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Operations of n-Wheel Graph via Topological Indices

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Abstract: In this paper, we discussed the Topological indices viz., Wiener, Harmonic, Geometric-Arithmetic(GA), first and second Zagreb indices of n-wheel graphs with bridges using operator techniques.

Key Words: n-wheel graph, subdivision operator, line graph, complement of n-wheel, wiener index, harmonic, GA, first and second zagreb indices.

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§1. Introduction

For vertices $u, v \in V(G)$, the distance between u and v in G, denoted by $d_G(u, v)$, is the length of a shortest (u, v)-path in G and let $d_G(v)$ be the degree of a vertex $v \in V(G)$. A topological index of a graph is a real number related to the graph; it does not depend on labeling or pictorial representation of a graph. In theoretical chemistry, molecular structure descriptors (also called topological indices) are used for modeling physicochemical, pharmacologic, toxicologic, biological and other properties of chemical compounds [4]. There exist several types of such indices, especially those based on vertex and edge distances. One of the most intensively studied topological indices is the Wiener index.

The Wiener index [17], defined as the sum of all distances between pairs of vertices u and v in a graph G is given by

$$W(G) = \sum_{uv \in E(G)} d(u, v)$$

Another few degree based topological indices are defined as follows:

The Harmonic index according to [13] is given by

$$H(G) = \sum_{uv \in E(G)} \frac{2}{d_u + d_v}$$

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The geometric-arithmetic index of a graph G [3], denoted by $GA(G)$ and is defined by

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u.d_v}}{d_u + d_v}$$

The first and second Zagreb indices [10] is defined as

$$M_1(G) = \sum_{u \in V(G)} d_u^2, \quad M_2(G) = \sum_{uv \in E(G)} d_u.d_v$$

Throughout paper, we have used n-wheel graph with standard operators.

The n-wheel graph is defined as the graph $K_1 + C_n$ where K_1 is the singleton graph and C_n is the cycle graph. The center of the wheel is called the hub and the edges joining the hub and vertices of C_n are called the spokes.

The well known operators are recalled [7, 9, 10].

Adding a additional edge on top most vertex of two or more graphs is defined as bridge operator.

The subdivision graph $S(G)$ of a graph G is the graph obtained by inserting an additional vertex into each edge of G.

The Line graph $L(G)$ of a graph G is the graph whose vertices correspond to the edges of G with two vertices being adjacent in $L(G)$ if and only if the corresponding edges in G are adjacent in G.

Complement graph or inverse of a graph G is a graph G' on the same vertices such that two distinct vertices of G' are adjacent if and only if they are not adjacent in G.

The paper is starting with the preliminaries needed for our study. Section 2, construction of bridge operator in a wheel graph results on different topological indices are discussed. Section 3, complement of wheel graph of constructed graph results are established. Section 4, Subdivision operator of constructed graph results are shown. Final section deals with the line graph of constructed graph results are highlighted.

§2. Distance and Degree-Based Indices of n-Wheel Graph with Bridge Operator

In this section, we constructed a n-wheel graph by attaching bridge at top most vertex of a graphs and established the results on different topological indices.

Here, we denote the edge set of n-wheel graph G_n, then $E_i = \{e = uv \in E(G) | d_u + d_v = i, \forall i = 1, 2, \cdots, n\}$.

Theorem 2.1 *Let G_n be attached wheel graphs of n vertices with bridge operator b_k, $k > 0$, then the harmonic index is*

$$H(G_n, b_k) = (b_k + 1)\left[\frac{n^3 + 8n^2 + 3n - 42}{3n^2 + 15n + 18}\right] + \frac{23b_k + 16}{28}.$$

Proof Consider two wheel graphs with 4 vertices ($n \geq 4$), if a bridge is attached, there are E_6, E_7, E_8 edges for $B(G_1, G_2) = B(G_1, G_2, ; v_1, v_2)$.

(i) The number of edges is $7b_k + 6$

The number of wheel graphs and bridges will increases in a graph G_4 then the harmonic index is:

$$H(G_4, b_k) = \frac{59b_k + 52}{28}.$$

Similarly, if we consider two wheel graphs with 5 vertices, if a bridge is attached, there are E_6, E_7, E_8 edges.

(ii) The number of edges is $9b_k + 8$.

(iii) The number of wheel graphs and bridges will increases in a graph G_5 then the harmonic index is

$$H(G_5, b_k) = \frac{218b_k + 197}{84}.$$

The total number of edges in G_n is $(2n-1)b_k + 2(n-1)$.

Computing for n vertices with b_k bridges of G_n the harmonic index is

$$H(G_n, b_k) = (b_k + 1)\left[\frac{n^3 + 8n^2 + 3n - 42}{3n^2 + 15n + 18}\right] + \frac{23b_k + 16}{28}. \qquad \square$$

Theorem 2.2 *The geometric-arithmetic index of a bridge operator formed by G_n is*

$$GA(G_n, b_k) = (b_k + 1)\left[\frac{\sqrt{3}(n-2)}{\sqrt{n-1}} + \frac{4\sqrt{n-1}}{2n-1} + n - 3\right] + \frac{8\sqrt{3}(b_k + 1)}{7} + 2b_k.$$

Proof If a bridge is formed for two wheel graphs with 4 vertices, having E_6, E_7, E_8 edges and using equation (i), then geometric-arithmetic index is

$$GA(G_4, b_k) = \frac{4(3\sqrt{3} + 7)(b_k + 1)}{7}$$

Similarly, For $n = 5$. Using (ii), then Geometric-Arithmetic index of G_4 is

$$GA(G_5, b_k) = 2(b_k + 1)\frac{4\sqrt{3} + 21}{7} + \frac{4\sqrt{3} + 7}{7} + 5b_k.$$

Computing for n vertices b_k bridges of G_n the geometric-arithmetic index is

$$GA(G_n, b_k) = (b_k + 1)\left[\frac{\sqrt{3}(n-2)}{\sqrt{n-1}} + \frac{4\sqrt{n-1}}{2n-1} + n - 3\right] + \frac{8\sqrt{3}(b_k + 1)}{7} + 2b_k. \qquad \square$$

Theorem 2.3 *The G_n of a bridge operator for wiener index is*

$$W(G_n, b_k) = n^2(b_k + 1) - 3n(b_k + 1) + 3b_k + 2.$$

Proof We adopted the proof technique of Theorem 2.1 and using equations (i) and (ii) in the wiener indices for $n = 4$ and $n = 5$ is

$$W(G_4, b_k) = 7b_k + 6.$$

and

$$W(G_5, b_k) = 13b_k + 12.$$

Computing for the wiener index of graph G_n is

$$W(G_n, b_k) = n^2(b_k + 1) - 3n(b_k + 1) + 3b_k + 2. \qquad \square$$

§3. Complement of a Constructed Graph

In this segment, a complement of a wheel graphs connected with the number of bridges b_k (constructed graph) for $n \geq 5$ with respect to different topological indices are established.

Theorem 3.1 *Let $G_n^{'}$ be a complement of constructed graph then harmonic index is*

$$H(G_n^{'}, b_k) = (b_k + 1)\left[\frac{(n-3)(n-4)^2}{2} + \frac{2(n-4)}{2n-7}\right] + \frac{b_k}{n-3}.$$

Proof In $G_n^{'}$ having $(n-1)$ vertices with E_2, E_3, E_4 edges. Therefore,

(iv) the total number of edges is $2(b_k + 1)$.

Hence, the harmonic index is

$$H(G_n^{'}, b_k) = (b_k + 1)\left[\frac{(n-3)(n-4)^2}{2} + \frac{2(n-4)}{2n-7}\right] + \frac{b_k}{n-3}. \qquad \square$$

Theorem 3.2 *The geometric-arithmetic index of $G_n^{'}$ is*

$$GA(G_n^{'}, b_k) = (b_k + 1)\left[\frac{(n-3)(n-4)}{2} + \frac{2(n-4)\sqrt{(n-4)(n-3)}}{2n-7}\right] + b_k.$$

Proof The proof technique is applied as in Theorem 3.1. Hence using equation (iv) we get the required result.

$$GA(G_n^{'}, b_k) = (b_k + 1)\left[\frac{(n-3)(n-4)}{2} + \frac{2(n-4)\sqrt{(n-4)(n-3)}}{2n-7}\right] + b_k. \qquad \square$$

Theorem 3.3 *Let $G_n^{'}$ of wiener index is*

$$W(G_n^{'}, b_k) = (b_k + 1)\left[\frac{(n-1)(n-4)}{2}\right] + b_k.$$

Proof Let $n = 4, 5, 6, 7, \cdots$ having distance b_k, $3b_k + 2$, $6b_k + 5, \cdots$, then the wiener index is

$$W(G_n^{'}, b_k) = (b_k + 1)\left[\frac{(n-1)(n-4)}{2}\right] + b_k. \qquad \square$$

Observation 3.3 If $n = 4$, $H(G_n^{'}, b_k)$=$GA(G_n^{'}, b_k)$=$W(G_n^{'}, b_k)$=b_k.

§4. Subdivision of Constructed Graph on Degree-Based Indices

In this section, the subdivision operator of constructed graph (G_n) are highlighted.

Theorem 4.1 *Let $S(G_n)$ be a subdivision operator of constructed graph then,*

(1) $H[S(G_n, b_k)] = 2(b_k + 1)\left[\dfrac{3n^2 + 2n - 11}{5(n+1)}\right] + \dfrac{15b_k + 12}{12}$;

(2) $GA[S(G_n, b_k)] = (b_k + 1)\left[\dfrac{6\sqrt{6}(n+1)(n-2) + 10(n-1)\sqrt{n+2} + 5\sqrt{2}(n+1)}{5(n+1)}\right] + \dfrac{b_k}{2}$;

(3) $M_1[S(G_n, b_k)] = (b_k + 1)(n^2 + 7n - 1)$;

(4) $M_2[S(G_n, b_k)] = (b_k + 1)(22n - 25) + 16b_k$.

Proof A subdivision G_n graph having $(3n - 2)$ vertices and $4(n - 1)$ edges among which $2(n-1)$ vertices are of degree 2, $(n-2)$ vertices are of degree 3, n vertices are of degree $(n-1)$ and by attaching bridge there exists $2b_k$ edges and b_k vertices having degree 2. Here also, adopted the similar proof techniques of earlier theorems we obtained the required results. $\square$

§5. Line Graph of Constructed Graph

In this section, the Line graph of bridge graph of n wheel graph related to different topological indices are discussed. The following results are observed.

Theorem 5.1 *Let $L(G_n)$ be the line graph of constructed graph then*

(1) $H[L(G_n, b_k)] = 2(b_k + 1)\left[\dfrac{(n-3)}{8} + \dfrac{2(n-2)}{(n+4)} + \dfrac{(n-1)(n-2)}{4n} + \dfrac{2}{(n+5)} + \dfrac{2}{9}\right] + \dfrac{b_k}{5}$;

(2) $GA[L(G_n, b_k)] = 2(b_k+1)\left[\dfrac{n-3}{2} + \dfrac{4\sqrt{n}(n-2)}{(n+4)} + \dfrac{(n-1)(n-2)}{4} + \dfrac{2\sqrt{5n}}{(n+5)} + \dfrac{4\sqrt{5}}{9}\right] + b_k$;

(3) $M_1[L(G_n, b_k)] = (b_k + 1)[n^3 - n^2 + 16n - 7]$;

(4) $M_2[L(G_n, b_k)] = \dfrac{(b_k + 1)}{2}\left[n^4 - 3n^3 + 18n^2 + 20n - 16\right] + 25b_k$.

Proof Consider $L(G_n)$ be the line graph of wheel graph using bridge graph. In G_n there are two copies of $2(n-1)$ vertices and total number of edges exists $6(n-1)$ and b_k when bridge is attached. Hence the results are proved by adopting same proof technique used in the earlier sections. $\square$

§6. Conclusion

In this paper, degree based and distance based indices for different types of operators on wheel graphs are studied. This type of relationships may be useful to connectivity between graph structures or chemical structures.

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