Extension to the Eratosthenes sieve

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Abstract

The Sieve of Eratosthenes finds all the prime numbers up to any given limit by eliminating all non-primes from the list of all natural numbers. A list of natural numbers containing no multiples of 2, 3 or 5 is created, it is obtained by 8 formulas. The elimination of non-primes is effectuated from this new list. It then appears that it is possible to express all the non-primes belonging to this list by 36 other formulas, the prime numbers being the numbers satisfying the 8 formulas but not the 36.

Introduction

A prime number is a natural number with only two divisors : 1 and itself. A non-prime number is a natural number with at least three divisors.

In mathematics, a sieve is an algorithmic method that allows to approach the size of sifted sets of integers. The sieve of Eratosthenes allows to find all prime numbers less than a given natural number.

It proceeds by eliminating all non-primes starting from 2 and stopping when the square of the smallest remaining number is upper than the biggest number remaining because then, all non-primes have already been eliminated. At the end, only the non-divisible by any number other than1 and themselves remain, they are prime numbers.

A list of natural numbers containing no multiples of 2, 3 or 5 is created, it is obtained by 8 formulas. The elimination of non-primes is effectuated from this new list. It then appears that it is possible to express all the non-primes belonging to this list by 36 other formulas, the prime numbers being the numbers satisfying the 8 formulas but not the 36.

1. The 8 formulas

Suppose n, n' and n'' are natural numbers (n ∈ N, n' ∈ N, n''' ∈ N) :

1.1. Elimination of multiples of 2

A multiple of 2 is written as : **2*n**

A non-multiple of **2** is written as **(2*n)+1**

Prime numbers can then be written as : **(2*n)+1**

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1.2. Elimination of multiples of 3

A multiple of **3** is written as : **3*n**

A non-multiple of **3** is written as : **(3*n)+1** and **(3*n)+2**

Prime numbers can then be written as : **(3*n)+1** and **(3*n)+2**

1.3. Elimination of multiples of each of 3 and 2

(3*n)+1 = **(2*n)+n+1** = **(2*n)+1+n**

(2*n)+1 is not multiple of **2** so if **n** is multiple of **2** then **((2*n)+1)+n** is not multiple of **2** and is written as : **(3*(2*n'))+1** i.e. **(6* n')+1**

(3*n)+2 = **(2*n)+n+2** = **(2*n)+2+n**

(2*n)+2 is multiple of **2** so if **n** is not multiple of **2** then **((2*n)+2)+n** is not multiple of **2** and is written as : **(3*((2*n')+1))+2** i.e. **(6* n')+5**

Prime numbers can then be written as : **(6*n)+1** and **(6*n)+5**

1.4. Elimination of multiples of 5

A multiple of **5** is written as : **5*n**

A non-multiple of **5** is written as: **(5*n)+1** ; **(5*n)+2** ; **(5*n)+3** and **(5*n)+4**

Prime numbers can then be written as : **(5*n)+1** ; **(5*n)+2** ; **(5*n)+3** and **(5*n)+4**

1.5. Elimination of multiples of each of 5, 3 and 2

(5*n)+1 = **(4*n)+n+1** = **(4*n)+1+n**

(4*n)+1 is not multiple of **2** so if **n** is multiple of **2** then **(4*n)+1+n** is not multiple of **2** and is written as : **(5*(2*n'))+1** i.e. **(10* n')+1**

(10* n')+1 = **(9* n')+ n'+1**

9* n' is multiple of **3** so if **n'+1** is not multiple of **3** then **(9* n')+ n'+1** is not multiple of **3**

If **n'+1** is not multiple of 3 then **n'+1** = **(3*n'')+1** i.e. **n'=3*n''** or **n'+1** = **(3*n'')+2** i.e. **n'=(3*n'')+1**. So **(10* n')+1** is written as : **(10* (3*n''))+1** i.e. **(30* n'')+1** and **(10* ((3*n'')+1))+1** i.e. **(30* n'')+11**

(5*n)+2 = **(4*n)+n+2** = **(4*n)+2+n**

(4*n)+2 is multiple of **2** so if **n** is not multiple of **2** then **(4*n)+2+n** is not multiple of **2** is written as : **(5*((2*n')+1))+2** i.e. **(10* n')+7**

(10* n')+7 = **(9* n')+ n'+6+1** = **(9* n')+6+ n'+1**

(9* n')+6 is multiple of **3** so if **n'+1** is not multiple of **3** then **(9* n')+6+ n'+1** is not multiple of **3**

If **n'+1** is not multiple of **3** then **n'+1** = **(3*n'')+1** i.e. **n'=3*n''** or **n'+1** = **(3*n'')+2** i.e. **n'=(3*n'')+1**. So **(10* n')+7** is written as : **(10* (3*n''))+7** .i.e. **(30* n'')+7** and **(10* ((3*n'')+1))+7** .i.e. **(30* n'')+17**

(5*n)+3 = **(4*n)+n+3** = **((4*n)+3)+n**

(4*n)+3 is not multiple of **2** so if **n** is multiple of **2** then **((4*n)+3)+n** is not multiple of **2** and is written as : **(5*(2*n'))+3** .i.e. **(10* n')+3**

(10* n')+3 = **((9* n')+3) +n'**

(9* n')+3 is multiple of **3** so if **n'** is not multiple of **3** then **((9* n')+3) +n'** is not multiple of **3**

If **n'** is not multiple of 3 then **n'** = **(3*n'')+1** or **n'** = **(3*n'')+2**. So **(10* n')+3** is written as : **(10* ((3*n'')+1))+3** .i.e. **(30* n'')+13** and **(10* ((3*n'')+2))+3** .i.e. **(30* n'')+23**

(5*n)+4 = **(4*n)+n+4** = **(4*n)+4+n**

(4*n)+4 is multiple of **2** so if **n** is not multiple of **2** then **(4*n)+4+n** is not multiple of **2** and is written as : **(5*((2*n')+1))+4** .i.e. **(10* n')+9**

(10* n')+9 = **(9* n')+ n'+9** = **((9* n')+9)+ n'**

(9* n')+9 is multiple of **3** si if **n'** is not multiple of **3** then **((9* n')+9)+ n'** is not multiple of **3**

If **n'** is not multiple of 3 then **n'** = **(3*n'')+1** or **n'= (3*n'')+2**. So **(10* n')+9** is written as : **(10*((3*n'')+1))+9** .i.e. **(30* n'')+19** and **(10* ((3*n'')+2))+9** .i.e. **(30* n'')+29**

Prime numbers can then be written as : **(30*n)+1** ; **(30*n)+7** ; **(30*n)+11** ; **(30*n)+13** ; **(30*n)+17** ; **(30*n)+19** ; **(30*n)+23** ; **(30*n)+29**

2. Illustration of the sieve

Tab.1. Illustration of the sieve showing the distribution of the numbers given by the 8 formulas among all natural numbers less than or equal to 90

1	**31**	**61**
2	32	62
3	33	63
4	34	64
5	35	65
6	36	66
7	**37**	**67**
8	38	68
9	39	69
10	40	70
11	**41**	**71**
12	42	72
13	**43**	**73**
14	44	74
15	45	75
16	46	76
17	**47**	**77**
18	48	78
19	**49**	**79**
20	50	80
21	51	81
22	52	82
23	**53**	**83**
24	54	84
25	55	85
26	56	86
27	57	87
28	58	88
29	**59**	**89**
30	60	90

Tab.2. Illustration of the sieve showing the distribution of all multiples among all the numbers given by the 8 formulas or equal to 1021

A	B	C	D	E	F	G	H	I
1	1	1	1	1	1	1	1	1
7	7	7	7	7	7	7	7	7
11	11	11	11	11	11	11	11	11
13	13	13	13	13	13	13	13	13
17	17	17	17	17	17	17	17	17
19	19	19	19	19	19	19	19	19
23	23	23	23	23	23	23	23	23
29	29	29	29	29	29	29	29	29
31	31	31	31	31	31	31	31	31
37	37	37	37	37	37	37	37	37
41	41	41	41	41	41	41	41	41
43	43	43	43	43	43	43	43	43
47	47	47	47	47	47	47	47	47
49	49	49	49	49	49	49	49	49
53	53	53	53	53	53	53	53	53
59	59	59	59	59	59	59	59	59
61	61	61	61	61	61	61	61	61
67	67	67	67	67	67	67	67	67
71	71	71	71	71	71	71	71	71
73	73	73	73	73	73	73	73	73
77	77	77	77	77	77	77	77	77
79	79	79	79	79	79	79	79	79
83	83	83	83	83	83	83	83	83
89	89	89	89	89	89	89	89	89
91	91	91	91	91	91	91	91	91
97	97	97	97	97	97	97	97	97
101	101	101	101	101	101	101	101	101
103	103	103	103	103	103	103	103	103
107	107	107	107	107	107	107	107	107
109	109	109	109	109	109	109	109	109
113	113	113	113	113	113	113	113	113
119	119	119	119	119	119	119	119	119
121	121	121	121	121	121	121	121	121
127	127	127	127	127	127	127	127	127
131	131	131	131	131	131	131	131	131
133	133	133	133	133	133	133	133	133
137	137	137	137	137	137	137	137	137
139	139	139	139	139	139	139	139	139
143	143	143	143	143	143	143	143	143
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161	161	161	161	161	161	161	161	161
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1013	1013	1013	1013	1013	1013	1013	1013	1013
1019	1019	1019	1019	1019	1019	1019	1019	1019
1021	1021	1021	1021	1021	1021	1021	1021	1021

The formulas (30*n)+1 ; (30*n)+7 ; (30*n)+11 ; (30*n)+13 ; (30*n)+17 ; (30*n)+19 ; (30*n)+23 and (30*n)+29 give a sequence of numbers that present respectively the following gaps : 6 ; 4 ; 2 ; 4 ; 2 ; 4 ; 6. When n increases with 1, the obtained number is the result of the addition of 2 to the previous number. The gaps are then : 6 ; 4 ; 2 ; 4 ; 2 ; 4 ; 6 ; 2. This organisation draws a pattern that is repeated indefinitely (tab.1).

Column A on tab.2 indicates all numbers given by the 8 formulas without showing the gaps existing between them (in fact, they are multiples of 1 among the involved numbers i.e. all of them).

Multiples of 7 are indicated in column B, they are the result of the multiplication of the 8 formulas by 7. The gaps are multiplied by 7, the pattern is extended 7 times. This pattern generates a complementary pattern representing numbers not being multiples of 7.

It is possible to represent in this way all multiples of all the numbers given by the 8 formulas, it leads to an infinity of columns whose superposition makes the prime numbers appear by elimination.

Prime numbers draw then a complementary pattern of a pattern resulting from the superposition of the infinity of columns.

The superimposition of each new column on the previous ones brings a changing right from the square of the number whose column indicates the multiples excepting the non-primes which do not bring any change due to the presence of their dividers.

The multiplication of the numbers given by the 8 formulas between them for a value of n corresponding to "k+1" (k ∈ N) repeats the same (but extented) pattern resulting from the multiplication of the numbers given by the 8 formulas between them for a value of n corresponding to "k", it is extended a calculable number of times. However, the patterns drawn by the numbers given by the 8 formulas for the value of "k" continue to emerge, which brings a changing to the extended pattern. This changing occurs at each increasing of n when n does not give 8 non-primes (which can happen but never definitively according to Euclid's prime number theorem).

The pattern drawn by prime numbers is then repeated neither as it is nor as extented.

3. The 36 formulas

Suppose m and m' are natural numbers (m ∈ N, m' ∈ N) :

O is the set of the numbers given by the 8 formulas, it excludes multiples of each one of 2, 3 and 5 and is consisting of two sets :

- Q : set of non-primes resulting from the multiplication of all the numbers given by the 8 formulas between them and 36 formulas are obtained (tab.3).
- P : set of prime numbers.

Tab.3. Formulas expressing the multiplication of all the numbers given by the 8 formulas between them

1	((30*m)+1)*((30*m')+1)	m ∈ N \ {0}	m' ∈ N \ {0}
2	((30*m)+1)*((30*m')+7)	m ∈ N \ {0}	m' ∈ N
3	((30*m)+1)*((30*m')+11)	m ∈ N \ {0}	m' ∈ N
4	((30*m)+1)*((30*m')+13)	m ∈ N \ {0}	m' ∈ N
5	((30*m)+1)*((30*m')+17)	m ∈ N \ {0}	m' ∈ N
6	((30*m)+1)*((30*m')+19)	m ∈ N \ {0}	m' ∈ N
7	((30*m)+1)*((30*m')+23)	m ∈ N \ {0}	m' ∈ N
8	((30*m)+1)*((30*m')+29)	m ∈ N \ {0}	m' ∈ N
9	((30*m)+7)*((30*m')+7)	m ∈ N	m' ∈ N
10	((30*m)+7)*((30*m')+11)	m ∈ N	m' ∈ N
11	((30*m)+7)*((30*m')+13)	m ∈ N	m' ∈ N
12	((30*m)+7)*((30*m')+17)	m ∈ N	m' ∈ N
13	((30*m)+7)*((30*m')+19)	m ∈ N	m' ∈ N
14	((30*m)+7)*((30*m')+23)	m ∈ N	m' ∈ N
15	((30*m)+7)*((30*m')+29)	m ∈ N	m' ∈ N
16	((30*m)+11)*((30*m')+11)	m ∈ N	m' ∈ N
17	((30*m)+11)*((30*m')+13)	m ∈ N	m' ∈ N
18	((30*m)+11)*((30*m')+17)	m ∈ N	m' ∈ N
19	((30*m)+11)*((30*m')+19)	m ∈ N	m' ∈ N
20	((30*m)+11)*((30*m')+23)	m ∈ N	m' ∈ N
21	((30*m)+11)*((30*m')+29)	m ∈ N	m' ∈ N
22	((30*m)+13)*((30*m')+13)	m ∈ N	m' ∈ N
23	((30*m)+13)*((30*m')+17)	m ∈ N	m' ∈ N
24	((30*m)+13)*((30*m')+19)	m ∈ N	m' ∈ N
25	((30*m)+13)*((30*m')+23)	m ∈ N	m' ∈ N
26	((30*m)+13)*((30*m')+29)	m ∈ N	m' ∈ N
27	((30*m)+17)*((30*m')+17)	m ∈ N	m' ∈ N
28	((30*m)+17)*((30*m')+19)	m ∈ N	m' ∈ N
29	((30*m)+17)*((30*m')+23)	m ∈ N	m' ∈ N
30	((30*m)+17)*((30*m')+29)	m ∈ N	m' ∈ N
31	((30*m)+19)*((30*m')+19)	m ∈ N	m' ∈ N
32	((30*m)+19)*((30*m')+23)	m ∈ N	m' ∈ N
33	((30*m)+19)*((30*m')+29)	m ∈ N	m' ∈ N
34	((30*m)+23)*((30*m')+23)	m ∈ N	m' ∈ N
35	((30*m)+23)*((30*m')+29)	m ∈ N	m' ∈ N
36	((30*m)+29)*((30*m')+29)	m ∈ N	m' ∈ N

Elements of P i.e. prime numbers are obtained by excluding elements of Q from those of O.

References

Euclide. Les Eléments, volume 1. Introduction générale et Livres I à IV. Presses Universitaires de France, Paris. Introduction générale par Maurice Caveing et traduction et commentaires du texte de Heiberg par Bernard Vitrac, 1990.

Horsley, Rev. Samuel, F. R. S., "Κόσκινον Ερατοσθένους or, The Sieve of Eratosthenes. Being an account of his method of finding all the Prime Numbers," Philosophical Transactions (1683–1775), Vol. 62. (1772), pp. 327–347.