A generalization of the Clifford algebra

A.Balan

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1 The Clifford algebra

The Clifford algebra may be define with generators in a vector space E and relations:

$$e \otimes f + f \otimes e = -2g(e, f)$$

2 The generalization of the Clifford algebra

We choose two endomorphisms $\psi, \phi \in End(E)$ and change the relations:

$$\psi(e) \otimes \phi(f) + \psi(f) \otimes \phi(e) = -g(\psi(e), \phi(f)) - g(\psi(f), \phi(e))$$

If we take $\psi = \phi = 1$, then we have the usual Clifford algebra. If we take $e = f$, then we deduce:

$$\psi(e) \otimes \phi(f) = -g(\psi(e), \phi(e))$$

so that we can have:

$$\psi(e) \otimes \phi(e) = 0$$

and the vectors $\psi(e), \phi(e)$ may be non inversible even if $\psi(e) \neq 0$ and $\phi(e) \neq 0$.

3 Applications

We consider the two Dirac operators:

$$D_\psi = \sum_i \psi(e_i) \nabla_{e_i}$$

and

$$D_\phi = \sum_i \phi(e_i) \nabla_{e_i}$$

for (e_i) an orthonormal basis. Then, we have the generalized Lichnerowicz formula:

$$D_\psi D_\phi = \Delta(\psi, \phi) + \alpha$$

with $\Delta(\psi,\phi)$ the Laplacian operator:

$$\sum_i g(\psi(e_i),\phi(e_j))[\nabla_{e_i}\nabla_{e_j} - \nabla_{\nabla_{e_i}e_j}]$$

and α a scalar.

References

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[L] P.Lounesto, ”Clifford Algebras and Spinors”, London Mathematical Society, Lecture Note Series 239, 1997.