Gravito-electromagnetism Derived Mathematically from Newton's Law

—Electromagnetism/Gravito-electromagnetism Duality

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Abstract

In this article, we utilize the mathematic approach, which has been utilized to re-derive Maxwell equation mathematically from the Gauss/Coulomb's law, to demonstrate that, by applying Newton's law, the magnetic-type gravitational field exists mathematically. Namely, since a stationary gravitational charge satisfies Newton's law, its motion generates inevitably an axial gravito-magnetic field and an axial gravito-electric field; call Gravito-electromagnetism. This mathematical derivation proves the famous sentence, "The world is built mathematically". The so derived Electromagnetism and Gravito-electromagnetism disclose mathematically an intrinsic duality between electromagnetic force and gravitational force, which are the only two long-range forces in the Nature. The Electromagnetism/Gravito-electromagnetism duality can serves as the base for the advanced dualities between the two and for the unification of the two.

Key words: magnetic-type gravitation, gravito-magnetic field, gravito-electric field, gravito-electromagnetism, Maxwell-type gravitation,

Contents

1. Introduction

Recently, a mathematical approach has been proposed and applied to re-derive mathematically Maxwell equations from Gauss/Coulomb's law [1]. One may say, "Ok, it is no big deal, Maxwell equations have been demonstrated experimentally over 150 years".

The significance of the approach is that it *mathematically* demonstrates that magnetic field is an inevitable result of the motion of electric charges without taking into account Special Relativity. Namely for an electric charge satisfying Gauss/Coulomb's law, or more generally speaking, inverse-square law, its motion generates inevitably axial magnetic fields and axial electric fields. This approach discloses a new explanation of the generation of magnetic fields and helps us to understand magnetic fields in a different perspective. Equally important, this approach discloses the mathematical origin of the electric/magnetic duality, which is the best-known symmetry of theoretical physics.

Secondly, this approach has been utilized to derive mathematically Maxwell-type equations for non-uniformly moving electric charges, which shows that the acceleration of electric charges generate new axial magnetic-type fields and new axial electric-type fields [2].

Thirdly, we will apply this approach to study gravitational fields (g-fields) in this article. The g-field of a stationary gravitational charge (g-charge) satisfies an inverse-square law, Newton's law. From mathematical perspective, Gauss/coulomb's law and Newton's law are equivalent. The derivation of rest Maxwell-type equations describing moving g-charges is pure mathematic calculation that is universally valid for any physical quantity. The Maxwell-type gravito-electromagnetism is established mathematically.

Fourthly, the duality between electromagnetic force and gravitational force is one of the most mystery dualities and has been studied in different perspectives, such as Ads/CFT duality and Gauge/Gravity Duality [3]. This approach helps one to understand why the electromagnetic force and gravitational force, the only two long-range forces in Nature, have their particular dualities. "It has been evident for some time that a new concept or theory is needed, which can provide the underlying basis for the various concepts of science, linking them in some way. And it turns out that most of the important concepts and theories of physics can be unified and understood by their common attribute of duality".

2. Review of Maxwell-type Theory of Gravitation

Let's review magnetism first. Ampere's law governs magnetic fields generated by electric current. Historically, Ampere's law was established based on experiments. When a charge is at rest, it generates an electric field determined by the Gauss/Coulomb's law, $\nabla \cdot \mathbf{E} = 4\pi Q$, and doesn't generate a magnetic field.

Thus we believe that the Gauss/Coulomb's law is the fundamental-level law and cannot be derived mathematically from any other law.

However, this is a remarkable phenomenon: an observer moving relative to the same charge will experience not only an electric field but also a magnetic field.

Thus we believe that the laws related with magnetic fields are secondary-level laws and thus, should be derivable from the fundamental-level law mathematically. Although the magnetic field is completely different from electric field in the following senses: (1) the way the fields generated, "$\nabla \cdot$" vs. "$\nabla\times$"; (2) the nature of the fields, vector field vs. axial vector field; (3) effects of the fields on charges, "q**E**" vs. "q$\mathbf{u}\times\mathbf{B}$".

A physics student may ask questions: Why a moving observer experiences such different magnetic field? Or why the motion of charges generates magnetic fields? A teacher's answer is that magnetism is the combination of electric field with Special Relativity, i.e., a magnetic field is generated by pure Lorentz transformation and *not related with the Gauss/Coulomb's law*.

We argue that, if the motion of either charges or observer inevitably generates magnetic field, *the equations describing the generation of magnetic fields must be derivable mathematically from the combination of Gauss/Coulomb's law and motion of source charges*. Namely, the rest of Maxwell equations must be derivable mathematically from the Gauss/Coulomb's law. An approach re-deriving mathematically Maxwell equations from the Gauss/Coulomb's law has been proposed [1].

Now we ask a further question: taking into account the Newton/Coulomb duality, does the motion of gravitational charges (g-charge) generate magnetic-type or gravito-magnetic g-fields inevitably? Historically, based on the similarity between the Coulomb's and Newton's laws, the magnetic-type axial vector g-fields has been postulated without mathematical derivation, as earlier as year 1865 [4] after Maxwell's theory was proposed (1861). The most severe issue was that vector and axial vector g-fields carries negative energy, and stopped further investigation. Recently, this issue has been addressed [5]. Many effects of magnetic-type g-field have been proposed [6][7]. One of them, Lense-thirring effect [8], has been explained as the effect of magnetic-type g-field of the rotating Earth [7], and the positive results have been obtained experimentally [9]. We argue that, theoretically, the detection of Gravitational waves (*GW*) [10] indicates the existence of time varying gravito-magnetic fields [11], just as "electromagnetic wave is equivalent to time varying magnetic field". Experimentally, gravito-magnetic force possibly exists.

Similar to the case of Gauss/Coulomb's law, we believe that the Newton's law is the fundamental-level law, while equations describing magnetic-type gravitational phenomena are secondary-level laws, and should be derivable from the fundamental-level law. The mathematical derivation is the task of this article.

3. Mathematic Preparation

We will utilize the Newton's law, which relates the divergences of a vector Newton g-field $\mathbf{g}$ to a g-charge Q_g,

$$\nabla \cdot \mathbf{g} = -4\pi Q_g. \tag{1}$$

The vector analysis formulas we will utilize are shown below,

$$\nabla \times (\mathbf{S} \times \mathbf{T}) = \mathbf{S}(\nabla \cdot \mathbf{T}) - \mathbf{T}(\nabla \cdot \mathbf{S}) + (\mathbf{T} \cdot \nabla)\mathbf{S} - (\mathbf{S} \cdot \nabla)\mathbf{T}, \tag{2}$$

$$\boldsymbol{\nabla}(\mathbf{S} \cdot \mathbf{T}) = (\mathbf{S} \cdot \boldsymbol{\nabla})\mathbf{T} + (\mathbf{T} \cdot \boldsymbol{\nabla})\mathbf{S} + \mathbf{S} \times (\boldsymbol{\nabla} \times \mathbf{T}) + \mathbf{T} \times (\boldsymbol{\nabla} \times \mathbf{S}), \tag{3}$$

$$\mathbf{S} \cdot (\mathbf{T} \times \mathbf{Z}) = \mathbf{T} \cdot (\mathbf{Z} \times \mathbf{S}) = \mathbf{Z} \cdot (\mathbf{S} \times \mathbf{T}). \tag{4}$$

where **S**, **T** and **Z** are arbitrary vectors.

Combining Eq. (2) and (3), we obtain,

$$\nabla \times (\mathbf{S} \times \mathbf{T}) = \mathbf{S}(\nabla \cdot \mathbf{T}) - \mathbf{T}(\nabla \cdot \mathbf{S}) + (\mathbf{T} \cdot \nabla)\mathbf{S} - \{\boldsymbol{\nabla}(\mathbf{S} \cdot \mathbf{T}) - (\mathbf{T} \cdot \boldsymbol{\nabla})\mathbf{S} - \mathbf{S} \times (\boldsymbol{\nabla} \times \mathbf{T}) - \mathbf{T} \times (\boldsymbol{\nabla} \times \mathbf{S})\}. \tag{5}$$

Eq. (2) is the most noteworthy formula because it indicates that the combination of gradient and divergence operations of two arbitrary vectors generates inevitably an axial vector. Specifically, there are two divergence terms, $(\nabla \cdot \mathbf{T})$ and $(\nabla \cdot \mathbf{S})$, combining with other terms, definitely generate an axial field $\nabla \times (\mathbf{S} \times \mathbf{T})$. Combining with Gauss/Coulomb's law, Eq. (2) gives a mathematical origin of magnetic field [1], which proves that Eq. (2) is perfect for the purpose of this article.

The basic concept of this article is that the combination of the Newton's law, the motion $\mathbf{v}$ of g-charges, and Eq. (2) must generate an axial magnetic-type g-field.

Let's consider: (1) The Newton's law is a divergence law describing g-charges and its g-field, thus one of vectors **S** and **T** should be the g-field $\mathbf{g}$; (2) we want to consider the effects of the motion of g-charges, thus another vector must by velocity $\mathbf{v}$. Therefore, let $\mathbf{S} = \mathbf{v}$, Then Eq. (2), (3) and (5) become, respectively,

$$\nabla \times (\mathbf{v} \times \mathbf{T}) = \mathbf{v}(\nabla \cdot \mathbf{T}) - \mathbf{T}(\nabla \cdot \mathbf{v}) + (\mathbf{T} \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{T}, \tag{2a}$$

$$\boldsymbol{\nabla}(\mathbf{v} \cdot \mathbf{T}) = (\mathbf{v} \cdot \boldsymbol{\nabla})\mathbf{T} + (\mathbf{T} \cdot \boldsymbol{\nabla})\mathbf{v} + \mathbf{v} \times (\boldsymbol{\nabla} \times \mathbf{T}) + \mathbf{T} \times (\boldsymbol{\nabla} \times \mathbf{v}), \tag{3a}$$

$$\nabla \times (\mathbf{v} \times \mathbf{T}) = \mathbf{v}(\nabla \cdot \mathbf{T}) - \mathbf{T}(\nabla \cdot \mathbf{v}) + (\mathbf{T} \cdot \nabla)\mathbf{v} - \{\boldsymbol{\nabla}(\mathbf{v} \cdot \mathbf{T}) - (\mathbf{T} \cdot \boldsymbol{\nabla})\mathbf{v} - \mathbf{v} \times (\boldsymbol{\nabla} \times \mathbf{T}) - \mathbf{T} \times (\boldsymbol{\nabla} \times \mathbf{v})\}. \tag{5a}$$

In Section 10, for showing the physical meaning of obtained gravito-electromagnetic equations clearly, we will simplify the obtained Maxwell-type gravitation equations by considering the most common situations that a g-charge moves with non-spatially-varying velocity $\mathbf{v}$. Thus, $(\mathbf{T} \cdot \nabla)\mathbf{v} = \boldsymbol{\nabla} \times \mathbf{v} = \nabla \cdot \mathbf{v} = 0$. For those situations, Eq. (2a), (3a) and (5a) become, respectively,

$$\nabla \times (\mathbf{v} \times \mathbf{T}) = \mathbf{v}(\nabla \cdot \mathbf{T}) - (\mathbf{v} \cdot \nabla)\mathbf{T}, \tag{6}$$

$$\boldsymbol{\nabla}(\mathbf{v} \cdot \mathbf{T}) = (\mathbf{v} \cdot \boldsymbol{\nabla})\mathbf{T} + \mathbf{v} \times (\boldsymbol{\nabla} \times \mathbf{T}). \tag{7}$$

$$\nabla \times (\mathbf{v} \times \mathbf{T}) = \mathbf{v}(\nabla \cdot \mathbf{T}) - \boldsymbol{\nabla}(\mathbf{v} \cdot \mathbf{T}) + \mathbf{v} \times (\boldsymbol{\nabla} \times \mathbf{T}). \tag{8}$$

For the case of $\mathbf{S} = \mathbf{T} = \mathbf{v}$, Eq. (4) gives

$$\mathbf{v} \cdot (\mathbf{v} \times \mathbf{Z}) = \mathbf{Z} \cdot (\mathbf{v} \times \mathbf{v}) = 0. \tag{9}$$

4. Ampere-type Gravito-magnetic Law

Let's apply those vector analysis formulas to derive Ampere-type gravito-magnetic law mathematically. The combination of the vector analysis formula and Newton's law, Eq. (1), leads us to let $\mathbf{T} = \mathbf{g}$. Then Eq. (2a) gives

$$\nabla \times (\mathbf{v} \times \mathbf{g}) = \mathbf{v}(\nabla \cdot \mathbf{g}) - \mathbf{g}(\nabla \cdot \mathbf{v}) + (\mathbf{g} \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{g}, \tag{10}$$

Define an axial field, $\mathbf{B}_g$,

$$\mathbf{B}_g \equiv \mathbf{v} \times \mathbf{g}, \tag{11}$$

Combining Eq. (10) and (11), we obtain,

$$\nabla\times\mathbf{B}_g = -4\pi Q_g\mathbf{v} - \mathbf{g}(\nabla\cdot\mathbf{v}) + (\mathbf{g}\cdot\nabla)\mathbf{v} - (\mathbf{v}\cdot\nabla)\mathbf{g}, \tag{12}$$

where Newton's law has been applied. Eq. (12) shows that the velocity $\mathbf{v}$ of a g-charge Q_g indeed mathematically generates inevitably an axial g-field $\mathbf{B}_g$. For an axial vector, we have mathematically,

$$\nabla\cdot\mathbf{B}_g = 0. \tag{13}$$

In electromagnetism, one has the Ampere's law

$$\nabla\times\mathbf{B} = 4\pi Q\mathbf{v}, \tag{14}$$

and Gauss's law for magnetic,

$$\nabla\cdot\mathbf{B} = 0. \tag{15}$$

Comparing Eq. (12 and 13) with Eq. (14 and 15), respectively, we call Eq. (12) the Ampere-type gravito-magnetic law; Eq. (13) the Gauss-type gravito-magnetic law; and the axial vector $\mathbf{B}_g$ the gravito-magnetic field. We will discuss the terms, $-\mathbf{g}(\nabla\cdot\mathbf{v}) + (\mathbf{g}\cdot\nabla)\mathbf{v} - (\mathbf{v}\cdot\nabla)\mathbf{g}$, in Section 10.

In the derivation above, the only physics law we applied is the Newton's law, which indicates that, for a g-charge satisfying the Newton's law, the motion of the g-charge (called g-current, $4\pi Q_g\mathbf{v}$**),** inevitable generates an axial magnetic-type g-field $\mathbf{B}_g$, in the same way an electric current $4\pi Q\mathbf{v}$ generates magnetic field $\mathbf{B}$. The generation of the axial magnetic-type g-field $\mathbf{B}_g$ is predetermined by the wonderful mathematical formula Eq. (2). Namely, the combination of the divergence of a vector g-field and the vector velocity of a g-charge must mathematically generate an axial magnetic-type g-field $\mathbf{B}_g$.

Although the concept and formulas of a magnetic-type g-field $\mathbf{B}_g$ has been postulated before, now, for first time, we demonstrate mathematically the existence of the magnetic-type g-field $\mathbf{B}_g$.

5. Faraday-type Gravito-electric Law (1)

Based on symmetry/duality, it is nature to expect that the velocity of g-charges generate an axial electric-type g-field as well. For this aim, let's apply the formula, Eq. (2), again, and let $\mathbf{T} = \mathbf{B}_g$, we obtain,

$$\nabla\times(\mathbf{v}\times\mathbf{B}_g) = -\mathbf{B}_g(\nabla\cdot\mathbf{v}) + (\mathbf{B}_g\cdot\nabla)\mathbf{v} - (\mathbf{v}\cdot\nabla)\mathbf{B}_g, \tag{16}$$

where Eq. (13) has been applied.

Define an axial g-field, $\mathbf{g}$**,**

$$\mathbf{g} \equiv -\mathbf{v}\times\mathbf{B}_g. \tag{17}$$

Substituting Eq. (17), into Eq. (16), we obtain,

$$\nabla\times\mathbf{g} = \mathbf{B}_g(\nabla\cdot\mathbf{v}) - (\mathbf{B}_g\cdot\nabla)\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{B}_g, \tag{18}$$

The magnetic-type g-field $\mathbf{B}_g$ and the velocity $\mathbf{v}$ of a g-charge indeed generate an electric-type g-field $\mathbf{g}$. Eq. (18) relates the $\mathbf{g}$ field, velocity $\mathbf{v}$, and magnetic-type g-field $\mathbf{B}_g$**.**

Now we calculate the term $(\mathbf{v}\cdot\nabla)\mathbf{B}_g$ of Eq. (18). Let $\mathbf{T} = \mathbf{B}_g$, Eq. (3a) gives

$$\nabla(\mathbf{v}\cdot\mathbf{B}_g) = (\mathbf{v}\cdot\nabla)\mathbf{B}_g + (\mathbf{B}_g\cdot\nabla)\mathbf{v} + \mathbf{v}\times(\nabla\times\mathbf{B}_g) + \mathbf{B}_g\times(\nabla\times\mathbf{v}). \tag{19}$$

Rearranging Eq. (19), we have

$$(\mathbf{v}\cdot\nabla)\mathbf{B}_g = \nabla(\mathbf{v}\cdot\mathbf{B}_g) - (\mathbf{B}_g\cdot\nabla)\mathbf{v} - \mathbf{v}\times(\nabla\times\mathbf{B}_g) - \mathbf{B}_g\times(\nabla\times\mathbf{v}). \tag{20}$$

Utilizing the definition of $\mathbf{B}_g$, Eq. (11) gives

$$\mathbf{v}\cdot\mathbf{B}_g = \mathbf{v}\cdot(\mathbf{v}\times\mathbf{g}) = \mathbf{g}\cdot(\mathbf{v}\times\mathbf{v}) = \mathbf{0}.$$

Eq. (20) becomes,

$$(\mathbf{v}\cdot\nabla)\mathbf{B}_g = -\mathbf{v}\times(\nabla\times\mathbf{B}_g) - (\mathbf{B}_g\cdot\nabla)\mathbf{v} - \mathbf{B}_g\times(\nabla\times\mathbf{v}). \tag{21}$$

Substituting Eq. (21) into Eq. (18), we obtain

$$\nabla\times\mathbf{g} = -\mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{B}_g) - \mathbf{2}(\mathbf{B}_g\cdot\boldsymbol{\nabla})\mathbf{v} - \mathbf{B}_g\times(\boldsymbol{\nabla}\times\mathbf{v}) + \mathbf{B}_g(\nabla\cdot\mathbf{v}) \tag{22}$$

We will come back to this equation.

6. Scalar and Vector Potentials

Let's introduce a gravito-magnetic potential $\mathbf{A_g}$ defined below,

$$\mathbf{B_g} = \nabla\times\mathbf{A_g}, \tag{23}$$

$$\mathbf{g} = -\nabla\varphi_g - \frac{\partial\mathbf{A_g}}{\partial t}. \tag{24}$$

Now a g-field $\mathbf{g}$ includes two parts, vector Newton g-field $\mathbf{g}_N$ and axial gravito-electric g-field $\mathbf{E_g}$,

$$\mathbf{g} = \mathbf{g_N} + \mathbf{E_g}. \tag{25}$$

In terms of scalar potential φ_g and gravito-magnetic potential $\mathbf{A_g}$, we have

$$\mathbf{E_g} = -\frac{\partial\mathbf{A_g}}{\partial t}, \tag{26}$$

$$\mathbf{g_N} = -\nabla\varphi_g, \tag{27}$$

$$\nabla\times\mathbf{g} = \nabla\times\mathbf{E_g}, \tag{28}$$

$$\frac{\partial\mathbf{g}}{\partial t} = \frac{\partial\mathbf{E_g}}{\partial t}. \tag{29}$$

Taking into account Eq. (25), the Newton's law, $\nabla\cdot\mathbf{g_N} = -4\pi Q_g$, can be extended to

$$\nabla\cdot\mathbf{g} = -4\pi Q_g. \tag{30}$$

7. Interpretation of axial $\mathbf{B_g}$ and Axial $\mathbf{E_g}$ Fields

Consider a g-charge Q_g and an observer. When the g-charge is at rest relative to the observer, it generates a vector Newton g-field $\mathbf{g_N}$ only; when the g-charge is moving with velocity relative to the observer, beside the g-field $\mathbf{g_N}$, the observer also experiences an axial gravito-magnetic field $\mathbf{B_g}$ and an axial gravito-electric field $\mathbf{E_g}$ generated by the velocity of the g-charge, $\mathbf{B_g} = \mathbf{v}\times\mathbf{g}$, and $\mathbf{g} = -\mathbf{v}\times\mathbf{B_g}$. Namely, the velocity of a g-charge generates two axial fields.

8. Ampere-Maxwell-type Gravito-magnetic Law

Now let's study the term, $(\mathbf{v}\cdot\nabla)\mathbf{g}$, of Eq. (12). Applying the vector analysis formula, Eq. (3a). Let $\mathbf{T} = \mathbf{g}$, Eq. (3a) gives

$$(\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{g} = \boldsymbol{\nabla}(\mathbf{v}\cdot\mathbf{g}) - (\mathbf{g}\cdot\boldsymbol{\nabla})\mathbf{v} - \mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{g}) - \mathbf{g}\times(\boldsymbol{\nabla}\times\mathbf{v}). \tag{31}$$

Since, $\mathbf{v}\cdot\mathbf{g} = -\mathbf{v}\cdot(\mathbf{v}\times\mathbf{B}_g) = \mathbf{0}$, Eq. (32) becomes,

$$(\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{g} = -\mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{g}) - (\mathbf{g}\cdot\boldsymbol{\nabla})\mathbf{v} - \mathbf{g}\times(\boldsymbol{\nabla}\times\mathbf{v}). \tag{32}$$

Substituting Eq. (32) into Eq. (12), we obtain,

$$\nabla\times(\mathbf{B}_g) = -4\pi Q_g\mathbf{v} - \mathbf{g}(\nabla\cdot\mathbf{v}) + 2(\mathbf{g}\cdot\nabla)\mathbf{v} + \mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{g}) + \mathbf{g}\times(\boldsymbol{\nabla}\times\mathbf{v}). \tag{33}$$

Next let's find $\mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{g})$. For this aim, utilizing Eq. (17), (23) and (24), we obtain

$$\mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{g}) = -\mathbf{v}\times\frac{\partial\mathbf{B_g}}{\partial t} = -\frac{\partial\mathbf{v}\times\mathbf{B_g}}{\partial t} + \dot{\mathbf{v}}\times\mathbf{B}_g = \frac{\partial\mathbf{g}}{\partial t} + \dot{\mathbf{v}}\times\mathbf{B}_g. \tag{34}$$

Then, substituting Eq. (34) into Eq. (33), we obtain,

$$\nabla\times\mathbf{B}_g = -4\pi Q_g\mathbf{v} + \frac{\partial\mathbf{g}}{\partial t} + \dot{\mathbf{v}}\times\mathbf{B}_g - \mathbf{g}(\nabla\cdot\mathbf{v}) + 2(\mathbf{g}\cdot\nabla)\mathbf{v} + \mathbf{g}\times(\boldsymbol{\nabla}\times\mathbf{v}). \tag{35}$$

From Eq. (29), Eq. (35) can be rewrite as,

$$\nabla\times\mathbf{B}_g = -4\pi Q_g\mathbf{v} + \frac{\partial\mathbf{E_g}}{\partial t} + \dot{\mathbf{v}}\times\mathbf{B}_g - \mathbf{g}(\nabla\cdot\mathbf{v}) + 2(\mathbf{g}\cdot\nabla)\mathbf{v} + \mathbf{g}\times(\boldsymbol{\nabla}\times\mathbf{v}) \tag{36}$$

Comparing Eq. (35) or (36) with the Ampere-Maxwell's equation,

$$\nabla\times\mathbf{B} = 4\pi Q\mathbf{v} + \frac{\partial\mathbf{E}}{\partial t}.$$

We call Eq. (35) and (36) the Ampere-Maxwell-type gravito-magnetic equation that is derived mathematically.

9. Faraday-type Gravito-electric Law (2)

Now let's substituting Eq. (35) into Eq. (22), we obtain,

$$\nabla\times\mathbf{g} = -\mathbf{v}\times\left\{-4\pi Q_g\mathbf{v} + \frac{\partial\mathbf{g}}{\partial t} + \dot{\mathbf{v}}\times\mathbf{B}_g - \mathbf{g}(\nabla\cdot\mathbf{v}) + 2(\mathbf{g}\cdot\nabla)\mathbf{v} + \mathbf{g}\times(\nabla\times\mathbf{v})\right\} - \\ -2(\mathbf{B}_g\cdot\nabla)\mathbf{v} - \mathbf{B}_g\times(\nabla\times\mathbf{v}) + \mathbf{B}_g(\nabla\cdot\mathbf{v}) \quad (37)$$

Utilizing Eq. (11), we have

$$\mathbf{v}\times\frac{\partial\mathbf{g}}{\partial t} = \frac{\partial\mathbf{v}\times\mathbf{g}}{\partial t} - \dot{\mathbf{v}}\times\mathbf{g} = \frac{\partial\mathbf{B}_g}{\partial t} - \dot{\mathbf{v}}\times\mathbf{g}.$$

Then Eq. (37) gives

$$\nabla\times\mathbf{g} = -\frac{\partial\mathbf{B}_g}{\partial t} + \dot{\mathbf{v}}\times\mathbf{g} - \mathbf{v}\times\left\{\dot{\mathbf{v}}\times\mathbf{B}_g - \mathbf{g}(\nabla\cdot\mathbf{v}) + 2(\mathbf{g}\cdot\nabla)\mathbf{v} + \mathbf{g}\times(\nabla\times\mathbf{v})\right\} - \\ -2(\mathbf{B}_g\cdot\nabla)\mathbf{v} - \mathbf{B}_g\times(\nabla\times\mathbf{v}) + \mathbf{B}_g(\nabla\cdot\mathbf{v}). \quad (38)$$

Comparing with the Faraday's law, we call Eq. (38) the Faraday-type gravito-electric law. We have mathematically derived Maxwell-type gravito-electromagnetic equations from the Newton's law.

10. Gravito-electromagnetism for Common Situations

For clearly showing the physical meaning of obtained gravitation equations, we simplify the obtained Ampere-Maxwell-type and Faraday-type gravitation equations. For general situations of gravitation, the velocities of g-charges are moving with non-spatially-varying velocity $\mathbf{v}$, i.e.,

$$\nabla\cdot\mathbf{v} = (\mathbf{Z}\cdot\nabla)\mathbf{v} = (\nabla\times\mathbf{v}) = 0, \quad (39)$$

Where $\mathbf{Z}$ represents either $\mathbf{g}$ or $\mathbf{B}_g$. Substituting the condition, Eq. (39), into Eq. (36) and Eq. (38), respectively, we obtain,

$$\nabla\times\mathbf{B}_g = -4\pi Q_g\mathbf{v} + \frac{\partial\mathbf{E}_g}{\partial t} + \dot{\mathbf{v}}\times\mathbf{B}_g, \quad (40)$$

$$\nabla\times\mathbf{g} = -\frac{\partial\mathbf{B}_g}{\partial t} + \dot{\mathbf{v}}\times\mathbf{g} - \mathbf{v}\times(\dot{\mathbf{v}}\times\mathbf{B}_g). \quad (41)$$

Furthermore, for uniformly moving g-charges, Eq. (40 and 41) become respectively,

$$\nabla\times\mathbf{B}_g = -4\pi Q_g\mathbf{v} + \frac{\partial\mathbf{g}}{\partial t}, \quad (42)$$

$$\nabla\times\mathbf{g} = -\frac{\partial\mathbf{B}_g}{\partial t}, \quad (43)$$

or

$$\nabla\times\mathbf{B}_g = -4\pi Q_g\mathbf{v} + \frac{\partial\mathbf{E}_g}{\partial t}, \quad (44)$$

$$\nabla\times\mathbf{E}_g = -\frac{\partial\mathbf{B}_g}{\partial t}. \quad (45)$$

Note it is straightforward to derive Eq. (40) and (42) directly from Eq. (8).

11. Gravito-electric /Gravito-magnetic Duality

Based on the Newton's law, for a moving g-charge, we derive mathematically the Maxwell-type gravitation equations, called Gravito-electromagnetism. The conclusion is that the motion of a g-charge must generate an axial vector magnetic-type g-field and an axial vector electric-type g-field. This fact is predetermined mathematically by Eq. (1) and (2). The magnetic-type gravitation has its mathematical origin.

For source free and uniform motion situation, under the transformation $\mathbf{B}_g \to \mathbf{E}_g$ and $\mathbf{E}_g \to -\mathbf{B}_g$, the axial gravito-electric field is the dual of the gravito-magnetic field as below,

1. The Ampere-Maxwell-type gravito-magnetic law, $\nabla\times\mathbf{B}_g = \frac{\partial\mathbf{E}_g}{\partial t}$, is the dual of Faraday-type

gravito-electric law, $\nabla\times\mathbf{E_g} = -\frac{\partial\mathbf{B_g}}{\partial t}$.

2. The extended Newton's law, $\nabla\cdot\mathbf{g} = -4\pi Q_g$, is the dual of Gauss-type gravito-magnetic law, $\nabla\cdot\mathbf{B}_g = 0$.
3. The scalar potential φ_g is the dual of the gravito-magnetic potential $\mathbf{A}_g$.

The underlying reason for this gravito-electric/gravito-magnetic duality is due to that both were derived by the same mathematical approach.

Table 1 shows the duality between gravito-magnetic and gravito-electric fields.

Table 1: gravito-magnetic/gravito-electric Duality

Gravito-electric g-ield	Correspondence	Gravito-magnetic g-field
$\mathbf{g} = -\mathbf{v}\times\mathbf{B_g}$	$\mathbf{g} \leftrightarrow \mathbf{B}_g$ $\mathbf{B_g} \leftrightarrow -\mathbf{g}$ $\mathbf{E}_g \leftrightarrow \mathbf{B_g}$ $\mathbf{B_g} \leftrightarrow -\mathbf{E}_g$	$\mathbf{B_g} = \mathbf{v}\times\mathbf{g}$
$\nabla\cdot\mathbf{g} = -4\pi Q_g$ $\nabla\cdot\mathbf{E}_g = 0$		$\nabla\cdot\mathbf{B}_g = 0$
$\nabla\times\mathbf{E_g} = -\frac{\partial\mathbf{B_g}}{\partial t}$ $\nabla\times\mathbf{g} = -\frac{\partial\mathbf{B_g}}{\partial t}$		$\nabla\times\mathbf{B}_g = -4\pi Q_g\mathbf{v} + \frac{\partial\mathbf{E_g}}{\partial t}$ $\nabla\times\mathbf{B}_g = -4\pi Q_g\mathbf{v} + \frac{\partial\mathbf{g}}{\partial t}$

12. Electromagnetism/Gravito-electromagnetism Duality

The most important discovery of this mathematical approach is the duality between Electromagnetism and gravitation (Gravito-magnetism). The fundamental duality is between the Gauss/Coulomb's law and the Newton's law (Table 2).

Table 2: Duality between Gauss/Coulomb's law and Newton's law

Gauss/Coulomb's law	Correspondence	Newton's law
$\nabla\cdot\mathbf{E} = 4\pi Q_e$	$Q_e \leftrightarrow -Q_g$ $\mathbf{E} \leftrightarrow \mathbf{g}$	$\nabla\cdot\mathbf{g} = -4\pi Q_g$

By applying the exactly same mathematical derivation, we obtain the Maxwell equations in reference [1] and Maxwell-type gravitation equations in this article, which leads to the duality between Electromagnetics and Gravito-electromagnetics (Table 3). The Electromagnetics/Gravito-electromagnetics duality is based on the Coulomb/Newton duality and thus, is the secondary duality.

Table 3: Electromagnetics/Gravito-electromagetics Duality

Electromagnetics	Correspondence	Gravito-Electromagnetics
$\nabla\times\mathbf{B} = 4\pi Q_e\mathbf{v} + \frac{\partial\mathbf{E}}{\partial t}$	$Q_e \leftrightarrow -Q_g$ $\mathbf{B} \leftrightarrow \mathbf{B}_g$ $\mathbf{E} \leftrightarrow \mathbf{g}$	$\nabla\times\mathbf{B}_g = -4\pi Q_g\mathbf{v} + \frac{\partial\mathbf{g}}{\partial t}$
$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$	$\mathbf{E} \leftrightarrow \mathbf{g}$ $\mathbf{B} \leftrightarrow \mathbf{B}_g$	$\nabla\times\mathbf{g} = -\frac{\partial\mathbf{B_g}}{\partial t}$
$\nabla\cdot\mathbf{B} = 0$	$\mathbf{B} \leftrightarrow \mathbf{B}_g$	$\nabla\cdot\mathbf{B}_g = 0$

This duality shows that under the conversion of

$$Q_e \leftrightarrow -Q_g,\ \mathbf{E} \leftrightarrow \mathbf{g},\ \mathbf{B} \leftrightarrow \mathbf{B}_g,$$

Electromagnetism converts to Gravito-electromagnetism and vice versa.

This duality shows the intrinsic similarity between electromagnetic and gravitation (Table 3).

The reason for this electromagnetism/gravito-electromagnetism duality is due to three factors:

(1) Both Electromagnetic force and gravitational force are only two long-range forces in Nature, therefore no non-linear term;

(2) The Coulomb's law is the dual of Newton's law, both are inverse-square law;

(3) Electromagnetism has been re-derived by a mathematic approach from the Coulomb's law [1]; Gravito-electromagnetism is derived by the exactly same mathematic approach from the Newton's law in this article.

13.Summary and Discussion

Based on the Newton's law, for a moving g-charge, we derive mathematically the Maxwell-type gravitation equations, called Gravito-electromagnetism. The conclusion is that the motion of a g-charge indeed generates an axial vector magnetic-type g-field and an axial vector electric-type g-field inevitably. This fact is predetermined mathematically by Eq. (1) and (2). The magnetic-type gravitation has its mathematical origin. The Electromagnetics/Gravito-electromagetics Duality is reached in the perspective of gravity as a physical field.

Lorentz Invariance. For approximate situations, the Maxwell-type gravito-electromagnetic equations reduce to the exactly same form as Maxwell equation, and satisfy the Lorentz transformation. Only for the uniformly moving electric charges and g-charges, Maxwell equations and Gravito-electromagnetic equations are Lorentz invariant.

Positive/negative gravitational charge duality. One may argue that there is the positive/negative electric charge duality, is there positive/negative g-charge duality? Yes, the positive/negative g-charge duality has been studied [12], and has been utilized to explain the accelerating universe and predict the jerking universe [13]. We argue that the reported observation that the acceleration of the expansion of the universe is accelerating [14] is an evidence of the existence of the negative g-charge.

Conservation of gravitational charge and internal symmetry. The total g-charge in an isolated system keeps constant. By analogy to that the conservation of electric charges leads to the internal U(1) symmetry, we ague that the conservation of g-charge leads to the U(1) internal symmetry [15] for gravity.

Quantization of Gravito-electromagnetics and unification with Electromagnetics. Since Electrodynamics has been quantized (QED), the Electromagnetics/Gravito-electromagetics Duality allows one to quantize Gravito-electromagnetics, denoted as QGD, by following the same procedure. Then QED and QGD can be unified with internal $\mathrm{U_g}(1) \times \mathrm{U_e}(1)$ group [15].

The mathematic approach utilized in this article and in reference [1] demonstrates once more the famous sentence, "The world is built mathematically".

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