Plithogenic Set, an Extension of Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Sets - Revisited

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Abstract. In this paper, we introduce the plithogenic set (as generalization of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets), which is a set whose elements are characterized by many attributes' values. An attribute value *v* has a corresponding (fuzzy, intuitionistic fuzzy, or neutrosophic) degree of appurtenance d(*x,v*) of the element *x*, to the set *P*, with respect to some given criteria. In order to obtain a better accuracy for the plithogenic aggregation operators in the plithogenic set, and for a more exact inclusion (partial order), a (fuzzy, intuitionistic fuzzy, or neutrosophic) contradiction (dissimilarity) degree is defined between each attribute value and the dominant (most important) attribute value. The plithogenic intersection and union are linear combinations of the fuzzy operators t_{norm} and t_{conorm}, while the plithogenic complement, inclusion (inequality), equality are influenced by the attribute values contradiction (dissimilarity) degrees. This article offers some examples and applications of these new concepts in our everyday life.

Keywords: Plithogeny; Plithogenic Set; Neutrosophic Set; Plithogenic Operators.

1 Informal Definition of Plithogenic Set

Plithogeny is the genesis or origination, creation, formation, development, and evolution of *new entities* from dynamics and organic fusions of contradictory and/or neutrals and/or non-contradictory *multiple old entities*.

While plithogenic means what is pertaining to plithogeny.

A plithogenic set *P* is a set whose elements are characterized by one or more attributes, and each attribute may have many values. Each attribute's value *v* has a corresponding degree of appurtenance d(*x,v*) of the element *x*, to the set *P*, with respect to some given criteria.

In order to obtain a better accuracy for the plithogenic aggregation operators, a contradiction (dissimilarity) degree is defined between each attribute value and the dominant (most important) attribute value.

{However, there are cases when such dominant attribute value may not be taking into consideration or may not exist [therefore it is considered zero by default], or there may be many dominant attribute values. In such cases, either the contradiction degree function is suppressed, or another relationship function between attribute values should be established.}

The plithogenic aggregation operators (intersection, union, complement, inclusion, equality) are based on contradiction degrees between attributes' values, and the first two are linear combinations of the fuzzy operators' t_{norm} and t_{conorm}.

Plithogenic set is a generalization of the crisp set, fuzzy set, intuitionistic fuzzy set, and neutrosophic set, since these four types of sets are characterized by a single attribute value (appurtenance): which has one value (membership) – for the crisp set and fuzzy set, two values (membership, and nonmembership) – for intuitionistic fuzzy set, or three values (membership, nonmembership, and indeterminacy) – for neutrosophic set.

2 Formal Definition of Single (Uni-Dimensional) Attribute Plithogenic Set

Let *U* be a universe of discourse, and *P* a non-empty set of elements, $P \subseteq U$.

2.1 Attribute Value Spectrum

Let $\mathscr{A}$ be a non-empty set of uni-dimensional attributes $\mathscr{A} = \{\alpha_1, \alpha_2, \ldots, \alpha_m\}$, $m \geq 1$; and $\alpha \in \mathscr{A}$ be a given attribute whose spectrum of all possible values (or states) is the non-empty set *S*, where *S* can be a finite discrete set, $S = \{s_1, s_2, \ldots, s_l\}$, $1 \leq l < \infty$, or infinitely countable set $S = \{s_1, s_2, \ldots, s_\infty\}$, or infinitely uncountable (continuum) set $S =]a, b[$, $a < b$, where $]\ldots[$ is any open, semi-open, or closed interval from the set of real numbers or from other general set.

2.2 Attribute Value Range

Let V be a non-empty subset of S, where V is the *range of all attribute's values* needed by the experts for their application. Each element $x \in P$ is characterized by all attribute's values in $V = \{v_1, v_2, \ldots, v_n\}$, for $n \geq 1$.

2.3 Dominant Attribute Value

Into the attribute's value set V, in general, there is a dominant attribute value, which is determined by the experts upon their application. Dominant attribute value means the most important attribute value that the experts are interested in.

{However, there are cases when such dominant attribute value may not be taking into consideration or not exist, or there may be many dominant (important) attribute values - when different approach should be employed.}

2.4 Attribute Value Appurtenance Degree Function

Each attributes value $v \in V$ has a corresponding degree of appurtenance $d(x, v)$ of the element x, to the set P, with respect to some given criteria.

The degree of appurtenance may be: a fuzzy degree of appurtenance, or intuitionistic fuzzy degree of appurtenance, or neutrosophic degree of appurtenance to the plithogenic set.

Therefore, the attribute value appurtenance degree function is:

$$\forall x \in P,\ d: P \times V \rightarrow \mathscr{P}([0, 1]^z), \quad (1)$$

so $d(x, v)$ is a subset of $[0, 1]^z$, and $\mathscr{P}([0, 1]^z)$ is the power set of the $[0, 1]^z$, where $z = 1$ (for fuzzy degree of appurtenance), $z = 2$ (for intuitionistic fuzzy degree of appurtenance), or $z = 3$ (for neutrosophic degree de appurtenance).

2.5 Attribute Value Contradiction (Dissimilarity) Degree Function

Let the cardinal $|V| \geq 1$.

Let $c: V \times V \rightarrow [0, 1]$ be the attribute value contradiction (dissimilarity) degree function (that we introduce now for the first time) between any two attribute values v_1 and v_2, denoted by

$c(v_1, v_2)$, and satisfying the following axioms:
$c(v_1, v_1) = 0$, the contradiction degree between the same attribute values is zero;
$c(v_1, v_2) = c(v_2, v_1)$, commutativity.

For simplicity, we use a fuzzy attribute value contradiction degree function (c as above, that we may denote by c_F in order to distinguish it from the next two), but an intuitionistic attribute value contradiction function ($c_{IF} : V \times V \rightarrow [0, 1]^2$), or more general a neutrosophic attribute value contradiction function ($c_N : V \times V \rightarrow [0, 1]^3$) may be utilized increasing the complexity of calculation but the accuracy as well.

We mostly compute the contradiction degree between *uni-dimensional attribute values*. For *multi-dimensional attribute values* we split them into corresponding uni-dimensional attribute values.

The attribute value contradiction degree function helps the plithogenic aggregation operators, and the plithogenic inclusion (partial order) relationship to obtain a more accurate result.

The attribute value contradiction degree function is designed in each field where plithogenic set is used in accordance with the application to solve. If it is ignored, the aggregations still work, but the result may lose accuracy.

Several examples will be provided into this paper.

Then (P, a, V, d, c) is called a plithogenic set:

- where "P" is a set, "a" is a (multi-dimensional in general) attribute, "V" is the range of the attribute's values, "d" is the degree of appurtenance of each element x's attribute value to the set P with respect to some given criteria ($x \in P$), and "d" stands for "d_F" or "d_{IF}" or "d_N", when dealing with fuzzy degree of appurtenance, intuitionistic fuzzy degree of appurtenance, or neutrosophic degree of appurtenance respectively of an element x to the plithogenic set P;
- and "c" stands for "c_F" or "c_{IF}" or "c_N", when dealing with fuzzy degree of contradiction, intuitionistic fuzzy degree of contradiction, or neutrosophic degree of contradiction between attribute values respectively.

The functions $d(\cdot,\cdot)$ and $c(\cdot,\cdot)$ are defined in accordance with the applications the experts need to solve.

One uses the notation: $x\big(d(x, V)\big)$, where $d(x, V) = \{d(x, v), \text{for all } v \in V\}, \forall x \in P$.

2.6 About the Plithogenic Aggregation Set Operators

The attribute value contradiction degree is calculated between each attribute value with respect to the dominant attribute value (denoted v_D) in special, and with respect to other attribute values as well.

The attribute value contradiction degree function *c* between the attribute's values is used into the definition of plithogenic aggregation operators {*Intersection (AND), Union (OR), Implication* ($\Rightarrow$), *Equivalence* ($\Leftrightarrow$), *Inclusion Relationship (Partial Order, or Partial Inequality)*, and other plithogenic aggregation operators that combine two or more attribute value degrees - that t_{norm} and t_{conorm} act upon}.

Most of the plithogenic aggregation operators are linear combinations of the fuzzy t_{norm} (denoted $\wedge_F$), and fuzzy t_{conorm} (denoted $\vee_F$), but non-linear combinations may as well be constructed.

If one applies the t_{norm} on dominant attribute value denoted by v_D, and the contradiction between v_D and v_2 is $c(v_D, v_2)$, then onto attribute value v_2 one applies:

$$[1 - c(v_D, v_2)]\cdot t_{norm}(v_D, v_2) + c(v_D, v_2)\cdot t_{conorm}(v_D, v_2), \quad (2)$$

Or, by using symbols:

$$[1 - c(v_D, v_2)]\cdot(v_D \wedge_F v_2) + c(v_D, v_2)\cdot(v_D \vee_F v_2). \quad (3)$$

Similarly, if one applies the t_{conorm} on dominant attribute value denoted by v_D, and the contradiction between v_D and v_2 is $c(v_D, v_2)$, then onto attribute value v_2 one applies:

$$[1 - c(v_D, v_2)]\cdot t_{conorm}(v_D, v_2) + c(v_D, v_2)\cdot t_{norm}(v_D, v_2), \quad (4)$$

Or, by using symbols:

$$[1 - c(v_D, v_2)]\cdot(v_D \vee_F v_2) + c(v_D, v_2)\cdot(v_D \wedge_F v_2). \quad (5)$$

3 Plithogenic Set as Generalization of other Sets

Plithogenic set is a generalization of the crisp set, fuzzy set, intuitionistic fuzzy set, and neutrosophic set, since these four types of sets are characterized by a single attribute (*appurtenance*): which has one value (membership) – for the crisp set and for fuzzy set, two values (membership, and nonmembership) – for intuitionistic fuzzy set, or three values (membership, nonmembership, and indeterminacy) – for neutrosophic set.

For examples:

Let U be a universe of discourse, and a non-empty set $P \subseteq U$. Let $x \in P$ be a generic element.

3.1 Crisp (Classical) Set (CCS)

The *attribute* is α = "*appurtenance*";
the *set of attribute values V = {membership, nonmembership}*, with cardinal $|V| = 2$;
the *dominant attribute value = membership*;
the *attribute value appurtenance degree function*:

$$d: P\times V \rightarrow \{0, 1\}, \quad (6)$$

$d(x, membership) = 1$, $d(x, nonmembership) = 0$,
and the *attribute value contradiction degree function*:

$$c: V\times V \rightarrow \{0, 1\}, \quad (7)$$

$c(membership, membership) = c(nonmembership, nonmembership) = 0$,
$c(membership, nonmembership) = 1$.

3.1.1 Crisp (Classical) Intersection

$$a \wedge b \in \{0, 1\} \quad (8)$$

3.1.2 Crisp (Classical) Union

$$a \vee b \in \{0, 1\} \quad (9)$$

3.1.3 Crisp (Classical) Complement (Negation)

$$\neg a \in \{0, 1\}. \quad (10)$$

3.2 Single-Valued Fuzzy Set (SVFS)

The *attribute* is α = "*appurtenance*";
the *set of attribute values V = {membership},* whose cardinal |*V*| = 1;
the *dominant attribute value = membership;*
the *appurtenance attribute value degree function:*

$$d: P\times V\to[0, 1], \quad (11)$$

with *d(x, membership)* ∈ [0, 1];
and the *attribute value contradiction degree function*:

$$c: V\times V\to[0, 1], \quad (12)$$

c(membership, membership) = 0.

3.2.1 Fuzzy Intersection

$$a \wedge_F b \in [0, 1] \quad (13)$$

3.2.2 Fuzzy Union

$$a \vee_F b \in [0, 1] \quad (14)$$

3.2.3 Fuzzy Complement (Negation)

$$\neg_F a = 1 - a \in [0, 1]. \quad (15)$$

3.3 Single-Valued Intuitionistic Fuzzy Set (SVIFS)

The *attribute* is α = "*appurtenance*";
the *set of attribute values V = {membership, nonmembership},* whose cardinal |*V*| = 2;
the *dominant attribute value = membership;*
the *appurtenance attribute value degree function*:

$$d: P\times V\to[0, 1], \quad (16)$$

d(x, membership) ∈ [0, 1], *d(x, nonmembership)* ∈ [0, 1],
with *d(x, membership)* + *d(x, nonmembership)* ≤ 1,
and the *attribute value contradiction degree function*:

$$c: V\times V\to[0, 1], \quad (17)$$

c(membership, membership) = *c(nonmembership, nonmembership)* = 0,
c(membership, nonmembership) = 1,
which means that for SVIFS aggregation operators' intersection (AND) and union (OR), if one applies the t_{norm} on membership degree, then one has to apply the t_{conorm} on nonmembership degree – and reciprocally.

Therefore:

3.3.1 Intuitionistic Fuzzy Intersection

$$(a_1, a_2) \wedge_{IFS} (b_1, b_2) = (a_1 \wedge_F b_1, a_2 \vee_F b_2) \quad (18)$$

3.3.2 Intuitionistic Fuzzy Union

$$(a_1, a_2) \vee_{IFS} (b_1, b_2) = (a_1 \vee_F b_1, a_2 \wedge_F b_2), \quad (19)$$

and

3.3.3 Intuitionistic Fuzzy Complement (Negation)

$$\neg_{IFS} (a_1, a_2) = (a_2, a_1). \quad (20)$$

where $\wedge_F$ and $\vee_F$ are the fuzzy t_{norm} and fuzzy t_{conorm} respectively.

3.3.4 Intuitionistic Fuzzy Inclusions (Partial Orders)

Simple Intuitionistic Fuzzy Inclusion (the most used by the intuitionistic fuzzy community):

$$(a_1, a_2) \leq_{IFS} (b_1, b_2) \quad (21)$$

iff $a_1 \leq b_1$ and $a_2 \geq b_2$.

Plithogenic (Complete) Intuitionistic Fuzzy Inclusion (that we now introduce for the first time):

$$(a_1, a_2) \leq_P (b_1, b_2) \quad (22)$$

iff $a_1 \leq (1-c_v)\cdot b_1, a_2 \geq (1-c_v)\cdot b_2$,
where $c_v \in [0, 0.5)$ is the contradiction degree between the attribute dominant value and the attribute value *v* { the last one whose degree of appurtenance with respect to Expert A is *(a₁, a₂)*, while with respect to Expert B is *(b₁, b₂)* }. If c_v does not exist, we take it by default as equal to zero.

3.4 Single-Valued Neutrosophic Set (SVNS)

The *attribute* is α = "*appurtenance*";
the *set of attribute values V = {membership, indeterminacy, nonmembership}*, whose cardinal |*V*| = 3;
the *dominant attribute value = membership*;
the *attribute value appurtenance degree function*:

$$d: P\times V\rightarrow[0, 1], \quad (23)$$

d(x, membership) ∈ [0, 1], *d(x, indeterminacy)* ∈ [0, 1],
d(x, nonmembership) ∈ [0, 1],
with 0 ≤ *d(x, membership)* + *d(x, indeterminacy)* + *d(x, nonmembership)* ≤ 3;
and the *attribute value contradiction degree function*:

$$c: V\times V\rightarrow[0, 1], \quad (24)$$

c(membership, membership) = *c(indeterminacy, indeterminacy)* =
c(nonmembership, nonmembership) = 0,
c(membership, nonmembership) = 1,
c(membership, indeterminacy) = *c(nonmembership, indeterminacy)* = 0.5,
which means that for the SVNS aggregation operators (Intersection, Union, Complement etc.), if one applies the t_{norm} on membership, then one has to apply the t_{conorm} on nonmembership {and reciprocally), while on indeterminacy one applies the average of t_{norm} and t_{conorm}, as follows:

3.4.1 Neutrosophic Intersection

Simple Neutrosophic Intersection (the most used by the neutrosophic community):

$$(a_1, a_2, a_3) \wedge_{NS} (b_1, b_2, b_3) = (a_1 \wedge_F b_1, a_2 \vee_F b_2, a_3 \vee_F b_3) \quad (25)$$

Plithogenic Neutrosophic Intersection:

$$(a_1, a_2, a_3) \wedge_P (b_1, b_2, b_3) = \left(a_1 \wedge_F b_1, \frac{1}{2}\left[(a_2 \wedge_F b_2) + (a_2 \vee_F b_2)\right], a_3 \vee_F b_3\right) \quad (26)$$

3.4.2 Neutrosophic Union

Simple Neutrosophic Union (the most used by the neutrosophic community):

$$(a_1, a_2, a_3) \vee_{NS} (b_1, b_2, b_3) = (a_1 \vee_F b_1, a_2 \wedge_F b_2, a_3 \wedge_F b_3) \quad (27)$$

Plithogenic Neutrosophic Union:

$$(a_1, a_2, a_3) \vee_P (b_1, b_2, b_3) = \left(a_1 \vee_F b_1, \frac{1}{2}\left[(a_2 \wedge_F b_2) + (a_2 \vee_F b_2)\right], a_3 \wedge_F b_3\right). \quad (28)$$

In other way, with respect to what one applies on the membership, one applies the opposite on non-membership, while on indeterminacy one applies the average between them.

3.4.3 Neutrosophic Complement (Negation)

$$\neg_{NS} (a_1, a_2, a_3) = (a_3, a_2, a_1). \quad (29)$$

3.4.4 Neutrosophic Inclusions (Partial-Orders)

Simple Neutrosophic Inclusion (the most used by the neutrosophic community):

$$(a_1, a_2, a_3) \leq_{NS} (b_1, b_2, b_3) \quad (30)$$

iff $a_1 \leq b_1$ and $a_2 \geq b_2$, $a_3 \geq b_3$.

Plithogenic Neutrosophic Inclusion (defined now for the first time):
Since the degrees of contradiction are

$$c(a_1, a_2) = c(a_2, a_3) = c(b_1, b_2) = c(b_2, b_3) = 0.5, \quad (31)$$

one applies: $a_2 \geq [1- c(a_1, a_2)]b_2$ or $a_2 \geq (1\text{-}0.5)b_2$ or $a_2 \geq 0.5 \cdot b_2$
while

$$c(a_1, a_3) = c(b_1, b_3) = 1 \quad (32)$$

{having $a_1 \leq b_1$ one does the opposite for $a_3 \geq b_3$},
whence

$$(a_1, a_2, a_3) \leq_P (b_1, b_2, b_3) \quad (33)$$

iff $a_1 \leq b_1$ and $a_2 \geq 0.5 \cdot b_2$, $a_3 \geq b_3$.

4 Classifications of the Plithogenic Set

4.1 First Classification

4.1.1 Refined Plithogenic Set

If at least one of the attribute's values $v_k \in V$ is split (refined) into two or more attribute sub-values: $v_{k1}, v_{k2}, \ldots \in V$, with the attribute sub-value appurtenance degree function: $d(x, v_{ki}) \in P([0, 1])$, for $i = 1, 2, \ldots$, then (P_r, α, V, d, c) is called a *Refined Plithogenic Set*, where "*r*" stands for "refined".

4.1.2 Plithogenic Overset / Underset / Offset

If for at least one of the attribute's values $v_k \in V$, of at least one element $x \in P$, has the attribute value appurtenance degree function $d(x, v_k)$ exceeding 1, then (P_o, α, V, d, c) is called a *Plithogenic Overset*, where "*o*" stands for "overset"; but if $d(x, v_k)$ is below 0, then (P_u, α, V, d, c) is called a *Plithogenic Underset*, where "*u*" stands for "underset"; while if $d(x, v_k)$ exceeds 1, and $d(y, s_j)$ is below 0 for the attribute values $v_k, v_j \in V$ that may be the same or different attribute values corresponding to the same element or to two different elements $x, y \in P$, then $(P_{off}, \alpha, V, d, c)$ is called a *Plithogenic Offset*, where "off" stands for "offset" (or plithogenic set that is both overset and underset).

4.1.3 Plithogenic Multiset

A plithogenic set P that has at least an element $x \in P$, which repeats into the set P with the same plithogenic components

$$x(a_1, a_2, \ldots, a_m), x(a_1, a_2, \ldots, a_m) \quad (34)$$

or with different plithogenic components

$$x(a_1, a_2, \ldots, a_m), x(b_1, b_2, \ldots, b_m), \quad (35)$$

then (P_m, α, V, d, c) is called a *Plithogenic Multiset*, where "*m*" stands for "multiset".

4.1.4 Plithogenic Bipolar Set

If $\forall x \in P$, d: $P \times V \to \{\mathcal{P}([-1, 0]) \times \mathcal{P}([0, 1])\}^z$, then (P_b, α, V, d, c) is called a *Plithogenic Bipolar Set*, since $d(x, v)$, for $v \in V$, associates an appurtenance negative degree (as a subset of $[-1, 0]$) and a positive degree (as a subset of $[0, 1]$) to the value v; where $z = 1$ for fuzzy degree, $z = 2$ for intuitionistic fuzzy degree, and $z = 3$ for neutrosophic fuzzy degree.

4.1.5-6 Plithogenic Tripolar Set & Plitogenic Multipolar Set

Similar definitions for *Plithogenic Tripolar Set* and *Plitogenic Multipolar Set* (extension from *Neutrosophic Tripolar Set* and respectively *Neutrosophic Multipolar Set* {[4], 123-125}.

4.1.7 Plithogenic Complex Set

If, for any $x \in$ P, d: P×V→ $(\mathcal{P}([0, 1]) \times \mathcal{P}([0, 1]))^z$, and for any v ∈ V, d(*x*, *v*) is a complex value, i.e. d(*x*, *v*) = $M_1 \cdot e^{jM_2}$, where $M_1 \subseteq [0, 1]$ is called *amplitude*, and $M_2 \subseteq [0, 1]$ is called *phase*, and the *appurtenance degree* may be fuzzy (*z = 1*), intuitionistic fuzzy (*z = 2*), or neutrosophic (*z = 3*), then (P_{com}, α, V, d, c) is called a *Plithogenic Complex Set*.

4.2 Second Classification

Upon the values of the appurtenance degree function, one has:

4.2.1 Single-Valued Plithogenic Fuzzy Set

If

$$\forall x \in P,\ d: P\times V\to[0, 1], \quad (36)$$

and $\forall v \in V$, $d(x, v)$ is a single number in $[0, 1]$.

4.2.2 Hesitant Plithogenic Fuzzy Set

If

$$\forall x \in P,\ d: P\times V\to \mathcal{P}([0, 1]), \quad (37)$$

and $\forall v \in V$, $d(x, v)$ is a discrete finite set of the form $\{n_1, n_2, ..., n_p\}$, where $1\leq p < \infty$, included in [0, 1].

4.2.3 Interval-Valued Plithogenic Fuzzy Set

If

$$\forall x \in P,\ d: P\times V\to \mathcal{P}([0, 1]), \quad (38)$$

and $\forall v \in V$, $d(x, v)$ is an (open, semi-open, closed) interval included in [0, 1].

5 Applications and Examples

5.1 Applications of Uni-Dimensional Attribute Plithogenic Single-Valued Fuzzy Set

Let *U* be a universe of discourse, and a non-empty plithogenic set $P \subseteq U$. Let $x \in P$ be a generic element.
For simplicity, we consider the uni-dimensional attribute and the single-valued fuzzy degree function.

5.1.1 Small Discrete-Set of Attribute-Values

If the *attribute* is "*color*", and we consider only a discrete *set of attribute values V*, formed by the following six pure colors:

V = *{violet, blue, green, yellow, orange, red}*,

the *attribute value appurtenance degree function*:

$$d: P\times V\to[0, 1], \quad (39)$$

$d(x, violet) = v \in [0, 1]$, $d(x, blue) = b \in [0, 1]$, $d(x, green) = g \in [0, 1]$,
$d(x, yellow) = y \in [0, 1]$, $d(x, orange) = o \in [0, 1]$, $d(x, red) = r \in [0, 1]$,

then one has: $x(v, b, g, y, o, r)$, where *v, b, g, y, o, r* are fuzzy degrees of violet, blue, green, yellow, orange, and red, respectively, of the object *x* with respect to the set of objects *P*, where $v, b, g, y, o, r \in [0, 1]$.
The cardinal of the set of attribute values *V* is *6*.
The other colors are blends of these pure colors.

5.1.2 Large Discrete-Set of Attribute-Values

If the *attribute* is still "*color*" and we choose a more refined representation of the color values as:

$x\{d_{390}, d_{391}, \ldots, d_{699}, d_{700}\}$,

measured in nanometers, then we have a discrete finite set of attribute values, whose cardinal is: $700 - 390 + 1 = 311$, where for each $j \in V = \{390, 391, ..., 699, 700\}$, d_j represents the degree to which the object *x*'s color, with respect to the set of objects *P*, is of "*j*" nanometers per wavelength, with $d_i \in [0, 1]$. A nanometer (nm) is a billionth part of a meter.

5.1.3 Infinitely-Uncountable-Set of Attribute-Values

But if the *attribute* is again "*color*", then one may choose a continuous representation:

$x(\mathrm{d}([390, 700]))$,

having $V = [390, 700]$ a closed real interval, hence an infinitely uncountable (continuum) set of attribute values. The cardinal of the V is ∞.

For each $j \in [390, 700]$, d_j represents the degree to which the object x's color, with respect to the set of objects P, is of "j" nanometers per wavelength, with $d_i \in [0, 1]$. And $d([390, 700]) = \{dj, j \in [390, 700]\}$.

The light, ranging from *390* (violet color) to *700* (red color) nanometers per wavelengths is visible to the eye of the human. The cardinal of the set of attribute values V is continuum infinity.

5.2 Example of Uni-Attribute (of 4-Attribute-Values) Plithogenic Single-Valued Fuzzy Set Complement (Negation)

Let's consider that the attribute "size" that has the following values: *small* (the dominant one), *medium*, *big*, *very big*.

Degrees of contradiction	0	0.50	0.75	1
Attribute values	small	medium	big	very big
Degrees of appurtenance	0.8	0.1	0.3	0.2

Table 1.

5.3 Example of Refinement and Negation of a Uni-Attribute (of 4-Attribute-Values) Plithogenic Single-Valued Fuzzy Set

As a refinement of the above table, let's add the attribute "bigger" as in the below table.

The opposite (negation) of the attribute value "big", which is 75% in contradiction with "small", will be an attribute value which is $1 - 0.75 = 0.25 = 25\%$ in contradiction with "small", so it will be equal to $\frac{1}{2}["small" + "medium"]$. Let's call it "less medium", whose degree of appurtenance is 1 – 0.3 = 0.7.

If the attribute "size" has other values, small being dominant value:

Degrees of contradiction	0	**0.14**	**0.25**	0.50	0.75	**0.86**	1
Attribute values	small	**above small (anti-bigger)**	**less medium (anti-big)**	medium	big	**bigger**	very big
Degrees of appurtenance	0.8	**0.6**	**0.7**	0.1	0.3	**0.4**	0.2

Table 2.

The opposite (negation) of "bigger" is 1 - 0.86 = 0.14 = 14% in contradiction degree with the dominant attribute value ("small"), so it is in between "small" and "medium", we may say it is included into the attribute-value interval *[small, medium]*, much closer to "small" than to "medium". Let's call is "above small", whose degree of appurtenance is 1 – 0.4 = 0.6.

5.4 Example of Multi-Attribute (of 24 Attribute-Values) Plithogenic Fuzzy Set Intersection, Union, and Complement

Let P be a plithogenic set, representing the students from a college. Let $x \in P$ be a generic student that is characterized by three attributes:

- altitude, whose values are {tall, short} $\stackrel{\text{def}}{=} \{a_1, a_2\}$;
- weight, whose values are {obese, fat, medium, thin} $\stackrel{\text{def}}{=} \{w_1, w_2, w_3, w_4\}$;
- hair color, whose values are {blond, reddish, brown} $\stackrel{\text{def}}{=} \{h_1, h_2, h_3\}$.

The multi-attribute of dimension *3* is

$V_3 = \{(a_i, w_j, h_k), \text{for all } 1 \le i \le 2, 1 \le j \le 4, 1 \le k \le 3\}$.

The cardinal of V_3 is $|V_3| = 2 \times 4 \times 3 = 24$.

The *uni-dimensional attribute contradiction degrees* are:
$c(a_1, a_2) = 1$;
$c(w_1, w_2) = \frac{1}{3}$, $c(w_1, w_3) = \frac{2}{3}$, $c(w_1, w_4) = 1$;
$c(h_1, h_2) = 0.5$, $c(h_1, h_3) = 1$.

Dominant attribute values are: a_1, w_1, and h_1 respectively for each corresponding uni-dimensional attribute.
Let's use the fuzzy $t_{norm} =$ a $\wedge_F$ b = *ab*, and fuzzy $t_{conorm} = a \vee_F b = a + b - ab$.

5.4.1 Tri-Dimensional Plithogenic Single-Valued Fuzzy Set Intersection and Union

Let

$$x_A = \left\{ \begin{matrix} d_A(x, a_i, w_j, h_k), \\ for\ all\ 1 \le i \le 2, 1 \le j \le 4, 1 \le k \le 3 \end{matrix} \right\} \tag{40}$$

and

$$x_B = \left\{ \begin{matrix} d_B(x, a_i, w_j, h_k), \\ for\ all\ 1 \le i \le 2, 1 \le j \le 4, 1 \le k \le 3 \end{matrix} \right\}. \tag{41}$$

Then:

$$x_A(a_i, w_j, h_k) \wedge_P x_B(a_i, w_j, h_k) = \left\{ \begin{array}{l} (1 - c(a_D, a_i)) \cdot [d_A(x, a_D) \wedge_F d_B(x, a_i)] \\ + c(a_D, a_i) \cdot [d_A(x, a_D) \vee_F d_B(x, a_i)], 1 \le i \le 2; \\ (1 - c(w_D, w_j)) \cdot [d_A(x, w_D) \wedge_F d_B(x, w_j)] \\ + c(w_D, w_j) \cdot [d_A(x, w_D) \vee_F d_B(x, w_j)], 1 \le j \le 4; \\ (1 - c(h_D, h_k)) \cdot [d_A(x, h_D) \wedge_F d_B(x, h_k)] \\ + c(h_D, h_k) \cdot [d_A(x, h_D) \vee_F d_B(x, h_k)], 1 \le k \le 3. \end{array} \right\} \tag{42}$$

and

$$x_A(a_i, w_j, h_k) \vee_P x_B(a_i, w_j, h_k) = \left\{ \begin{array}{l} (1 - c(a_D, a_i)) \cdot [d_A(x, a_D) \vee_F d_B(x, a_i)] \\ + c(a_D, a_i) \cdot [d_A(x, a_D) \wedge_F d_B(x, a_i)], 1 \le i \le 2; \\ (1 - c(w_D, w_j)) \cdot [d_A(x, w_D) \vee_F d_B(x, w_j)] \\ + c(w_D, w_j) \cdot [d_A(x, w_D) \wedge_F d_B(x, w_j)], 1 \le j \le 4; \\ (1 - c(h_D, h_k)) \cdot [d_A(x, h_D) \vee_F d_B(x, h_k)] \\ + c(h_D, h_k) \cdot [d_A(x, h_D) \wedge_F d_B(x, h_k)], 1 \le k \le 3. \end{array} \right\} \tag{43}$$

Let's have

$x_A(d_A(a_1) = 0.8, d_A(w_2) = 0.6, d_A(h_3) = 0.5)$

and

$x_B(d_B(a_1) = 0.4, d_B(w_2) = 0.1, d_B(h_3) = 0.7)$.

We take only one 3-attribute value: (a_1, w_2, h_3), for the other 23 3-attribute values it will be analougsly.

For $x_A \wedge_p x_B$ we calculate for each uni-dimensional attribute separately:

$$[1 - c(a_D, a_1)] \cdot [0.8 \wedge_F 0.4] + c(a_D, a_1) \cdot [0.8 \vee_F 0.4] = (1 - 0) \cdot [0.8(0.4)] + 0 \cdot [0.8 \vee_F 0.4] = 0.32;$$

$$\left[1 - c[w_D, w_2] \cdot [0.6 \wedge_F 0.1] + c(w_D, w_2) \cdot [0.6 \vee_F 0.1]\right] = \left(1 - \frac{1}{3}\right)[0.6(0.1)] + \frac{1}{3}[0.6 + 0.1 - 0.6(0.1)]$$
$$= \frac{2}{3}[0.06] + \frac{1}{3}[0.64] = \frac{0.76}{3} \approx 0.25;$$

$$[1-c(h_D,h_3)]\cdot\left[0.5\wedge_F 0.7\right]+c(h_D,h_3)\cdot\left[0.5\vee_F 0.7\right]=[1-1]\cdot[0.5(0.7)]+1\cdot[0.5+0.7-0.5(0.7)]$$
$$=0\cdot[0.35]+0.85=0.85.$$

Whence $x_A \wedge_p x_B(a_1,w_2,h_3)\approx(0.32,0.25,0.85)$.

For $x_A \vee_p x_B$ we do similarly:

$$[1-c(a_D,a_1)]\cdot\left[0.8\vee_F 0.4\right]+c(a_D,a_1)\cdot\left[0.8\wedge_F 0.4\right]=(1-0)\cdot[0.8+0.4-0.8(0.4)]+0\cdot[0.8(0.4)]$$
$$=1\cdot[0.88]+0=0.88;$$
$$\left[1-c[w_D,w_2]\cdot\left[0.6\vee_F 0.1\right]+c(w_D,w_2)\cdot\left[0.6\wedge_F 0.1\right]\right]=\left(1-\frac{1}{3}\right)[0.6+0.1-0.6(0.1)]+\frac{1}{3}[0.6(0.1)]$$
$$=\frac{2}{3}[0.64]+\frac{1}{3}[0.06]=\frac{1.34}{3}\approx 0.44;$$
$$[1-c(h_D,h_3)]\cdot\left[0.5\vee_F 0.7\right]+c(h_D,h_3)\cdot\left[0.5\wedge_F 0.7\right]=[1-1]\cdot[0.5+0.7-0.5(0.7)]+1\cdot[0.5(0.7)]$$
$$=0+0.35=0.35.$$

Whence $x_A \vee_p x_B(a_1,w_2,h_3)\approx(0.88,0.44,0.35)$.

For $\neg_p x_A(a_1,w_2,h_3)=(d_A(a_2)=0.8, d_A(w_3)=0.6, d_A(h_1)=0.5)$, since the opposite of a_1 is a_2, the opposite of w_2 is w_3, and the opposite of h_3 is h_1.

5.5 Another Example of Multi-Attribute (of 5 Attribute-Values) Plithogenic Fuzzy Set Complement and Refined Attribute-Value Set

The 5-attribute values plithogenic fuzzy complement (negation) of

$$x\begin{pmatrix} 0 & 0.50 & 0.75 & 0.86 & 1 \\ \text{small}, & \text{medium}, & \text{big}\ , & \text{bigger}, & \text{very big} \\ 0.8 & 0.1 & 0.3 & 0.4 & 0.2 \end{pmatrix}$$

Is:

$$\neg_p x\begin{pmatrix} 1-1 & 1-0.86 & 1-0.75 & 1-0.50 & 1-0 \\ \text{anti}-\text{very big}, & \text{anti}-\text{bigger}, & \text{anti}-\text{big}, & \text{anti}-\text{medium}, & \text{anti}-\text{small} \\ 0.2 & 0.4 & 0.3 & 0.1 & 0.8 \end{pmatrix}$$
$$=\neg_p x\begin{pmatrix} 0 & 0.14 & 0.25 & 0.50 & 1 \\ \text{small}, & \text{anti}-\text{bigger}, & \text{anti}-\text{big}, & \text{medium}, & \text{very big} \\ 0.2 & 0.4 & 0.3 & 0.1 & 0.8 \end{pmatrix}$$
$$=\neg_p x\begin{pmatrix} 0 & 0.14 & 0.25 & 0.50 & 1 \\ \text{small}, & \text{above small}, & \text{below medium}, & \text{medium}, & \text{very big} \\ 0.2 & 0.4 & 0.3 & 0.1 & 0.8 \end{pmatrix}.$$

Therefore, the original attribute-value set

V = {small, medium, big, bigger, very big}

has been partially refined into:

RefinedV = {small, above small, below medium, medium, very big},

where *above small, below medium* ∈ *[small, medium]*.

5.6 Application of Bi-Attribute Plithogenic Single-Valued Set

Let $\mathcal{U}$ be a universe of discourse, and $P\subset\mathcal{U}$ a plithogenic set.

In a plithogenic set P, each element (object) $x\in P$ is characterized by $m\geq 1$ attributes $\alpha_1,\alpha_2,\dots,\alpha_m$, and each attribute $\alpha_i, 1\leq i\leq m$, has $r_i\geq 1$ values:

$$V_i=\{v_{i1},v_{i2},\dots,v_{ir_i}\}.$$

Therefore, the element x is characterized by $r=r_1\times r_2\times\dots\times r_m$ attributes' values.

For example, if the attributes are "color" and "height", and their values (required by the application the experts want to do) are:

$$Color=\{green, yellow, red\}$$

and

$Height = \{tall, medium\}$,

then the object $x \in P$ is characterized by the Cartesian product

$$Color \times Height = \left\{ \begin{matrix} (green, tall), (green, medium), (yellow, tall), \\ (yellow, medium), (red, tall), (red, medium) \end{matrix} \right\}.$$

Let's consider the dominant (i.e. the most important, or reference) value of attribute "*color*" be "*green*", and of attribute "*height*" be "*tall*".

The attribute value contradiction fuzzy degrees are:

$c(green, green) = 0$,
$c(green, yellow) = \frac{1}{3}$,
$c(green, red) = \frac{2}{3}$,
$c(tall, tall) = 0$,
$c(tall, medium) = \frac{1}{2}$.

Suppose we have two experts *A* and *B*. Further on, we consider (fuzzy, intuitionistic fuzzy, or neutrosophic) degrees of appurtenance of each attribute value to the set *P* with respect to experts' criteria.

We consider the single value number fuzzy degrees, for simplicity of the example.

Let v_i be a uni-attribute value and its degree of contradiction with respect to the dominant uni-attribute value v_D be $c(v_D, v_i) \stackrel{\text{def}}{=} c_i$.

Let $d_A(x, v_i)$ be the appurtenance degree of the attribute value v_i of the element x with respect to the set A. And similarly for $d_B(x, v_i)$. Then, we recall the plithogenic aggregation operators with respect to this attribute value v_i that will be employed:

5.6.1 One-Attribute Value Plithogenic Single-Valued Fuzzy Set Intersection

$$d_A(x, v_i) \wedge_p d_B(x, v_i) = (1 - c_i) \cdot [d_A(x, v_i) \wedge_F d_B(x, v_i)] + c_i \cdot [d_A(x, v_i) \vee_F d_B(x, v_i)] \quad (44)$$

5.6.2 One-Attribute Value Plithogenic Single-Valued Fuzzy Set Union

$$d_A(x, v_i) \vee_p d_B(x, v_i) = (1 - c_i) \cdot [d_A(x, v_i) \vee_F d_B(x, v_i)] + c_i \cdot [d_A(x, v_i) \wedge_F d_B(x, v_i)] \quad (45)$$

5.6.3 One Attribute Value Plithogenic Single-Valued Fuzzy Set Complement (Negation)

$$\neg_p v_i = anti(v_i) = (1 - c_i) \cdot v_i \quad (46)$$

$$\neg_p d_A(x, (1 - c_i) v_i) = d_A(x, v_i) \quad (47)$$

5.7 Singe-Valued Fuzzy Set Degrees of Appurtenance

According to Expert A: $d_A: \{green, yellow, red; tall, medium\} \rightarrow [0, 1]$.
One has:

$d_A(green) = 0.6$,
$d_A(yellow) = 0.2$,
$d_A(red) = 0.7$;
$d_A(tall) = 0.8$,
$d_A(medium) = 0.5$.

We summarize as follows:

According to Expert A:

Contradiction Degrees	0	$\frac{1}{3}$	$\frac{2}{3}$		0	$\frac{1}{2}$
Attributes' Values	green	yellow	red		tall	medium
Fuzzy Degrees	0.6	0.2	0.7		0.8	0.5

Table 3.

According to Expert B:

Contradiction Degrees	0	$\frac{1}{3}$	$\frac{2}{3}$		0	$\frac{1}{2}$
Attributes' Values	green	yellow	red		tall	medium
Fuzzy Degrees	0.7	0.4	0.6		0.6	0.4

Table 4.

The element

x{ (green, tall), (green, medium), (yellow, tall), (yellow, medium), (red, tall), (red, medium) } ∈ *P*

with respect to the two experts as above is represented as:

$x_A\{(0.6, 0.8), (0.6, 0.5), (0.2, 0.8), (0.2, 0.5), (0.7, 0.8), (0.7, 0.5)\}$

and

$x_B\{(0.7, 0.6), (0.7, 0.4), (0.4, 0.6), (0.4, 0.4), (0.6, 0.6), (0.6, 0.4)\}$.

In order to find the optimal representation of x, we need to intersect x_A and x_B, each having six duplets. Actually, we separately intersect the corresponding duplets.

In this example, we take the fuzzy t_{norm}: $a \wedge_F b = ab$ and the fuzzy t_{conorm}: $a \vee_F b = a + b - ab$.

5.7.1 Application of Uni-Attribute Value Plithogenic Single-Valued Fuzzy Set Intersection

Let's compute $x_A \wedge_p x_B$.

$$\begin{matrix}0 & 0 & & 0 & 0\end{matrix}\ \{\textit{degrees of contradictions}\}$$
$$(0.6, 0.8) \wedge_p (0.7, 0.6) = (0.6 \wedge_p 0.7, 0.8 \wedge_p 0.6) = (0.6 \cdot 0.7, 0.8 \cdot 0.6) = (0.42, 0.48),$$

where above each duplet we wrote the degrees of contradictions of each attribute value with respect to their correspondent dominant attribute value. Since they were zero, $\wedge_p$ coincided with $\wedge_F$.

{*the first raw below 0 ½ and again 0 ½ represents the contradiction degrees*}

$$\begin{pmatrix}0 & \frac{1}{2}\\ 0.6, & 0.5\end{pmatrix} \wedge_p \begin{pmatrix}0 & \frac{1}{2}\\ 0.7, & 0.4\end{pmatrix} = (0.6 \wedge_p 0.7, 0.5 \wedge_p 0.4) = (0.6 \cdot 0.7, (1 - 0.5) \cdot [0.5 \wedge_F 0.4] + 0.5 \cdot [0.5 \vee_F 0.4])$$
$$= (0.42, 0.5[0.2] + 0.5[0.5 + 0.4 - 0.5 \cdot 0.4]) = (0.42, 0.45).$$

$$\begin{pmatrix}\frac{1}{3} & 0\\ 0.2, & 0.8\end{pmatrix} \wedge_p \begin{pmatrix}\frac{1}{3} & 0\\ 0.4, & 0.6\end{pmatrix} = (0.2 \wedge_p 0.4, 0.8 \wedge_p 0.6) = \left(\left\{1 - \frac{1}{3}\right\} \cdot [0.2 \wedge_F 0.4] + \left\{\frac{1}{3}\right\} \cdot [0.2 \vee_F 0.4], 0.8 \cdot 0.6\right)$$
$$\approx (0.23, 0.48).$$

$$\begin{pmatrix}\frac{1}{3} & \frac{1}{2}\\ 0.2, & 0.5\end{pmatrix} \wedge_p \begin{pmatrix}\frac{1}{3} & \frac{1}{2}\\ 0.4, & 0.4\end{pmatrix} = (0.2 \wedge_p 0.4, 0.5 \wedge_p 0.4)$$

(they were computed above)

$$\approx (0.23, 0.45).$$

$$\begin{pmatrix}\frac{2}{3} & 0\\ 0.7, & 0.8\end{pmatrix} \wedge_p \begin{pmatrix}\frac{2}{3} & 0\\ 0.6, & 0.6\end{pmatrix} = (0.7 \wedge_p 0.8, 0.8 \wedge_p 0.6) = \left(\left\{1 - \frac{2}{3}\right\} \cdot [0.7 \wedge_F 0.6] + \left\{\frac{2}{3}\right\} \cdot [0.7 \vee_F 0.6], 0.48\right)$$

(the second component was computed above)

$$= \left(\frac{1}{3}[0.7 \cdot 0.6] + \frac{2}{3}[0.7 + 0.6 - 0.7 \cdot 0.6], 0.48\right) \approx (0.73, 0.48).$$

And the last duplet:

$$\begin{pmatrix}\frac{2}{3} & \frac{1}{2}\\ 0.7, & 0.5\end{pmatrix} \wedge_p \begin{pmatrix}\frac{2}{3} & \frac{1}{2}\\ 0.6, & 0.4\end{pmatrix} = (0.7 \wedge_p 0.6, 0.5 \wedge_p 0.4)$$
$$\approx (0.73, 0.45)$$

(they were computed above).

Finally:

$$x_A \wedge_p x_B \approx \begin{Bmatrix} (0.42, 0.48), (0.42, 0.45), (0.23, 0.48), (0.23, 0.45), \\ (0.73, 0.48), (0.73, 0.45) \end{Bmatrix},$$

or, after the intersection of the experts' opinions $A\wedge_P B$, we summarize the result as:

Contradiction Degrees	0	$\frac{1}{3}$	$\frac{2}{3}$		0	$\frac{1}{2}$
Attributes' Values	green	yellow	red		tall	medium
Fuzzy Degrees of Expert A for x	0.6	0.2	0.7		0.8	0.5
Fuzzy Degrees of Expert B for x	0.7	0.4	0.6		0.6	0.4
Fuzzy Degrees of $x_A \wedge_p x_B$	0.42	0.23	0.73		0.48	0.45
Fuzzy Degrees of $x_A \vee_p x_B$	0.88	0.37	0.57		0.92	0.45

Table 5.

5.7.2 Application of Uni-Attribute Value Plithogenic Single-Valued Fuzzy Set Union

We separately compute for each single attribute value:

$$d_A^F(x, green) \vee_p d_B^F(x, green) = 0.6 \vee_p 0.7 = (1-0)\cdot[0.6 \vee_F 0.7] + 0\cdot[0.6 \wedge_F 0.7]$$
$$= 1\cdot[0.6 + 0.7 - 0.6\cdot 0.7] + 0 = 0.88.$$

$$d_A^F(x, yellow) \vee_p d_B^F(x, yellow) = 0.2 \vee_p 0.4 = \left(1-\frac{1}{3}\right)\cdot[0.2 \vee_F 0.4] + \frac{1}{3}\cdot[0.2 \wedge_F 0.4]$$
$$= \frac{2}{3}\cdot(0.2 + 0.4 - 0.2\cdot 0.4) + \frac{1}{3}(0.2\cdot 0.4) \approx 0.37.$$

$$d_A^F(x, red) \vee_p d_B^F(x, red) = 0.7 \vee_p 0.6 = \left\{1-\frac{2}{3}\right\}\cdot[0.7 \vee_F 0.6] + \frac{2}{3}\cdot[0.7 \wedge_F 0.6]$$
$$= \frac{1}{3}\cdot(0.7 + 0.6 - 0.7\cdot 0.6) + \frac{2}{3}(0.7\cdot 0.6) \approx 0.57.$$

$$d_A^F(x, tall) \vee_p d_B^F(x, tall) = 0.8 \vee_p 0.6 = (1-0)\cdot(0.8 + 0.6 - 0.8\cdot 0.6) + 0\cdot(0.8\cdot 0.6) = 0.92.$$

$$d_A^F(x, medium) \vee_p d_B^F(x, medium) = 0.5 \vee_p 0.4 = \frac{1}{2}(0.5 + 0.4 - 0.5\cdot 0.4) + \frac{1}{2}\cdot(0.5\cdot 0.4) = 0.45.$$

5.7.3 Properties of Plithogenic Single-Valued Set Operators in Applications

1) When the attribute value contradiction degree with respect to the corresponding dominant attribute value is 0 (zero), one simply use the fuzzy intersection:

$$d_{A\wedge_p B}(x, green) = d_A(x, green) \wedge_F d_B(x, green) = 0.6\cdot 0.7 = 0.42,$$

and

$$d_{A\wedge_p B}(x, tall) = d_A(x, tall) \wedge_F d_B(x, tall) = 0.8\cdot 0.6 = 0.48.$$

2) But, if the attribute value contradiction degree with respect to the corresponding dominant attribute value is different from 0 and from 1, the result of the plithogenic intersection is between the results of fuzzy t_{norm} and fuzzy t_{conorm}.

Examples:

$$d_A(x, yellow) \wedge_F d_B(x, yellow) = 0.2 \wedge_F 0.4 = 0.2\cdot 0.4 = 0.08\ (t_{norm}),$$
$$d_A(x, yellow) \vee_F d_B(x, yellow) = 0.2 \vee_F 0.4 = 0.2 + 0.4 - 0.2\cdot 0.4 = 0.52\ (t_{conorm});$$

while

$$d_A(x, yellow) \wedge_p d_B(x, yellow) = 0.23 \in [0.08, 0.52]$$

{or 0.23 ≈ 0.2266… = (2/3)×0.08 + (1/3)×0.52, i.e. a linear combination of t_{norm} and t_{conorm}}.

Similarly:

$$d_A(x, red) \wedge_p d_B(x, red) = 0.7 \wedge_F 0.6 = 0.7 \cdot 0.6 = 0.42\ (t_{norm}),$$
$$d_A(x, red) \vee_p d_B(x, red) = 0.7 \vee_F 0.6 = 0.7 + 0.6 - 0.7 \cdot 0.6 = 0.88\ (t_{conorm});$$

while

$$d_A(x, red) \wedge_p d_B(x, red) = 0.57 \in [0.42, 0.88]$$

{linear combination of t_{norm} and t_{conorm}}.

And

$$d_A(x, medium) \wedge_F d_B(x, medium) = 0.5 \wedge_F 0.4 = 0.5 \cdot 0.4 = 0.20,$$
$$d_A(x, medium) \vee_F d_B(x, medium) = 0.5 \vee_F 0.4 = 0.5 + 0.4 - 0.5 \cdot 0.4 = 0.70,$$

while

$$d_A(x, medium) \wedge_p d_B(x, medium) = 0.45,$$

which is just in the middle (because "medium" contradiction degree is $\frac{1}{2}$) of the interval $[0.20, 0.70]$.

Conclusion & Future Research

As generalization of dialectics and neutrosophy, plithogeny will find more use in blending diverse philosophical, ideological, religious, political and social ideas. After the extension of fuzzy set, intuitionistic fuzzy set, and neutrosophic set to the plithogenic set; the extension of classical logic, fuzzy logic, intuitionistic fuzzy logic and neutrosophic logic to plithogenic logic; and the extension of classical probability, imprecise probability, and neutrosophic probability to plithogenic probability [12] – more applications of the plithogenic set/logic/probability/statistics in various fields should follow. The classes of plithogenic implication operators and their corresponding sets of plithogenic rules are to be constructed in this direction. Also, exploration of non-linear combinations of t_{norm} and t_{conorm}, or of other norms and conorms, in constructing of more sophisticated plithogenic set, logic and probabilistic aggregation operators, for a better modeling of real life applications.

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