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On r-Dynamic Coloring of the Triple Star Graph Families

T.Deepa and M. Venkatachalam

(Department of Mathematics, Kongunadu Arts and Science College, Coimbatore - 641 029,Tamilnadu, India)

E-mail: deepathangavelu88@gmail.com, venkatmaths@gmail.com

Abstract: An r-dynamic coloring of a graph G is a proper coloring c of the vertices such that $|c(N(v))| \geq \min\{r, d(v)\}$, for each $v \in V(G)$. The r-dynamic chromatic number of a graph G is the minimum k such that G has an r-dynamic coloring with k colors. In this paper we investigate the r-dynamic chromatic number of the central graph, middle graph, total graph and line graph of the triple star graph $K_{1,n,n,n}$ denoted by $C(K_{1,n,n,n})$, $M(K_{1,n,n,n})$, $T(K_{1,n,n,n})$ and $L(K_{1,n,n,n})$ respectively.

Key Words: Smarandachely r-dynamic coloring, r-dynamic coloring, triple star graph, central graph, middle graph, total graph and line graph.

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§1. Introduction

Graphs in this paper are simple and finite. For undefined terminologies and notations see [5, 17]. Thus for a graph $G, \delta(G), \Delta(G)$ and $\chi(G)$ denote the minimum degree, maximum degree and chromatic number of G respectively. When the context is clear we write, δ, Δ and χ for brevity. For $v \in V(G)$, let $N(v)$ denote the set of vertices adjacent to v in G and $d(v) = |N(v)|$. The r-dynamic chromatic number was first introduced by Montgomery [14].

An r-dynamic coloring of a graph G is a map c from $V(G)$ to the set of colors such that (i) if $uv \in E(G)$, then $c(u) \neq c(v)$ and (ii) for each vertex $v \in V(G), |c(N(v))| \geq min\{r, d(v)\}$, where $N(v)$ denotes the set of vertices adjacent to v, $d(v)$ its degree and r is a positive integer. Generally, for a subgraph $G' \prec G$ and a coloring c on G if $|c(N(v))| \geq min\{r, d(v)\}$ for $v \in V(G \setminus G')$ but $|c(N(v))| \leq min\{r, d(v)\}$ for $u \in V(G')$, such a r coloring is called a *Smarandachely r-dynamic coloring* on G. Clearly, if $G' = \emptyset$, a Smarandachely r-dynamic coloring is nothing else but the r-dynamic coloring.

The first condition characterizes proper colorings, the adjacency condition and second condition is double-adjacency condition. The r-dynamic chromatic number of a graph G, written $\chi_r(G)$, is the minimum k such that G has an r-dynamic proper k-coloring. The 1-dynamic chromatic number of a graph G is equal to its chromatic number. The 2-dynamic chromatic number of a graph has been studied under the name dynamic chromatic number denoted by $\chi_d(G)$ [1-4, 8]. By simple observation, we can show that $\chi_r(G) \leq \chi_{r+1}(G)$, however $\chi_{r+1}(G) - \chi_r(G)$ can

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be arbitrarily large, for example $\chi(Petersen) = 2$, $\chi_d(Petersen) = 3$, but $\chi_3(Petersen) = 10$. Thus, finding an exact values of $\chi_r(G)$ is not trivially easy.

There are many upper bounds and lower bounds for $\chi_d(G)$ in terms of graph parameters. For example, for a graph G with $\Delta(G) \geq 3$, Lai et al. [8] proved that $\chi_d(G) \leq \Delta(G) + 1$. An upper bound for the dynamic chromatic number of a d-regular graph G in terms of $\chi(G)$ and the independence number of G, $\alpha(G)$, was introduced in [7]. In fact, it was proved that $\chi_d(G) \leq \chi(G) + 2log_2\alpha(G) + 3$. Taherkhani gave in [15] an upper bound for $\chi_2(G)$ in terms of the chromatic number, the maximum degree Δ and the minimum degree δ. i.e., $\chi_2(G) - \chi(G) \leq \lceil (\Delta e)/\delta log\left(2e\left(\Delta^2+1\right)\right)\rceil$.

Li et al. proved in [10] that the computational complexity of $\chi_d(G)$ for a 3-regular graph is an NP-complete problem. Furthermore, Li and Zhou [9] showed that to determine whether there exists a 3-dynamic coloring, for a claw free graph with the maximum degree 3, is NP-complete.

N.Mohanapriya et al. [11, 12] studied the dynamic chromatic number for various graph families. Also, it was proven in [13] that the $r-$ dynamic chromatic number of line graph of a helm graph H_n is

$$\chi_r(L(H_n)) = \begin{cases} n-1, & \delta \leq r \leq n-2, \\ n+1, & r = n-1, \\ n+2, & r = n \ \ and \ \ n \equiv 1 \ \ mod \ \ 3, \\ n+3, & r = n \ \ and \ \ n \not\equiv 1 \ \ mod \ \ 3, \\ n+4, & r = n+1 = \Delta, n \geq 6 \ \ and \ \ 2n-2 \equiv 0 \ \ mod \ \ 5, \\ n+5, & r = n+1 = \Delta, n \geq 6 \ \ and \ \ 2n-2 \not\equiv 0 \ \ mod \ \ 5. \end{cases}$$

In this paper, we study $\chi_r(G)$, the r- dynamic chromatic number of the middle, central, total and line graphs of the triple star graphs are discussed.

§2. Preliminaries

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The middle graph [6] of G, denoted by $M(G)$ is defined as follows. The vertex set of $M(G)$ is $V(G) \cup E(G)$. Two vertices x, y of $M(G)$ are adjacent in $M(G)$ in case one of the following holds: (i) x, y are in $E(G)$ and x, y are adjacent in G. (ii) x is in $V(G)$, y is in $E(G)$, and x, y are incident in G.

The central graph [16] $C(G)$ of a graph G is obtained from G by adding an extra vertex on each edge of G, and then joining each pair of vertices of the original graph which were previously non-adjacent.

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The total graph [6, 16] of G, denoted by $T(G)$ is defined in the following way. The vertex set of $T(G)$ is $V(G) \cup E(G)$. Two vertices x, y of $T(G)$ are adjacent in $T(G)$ in case one of the following holds: (i) x, y are in $V(G)$ and x is adjacent to y in G. (ii) x, y are in $E(G)$ and x, y are adjacent in G. (iii) x is in $V(G), y$ is in $E(G)$, and x, y are incident in G.

The line graph [13] of G denoted by $L(G)$ is the graph with vertices are the edges of G

with two vertices of $L(G)$ adjacent whenever the corresponding edges of G are adjacent.

Theorem 2.1 *For any triple star graph $K_{1,n,n,n}$, the r-dynamic chromatic number*

$$\chi_r(C(K_{1,n,n,n})) = \begin{cases} 2n+1, & r=1 \\ 3n+1, & 2 \le r \le \Delta - 1 \\ 4n+1, & r \ge \Delta \end{cases}$$

Proof First we apply the definition of central graph on $K_{1,n,n,n}$. Let the edge vv_i, v_iw_i and w_iu_i be subdivided by the vertices $e_i(1 \le i \le n)$, $e'_i(1 \le i \le n)$ and $e''_i(1 \le i \le n)$ in $K_{1,n,n,n}$.

Clearly $V(C(K_{1,n,n,n})) = \{v\} \bigcup \{v_i : 1 \le i \le n\} \bigcup \{w_i : 1 \le i \le n\} \bigcup \{u_i : 1 \le i \le n\} \bigcup \{e_i : 1 \le i \le n\} \bigcup \{e'_i : 1 \le i \le n\} \bigcup \{e''_i : 1 \le i \le n\}$. The vertices $v_i(1 \le i \le n)$ induce a clique of order n (say K_n) and the vertices $v, u_i(1 \le i \le n)$ induce a clique of order $n+1$ (say K_{n+1}) in $C(K_{1,n,n,n})$ respectively. Thus, we have $\chi_r(C(K_{1,n,n,n})) \ge n+1$.

Case 1. $r = 1$.

Consider the color class $C_1 = \{c_1, c_2, c_3, \cdots, c_{(2n+1)}\}$ and assign the r-dynamic coloring to $C(K_{1,n,n,n})$ by Algorithm 2.1.1. Thus, an easy check shows that the $r-$ adjacency condition is fulfilled. Hence, $\chi_r(C(K_{1,n,n,n})) = 2n+1$.

Case 2. $2 \le r \le \Delta - 1$.

Consider the color class $C_2 = \{c_1, c_2, c_3, \cdots, c_{(3n+1)}\}$ and assign the r-dynamic coloring to $C(K_{1,n,n,n})$ by Algorithm 2.1.2. Thus, an easy check shows that the $r-$ adjacency condition is fulfilled. Hence, $\chi_r(C(K_{1,n,n,n})) = 3n+1$.

Case 3. $r \ge \Delta$.

Consider the color class $C_3 = \{c_1, c_2, c_3, \cdots, c_{(4n+1)}\}$ and assign the r-dynamic coloring to $C(K_{1,n,n,n})$ by Algorithm 2.1.3. Thus, an easy check shows that the $r-$ adjacency condition is fulfilled. Hence $\chi_r(C(K_{1,n,n,n})) = 4n+1$. □

Algorithm 2.1.1

Input: The number "n" of $K_{1,n,n,n}$.

Output: Assigning r-dynamic coloring for the vertices in $C(K_{1,n,n,n})$.

begin

for $i = 1$ to n

{
$V_1 = \{e_i\}$;
$C(e_i) = i$;
}
$V_2 = \{v\}$;
$C(v) = n+1$;

for $i = 1$ to n
{
$V_3 = \{v_i\};$
$C(v_i) = n + i + 1;$
}
for $i = 1$ to n
{
$V_4 = \{e'_i\};$
$C(e'_i) = n + 1;$
}
for $i = 1$ to n
{
$V_5 = \{w_i\};$
$C(w_i) = i;$
}
for $i = 1$ to n
{
$V_6 = \{e''_i\};$
$C(e''_i) = n + 1;$
}
for $i = 1$ to n
{
$V_7 = \{u_i\};$
$C(u_i) = i;$
}
$V = V_1 \bigcup V_2 \bigcup V_3 \bigcup V_4 \bigcup V_5 \bigcup V_6 \bigcup V_7;$
end

Algorithm 2.1.2

Input: The number "n" of $K_{1,n,n,n}$.

Output: Assigning r-dynamic coloring for the vertices in $C(K_{1,n,n,n})$.

begin
for $i = 1$ to n
{
$V_1 = \{u_i\};$
$C(u_i) = i;$
}
for $i = 1$ to n
{
$V_2 = \{e''_i\};$
$C(e''_i) = n + 1;$
}

for $i = 1$ to n
{
$V_3 = \{w_i\}$;
$C(w_i) = n + i + 1$;
}
for $i = 1$ to n
{
$V_4 = \{e'_i\}$;
$C(e'_i) = i$;
}
for $i = 1$ to n
{
$V_5 = \{v_i\}$;
$C(v_i) = 2n + i + 1$;
}
for $i = 1$ to $n - 1$
{
$V_6 = \{e_i\}$;
$C(e_i) = 2n + i + 2$;
}
$C(e_n) = 2n + 2$;
$V_7 = \{v\}$;
$C(v) = n + 1$;
$V = V_1 \bigcup V_2 \bigcup V_3 \bigcup V_4 \bigcup V_5 \bigcup V_6 \bigcup V_7$;
end

Algorithm 2.1.3

Input: The number "n" of $K_{1,n,n,n}$.

Output: Assigning r-dynamic coloring for the vertices in $C(K_{1,n,n,n})$.

begin
for $i = 1$ to n
{
$V_1 = \{u_i\}$;
$C(u_i) = i$;
}
$V_2 = \{v\}$;
$C(v) = n + 1$;
for $i = 1$ to n
{
$V_3 = \{w_i\}$;
$C(w_i) = n + i + 1$;
}

for $i = 1$ to n
{
$V_4 = \{v_i\}$;
$C(v_i) = 2n + i + 1$;
}
for $i = 1$ to n
{
$V_5 = \{e_i\}$;
$C(e_i) = 3n + i + 1$;
}
for $i = 1$ to n
{
$V_6 = \{e'_i\}$;
$C(e'_i) = i$;
}
for $i = 1$ to n
{
$V_7 = \{e''_i\}$;
$C(e''_i) = 3n + 2$;
}
$V = V_1 \bigcup V_2 \bigcup V_3 \bigcup V_4 \bigcup V_5 \bigcup V_6 \bigcup V_7$;
end

Theorem 2.2 *For any triple star graph $K_{1,n,n,n}$, the r-dynamic chromatic number*

$$\chi_r(M(K_{1,n,n,n})) = \begin{cases} n+1, & 1 \leq r \leq n \\ n+2, & r = n+1 \\ n+3, & r \geq \Delta \end{cases}$$

Proof By definition of middle graph, each edge vv_i, v_iw_i and w_iu_i be subdivided by the vertices $e_i (1 \leq i \leq n)$, $e'_i (1 \leq i \leq n)$ and $e''_i (1 \leq i \leq n)$ in $K_{1,n,n,n}$ and the vertices v, e_i induce a clique of order $n+1$(say K_{n+1}) in $M(K_{1,n,n,n})$. i.e.,$V(M(K_{1,n,n,n})) = \{v\} \bigcup \{v_i : 1 \leq i \leq n\} \bigcup \{w_i : 1 \leq i \leq n\} \bigcup \{u_i : 1 \leq i \leq n\} \bigcup \{e_i : 1 \leq i \leq n\} \bigcup \{e'_i : 1 \leq i \leq n\} \bigcup \{e''_i : 1 \leq i \leq n\}$. Thus we have $\chi_r(M(K_{1,n,n,n})) \geq n+1$.

Case 1. $1 \leq r \leq n$.

Consider the color class $C_1 = \{c_1, c_2, c_3, \cdots, c_{(n+1)}\}$ and assign the r-dynamic coloring to $M(K_{1,n,n,n})$ by Algorithm 2.2.1. Thus, an easy check shows that the $r-$ adjacency condition is fulfilled. Hence, $\chi_r(M(K_{1,n,n,n})) = n+1$, for $1 \leq r \leq n$.

Case 2. $r = n+1$.

Consider the color class $C_2 = \{c_1, c_2, c_3, \cdots, c_{(n+1)}, c_{(n+2)}\}$ and assign the r-dynamic coloring to $M(K_{1,n,n,n})$ by Algorithm 2.2.2. Thus, an easy check shows that the $r-$ adjacency condition is fulfilled. Hence, $\chi_r(M(K_{1,n,n,n})) = n + 2$, for $r = n + 1$.

Case 3. $r = \Delta$.

Consider the color class $C_3 = \{c_1, c_2, c_3, \cdots, c_n, c_{(n+1)}, c_{(n+2)}, c_{(n+3)}\}$ and assign the r-dynamic coloring to $M(K_{1,n,n,n})$ by Algorithm 2.2.3. Thus, an easy check shows that the $r-$ adjacency condition is fulfilled. Hence, $\chi_r(M(K_{1,n,n,n})) = n + 3$, for $r \geq \Delta$. □

Algorithm 2.2.1

Input: The number "n" of $K_{1,n,n,n}$.

Output: Assigning r-dynamic coloring for the vertices in $M(K_{1,n,n,n})$.

begin

for $i = 1$ to n

{

$V_1 = \{e_i\}$;

$C(e_i) = i$;

}

$V_2 = \{v\}$;

$C(v) = n + 1$;

for $i = 1$ to n

{

$V_3 = \{v_i\}$;

$C(v_i) = n + 1$;

}

for $i = 1$ to $n - 1$

{

$V_4 = \{e'_i\}$;

$C(e'_i) = i + 1$;

}

$C(e'_n) = 1$;

for $i = 1$ to $n - 2$

{

$V_5 = \{w_i\}$;

$C(w_i) = i + 2$;

}

$C(w_{n-1}) = 1$;

$C(w_n) = 2$;

for $i = 1$ to n

{

$V_6 = \{e''_i\}$;

$C(e''_i) = n + 1$;

}
for $i = 1$ to n
{
$V_7 = \{u_i\}$;
$C(u_i) = i$;
}
$V = V_1 \bigcup V_2 \bigcup V_3 \bigcup V_4 \bigcup V_5 \bigcup V_6 \bigcup V_7$;
end

Algorithm 2.2.2

Input: The number "n" of $K_{1,n,n,n}$.

Output: Assigning r-dynamic coloring for the vertices in $M(K_{1,n,n,n})$.

begin
for $i = 1$ to n
{
$V_1 = \{e_i\}$;
$C(e_i) = i$;
}
$V_2 = \{v\}$;
$C(v) = n + 1$;
for $i = 1$ to n
{
$V_3 = \{v_i\}$;
$C(v_i) = n + 2$;
}
for $i = 1$ to n
{
$V_4 = \{e'_i\}$;
$C(e'_i) = n + 1$;
}
for $i = 1$ to $n - 1$
{
$V_5 = \{w_i\}$;
$C(w_i) = i + 1$;
}
$C(w_n) = 1$;
for $i = 1$ to $n - 2$
{
$V_6 = \{e''_i\}$;
$C(e''_i) = i + 2$;
}
$C(e''_{n-1}) = 1$;

$C(e''_n) = 2;$
for $i = 1$ to n
{
$V_7 = \{u_i\};$
$C(u_i) = n + 1;$
}
$V = V_1 \bigcup V_2 \bigcup V_3 \bigcup V_4 \bigcup V_5 \bigcup V_6 \bigcup V_7;$
end

Algorithm 2.2.3

Input: The number "n" of $K_{1,n,n,n}$.

Output: Assigning r-dynamic coloring for the vertices in $M(K_{1,n,n,n})$.

begin
for $i = 1$ to n
{
$V_1 = \{e_i\};$
$C(e_i) = i;$
}
$V_2 = \{v\};$
$C(v) = n + 1;$
for $i = 1$ to n
{
$V_3 = \{v_i\};$
$C(v_i) = n + 2;$
}
for $i = 1$ to n
{
$V_4 = \{e'_i\};$
$C(e'_i) = n + 3;$
}
for $i = 1$ to n
{
$V_5 = \{w_i\};$
$C(w_i) = n + 1;$
}
for $i = 1$ to $n - 1$
{
$V_6 = \{e''_i\};$
$C(e''_i) = i + 1;$
}
$C(e''_n) = 1;$
for $i = 1$ to n

{
$V_7 = \{u_i\}$;
$C(u_i) = n + 2$;
}
$V = V_1 \bigcup V_2 \bigcup V_3 \bigcup V_4 \bigcup V_5 \bigcup V_6 \bigcup V_7$;
end

Theorem 2.3 *For any triple star graph $K_{1,n,n,n}$, the r-dynamic chromatic number,*

$$\chi_r(T(K_{1,n,n,n})) = \begin{cases} n+1, & 1 \leq r \leq n \\ r+1, & n+1 \leq r \leq \Delta - 2 \\ 2n, & r = \Delta - 1 \\ 2n+1, & r \geq \Delta \end{cases}$$

Proof By definition of total graph, each edge vv_i, v_iw_i and w_iu_i be subdivided by the vertices $e_i(1 \leq i \leq n)$, $e'_i(1 \leq i \leq n)$ and $e''_i(1 \leq i \leq n)$ in $K_{1,n,n,n}$ and the vertices v, e_i induce a clique of order $n+1$(say K_{n+1}) in $T(K_{1,n,n,n})$. i.e.,$V(T(K_{1,n,n,n})) = \{v\} \bigcup \{v_i : 1 \leq i \leq n\} \bigcup \{w_i : 1 \leq i \leq n\} \bigcup \{u_i : 1 \leq i \leq n\} \bigcup \{e_i : 1 \leq i \leq n\} \bigcup \{e'_i : 1 \leq i \leq n\} \bigcup \{e''_i : 1 \leq i \leq n\}$. Thus, we have $\chi_r(T(K_{1,n,n,n})) \geq n+1$.

Case 1. $1 \leq r \leq n$.

Consider the color class $C_1 = \{c_1, c_2, c_3, \cdots, c_{(n+1)}\}$ and assign the r-dynamic coloring to $T(K_{1,n,n,n})$ by Algorithm 2.3.1. Thus, an easy check shows that the $r-$ adjacency condition is fulfilled. Hence, $\chi_r(T(K_{1,n,n,n})) = n+1$, for $1 \leq r \leq n$.

Case 2. $n+1 \leq r \leq \Delta - 2$.

Consider the color class $C_2 = \{c_1, c_2, c_3, \cdots, c_{(2n-1)}\}$ and assign the r-dynamic coloring to $T(K_{1,n,n,n})$ by Algorithm 2.3.2. Thus, an easy check shows that the $r-$ adjacency condition is fulfilled. Hence, $\chi_r(T(K_{1,n,n,n})) = r+1$, for $n+1 \leq r \leq \Delta - 2$.

Case 3. $r = \Delta - 1$.

Consider the color class $C_3 = \{c_1, c_2, c_3, \cdots, c_{2n}\}$ if $r = \Delta - 1$ and assign the r-dynamic coloring to $T(K_{1,n,n,n})$ by Algorithm 2.3.3. Thus, an easy check shows that the $r-$ adjacency condition is fulfilled. Hence, $\chi_r(T(K_{1,n,n,n})) = 2n$ for $r = \Delta - 1$.

Case 4. $r = \Delta$.

Consider the color class $C_4 = \{c_1, c_2, c_3, \cdots, c_{2n+1}\}$ if $r = \Delta$ and assign the r-dynamic coloring to $T(K_{1,n,n,n})$ by Algorithm 2.3.4. Thus, an easy check shows that the $r-$ adjacency condition is fulfilled. Hence, $\chi_r(T(K_{1,n,n,n})) = 2n+1$ for $r \geq \Delta$. □

Algorithm 2.3.1

Input: The number ”n” of $K_{1,n,n,n}$.

Output: Assigning r-dynamic coloring for the vertices in $T(K_{1,n,n,n})$.

begin

for $i = 1$ to n

{
$V_1 = \{e_i\}$;
$C(e_i) = i$;
}
$V_2 = \{v\}$;
$C(v) = n + 1$;

for $i = 1$ to $n - 3$

{
$V_3 = \{v_i\}$;
$C(v_i) = i + 3$;
}
$C(v_{n-2}) = 1$;
$C(v_{n-1}) = 2$;
$C(v_n) = 3$;

for $i = 1$ to $n - 2$

{
$V_4 = \{e'_i\}$;
$C(e'_i) = i + 2$;
}
$C(e'_{n-1}) = 1$;
$C(e'_n) = 2$;

for $i = 1$ to $n - 1$

{
$V_5 = \{w_i\}$;
$C(w_i) = i + 1$;
}
$C(w_n) = 1$;

for $i = 1$ to n

{
$V_6 = \{e''_i\}$;
$C(e''_i) = n + 1$;
}

for $i = 1$ to n

{
$V_7 = \{u_i\}$;
$C(u_i) = i$;
}

$V = V_1 \bigcup V_2 \bigcup V_3 \bigcup V_4 \bigcup V_5 \bigcup V_6 \bigcup V_7$;

end

Algorithm 2.3.2

Input: The number "n" of $K_{1,n,n,n}$.

Output: Assigning r-dynamic coloring for the vertices in $T(K_{1,n,n,n})$.

begin

for $i = 1$ to n

{

$V_1 = \{e_i\}$;

$C(e_i) = i$;

}

$V_2 = \{v\}$;

$C(v) = n + 1$;

for $i = 1$ to $n - 2$

{

$V_3 = \{v_i\}$;

$C(v_i) = r + 1$;

}

$C(v_{n-1}) = n + 2$;

$C(v_n) = n + 3$;

for $i = 1$ to $n - 3$

{

$V_4 = \{e'_i\}$;

$C(e'_i) = n + i + 2$;

}

$C(e'_{n-2}) = n + 2$;

$C(e'_{n-1}) = n + 3$;

$C(e'_n) = n + 4$;

for $i = 1$ to $n - 1$

{

$V_5 = \{w_i\}$;

$C(w_i) = i + 1$;

}

$C(w_n) = 1$;

for $i = 1$ to n

{

$V_6 = \{e''_i\}$;

$C(e''_i) = n + 1$;

}

for $i = 1$ to n

{
$V_7 = \{u_i\}$;
$C(u_i) = i$;
}
$V = V_1 \bigcup V_2 \bigcup V_3 \bigcup V_4 \bigcup V_5 \bigcup V_6 \bigcup V_7$;
end

Algorithm 2.3.3

Input: The number "n" of $K_{1,n,n,n}$.

Output: Assigning r-dynamic coloring for the vertices in $T(K_{1,n,n,n})$.

begin
for $i = 1$ to n
{
$V_1 = \{e_i\}$;
$C(e_i) = i$;
}
$V_2 = \{v\}$;
$C(v) = n + 1$;
for $i = 1$ to $n - 1$
{
$V_3 = \{v_i\}$;
$C(v_i) = n + i + 1$;
}
$C(v_n) = n + 2$;
for $i = 1$ to $n - 2$
{
$V_4 = \{e'_i\}$;
$C(e'_i) = n + i + 2$;
}
$C(e'_{n-1}) = n + 2$;
$C(e'_n) = n + 3$;
for $i = 1$ to $n - 1$
{
$V_5 = \{w_i\}$;
$C(w_i) = i + 1$;
}
$C(w_n) = 1$;
for $i = 1$ to n
{
$V_6 = \{e''_i\}$;
$C(e''_i) = n + 1$;
}

for $i = 1$ to n
{
$V_7 = \{u_i\}$;
$C(u_i) = i$;
}
$V = V_1 \bigcup V_2 \bigcup V_3 \bigcup V_4 \bigcup V_5 \bigcup V_6 \bigcup V_7$;
end

Algorithm 2.3.4

Input: The number "n" of $K_{1,n,n,n}$.
Output: Assigning r-dynamic coloring for the vertices in $T(K_{1,n,n,n})$.

begin
for $i = 1$ to n
{
$V_1 = \{e_i\}$;
$C(e_i) = i$;
}
$V_2 = \{v\}$;
$C(v) = n + 1$;
for $i = 1$ to n
{
$V_3 = \{v_i\}$;
$C(v_i) = n + i + 1$;
}
for $i = 1$ to $n - 1$
{
$V_4 = \{e'_i\}$;
$C(e'_i) = n + i + 2$;
}
$C(e'_n) = n + 2$;
for $i = 1$ to $n - 1$
{
$V_5 = \{w_i\}$;
$C(w_i) = i + 1$;
}
$C(w_n) = 1$;
for $i = 1$ to n
{
$V_6 = \{e''_i\}$;
$C(e''_i) = n + 1$;
}
for $i = 1$ to n

{
$V_7 = \{u_i\}$;
$C(u_i) = i$;
}
$V = V_1 \bigcup V_2 \bigcup V_3 \bigcup V_4 \bigcup V_5 \bigcup V_6 \bigcup V_7$;
end

Theorem 2.4 *For any triple star graph $K_{1,n,n,n}$, the r-dynamic chromatic number,*

$$\chi_r(L(K_{1,n,n,n})) = \begin{cases} n, & 1 \le r \le n-1 \\ n+1, & r \ge \Delta \end{cases}$$

Proof First we apply the definition of line graph on $K_{1,n,n,n}$. By the definition of line graph, each edge of $K_{1,n,n,n}$ taken to be as vertex in $L(K_{1,n,n,n})$.The vertices e_1, e_2,$\cdots$, e_n induce a clique of order n in $L(K_{1,n,n,n})$. i.e., $V(L(K_{1,n,n,n})) = E(K_{1,n,n,n}) = \{e_i : 1 \le i \le n\} \bigcup \{e'_i : 1 \le i \le n\} \bigcup \{e''_i : 1 \le i \le n\}$. Thus, we have $\chi_r(L(K_{1,n,n,n})) \ge n$.

Case 1. $1 \le r \le \Delta - 1$.

Now consider the vertex set $V(L(K_{1,n,n,n}))$ and color class $C_1 = \{c_1, c_2, \cdots, c_n\}$, assign r dynamic coloring to $L(K_{1,n,n,n})$ by Algorithm 2.4.1. Thus, an easy check shows that the $r-$ adjacency condition is fulfilled. Hence, $\chi_r(L(K_{1,n,n,n})) = n$, for $1 \le r \le \Delta - 1$.

Case 2. $r \ge \Delta$.

Now consider the vertex set $V(L(K_{1,n,n}))$ and color class $C_2 = \{c_1, c_2, \cdots, c_n, c_{n+1}\}$, assign r dynamic coloring to $L(K_{1,n,n,n})$ by Algorithm 2.4.2. Thus, an easy check shows that the $r-$ adjacency condition is fulfilled. Hence, $\chi_r(L(K_{1,n,n,n})) = n + 1$ for $r \ge \Delta$. □

Algorithm 2.4.1

Input: The number "n" of $K_{1,n,n,n}$.

Output: Assigning r-dynamic coloring for the vertices in $L(K_{1,n,n,n})$.

begin

for $i = 1$ to n
{
$V_1 = \{e_i\}$;
$C(e_i) = i$;
}
for $i = 1$ to $n - 1$
{
$V_2 = \{e'_i\}$;
$C(e'_i) = i + 1$;
}
$C(e'_n) = 1$;

for $i = 1$ to $n - 2$
{
$V_3 = \{e''_i\}$;
$C(e''_i) = i + 2$;
}
$C(e''_{n-1}) = 1$;
$C(e''_n) = 2$;
$V = V_1 \bigcup V_2 \bigcup V_3$;
end

Algorithm 2.4.2

Input: The number "n" of $K_{1,n,n,n}$.

Output: Assigning r-dynamic coloring for the vertices in $L(K_{1,n,n,n})$.

begin
for $i = 1$ to n
{
$V_1 = \{e_i\}$;
$C(e_i) = i$;
}
for $i = 1$ to n
{
$V_2 = \{e'_i\}$;
$C(e'_i) = n + 1$;
}
for $i = 1$ to $n - 1$
{
$V_3 = \{e''_i\}$;
$C(e''_i) = i + 1$;
}
$C(e''_n) = 1$;
$V = V_1 \bigcup V_2 \bigcup V_3$;
end

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