The Ellipse - Circle - Ellipse property of projectile motion

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Abstract

Recently we gave an alternate method to show the presence of an ellipse in the paths of projectiles, to the method given by Fernandez-Chapou and coworkers. Our method demonstrated that particles dropped from points of a circle in a vertical plane on to the horizontal diameter and reflected through 90 degrees by mirrors halfway on their paths, follow parabolic trajectories and that the apexes of these parabolic paths lie on an ellipse. In this article we give a similar procedure to show that particles dropped from points of an ellipse in a vertical plane to the horizontal (minor) axis but reflected through 90 degrees by mirrors halfway on their paths, follow parabolic trajectories and that the apexes of these parabolic paths lie on a circle. From this circle we can generate another ellipse by a similar procedure as above. This 'circle-ellipse-circle–ellipse-…' cycle is a representation of the 'birth-death-birth-death-…' cycle. It is a representation of the 'parent-child-parent-child-…' cycle of living organisms. These cycles have no beginning and no end. They exhibit the beauty of symmetry of Natural law. We don't use vectors or calculus or Newton's laws in our method. We follow Galileo's tradition, use geometry principles in our analysis.

Key words: Projectiles, Circle in parabolic paths, circle-ellipse-circle cycle, 'birth-death-birth-…' cycle

Introduction

In an earlier article[1] we presented an alternate method of finding the ellipse hidden in a set of parabolas corresponding to a family of projectile paths, to the method of Fernandez-Chapou and coworkers[2]. We present in this article the parent-child-parent relationship between a circle, its (daughter/child) ellipse, and its (daughter/child) circle. An interesting result is that from one generation to the next, the ellipse not only changes its size, but changes its orientation through ninety degrees so as to yield the next generation circle. In fact, the circle also changes its size and changes its orientation through ninety degrees, but change in the rotation of a circle goes undetected. We emphasize, that we don't use vectors or calculus or Newton's laws in our analysis. We follow the traditions of Galileo[3] – we use geometry principles.

The circle hidden in the maxima of projectile paths

Let us consider an ellipse in a vertical plane with major axis in the vertical direction and the minor axis along the horizontal. Notice that we are considering the ellipse we presented in reference 1, now rotated through 90 degrees (see Fig.1). (The circle and the ellipse in dashed lines are the ones from reference 1). We do this in order to keep the direction of acceleration along the vertical. This gives the feel that particles released from points on the ellipse in the first quadrant move with uniform acceleration in the vertically downward direction. (We stress the fact that instead of rotating the ellipse, if we rotate the direction of acceleration to the horizontal, our results remain the same). From points A_i on the ellipse in the first quadrant we draw vertical lines $A_i O_i$ on to the minor axis (see Fig.1). At the mid points♠ m_i, of A_iO_i we place mirrors at an angle of 45 degrees to the horizontal. We let particles fall from rest vertically down from points A_i. At m_i the particles hit the mirrors and get reflected to move along the horizontal direction with the speed v_i, acquired in falling through A_im_i. (The speed v_i is equal to half the speed in

♠ Choosing mid points has significance – the speed at mid points gives the time average speed of fall through $A_i O_i$.

falling through A_iO_i. It is also the time average speed in falling through A_iO_i). As they do so, the particles are subjected to uniform acceleration in the vertically down-ward direction. Thus the particles are subjected to a uniform motion of constant speed along the horizontal and uniformly accelerated motion along the vertically down-ward direction.

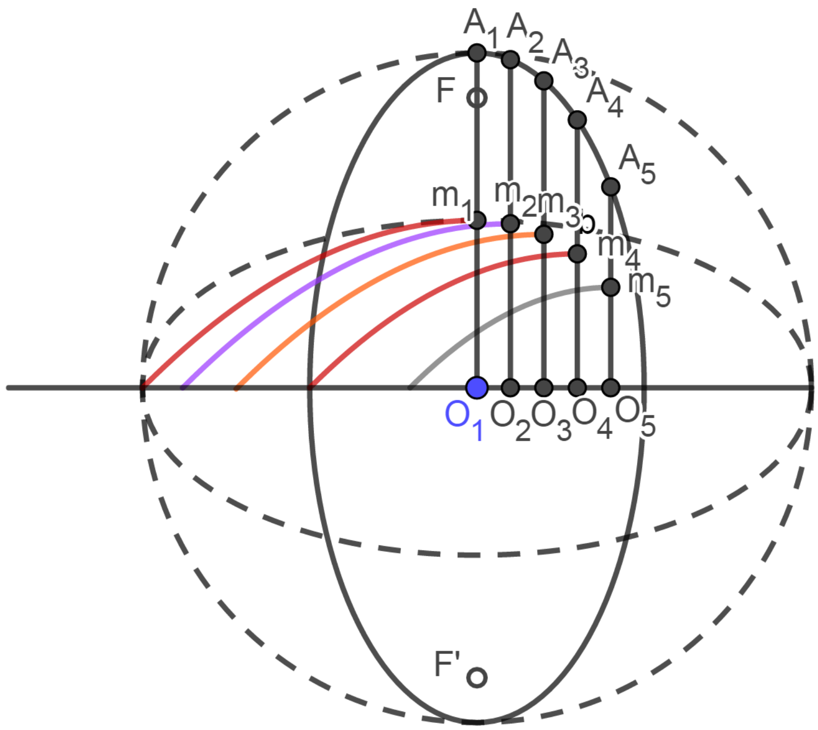

Fig.1 Family of parabolic paths of projectiles launched from mid points of A_iO_i.

As a result of these simultaneous motions in orthogonal directions, the particles execute a semi-parabolic motion in the vertical plane. We can get the corresponding paths in the second quadrant from symmetry considerations (here, by reflection). The starting points of these parabolic motions (positions of mirrors) are the vertices of the parabolic paths. The locus of these vertices gives the upper half of a circle. A reflection of this in the horizontal gives the lower half of the circle. This narration is depicted in Fig. 2. It is easy to show that the location of mirrors which correspond to the vertices of the parabolas lie on a circle.

Aside

The equation of the ellipse with its major axis in the horizontal direction and minor axis in the vertical direction (shown in dashed lines in Fig. 1) as was shown in reference 1 is,

$$\left(\frac{x'}{a}\right)^2 + \left(\frac{y'}{b}\right)^2 = 1\,, \qquad a = 2b \tag{1}$$

Since we rotated it through 90 degrees for the analysis in this paper, the equation of the ellipse with its minor axis in the horizontal direction and major axis in the vertical direction changes to (2), below.

End of aside

The equation of the ellipse with major axis in the vertical direction (solid line in Fig. 2), is given by

$$\left(\frac{x'}{b}\right)^2 + \left(\frac{y'}{a}\right)^2 = 1 \tag{2}$$

Where x', y' are the coordinates of any point on the ellipse with its center at the origin. Let the coordinates of the locations of the mirrors be x'' and y''. The pairs x', y' and x'', y'' are related as,

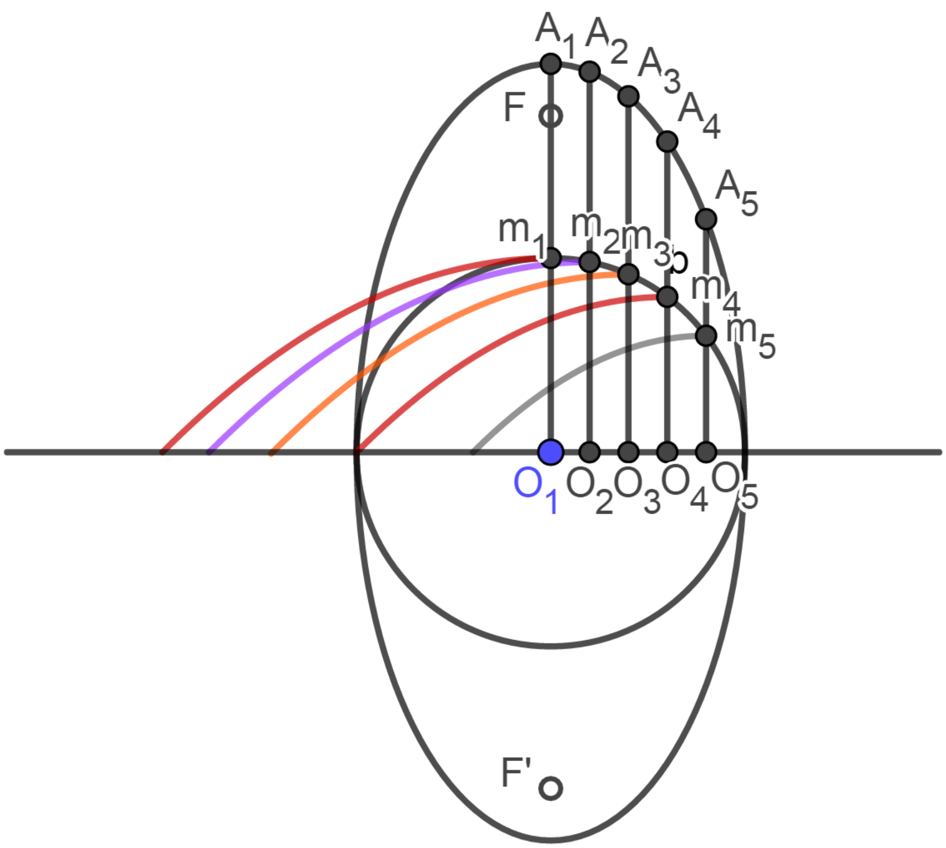

Fig. 2. Hidden circle in maxima of a family of projectile paths

$$x'' = x' \ \ and\, y'' = \frac{y'}{2}, \ \ and \ \ a = 2b \tag{3}$$

From (2) and (3) we get

$$x''^2 + \ y''^2 = \ b^2 \tag{4}$$

Equation (4) is an equation of a circle centered at the origin and having a radius ‘b’ (see Fig. 2).

Thus we see, a circle serves to generate an ellipse offspring (as shown in our earlier article[1]) which in turn serves to generate a next generation circle (as shown above), which in turn serves to generate an ellipse to continue the cycle endlessly (see Fig. 3). To generate the circle from an ellipse keeping acceleration in vertical direction we need the major axis of the ellipse in vertical direction. Therefore, we rotated the ellipse of reference 1 through 90 degrees, to give the feel of free fall in the vertical direction.

This ‘circle – ellipse – circle’ cycle continues indefinitely in both forward and reverse directions as can be seen from figure 3. This is the ‘parent – child – parent’ like relation that has no beginning and no end. It is the ‘birth – death – birth cycle. It is the statement of symmetry. It is the law of Nature.

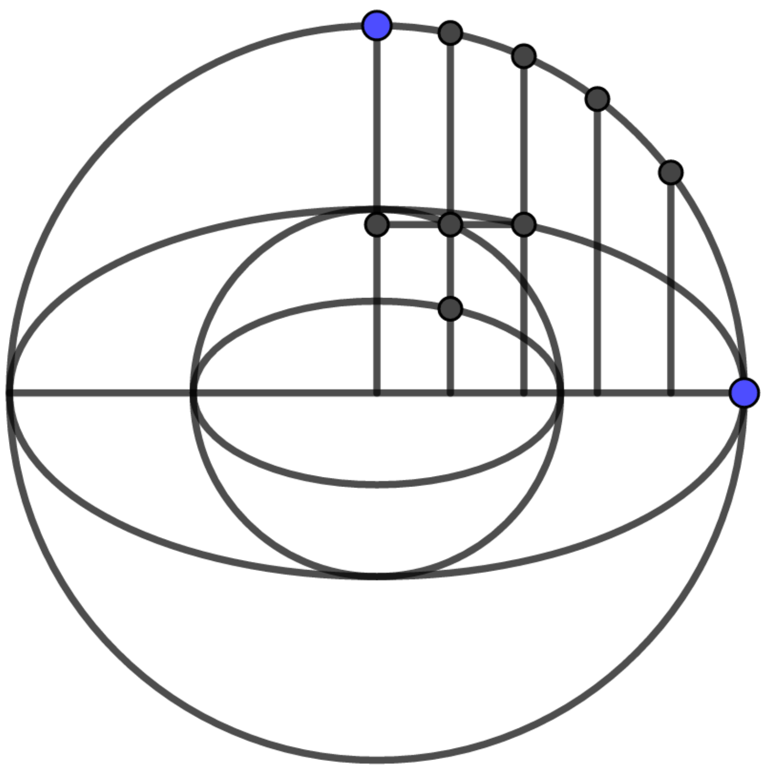

Fig. 3. Circle-ellipse-circle-ellipse- .... cycle

Discussion

The circle-ellipse- circle cycle may be viewed as representing not only birth-death-birth cycle but many other similar cycles, for example the cycle of breathing. If the change of a given circle via ellipse to a smaller circle is treated as representing exhalation then the reverse change of small circle to a bigger circle, via ellipse represents inhalation. The change from big circle to small circle corresponds to half cycle and the reverse process corresponds to the second half cycle. Similarly, if we imagine a pendulum drawn to a point, on one side and released to start a cycle of oscillation, then that initial position may be treated as representing the small circle, the vertical position represents the ellipse (with its major axis in the vertical direction) and the extreme point on other side represents the big circle to complete the oscillation. The vertical position on the reverse swing represents the ellipse with its minor axis in the horizontal direction.

These conics (including hyperbolas, not shown in the figures) in our analysis are members of families of central conics.

In our future articles we show the relation between the ellipse we considered, and the Newton's ellipse. We will show that Newton's ellipse is a component ellipse, of the ellipse we considered. Two such (component) Newton's ellipses combine to form '*the resultant ellipse*'. Many other properties of these ellipses, circles, parabolas, hyperbolas follow from consideration of the geometrical relations.

Dedication: I dedicate this article with reverence to my parents: Nagaratnamma and Adinarayana, Padyala.

Acknowledgement

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