

For the t component, we have

$$\partial_r (r^2 F^{10}) = \partial_r (r^2 g^{11} g^{00} F_{10}) = \partial_r (r^2 f) = 0 \quad (54)$$

Therefore,

$$f(r) = \frac{const.}{r^2} \quad (55)$$

The Gauss flux theorem gives $. = Q/\sqrt{4\pi}$, where Q is the total electric charge of a black hole. Finally, the electromagnetic field strength tensor is obtained as

$$F_{\alpha\beta} = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} 0 & Qr^{-2} & 0 & 0 \\ -Qr^{-2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (56)$$

Only one unknown variable is left, $\nu(r)$, which is obtained by Eq. (49). To solve this, one equation is enough to determine the unknown. Let us consider

$$\lim_{a\to 0} R_{22} = \lim_{a\to 0} 8\pi T_{22} \quad (57)$$

We also need the following limit conditions

$$\lim_{a\to 0} h = r^2 , \lim_{a\to 0} \rho = r \quad (58)$$

$$\lim_{a\to 0} R_{22} = e^{-2\lambda}(r(\partial_1 \lambda - \partial_1 \nu) - 1) + 1 = e^{2\nu}(-2r\partial_1 \nu - 1) + 1 \quad (59)$$

$$\lim_{a\to 0} 8\pi T_{22} = \frac{8\pi}{2} r^2 \left(\frac{Q}{\sqrt{4\pi} r^2}\right)^2 = \frac{Q^2}{r^2} \quad (60)$$

Combining Eqs. (59) and (60), we obtain the following equation

$$\partial_r (r e^{2\nu}) = 1 - \frac{Q^2}{r^2} \quad (61)$$

After integrating and solving Eq.(61), we get

$$\lim_{a\to 0} e^{2\nu} = 1 + \frac{C}{r} + \frac{Q^2}{r^2}, let\ C = -2M \quad (62)$$

$$\lim_{a\to 0} e^{2\nu} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = \frac{r^2 - 2Mr + Q^2}{r^2} \quad (63)$$

$$\lim_{Q\to 0} e^{2\nu} = \frac{r^2 - 2Mr + a^2}{r^2 + a^2 \cos^2\theta} \quad (64)$$

$$e^{2\nu} = \frac{r^2 - 2Mr + a^2 + Q^2}{r^2 + a^2 \cos^2\theta} \equiv \frac{\Delta_{KN}}{\rho^2},\ e^{2\lambda} = e^{-2\nu} \equiv \frac{\rho^2}{\Delta_{KN}} \quad (65)$$

Where, $\Delta_{KN} \equiv r^2 - 2Mr + a^2 + Q^2$ is the delta function of the Kerr–Newman metric.

Finally, we obtain the Kerr–Newman metric through

$$\begin{aligned} ds^2 = & \frac{r^2 - 2Mr + a^2 + Q^2}{r^2 + a^2\cos^2\theta}(dt - a\sin^2\theta\, d\phi)^2 \\ & - \frac{r^2 + a^2\cos^2\theta}{r^2 - 2Mr + a^2 + Q^2} dr^2 - \rho^2 d\theta^2 \\ & - \frac{(r^2 + a^2)^2 \sin^2\theta}{\rho^2}\left(d\phi - \frac{a}{r^2 + a^2} dt\right)^2 \end{aligned} \quad (66)$$

8. DISCUSSION

Two axi-symmetric rotation solutions of general relativity, the Kerr metric and Kerr–Newman metric, have the same ellipsoid orthogonal coordinates embedded delta function (Δ_K and Δ_{KN}). When mass M and charge Q approach zero, both delta functions degenerate to $r^2 + a^2$, and two rotation solutions degenerate to vacuum ellipsoidal coordinates, as shown in Eq. (20).

From the aforementioned discussion, we can see that the Kerr metric and Kerr–Newman metric have an ellipsoid, space-time geometry. This geometry can be obtained through the proposed ellipsoid coordinate transformation or the Newman–Janis algorithm complex transformation coordinates [23]. In 1965, Newman used this algorithm to obtain the Kerr–Newman metric. This close association led us to speculate that all axi-symmetric rotation solutions can be represented in the same ellipsoidal coordinate.

In summary, the coupled Einstein–Maxwell equations have been solved and a metric has been presented that describes the geometry of the space-time surrounding a rotating black hole with a static electric charge. The proposed metric is a straightforward derivation, which deserves further study to determine whether this method could be extended to other rotating Einstein field equations.

ACKNOWLEDGEMENTS

I would like to specially thank C.E.O. Ruby of Health 101 clinic, Dr. Simon Lin of Academia Sinica, and Prof. Anthozy Zee. I feel also indebted to my sister, Shanrong, for refining my writing. This manuscript was edited by Wallace Academic Editing.

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

1. Schwarzschild K. On the gravitational field of a point-mass, according to Einstein's theory (English translation). Abraham Zelmanov J. 1916;1:10-19.
2. Reissner H. Über die eigengravitation des elektrischen feldes nach der einsteinschen theorie. Annalen der Physik (in German). 1916;50:106-120.
3. Nordström G. On the energy of the gravitational field in einstein's theory verhandl. Kon. Ned. Akad. Wet. Afdel. Natuurk., Amsterdam. 1918;26:1201-1208.
4. Kerr RP. Gravitational field of a spinning mass as an example of algebraically special metrics. Phys. Rev. Lett. 1963;11:237-238.
5. Newman E, Couch E, Chinnapared K, Exton A, Prakash A, Torrence R. Metric of a rotating, charged mass. J Math Phys. 1965;6(6):918-919.
6. MacCallum MAH. Finding and using exact solutions of the Einstein equations. AIP Conference Proceedings. 2006;841:129-143.
7. Rendall AD. Local and global existence theorems for the Einstein equations. Living Rev. Relativ. 2000;3:1.
   Available:https://doi.org /10.12942/lrr-2000-1
8. Adamo T, Newman ET. The Kerr-Newman metric: A review. Scholarpedia. 2014;9:31791.
9. Brauer O, Camargo HA, Socolovsky M. Newman-Janis Algorithm revisited. Int J Theor Phys. 2015;54:302.
   Available:https://doi.org/10.1007/ s10773-014-2225-3
10. Penrose R, Rindler W. Spinors and space-time. Cambridge U.P, Cambridge; 1984.
11. Nerozzi A. A new approach to the Newman-Penrose formalism. arXiv:1109.4400 [gr-qc]; 2011.
12. Kramer D, Stephani H, MacCallum M, Herlt E. Exact solutions of Einstein's equations. Cambridge U.P., Cambridge; 1981.
13. Harrison K. Bäcklund transformation for the Ernst equation of general relativity. Phys. Rev. Lett. 1978;41:1835.
14. Meinel R. A physical derivation of the Kerr-Newman black hole solution. arXiv:1310.0640 [gr-qc]; 2013.
15. Kramer D. The ernst equation in general relativity Czech. J. Phys. 1987;37:350.
    Available:https://doi.org/10.1007/BF01597261
16. Chandrasekhar S. The mathematical theory of black holes. Oxford U.P., New York. 1983; 273-318.
17. Krasinski A. Ellipsoidal space-time, sources for the Kerr Metric. Ann. Phys. 1978;112(1): 22-40.
18. Enderlein J. A heuristic way of obtaining the Kerr metric. Am. J. Phys. 1997;65(9):897-902.
19. Branislav N, Milan P. A possible intuitive derivation of the Kerr Metric in orthogonal form based on ellipsoidal metric ansatz. arXiv:1210.5922 [gr-qc]; 2012.
20. Yu-Ching C. A derivation of the Kerr metric by ellipsoid coordinate transformation. Int. J. Phys. Sci. IJPS. 2017;12(11):130.
21. O'Neil B. The geometry of Kerr black holes. A. K. Peters, Wellesley, Massachusetts. 1995;69.
22. Landau L, Lifshitz E. The classical theory of relativity. Pergamon Press. 1987;2.
23. Newman E, Janis A. Note on the Kerr spinning-particle metric. J. Math. Phys. 1965;6: 915.

Appendix A Derivation of Eq. (6)

Apply $g^{\alpha\beta}$ to Einstein's field equation of general relativity to obtain

$$g^{\alpha\beta}\left(R_{\alpha\beta}-\frac{1}{2}g_{\alpha\beta}R\right)= 8\pi T_{\alpha\beta}g^{\alpha\beta} \tag{A.1}$$
$$R-\frac{1}{2}(4R)=8\pi T \tag{A.2}$$
$$-R=8\pi T \tag{A.3}$$

Because the trace $T=0$, Therefore

$$R_{\alpha\beta}=\frac{1}{2}g_{\alpha\beta}R+8\pi T_{\alpha\beta} \tag{A.4}$$
$$=\frac{1}{2}g_{\alpha\beta}(-8\pi T)+8\pi T_{\alpha\beta} \tag{A.5}$$
$$=8\pi\left(T_{\alpha\beta}-\frac{1}{2}g_{\alpha\beta}T\right) \tag{A.6}$$
$$=8\pi T_{\alpha\beta} \tag{A.7}$$