Maxwell-type Theory for Non-uniformly Moving Electric Charge

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Abstract

In this article, we propose a mathematic approach to study the effects of accelerating charges. We postulate and demonstrate mathematically that the acceleration of electric charges generate new **N** and **M** fields. The **N** and **M** field equations are supplement to Maxwell equations, and have the similar forms as that of Maxwell equations. The effect of the **N** and **M** fields is to vary the Lorentz force with time, and have the similar forms as that of Lorentz force. The **N** and **M** fields propagate as waves. The **N** and **M** fields due to accelerating charges exist mathematically. We suggest that those new fields are worth to be systematically studied experimentally. The study of those new fields might open a new window.

Key words: Maxwell equations, Lorentz force, electrodynamics, wave, electromagnetic experiment, non-uniform moving electric charge

1. Introduction

Physics laws describing phenomena that relate with uniform motion include:

(1) Maxwell equations govern electromagnetic fields generated by stationary charges and steady current.

(2) Special Relativity postulates that physics laws are invariant in all inertial frames of reference.

(3) de Broglie wavelength $\lambda = {}^{h}/_{p}$ of uniformly moving particles.

In the 1998, scientists reported a revolutionary discovery that the expansion of the universe is accelerating [1], which implies that everything in the universe is accelerating.

A task is: modify physics laws to describe acceleration related phenomena, namely, to include the terms of acceleration in physics laws. Recently Hubble's law and Doppler's law have been extended to contain terms of acceleration for describing phenomena caused by acceleration [2]. Indeed acceleration does cause significant differences in those phenomena.

Galileo transformation and Lorentz transformation are between inertial frames with low and high velocity respectively. For accelerating frames, both transformations no long hold. There is no appropriate transformation either between an inertial frame and a non-inertial frame or between non-inertial systems.

At quantum level, the phenomenon of wavefunction collapse is explained as due to the acceleration of particle [3].

In this article, we study what effect will the acceleration of an electric charge induce. We demonstrate mathematically: (1) the acceleration of a charge generates new fields; (2) those new fields affect stationary, uniformly moving, and accelerating test charges differently; (3) new fields propagate as wave.

2. Review of Maxwell Equations

First let's review Maxwell equations and the Lorentz force. Maxwell equations are invariant under Lorentz transformation and describe the fields generated by stationary charges and steady current. There is an interesting phenomenon: When a charge is at rest, it generates an electric field, and doesn't generate a magnetic field. However, an observer moving uniformly relative to the same charge observes not only an electric field but also a magnetic field. The magnetic field is completely different from electric field in the following senses: (1) the way the fields generated; (2) the nature of the fields; (3) effects of the fields on charges.

Now we ask: What will an accelerating observer experience, namely, what new fields will the acceleration, instead of the velocity, of a charge generate? What are the effects of new fields on stationary, uniformly moving, and accelerating test charges?

The non-uniform motion of a charge is characterized by velocity and acceleration. At a given instant, the non-uniform motion can be considered as uniform motion, and the instant velocity will generate magnetic field. The question is: are there new fields generated by the acceleration? The term "acceleration" means either acceleration or deceleration.

Let's review Maxwell equations,

$$\nabla\times\mathbf{B} = 4\pi\rho\mathbf{v} + \frac{\partial\mathbf{E}}{\partial t}, \qquad \nabla\cdot\mathbf{B} = 0,$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}, \qquad \nabla\cdot\mathbf{E} = 4\pi\rho.$$

Which are invariant under Lorentz transformation. The velocity of the source, $\mathbf{v}$, is constant. Taking the curl of Maxwell equations, one obtains the **E** and **B** field wave equations,

$$\nabla^2\mathbf{E}-\frac{\partial^2\mathbf{E}}{\partial t^2}=4\pi\frac{\partial\rho\mathbf{v}}{\partial t}+4\pi\nabla\rho, \tag{1}$$

$$\nabla^2\mathbf{B}-\frac{\partial^2\mathbf{B}}{\partial t^2}=-4\pi\nabla\times(\rho\mathbf{v}). \tag{2}$$

Now we face two cases.

First case: The ρ and $\rho\mathbf{v}$ in Maxwell equations are constant. For constant density of charge and constant current,

$$\nabla\rho=\frac{\partial\rho\mathbf{v}}{\partial t}=\nabla\times(\rho\mathbf{v})=0,$$

Eq. (1) and (2) reduce to the plane **E** and **B** field wave equations without source,

$$\nabla^2\mathbf{E}-\frac{\partial^2\mathbf{E}}{\partial t^2}=0, \tag{1a}$$

$$\nabla^2\mathbf{B}-\frac{\partial^2\mathbf{B}}{\partial t^2}=0. \tag{2a}$$

Which implies that the constant current and constant density of charge do not generate the **E** and **B** waves. Eq. (1a) and (2a) are Lorentz invariant even one takes a curl operation of Maxwell equation.

The densities of charge and current have been extended to be space-time functions as $\rho\,(x,t)$ and $\rho\mathbf{v}=\mathbf{J}(x,t)$, which means that charges move with acceleration.

Second case: the density of charge and current are time-varying and special-dependent,

$$\nabla\rho\neq 0,\ \frac{\partial\rho\mathbf{v}}{\partial t}\neq 0, \text{ and } \nabla\times(\rho\mathbf{v})\neq 0.$$

Then, terms of $\nabla\rho$, $\frac{\partial\rho\mathbf{v}}{\partial t}$, and $\nabla\times(\rho\mathbf{v})$ perform as the sources of the **E** and **B** field wave.

For this case, *Eq. (1) and (2) are not Lorentz invariant*. The curl operation on Maxwell equations and the non-constant density of charge/current break down the Lorentz invariant.

For this case, the continuity equation,

$$\dot{\rho}+\nabla\cdot(\rho\mathbf{v})=0, \tag{3}$$

is not sufficient, since the density of charge and current varies with time and space. Where the dot " · " on the top of a variable means the time derivative of the variable. Thus we should extend to space-time-varying continuity equations, as a supplement of the original continuity equation, Eq. (3), as,

$$\ddot{\rho}+\nabla\cdot(\rho\dot{\mathbf{v}}+\dot{\rho}\mathbf{v})=0, \tag{4}$$

and

$$\nabla\dot{\rho}+\nabla\{\nabla\cdot(\rho\mathbf{v})\}=0, \tag{5}$$

for describing completely the conservation of charge density.

When one substitutes instantaneous values of $\rho\,(x,t_1)$ and $\rho\mathbf{v}=\mathbf{J}(x,t_1)$ into Maxwell equations, Maxwell equations describe the $\mathbf{E}_1$ and $\mathbf{B}_1$ fields for the given instant t_1; for next instant t_2, $\rho\,(x,t_1)$ and $\mathbf{J}(x,t_1)$ change to $\rho\,(x,t_2)$ and $\mathbf{J}(x,t_2)$, and thus the $\mathbf{E}_1$ and $\mathbf{B}_1$ field are changed to the $\mathbf{E}_2$ and $\mathbf{B}_2$.

The **E** and **B** fields due to constant charge/current have been extended to that due to an accelerating charge by using the Lienard-Wiechert potentials. Jefimenko [4] gives the **E** and **B** fields induced by an arbitrary charge/current distribution,

$$\mathbf{E}=\int\left\{\left[\frac{\rho(r',t_{ret})}{R^3}+\frac{1}{R^2}\frac{\partial\rho(r',t_{ret})}{\partial t}\right]\mathbf{R}-\frac{1}{R}\frac{\partial\mathbf{J}(r',t_{ret})}{\partial t}\right\}d^3r'. \tag{6}$$

$$\mathbf{B}=\int\left\{\left[\frac{\mathbf{J}(r',t_{ret})}{R^3}+\frac{1}{R^2}\frac{\partial\mathbf{J}(r',t_{ret})}{\partial t}\right]\times\mathbf{R}\right\}d^3r'. \tag{7}$$

where $\mathbf{R}=\mathbf{r}-\mathbf{r}'$, $R=|\mathbf{r}-\mathbf{r}'|$; the retarded time, $t_{ret}=t-R/c$, c is the speed of light, we set $c=1$.

In Jefimenko's solutions, the accelerating motion of source charges is involved in $\frac{\partial\mathbf{J}(r',t_{ret})}{\partial t}$.

For a point charge Q, Feynman's expression [5] for the solution of Eq. (1) is,

$$\mathbf{E}=Q\left\{\left[\frac{\mathbf{R}}{R^3}\right]_{ret}+\frac{[R]_{ret}}{c}\frac{\partial}{\partial t}\left[\frac{\mathbf{R}}{R^3}\right]_{ret}+\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left[\frac{\mathbf{R}}{R}\right]_{ret}\right\}, \tag{8}$$

while Heaviside's expression of the solution of Eq. (2) is,

$$\mathbf{B} = \mathrm{Q}\left\{\left[\frac{\mathbf{v}\times\mathbf{R}}{\kappa^2\mathrm{R}^3}\right]_{\mathrm{ret}} + \frac{1}{c[\mathrm{R}]_{\mathrm{ret}}}\frac{\partial}{\partial \mathrm{t}}\left[\frac{\mathbf{v}\times\mathbf{R}}{\kappa R}\right]_{\mathrm{ret}}\right\}, \tag{9}$$

where $[\mathrm{f}]_{\mathrm{ret}}$ implies that the time is to be calculated at the retarded time, $\mathrm{t}_{\mathrm{ret}}$; $\kappa = 1 - \mathbf{v}\cdot\widehat{\mathbf{R}}/c$ is a retardation factor, $\widehat{\mathbf{R}} = \mathbf{R}/\mathrm{R}$. The solution of the **E** field, Eq. (8), contains 3 terms, a static charge, the velocity of the charge, and the acceleration of the charge; while the solution of the **B** field, Eq. (9), contains two terms, the velocity of the charge and the acceleration of the charge.

The non-Lorentz-invariant of the **B** and **E** wave equations can be avoided by going to potential wave by substituting $\mathbf{B} = \nabla\times\mathbf{A}$ and $\mathbf{E} = -\nabla\varphi - \frac{\partial\mathbf{A}}{\partial \mathrm{t}}$ into Maxwell equations. One obtains,

$$\frac{\partial^2\mathbf{A}}{\partial \mathrm{t}^2} - \nabla^2\mathbf{A} = 4\pi\rho\mathbf{v}, \tag{10}$$

$$\frac{\partial^2\varphi}{\partial \mathrm{t}^2} - \nabla^2\varphi = 4\pi\rho. \tag{11}$$

Where Lorentz gauge,

$$\frac{\partial\varphi}{\partial \mathrm{t}} + \nabla\cdot\mathrm{A} = 0,$$

has been applied.

Combining Eq. (10) and (11), one obtain,

$$\left[\frac{\partial^2}{\partial \mathrm{t}^2} - \nabla^2\right]\mathrm{A}_{\mathrm{i}} = 4\pi\mathrm{J}_{\mathrm{i}}, \tag{12}$$

where $\mathrm{J}_{\mathrm{i}} = (\rho, \rho\mathbf{v})$ and $\mathrm{A}_{\mathrm{i}} = (\varphi, \mathbf{A})$.

The wave equations, Eq. (10), (11), and (12), keep Lorentz invariant. The reasons are: first, in derivation of Eq. (10), (11), and (12), one did not apply additional operation, such as the "$\nabla\times$" in the derivation of Eq. (1) and (2); second, thus the density of charge and current keep the same as in Maxwell equations.

To summarize: as long as the density of charge and current are constant, the **E** and **B** fields wave equations, and φ and $\mathbf{A}$ potentials wave equations are Lorentz invariant, it does not matter whether there is an additional curl operation or not. On the contrary, when the density of charge and current are not constant, Lorentz invariant no longer holds for the **E** and **B** wave equations.

However, in Eq. (8), (9) and (10), the ρ and $\rho\mathbf{v}$ coming from Maxwell equations are steady, and does not generate wave. One way to resolve this issue is to extend the density of charge and current to be space-time functions as $\rho\,(\mathrm{x},\mathrm{t})$ and $\rho\mathbf{v} = \mathbf{J}(\mathrm{x},\mathrm{t})$, which means that charges move with acceleration. Electromagnetic waves can be obtained from the Lienard-Wiechert potentials.

3. New Fields Generated by Non-uniform Motion of Charge

In this article we propose a different approach to deal with effects of acceleration of charges. Maxwell equations, except the displacement current, were established based on series experiments. Up to now, as I know of, there is no experiment(s) to study systematically the effects of the acceleration of charges.

To answer those questions mentioned above, we propose a mathematic approach, and show that the acceleration of a charge generates new fields. According to the concept of the field, new fields should mediate new effects. Moreover, we postulate and show that the acceleration of a charge generates new fields in the same way as that either the velocity of a charge generates magnetic field or the time-varying electric/magnetic field generates the induced magnetic/electric field.

The basic concept is that, since the uniform velocity of a charge generates magnetic field that is axial field, the acceleration of a charge should generates axial fields like magnetic field. The new field equations should be supplement to Maxwell equations and can be utilized to investigate phenomena caused by the acceleration of a charge at the level of the field strengths [6].

Note that those new field equations don't obey Lorentz transformation. In general, a physics law containing term of acceleration will not obey Lorentz transformation.

3.1. Magnetic-type N Field

Let's start the mathematic approach. The very first step is to find mathematically how the acceleration of charges generates an axial field. There is a formula in vector analysis,

$$\nabla\times(\mathbf{S}\times\mathbf{T}) = \mathbf{S}(\nabla\cdot\mathbf{T}) - \mathbf{T}(\nabla\cdot\mathbf{S}) + (\mathbf{T}\cdot\nabla)\mathbf{S} - (\mathbf{S}\cdot\nabla)\mathbf{T}. \quad (13)$$

What is significant of this formula is that the combinations of the divergence and the gradient of two either non- axial or axial vectors, **S** and **T**, induce inevitable an axial vector, $\mathbf{S}\times\mathbf{T}$. Namely an axial vector is inevitable born of two arbitrary vectors.

Coulomb's law gives the relation between the divergence of a non-axial vector electric field to charge, $\nabla\cdot\mathbf{E} = 4\pi Q$. So the combination of the vector analysis formula and Coulomb's law leads us to let $\mathbf{S} = \dot{\mathbf{v}}$, $\mathbf{T} = \mathbf{E}$, where $\dot{\mathbf{v}}$ is acceleration. Eq. (13) gives

$$\nabla\times(\dot{\mathbf{v}}\times\mathbf{E}) = \dot{\mathbf{v}}(\nabla\cdot\mathbf{E}) - \mathbf{E}(\nabla\cdot\dot{\mathbf{v}}) + (\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} - (\dot{\mathbf{v}}\cdot\nabla)\mathbf{E}. \quad (14)$$

Define a new field, **N**,

$$\mathbf{N} \equiv \dot{\mathbf{v}}\times\mathbf{E}, \quad (15)$$

Note the **N** field is an acceleration correspondence of magnetic field **B**,

$$\mathbf{B} \equiv \mathbf{v}\times\mathbf{E}. \quad (16)$$

Therefore we call the **N** field the magnetic-type field.

For a constant electric field, the **N** and **B** fields are simply related,

$$\mathbf{N} = \frac{\partial\mathbf{B}}{\partial t}. \quad (17)$$

Combining Eq. (14 and 15), we obtain,

$$\nabla\times\mathbf{N} = 4\pi Q\dot{\mathbf{v}} - \mathbf{E}(\nabla\cdot\dot{\mathbf{v}}) + (\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} - (\dot{\mathbf{v}}\cdot\nabla)\mathbf{E}, \quad (18)$$

where Maxwell equation, $\boldsymbol{\nabla}\cdot\mathbf{E} = 4\pi Q$, has been applied.

Eq. (18) describes a new **N** field generated by the acceleration of a charge.

For non-spatially-varying acceleration, $\mathbf{E}(\nabla\cdot\dot{\mathbf{v}}) = (\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} = \mathbf{0}$, Eq. (18) reduces to

$$\nabla\times\mathbf{N} = 4\pi Q\dot{\mathbf{v}} - (\dot{\mathbf{v}}\cdot\nabla)\mathbf{E}. \quad (19)$$

The acceleration of a charge indeed generates inevitably an axial field **N** like the velocity of a charge generates a axial field **B**. Eq. (18) and (19) show the relation between the **N** field, acceleration $\dot{\mathbf{v}}$, and electric field **E**.

3.2. Electric-type M field

Based on symmetry, it is nature to expect that the acceleration of charges generate an electric-type field as well. For this aim, let's apply the formula, Eq. (13), again. Let $\mathbf{S} = \dot{\mathbf{v}}$, $\mathbf{T} = \mathbf{B}$, where **B** is magnetic field, we obtain,

$$\nabla\times(\dot{\mathbf{v}}\times\mathbf{B}) = -\mathbf{B}(\nabla\cdot\dot{\mathbf{v}}) + (\mathbf{B}\cdot\nabla)\dot{\mathbf{v}} - (\dot{\mathbf{v}}\cdot\nabla)\mathbf{B}, \quad (20)$$

where Maxwell equation, $\boldsymbol{\nabla}\cdot\mathbf{B} = 0$, has been utilized.

Define a new axial field, **M**,

$$\mathbf{M} \equiv -\dot{\mathbf{v}}\times\mathbf{B}. \quad (21)$$

Note the **M** field is an acceleration correspondence of electric field **E**,

$$\mathbf{E} \equiv -\mathbf{v}\times\mathbf{B}. \quad (22)$$

Thus call the **M** field the electric-type field.

For a constant magnetic field, the **M** and **E** fields are simply related as,

$$\mathbf{M} = \frac{\partial \mathbf{E}}{\partial t}. \tag{23}$$

Substituting the **M** field, Eq. (21), into Eq. (20), we obtain,

$$\nabla\times\mathbf{M} = \mathbf{B}(\nabla\cdot\dot{\mathbf{v}}) - (\mathbf{B}\cdot\nabla)\dot{\mathbf{v}} + (\dot{\mathbf{v}}\cdot\nabla)\mathbf{B}. \tag{24}$$

For non-spatially-varying acceleration, $\mathbf{B}(\nabla\cdot\dot{\mathbf{v}}) = (\mathbf{B}\cdot\nabla)\dot{\mathbf{v}} = \mathbf{0}$, we obtain the **M** field equation,

$$\nabla\times\mathbf{M} = (\dot{\mathbf{v}}\cdot\nabla)\mathbf{B}. \tag{25}$$

The acceleration of a charge indeed generates a axial field **M**. Eq. (24) and (25) relate the **M** field, acceleration $\dot{\mathbf{v}}$, and magnetic field **B.**

3.3. Interpretation of the N and M Fields

The physical meaning of the **N** and **M** fields can be interpreted as the following.

Consider a charge Q and an observer O. When the charge is at rest relative to the observer, it generates an electro-static field; when the charge is moving with a constant velocity relative to the observer, beside the electric field, the observer also experiences a magneto-static field generated by the uniform motion of the charge; when the charge is moving with acceleration $\dot{\mathbf{v}}$, beside the electric and magnetic field, the observer will observe both a **N** field, $\mathbf{N} = \dot{\mathbf{v}}\times\mathbf{E}$, and a **M** field, $\mathbf{M} = -\dot{\mathbf{v}}\times\mathbf{B}$. Namely, the constant velocity of a charge generates one axial field; the acceleration of a charge generates two additional axial fields (Table 2).

Table 2: Fields generated by stationary charge and the motion of charges

	Generated Field
Stationary Charge: $\mathbf{v} = \mathbf{0}$	$\mathbf{E}$
Uniform motion of charge: $\mathbf{v} \neq \mathbf{0},\ \dot{\mathbf{v}} = \mathbf{0}$	$\mathbf{B} = \mathbf{v}\times\mathbf{E}$
Non-uniform motion of charge: $\mathbf{v} \neq \mathbf{0},\ \dot{\mathbf{v}} \neq \mathbf{0}$	$\mathbf{N} = \dot{\mathbf{v}}\times\mathbf{E}$ $\mathbf{M} = -\dot{\mathbf{v}}\times\mathbf{B}$

3.4. Correlations between the N and M Fields

3.4.1. Ampere-law-type Correlation

Eq. (18 and 19) and (24 and 25) show the correlations between the **N** field, acceleration $\dot{\mathbf{v}}$, and electric field **E,** and between the **M** field, acceleration $\dot{\mathbf{v}}$, and magnetic field **B,** respectively. Now we need to find the correlation between the **N** and **M** fields. For this aim, we need to re-express the term of $(\dot{\mathbf{v}}\cdot\nabla)\mathbf{E}$ in Eq. (18 and 19) and $(\dot{\mathbf{v}}\cdot\nabla)\mathbf{B}$ in Eq. (24 and 25).

Applying another vector analysis formula,

$$\nabla(\mathbf{S}\cdot\mathbf{T}) = (\mathbf{S}\cdot\nabla)\mathbf{T} + (\mathbf{T}\cdot\nabla)\mathbf{S} + \mathbf{S}\times(\nabla\times\mathbf{T}) + \mathbf{T}\times(\nabla\times\mathbf{S}). \tag{26}$$

Let $\mathbf{S} = \dot{\mathbf{v}}$ and $\mathbf{T} = \mathbf{E}$, where **E** is electric field, Eq. (26) gives

$$\nabla(\dot{\mathbf{v}}\cdot\mathbf{E}) = (\dot{\mathbf{v}}\cdot\nabla)\mathbf{E} + (\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} + \dot{\mathbf{v}}\times(\nabla\times\mathbf{E}) + \mathbf{E}\times(\nabla\times\dot{\mathbf{v}}).$$

Or rewrite as

$$(\dot{\mathbf{v}}\cdot\nabla)\mathbf{E} = \nabla(\dot{\mathbf{v}}\cdot\mathbf{E}) - (\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} - \dot{\mathbf{v}}\times(\nabla\times\mathbf{E}) - \mathbf{E}\times(\nabla\times\dot{\mathbf{v}}). \tag{27}$$

Substituting Eq. (27) into Eq. (18), we obtain

$$\nabla\times\mathbf{N} = 4\pi Q\dot{\mathbf{v}} - \mathbf{E}(\nabla\cdot\dot{\mathbf{v}}) + 2(\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} - \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) + \dot{\mathbf{v}}\times(\boldsymbol{\nabla}\times\mathbf{E}) + \mathbf{E}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}). \qquad (28)$$

For non-spatially-varying acceleration, $(\mathbf{E}\cdot\boldsymbol{\nabla})\dot{\mathbf{v}} = (\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} = \mathbf{E}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}) = \mathbf{0},$ Eq. (28) becomes,

$$\nabla\times\mathbf{N} = 4\pi Q\dot{\mathbf{v}} - \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) + \dot{\mathbf{v}}\times(\boldsymbol{\nabla}\times\mathbf{E}). \qquad (29)$$

Next let's find $\dot{\mathbf{v}}\times(\boldsymbol{\nabla}\times\mathbf{E})$ in term of the **M** field. For this aim, taking time derivative of the **M** field, we obtain,

$$\frac{\partial\mathbf{M}}{\partial t} = -\frac{\partial}{\partial t}(\dot{\mathbf{v}}\times\mathbf{B}) = -\ddot{\mathbf{v}}\times\mathbf{B} - \dot{\mathbf{v}}\times\frac{\partial\mathbf{B}}{\partial t} = -\ddot{\mathbf{v}}\times\mathbf{B} + \dot{\mathbf{v}}\times[\nabla\times\mathbf{E}]. \qquad (30)$$

Where Maxwell equation, $\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$, is used; " $\ddot{\mathbf{v}}$ " is jerk and has four cases: (1) accelerating the acceleration; (2) decelerating the acceleration; (3) accelerating the deceleration; and (4) decelerating the deceleration. In this article we just use letter " $\ddot{\mathbf{v}}$ " to represent all of four cases.

Substituting Eq. (30) into Eq. (28) and (29), respectively, we obtain Ampere-law-type correlation,

$$\nabla\times\mathbf{N} = 4\pi Q\dot{\mathbf{v}} + \frac{\partial\mathbf{M}}{\partial t} + \ddot{\mathbf{v}}\times\mathbf{B} - \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) - \mathbf{E}(\nabla\cdot\dot{\mathbf{v}}) + 2(\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} + \mathbf{E}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}), \qquad (31)$$

and

$$\nabla\times\mathbf{N} = 4\pi Q\dot{\mathbf{v}} + \frac{\partial\mathbf{M}}{\partial t} + \ddot{\mathbf{v}}\times\mathbf{B} - \nabla(\dot{\mathbf{v}}\cdot\mathbf{E}). \qquad (32)$$

For constant acceleration, $\ddot{\mathbf{v}} = 0$, Eq. (31) and (32) become respectively,

$$\nabla\times\mathbf{N} = 4\pi Q\dot{\mathbf{v}} + \frac{\partial\mathbf{M}}{\partial t} - \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) - \mathbf{E}(\nabla\cdot\dot{\mathbf{v}}) + 2(\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} + \mathbf{E}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}), \qquad (33)$$

and

$$\nabla\times\mathbf{N} = 4\pi Q\dot{\mathbf{v}} + \frac{\partial\mathbf{M}}{\partial t} - \nabla(\dot{\mathbf{v}}\cdot\mathbf{E}), \qquad (34)$$

which is the acceleration correspondence of the Ampere's law,

$$\nabla\times\mathbf{B} = 4\pi Q\mathbf{v} + \frac{\partial\mathbf{E}}{\partial t}.$$

Eq. (31) −(34) are the Ampere-law-type correlation of the **N** and **M** fields for different approximations.

Next let's calculate $\nabla(\dot{\mathbf{v}}\cdot\mathbf{E})$. Taking Eq. (22),

$$\nabla(\dot{\mathbf{v}}\cdot\mathbf{E}) = \nabla[\dot{\mathbf{v}}\cdot(-\mathbf{v}\times\mathbf{B})] = -\nabla[\mathbf{B}\cdot(\dot{\mathbf{v}}\times\mathbf{v})].$$

For the case of $\dot{\mathbf{v}}//\mathbf{v}$,

$$\nabla(\dot{\mathbf{v}}\cdot\mathbf{E}) = \mathbf{0}.$$

Substituting it into Eq. (34), we obtain,

$$\nabla\times\mathbf{N} = 4\pi Q\dot{\mathbf{v}} + \frac{\partial\mathbf{M}}{\partial t}. \qquad (35)$$

Eq. (34a) has the same form as that of Ampere-Maxwell's law.

3.4.2. Faraday-law-type Correlation

Based on symmetry, we postulate another correlation between the **N** and **M** fields.

Let $\mathbf{S} = \dot{\mathbf{v}}$ and $\mathbf{T} = \mathbf{B},$ where **B** is magnetic field. Substituting into Eq. (26), gives

$$\boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B}) = (\dot{\mathbf{v}}\cdot\boldsymbol{\nabla})\mathbf{B} + (\mathbf{B}\cdot\boldsymbol{\nabla})\dot{\mathbf{v}} + \dot{\mathbf{v}}\times(\boldsymbol{\nabla}\times\mathbf{B}) + \mathbf{B}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}).$$

Or rewrite as

$$(\dot{\mathbf{v}}\cdot\boldsymbol{\nabla})\mathbf{B} = \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B}) - (\mathbf{B}\cdot\boldsymbol{\nabla})\dot{\mathbf{v}} - \dot{\mathbf{v}}\times(\boldsymbol{\nabla}\times\mathbf{B}) - \mathbf{B}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}). \qquad (36)$$

Substituting Eq. (36) into Eq. (24), we obtain,

$$\nabla\times\mathbf{M} = \mathbf{B}(\nabla\cdot\dot{\mathbf{v}}) - 2(\mathbf{B}\cdot\nabla)\dot{\mathbf{v}} + \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B}) - \dot{\mathbf{v}}\times(\boldsymbol{\nabla}\times\mathbf{B}) - \mathbf{B}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}). \qquad (37)$$

For non-spatially-varying acceleration, $\mathbf{B}(\nabla\cdot\dot{\mathbf{v}}) = (\mathbf{B}\cdot\boldsymbol{\nabla})\dot{\mathbf{v}} = \mathbf{B}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}) = \mathbf{0},$ Eq. (37) becomes,

$$\nabla\times\mathbf{M} = \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B}) - \dot{\mathbf{v}}\times(\boldsymbol{\nabla}\times\mathbf{B}). \qquad (38)$$

Next let's find $\dot{\mathbf{v}}\times(\boldsymbol{\nabla}\times\mathbf{B})$ in term of **N** field. Taking time derivative of the **N** field, we obtain,

$$\frac{\partial\mathbf{N}}{\partial t} = \frac{\partial}{\partial t}(\dot{\mathbf{v}}\times\mathbf{E}) = \ddot{\mathbf{v}}\times\mathbf{E} + \dot{\mathbf{v}}\times(\nabla\times\mathbf{B}) - 4\pi Q\dot{\mathbf{v}}\times\mathbf{v}, \qquad (39)$$

where Maxwell equation, $(\nabla\times\mathbf{B}) = 4\pi Q\mathbf{v} + \dot{\mathbf{E}}$, has been used. Substituting Eq. (39) into Eq. (37), we obtain the correlation equation of the **N** and **M** fields,

$$\nabla\times\mathbf{M} = -\frac{\partial\mathbf{N}}{\partial t} + \mathbf{B}(\nabla\cdot\dot{\mathbf{v}}) - 2(\mathbf{B}\cdot\nabla)\dot{\mathbf{v}} + \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B}) + \ddot{\mathbf{v}}\times\mathbf{E} - 4\pi Q\dot{\mathbf{v}}\times\mathbf{v} - \mathbf{B}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}). \quad (40)$$

For non-spatially-varying acceleration, $\mathbf{B}(\nabla\cdot\dot{\mathbf{v}}) = (\mathbf{B}\cdot\boldsymbol{\nabla})\dot{\mathbf{v}} = \mathbf{B}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}) = \mathbf{0}$, Eq. (40) becomes,

$$\nabla\times\mathbf{M} = -\frac{\partial\mathbf{N}}{\partial t} + \ddot{\mathbf{v}}\times\mathbf{E} + \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B}) - 4\pi Q\dot{\mathbf{v}}\times\mathbf{v}. \quad (41)$$

For the case of $\dot{\mathbf{v}}//\mathbf{v}$, E. (40) and (41) give, respectively,

$$\nabla\times\mathbf{M} = -\frac{\partial\mathbf{N}}{\partial t} + \mathbf{B}(\nabla\cdot\dot{\mathbf{v}}) - 2(\mathbf{B}\cdot\nabla)\dot{\mathbf{v}} + \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B}) + \ddot{\mathbf{v}}\times\mathbf{E} - \mathbf{B}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}). \quad (42)$$

$$\nabla\times\mathbf{M} = -\frac{\partial\mathbf{N}}{\partial t} + \ddot{\mathbf{v}}\times\mathbf{E} + \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B}). \quad (43)$$

For constant acceleration, $\ddot{\mathbf{v}} = \mathbf{0}$, Eq. (42) and (43) simplify further, respectively,

$$\nabla\times\mathbf{M} = -\frac{\partial\mathbf{N}}{\partial t} + \mathbf{B}(\nabla\cdot\dot{\mathbf{v}}) - 2(\mathbf{B}\cdot\nabla)\dot{\mathbf{v}} + \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B}) - \mathbf{B}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}). \quad (44)$$

$$\nabla\times\mathbf{M} = -\frac{\partial\mathbf{N}}{\partial t} + \nabla(\dot{\mathbf{v}}\cdot\mathbf{B}), \quad (45)$$

Now let's calculate $\nabla(\dot{\mathbf{v}}\cdot\mathbf{B})$.

$$\nabla(\dot{\mathbf{v}}\cdot\mathbf{B}) = \nabla[\dot{\mathbf{v}}\cdot(\mathbf{v}\times\mathbf{E})] = \nabla[\mathbf{E}\cdot(\dot{\mathbf{v}}\times\mathbf{v})].$$

For the case of $\dot{\mathbf{v}}//\mathbf{v}$, we have

$$\nabla(\dot{\mathbf{v}}\cdot\mathbf{B}) = 0.$$

Then we obtain,

$$\nabla\times\mathbf{M} = -\frac{\partial\mathbf{N}}{\partial t}. \quad (46)$$

which is the acceleration correspondences of the Faraday's law,

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}.$$

Eq. (40)--(46) are the Faraday-law-type Correlation of the **N** and **M** fields for different approximations.

3.5. Divergence of the Axial N and Axial M fields

Since both the **N** and **M** fields are axial fields, we have,

$$\nabla\cdot\mathbf{N} = 0, \quad (47)$$

$$\nabla\cdot\mathbf{M} = 0. \quad (48)$$

3.6. Vector Potential of N and M Fields

By analogy to vector potential in electrodynamics,

$$\mathbf{B} = \boldsymbol{\nabla}\times\mathbf{A},$$

$$\mathbf{E} = -\frac{\partial\mathbf{A}}{\partial\mathbf{t}} - \nabla\varphi,$$

we postulate

$$\mathbf{N} \equiv \nabla\times\mathbf{P}, \quad (49)$$

$$\mathbf{M} \equiv -\frac{\partial\mathbf{P}}{\partial t}, \quad (50)$$

where **P** is a vector potential.

Substituting Eq. (49) and (50) into Eq. (33), we obtain the **P** potential wave equation,

$$\frac{\partial^2\mathbf{P}}{\partial t^2} - \nabla^2\mathbf{P} = 4\pi Q\dot{\mathbf{v}} - \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) - \mathbf{E}(\nabla\cdot\dot{\mathbf{v}}) + 2(\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} + \mathbf{E}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}). \quad (51)$$

For non-spatially-varying acceleration, $(\mathbf{E}\cdot\boldsymbol{\nabla})\dot{\mathbf{v}} = (\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} = \mathbf{E}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}) = \mathbf{0}$, Eq. (51) becomes,

$$\frac{\partial^2\mathbf{P}}{\partial t^2} - \nabla^2\mathbf{P} = 4\pi Q\dot{\mathbf{v}} - \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}), \quad (52)$$

which implies that acceleration of charges generates the potential **P** wave.

Next let's derive relations between acceleration $\dot{\mathbf{v}}$, **E** field, and **B** field. Substituting Eq. (49) and (50) into Eq. (40), we obtain,

$$-\nabla\times\frac{\partial\mathbf{P}}{\partial t} = -\frac{\partial}{\partial t}(\nabla\times\mathbf{P}) + \mathbf{B}(\nabla\cdot\dot{\mathbf{v}}) - 2(\mathbf{B}\cdot\nabla)\dot{\mathbf{v}} + \nabla(\dot{\mathbf{v}}\cdot\mathbf{B}) + \ddot{\mathbf{v}}\times\mathbf{E} - 4\pi Q\dot{\mathbf{v}}\times\mathbf{v} - \mathbf{B}\times(\nabla\times\dot{\mathbf{v}}.$$

Rearrange this equation; we obtain the relation between $\mathbf{v}$, $\dot{\mathbf{v}}$, $\ddot{\mathbf{v}}$, and **B,**

$$\mathbf{B}(\nabla\cdot\dot{\mathbf{v}}) - 2(\mathbf{B}\cdot\nabla)\dot{\mathbf{v}} + \nabla(\dot{\mathbf{v}}\cdot\mathbf{B}) + \ddot{\mathbf{v}}\times\mathbf{E} - 4\pi Q\dot{\mathbf{v}}\times\mathbf{v} - \mathbf{B}\times(\nabla\times\dot{\mathbf{v}}) = 0. \tag{53}$$

For non-spatially-varying acceleration, $\mathbf{B}(\nabla\cdot\dot{\mathbf{v}}) = (\mathbf{B}\cdot\nabla)\dot{\mathbf{v}} = \mathbf{B}\times(\nabla\times\dot{\mathbf{v}}) = \mathbf{0}$, Eq. (53) becomes

$$\ddot{\mathbf{v}}\times\mathbf{E} + \nabla(\dot{\mathbf{v}}\cdot\mathbf{B}) - 4\pi Q\dot{\mathbf{v}}\times\mathbf{v} = 0. \tag{54}$$

For the case of $\dot{\mathbf{v}}//\mathbf{v}$, Eq. (54) give,

$$\ddot{\mathbf{v}}\times\mathbf{E} + \nabla(\dot{\mathbf{v}}\cdot\mathbf{B}) = 0. \tag{55}$$

For constant acceleration, $\ddot{\mathbf{v}} = \mathbf{0}$, Eq. (55) simplify further,

$$\nabla(\dot{\mathbf{v}}\cdot\mathbf{B}) = 0. \tag{56}$$

We obtain this relation again.

3.7. Relation between vector Potential P and Potentials A and φ

The correlations between potential **P** and potentials **A** and φ are

$$\nabla\times\mathbf{P} = -\dot{\mathbf{v}}\times\left[\nabla\varphi + \frac{\partial\mathbf{A}}{\partial t}\right], \tag{57}$$

$$\frac{\partial\mathbf{P}}{\partial t} = \dot{\mathbf{v}}\times\nabla\times\mathbf{A}. \tag{58}$$

Note, without acceleration, there is no **P** potential.

3.8. Comparison Between Maxwell Equation and the N and M field Equation

By applying mathematic formulas and Maxwell equations, we derived the **N** and **M** field equations for the accelerating charges.

Table 3: Comparision between Maxwell equation and **N** and **M** Field equation

Maxwell Equations $\mathbf{v}\neq 0,\ \dot{\mathbf{v}} = 0$	**N** and **M** Field Equations $\mathbf{v}\neq 0,\ \dot{\mathbf{v}}\neq 0,\ \ddot{\mathbf{v}} = 0$
$\nabla\times\mathbf{B} = 4\pi Q\mathbf{v} + \frac{\partial\mathbf{E}}{\partial t}$	$\nabla\times\mathbf{N} = 4\pi Q\dot{\mathbf{v}} + \frac{\partial\mathbf{M}}{\partial t}$
$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$	$\nabla\times\mathbf{M} = -\frac{\partial\mathbf{N}}{\partial t}$
$\nabla\cdot\mathbf{B} = 0$	$\nabla\cdot\mathbf{N} = 0$
$\nabla\cdot\mathbf{E} = 4\pi Q$	$\nabla\cdot\mathbf{M} = 0$

Those equation leads us to predict that the acceleration of charges generate new **N** and **M** fields. The **N** and **M** field equations are summarizes and compares with Maxwell equations in Table 3.

The combination of Maxwell equations and the **N** and **M** field equations is a complete set of field equations to describe fields generated by accelerating charges.

4. Equation of Motion

In this section, we study effects of the **N** and **M** fields. We postulate that the effects of the **N** and **M** fields are to change the Lorentz force with the time. For demonstration, let's start with Lorentz force,

$$\mathbf{F} = q\mathbf{E} + q\mathbf{u}\times\mathbf{B}. \tag{59}$$

Considering two frames of reference, S and S'. The sources of electric field $\mathbf{E}'$ and magnetic field $\mathbf{B}'$ are in S' frame. An observer and a test charge q are in S frame. The S' frame moves relative to the S frame with velocity **v** and acceleration $\dot{\mathbf{v}}$. The $\mathbf{u}$ is the velocity of the test charge q in the S frame. We assume: (1) speed is slow; (2) at a given instant, the S' frame is approximately an inertial frame. Thus we have transformation at the given instant [5],

$$\mathbf{E}' \approx \mathbf{E} + \mathbf{v}\times\mathbf{B}, \tag{60}$$

$$\mathbf{B}' \approx \mathbf{B} - \mathbf{v}\times\mathbf{E}. \tag{61}$$

The **E** and **B** are measured in the S frame at the given instant. For next instant, the **E** and **B** field are changed due to the accelerating motion, thus the force is changed. From Eq. (58), the force varies with time as

$$\dot{\mathbf{F}} = \mathrm{q}\dot{\mathbf{E}} + \mathrm{q}\frac{\mathrm{d}(\mathbf{u}\times\mathbf{B})}{\mathrm{dt}} = \mathrm{q}\dot{\mathbf{E}} + \mathrm{q}\dot{\mathbf{u}}\times\mathbf{B} + \mathrm{q}\mathbf{u}\times\dot{\mathbf{B}}. \tag{62}$$

Next let's calculate $\dot{\mathbf{E}}$ and $\dot{\mathbf{B}}$, respectively. Taking time derivative of Eq. (60) and (61) respectively, we have,

$$\dot{\mathbf{E}}' = \dot{\mathbf{E}} + \dot{\mathbf{v}}\times\mathbf{B} + \mathbf{v}\times\dot{\mathbf{B}}, \tag{63}$$

$$\dot{\mathbf{B}}' = \dot{\mathbf{B}} - \dot{\mathbf{v}}\times\mathbf{E} - \mathbf{v}\times\dot{\mathbf{E}}. \tag{64}$$

Without losing generality, assume that in the S' frame, the electric and magnetic fields are static, $\dot{\mathbf{E}}' = \dot{\mathbf{B}}' = 0$. Eq. (63) and (64) become,

$$\dot{\mathbf{E}} = -\dot{\mathbf{v}}\times\mathbf{B} - \mathbf{v}\times\dot{\mathbf{B}}, \tag{65}$$

$$\dot{\mathbf{B}} = \dot{\mathbf{v}}\times\mathbf{E} + \mathbf{v}\times\dot{\mathbf{E}}. \tag{66}$$

Representing Eq. (65) and (66) in terms of the **N** and **M** fields, we obtain

$$\dot{\mathbf{E}} = \mathbf{M} - \mathbf{v}\times\dot{\mathbf{B}}, \tag{67}$$

$$\dot{\mathbf{B}} = \mathbf{N} + \mathbf{v}\times\dot{\mathbf{E}}. \tag{68}$$

Substituting Eq. (67 and 68) into Eq. (62), we obtain

$$\dot{\mathbf{F}} = \mathrm{q}\mathbf{M} - \mathrm{q}\mathbf{v}\times\dot{\mathbf{B}} + \mathrm{q}\mathbf{u}\times\mathbf{N} + \mathrm{q}\mathbf{u}\times[\mathbf{v}\times\dot{\mathbf{E}}] + \mathrm{q}\dot{\mathbf{u}}\times\mathbf{B}. \tag{69}$$

Eq. (69) shows the following: first, the effect of the **N** and **M** fields is to change the Lorentz force with time; second, the time-varying of the Lorentz force affects a stationary test charge via $[\mathrm{q}\mathbf{M} - \mathrm{q}\mathbf{v}\times\dot{\mathbf{B}}]$, uniform moving test charge via $[\mathrm{q}\mathbf{u}\times\mathbf{N} + \mathrm{q}\mathbf{u}\times[\mathbf{v}\times\dot{\mathbf{E}}]]$.

Note the acceleration of a test charge is also contributes to the time varying of the Lorentz force via $[\mathrm{q}\dot{\mathbf{u}}\times\mathbf{B}]$.

At a given instant, the non-uniform motions of a source charge can be considered as uniform motions, and generate regular electromagnetic fields. The acceleration of the source charge generates new **N** and **M** fields. At the same instant, a non-uniformly moving test charge can be considered as a uniformly moving charge and under the influence of the electromagnetic fields and **N** and **M** fields, which are govern by Lorentz force law and Eq. (68). The acceleration of the test charge also reacts with magnetic field. Eq. (68) is a supplement formula to the Lorentz force Eq. (58). We need both Eq. (58) and Eq. (68) to describe the motion of an accelerating test charge under the influence of the **E, B, N,** and **M** fields generated by accelerating source charges.

There is correspondence between Eq. (59) and Eq. (69) (Table 4):

Table 4: Comparison of Effects of **E, B, M,** and **N** Fields

Electromagnetic fields: **E, B**	**N** and **M** fields
Force on test charge $\mathbf{F} = q\mathbf{E} + q\mathbf{u}\times\mathbf{B}$	Time-varying of force on test charge $\dot{\mathbf{F}} = q\mathbf{M} + q\mathbf{u}\times\mathbf{N} - q\mathbf{v}\times\dot{\mathbf{B}} + q\dot{\mathbf{u}}\times\mathbf{B} + q\mathbf{u}\times[\mathbf{v}\times\dot{\mathbf{E}}]$

5. Wave Equations of N and M Fields

We postulate that the **N** and **M** fields propagate as wave. Let's derive wave equations of the new **N** and **M** fields.

5.1. Wave Equation of N field

For simplicity and without loss of generality, we start with Eq. (31). Taking curl of Eq. (31),

$$\nabla\times\nabla\times\mathbf{N} = 4\pi Q\boldsymbol{\nabla}\times\dot{\mathbf{v}} + \frac{\partial\nabla\times\mathbf{M}}{\partial t} + \boldsymbol{\nabla}\times(\ddot{\mathbf{v}}\times\mathbf{B}) - \boldsymbol{\nabla}\times\boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) - \boldsymbol{\nabla}\times[\mathbf{E}(\nabla\cdot\dot{\mathbf{v}})]$$
$$+2\boldsymbol{\nabla}\times[(\mathbf{E}\cdot\nabla)\dot{\mathbf{v}}] + \nabla\times[\mathbf{E}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}})], \tag{70}$$

and substituting Eq. (40) into Eq. (70), we obtain

$$\nabla^2\mathbf{N} - \frac{\partial^2\mathbf{N}}{\partial t^2} = -4\pi Q\boldsymbol{\nabla}\times\dot{\mathbf{v}} - \boldsymbol{\nabla}\times(\ddot{\mathbf{v}}\times\mathbf{B}) + \boldsymbol{\nabla}\times\boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) + \boldsymbol{\nabla}\times[\mathbf{E}(\nabla\cdot\dot{\mathbf{v}})]$$
$$-2\boldsymbol{\nabla}\times[(\mathbf{E}\cdot\nabla)\dot{\mathbf{v}}] - \nabla\times[\mathbf{E}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}})]$$
$$-\frac{\partial}{\partial t}\{\mathbf{B}(\nabla\cdot\dot{\mathbf{v}}) - 2(\mathbf{B}\cdot\nabla)\dot{\mathbf{v}} + \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B}) + \ddot{\mathbf{v}}\times\mathbf{E} - 4\pi Q\dot{\mathbf{v}}\times\mathbf{v} - \mathbf{B}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}})\}. \tag{71}$$

which implies that the $\dot{\mathbf{v}}$, $\ddot{\mathbf{v}}$ and $\dddot{\mathbf{v}}$ generate the **N** wave. As a comparison, the $\nabla\times\mathbf{v}$ is the source of **B** wave,

$$\nabla^2\mathbf{B} - \frac{\partial^2\mathbf{B}}{\partial t^2} = 4\pi Q\nabla\times\mathbf{v}.$$

For non-spatially-varying constant acceleration,

$$\boldsymbol{\nabla}\times\dot{\mathbf{v}} = \ddot{\mathbf{v}} = (\nabla\cdot\dot{\mathbf{v}}) = (\mathbf{B}\cdot\boldsymbol{\nabla})\dot{\mathbf{v}} = (\boldsymbol{\nabla}\times\dot{\mathbf{v}}) = \mathbf{0},\ \ \boldsymbol{\nabla}\times\boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) = \mathbf{0},$$

Eq. (71) becomes,

$$\nabla^2\mathbf{N} - \frac{\partial^2\mathbf{N}}{\partial t^2} = -\frac{\partial}{\partial t}[\boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B}) - 4\pi Q\dot{\mathbf{v}}\times\mathbf{v}]. \tag{72}$$

For $\dot{\mathbf{v}}//\mathbf{v}$, Eq. (72) is simplified further as,

$$\nabla^2\mathbf{N} - \frac{\partial^2\mathbf{N}}{\partial t^2} = \mathbf{0}. \tag{73}$$

Eq. (71), (72), and (73) are the wave equations of the **N** field.

5.2. Wave Equation of M field

Let's take curl of Eq. (40),

$$\nabla\times\nabla\times\mathbf{M} = -\frac{\partial\nabla\times\mathbf{N}}{\partial t} + \nabla\times[\mathbf{B}(\nabla\cdot\dot{\mathbf{v}})] - 2\nabla\times[(\mathbf{B}\cdot\nabla)\dot{\mathbf{v}}] + \nabla\times(\ddot{\mathbf{v}}\times\mathbf{E})$$
$$-4\pi Q\nabla\times[\dot{\mathbf{v}}\times\mathbf{v}] - \nabla\times[\mathbf{B}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}})]. \tag{74}$$

Where $\nabla\times\boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B}) = 0$ have be used. Substituting Eq. (31) into Eq. (74), we obtain,

$$\nabla\times\nabla\times\mathbf{M} = -\frac{\partial}{\partial t}\left\{4\pi Q\dot{\mathbf{v}} + \frac{\partial\mathbf{M}}{\partial t} + \ddot{\mathbf{v}}\times\mathbf{B} - \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) - \mathbf{E}(\nabla\cdot\dot{\mathbf{v}}) + 2(\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} + \mathbf{E}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}})\right\}$$
$$+\nabla\times[\mathbf{B}(\nabla\cdot\dot{\mathbf{v}})] - 2\nabla\times[(\mathbf{B}\cdot\nabla)\dot{\mathbf{v}}] + \nabla\times(\ddot{\mathbf{v}}\times\mathbf{E}) - 4\pi Q\nabla\times[\dot{\mathbf{v}}\times\mathbf{v}] - \nabla\times[\mathbf{B}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}})].$$

Or rearrange it as

$$\nabla^2\mathbf{M} - \frac{\partial^2\mathbf{M}}{\partial t^2} = 4\pi Q\ddot{\mathbf{v}} + \frac{\partial}{\partial t}\{\ddot{\mathbf{v}}\times\mathbf{B} - \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) - \mathbf{E}(\nabla\cdot\dot{\mathbf{v}}) + 2(\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} + \mathbf{E}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}})\} - \nabla\times[\mathbf{B}(\nabla\cdot\dot{\mathbf{v}})]$$
$$+2\nabla\times[(\mathbf{B}\cdot\nabla)\dot{\mathbf{v}}] - \nabla\times(\ddot{\mathbf{v}}\times\mathbf{E}) + 4\pi Q\nabla\times[\dot{\mathbf{v}}\times\mathbf{v}] + \nabla\times[\mathbf{B}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}})]. \tag{75}$$

Which is the **M** wave equation and implies that the $\dot{\mathbf{v}}$, $\ddot{\mathbf{v}}$ and $\dddot{\mathbf{v}}$ are the sour of the **M** wave. As a comparison, the acceleration of charge is the source of **E** wave,

$$\nabla^2\mathbf{E} - \frac{\partial^2\mathbf{E}}{\partial t^2} = 4\pi Q\dot{\mathbf{v}}.$$

For non-spatially-varying acceleration,

$$\boldsymbol{\nabla}\times\dot{\mathbf{v}} = (\nabla\cdot\dot{\mathbf{v}}) = (\mathbf{E}\cdot\boldsymbol{\nabla})\dot{\mathbf{v}} = (\boldsymbol{\nabla}\times\dot{\mathbf{v}}) = 0, \text{ and } \nabla\times\nabla(\dot{\mathbf{v}}\cdot\mathbf{E}) = \mathbf{0},$$

Eq. (75) becomes,

$$\nabla^2\mathbf{M} - \frac{\partial^2\mathbf{M}}{\partial t^2} = 4\pi Q\ddot{\mathbf{v}} + \frac{\partial}{\partial t}\{\ddot{\mathbf{v}}\times\mathbf{B} - \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E})\} - \nabla\times(\ddot{\mathbf{v}}\times\mathbf{E}) + 4\pi Q\nabla\times[\dot{\mathbf{v}}\times\mathbf{v}]. \quad (76)$$

For $\dot{\mathbf{v}}//\mathbf{v}$ and $\dddot{\mathbf{v}} = \mathbf{0}$, Eq. (76) is simplified further as,

$$\nabla^2\mathbf{M} - \frac{\partial^2\mathbf{M}}{\partial t^2} = 4\pi Q\ddot{\mathbf{v}} + \ddot{\mathbf{v}}\times\dot{\mathbf{B}} - \nabla\times(\ddot{\mathbf{v}}\times\mathbf{E}). \quad (77)$$

Table 5: Comparision between Wave Equations of **E, B, N, M** fields

E Wave $\mathbf{v}\neq 0,\ \dot{\mathbf{v}}\neq 0$	$\nabla^2\mathbf{E} - \frac{\partial^2\mathbf{E}}{\partial t^2} = 4\pi Q\dot{\mathbf{v}}$
M Wave $\mathbf{v}\neq 0,\ \dot{\mathbf{v}}\neq 0,\ \ddot{\mathbf{v}}\neq 0$	$\nabla^2\mathbf{M} - \frac{\partial^2\mathbf{M}}{\partial t^2} = 4\pi Q\ddot{\mathbf{v}} + \ddot{\mathbf{v}}\times\dot{\mathbf{B}} - \nabla\times(\ddot{\mathbf{v}}\times\mathbf{E})$
B Wave $\mathbf{v}\neq 0,\ \dot{\mathbf{v}} = 0$	$\nabla^2\mathbf{B} - \frac{\partial^2\mathbf{B}}{\partial t^2} = -4\pi Q\nabla\times\mathbf{v}$
N Wave $\mathbf{v}\neq 0,\ \dot{\mathbf{v}}\neq 0,\ \ddot{\mathbf{v}}\neq 0$	$\nabla^2\mathbf{N} - \frac{\partial^2\mathbf{N}}{\partial t^2} = -4\pi Q\boldsymbol{\nabla}\times\dot{\mathbf{v}} - \ddot{\mathbf{v}}\times\dot{\mathbf{E}} - \nabla\times(\ddot{\mathbf{v}}\times\mathbf{B})$

6. Summary and Discussion

We postulate and demonstrate mathematically that the acceleration of an electric charge generates new **N** and **M** fields. The new **N** and **M** field equations have the similar forms as that of Maxwell equations, and are supplement to Maxwell equations. The combination of Maxwell equations and the **N** and **M** field equations is a complete set of equations to describe fields generated by accelerating charges. The new **N** and **M** fields vary the Lorentz force with time. The new **N** and **M** fields propagate as waves.

The **N** and **M** fields due to accelerating charges exist mathematically.

We suggest that the new **N** and **M** fields are worth to be systematically studied experimentally, and might open a new window.

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Appendix (A): Summary

Maxwell equation and **N** and **M** Field equation

Maxwell Equation $\mathbf{v} \neq 0,\ \dot{\mathbf{v}} = 0$	**N** and **M** Field Equation $\mathbf{v} \neq 0,\ \dot{\mathbf{v}} \neq 0,\ \ddot{\mathbf{v}} \neq 0$
$\mathbf{E} = -\dot{\mathbf{A}} - \nabla\varphi$	$\mathbf{M} = -\dot{\mathbf{P}}, \quad \mathbf{M} \equiv -\dot{\mathbf{v}}\times\mathbf{B}$
$\mathbf{B} = \nabla\times\mathbf{A}$	$\mathbf{N} = \nabla\times\mathbf{P}, \quad \mathbf{N} \equiv \dot{\mathbf{v}}\times\mathbf{E}$
Relation between: **P, A**, φ, and $\dot{\mathbf{v}}$: $\nabla\times\mathbf{P} = -\dot{\mathbf{v}}\times\left[\nabla\varphi + \frac{\partial \mathbf{A}}{\partial t}\right], \qquad \frac{\partial \mathbf{P}}{\partial t} = \dot{\mathbf{v}}\times\nabla\times\mathbf{A}.$	
$\frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla^2\mathbf{A} = 4\pi Q\mathbf{v}$	$\frac{\partial^2 \mathbf{P}}{\partial t^2} - \nabla^2\mathbf{P} = 4\pi Q\dot{\mathbf{v}} - \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E})$
$\frac{\partial^2 \varphi}{\partial t^2} - \nabla^2\varphi = 4\pi Q$	
$\nabla\cdot\mathbf{E} = 4\pi Q$	$\nabla\cdot\mathbf{M} = 0$
$\nabla\times\mathbf{B} = 4\pi Q\mathbf{v} + \dot{\mathbf{E}}$	$\nabla\times\mathbf{N} = 4\pi Q\dot{\mathbf{v}} + \dot{\mathbf{M}} - \nabla(\dot{\mathbf{v}}\cdot\mathbf{E}) + \ddot{\mathbf{v}}\times\mathbf{B}$
$\nabla\times\mathbf{E} = -\dot{\mathbf{B}}$	$\nabla\times\mathbf{M} = -\dot{\mathbf{N}} + \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B}) + \ddot{\mathbf{v}}\times\mathbf{E}$
$\nabla\cdot\mathbf{B} = 0$	$\nabla\cdot\mathbf{N} = 0$
$\nabla^2\mathbf{E} - \frac{\partial^2 \mathbf{E}}{\partial t^2} = 4\pi Q\dot{\mathbf{v}}$	$\nabla^2\mathbf{M} - \frac{\partial^2 \mathbf{M}}{\partial t^2} = 4\pi Q\ddot{\mathbf{v}} + \ddot{\mathbf{v}}\times\dot{\mathbf{B}} - \nabla\times(\ddot{\mathbf{v}}\times\mathbf{E})$
$\nabla^2\mathbf{B} - \frac{\partial^2 \mathbf{B}}{\partial t^2} = -4\pi Q\nabla\times\mathbf{v}$	$\nabla^2\mathbf{N} - \frac{\partial^2 \mathbf{N}}{\partial t^2} = -4\pi Q\boldsymbol{\nabla}\times\dot{\mathbf{v}} - \ddot{\mathbf{v}}\times\dot{\mathbf{E}} - \nabla\times(\ddot{\mathbf{v}}\times\mathbf{B})$
$\mathbf{F} = q\mathbf{E} + q\mathbf{u}\times\mathbf{B}$	$\dot{\mathbf{F}} = q\mathbf{M} + q\mathbf{u}\times\mathbf{N} - q\mathbf{v}\times\dot{\mathbf{B}} + q\dot{\mathbf{u}}\times\mathbf{B} + q\mathbf{u}\times[\mathbf{v}\times\dot{\mathbf{E}}]$

Appendix (B): Re-derive Maxwell Equations

1. Purpose

The purpose of Appendix (B) is to demonstrate the validity of the approach used in this article by using the same approach to re-derive Maxwell equations [7].

2. Mathematic Preparation

We will utilize the vector analysis formulas used in this article and Gauss/Coulomb's law to re-derive Maxwell equations.

Gauss/Coulomb's law relates the divergences of a vector electric field to a charge,

$$\nabla \cdot \mathbf{E} = 4\pi Q. \tag{B1}$$

The vector analysis formulas we will utilize are shown below,

$$\nabla \times (\mathbf{S} \times \mathbf{T}) = \mathbf{S}(\nabla \cdot \mathbf{T}) - \mathbf{T}(\nabla \cdot \mathbf{S}) + (\mathbf{T} \cdot \nabla)\mathbf{S} - (\mathbf{S} \cdot \nabla)\mathbf{T}, \tag{B2}$$

$$\boldsymbol{\nabla}(\mathbf{S} \cdot \mathbf{T}) = (\mathbf{S} \cdot \boldsymbol{\nabla})\mathbf{T} + (\mathbf{T} \cdot \boldsymbol{\nabla})\mathbf{S} + \mathbf{S} \times (\boldsymbol{\nabla} \times \mathbf{T}) + \mathbf{T} \times (\boldsymbol{\nabla} \times \mathbf{S}), \tag{B3}$$

$$\mathbf{S} \cdot (\mathbf{T} \times \mathbf{Z}) = \mathbf{T} \cdot (\mathbf{Z} \times \mathbf{S}) = \mathbf{Z} \cdot (\mathbf{S} \times \mathbf{T}). \tag{B4}$$

where **S**, **T** and **Z** are arbitrary vectors.

Eq. (B2) is the most noteworthy formula because that it indicates that the combination of gradient and divergence operations of two arbitrary vectors generates inevitably an axial vector. Specifically, there are two divergence terms, $(\nabla \cdot \mathbf{T})$ and $(\nabla \cdot \mathbf{S})$, which definitely generate an axial field $\nabla \times (\mathbf{S} \times \mathbf{T})$. The Coulomb's law is a divergence law describing charges and its field. The basic concept of this article is that the combination of the Coulomb's law, the uniform motion of charges, and Eq. (B2) must generate an axial field, magnetic field.

We only consider the situations of charges moving with non-spatially-varying and uniform velocity $\mathbf{v}$, thus, $\dot{\mathbf{v}} = 0$, $\nabla \cdot \mathbf{v} = (\mathbf{X} \cdot \nabla)\mathbf{v} = \boldsymbol{\nabla} \times \mathbf{v} = 0$, where **X** represents either **S** or **T** vector. Therefore, let $\mathbf{S} = \mathbf{v}$, Then Eq. (B2) and (B3) become,

$$\nabla \times (\mathbf{v} \times \mathbf{T}) = \mathbf{v}(\nabla \cdot \mathbf{T}) - (\mathbf{v} \cdot \nabla)\mathbf{T}, \tag{B5}$$

$$\boldsymbol{\nabla}(\mathbf{v} \cdot \mathbf{T}) = (\mathbf{v} \cdot \boldsymbol{\nabla})\mathbf{T} + \mathbf{v} \times (\boldsymbol{\nabla} \times \mathbf{T}). \tag{B6}$$

Let $\mathbf{S} = \mathbf{T} = \mathbf{v}$, Eq. (B4) gives

$$\mathbf{v} \cdot (\mathbf{v} \times \mathbf{Z}) = \mathbf{Z} \cdot (\mathbf{v} \times \mathbf{v}) = 0. \tag{B7}$$

3. Magnetic Field induced Inevitably by Velocity of Charges

Let's apply this approach to re-derive Ampere's law mathematically. The combination of the vector analysis formula and Coulomb's law leads us to let $\mathbf{T} = \mathbf{E}$. Then Eq. (B5) gives

$$\nabla \times (\mathbf{v} \times \mathbf{E}) = \mathbf{v}(\nabla \cdot \mathbf{E}) - (\mathbf{v} \cdot \nabla)\mathbf{E}. \tag{B8}$$

Define a field, **Y,**

$$\mathbf{Y} \equiv \mathbf{v} \times \mathbf{E}, \tag{B9}$$

Combining Eq. (B8) and (B9), we obtain,

$$\nabla \times \mathbf{Y} = 4\pi Q\mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{E}, \tag{B10}$$

where Coulomb's law has been applied. Eq. (B10) shows that the velocity $\mathbf{v}$ of a charge indeed mathematically generates inevitably an axial field **Y**. For an axial field, we have mathematically,

$$\nabla \cdot \mathbf{Y} = 0. \tag{B11}$$

The Ampere's law is,

$$\nabla \times \mathbf{B} = 4\pi Q\mathbf{v}. \tag{B12}$$

Comparing Eq. (B10) and (B12), we assume that the same current, $4\pi Q\mathbf{v}$, should generates the same fields. Without the term, $(\mathbf{v} \cdot \nabla)\mathbf{E}$, Eq. (B10) is identical to the Ampere's law. Therefore, we identify the axial field **Y** as magnetic field **B**. Eq. (B10) and (B11) can be rewritten as,

$$\nabla\times\mathbf{B} = 4\pi Q\mathbf{v} - (\mathbf{v}\cdot\nabla)\mathbf{E}, \tag{B13}$$

$$\nabla\cdot\mathbf{B} = 0. \tag{B14}$$

We will discuss the term, $(\mathbf{v}\cdot\nabla)\mathbf{E}$, later.

In the derivation above, the only physics law we applied is the Coulomb's law, which indicates that, for a charge satisfying Coulomb's law, the uniform motion of the charge inevitable generates an axial magnetic **B** field. The generation of the magnetic **B** field is predetermined by the wonderful mathematical formula Eq. (B2 and B5). Namely, the combination of gradient and divergence operations of a vector electric field and vector velocity of charges must generate an axial vector magnetic field.

4. Faraday's Law

Based on symmetry, it is nature to expect that the velocity of charges generate an axial electric field as well. For this aim, let's apply the formula, Eq. (B5), again, and let $\mathbf{T} = \mathbf{B},$ where $\mathbf{B}$ is magnetic field derived in Section 3, we obtain,

$$\nabla\times(\mathbf{v}\times\mathbf{B}) = -(\mathbf{v}\cdot\nabla)\mathbf{B}. \tag{B15}$$

Define an axial field, **Z,**

$$\mathbf{Z} \equiv -\mathbf{v}\times\mathbf{B}. \tag{B16}$$

Note the **Z** field is an axial field related with velocity **v** and magnetic field **B**. Substituting Eq. (B16), into Eq. (B15), we obtain,

$$\nabla\times\mathbf{Z} = (\mathbf{v}\cdot\nabla)\mathbf{B}. \tag{B17}$$

The magnetic field **B** and the velocity of a charge indeed generates an axial field **Z**. Eq. (B17) relate the **Z** field, velocity **v**, and magnetic field **B.**

We introduce vector potential **A** defined below,

$$\mathbf{B} = \nabla\times\mathbf{A}, \tag{B18}$$

$$\mathbf{Z} = -\frac{\partial\mathbf{A}}{\partial t}. \tag{B 19}$$

Taking time derivative of Eq. (B 18) or taking curl of Eq. (B 19), we obtain,

$$\frac{\partial\mathbf{B}}{\partial\mathbf{t}} = \nabla\times\frac{\partial\mathbf{A}}{\partial t} = -\nabla\times\mathbf{Z},$$

or

$$\nabla\times\mathbf{Z} = -\frac{\partial\nabla\times\mathbf{A}}{\partial\mathbf{t}} = -\frac{\partial\mathbf{B}}{\partial t}. \tag{B20}$$

Comparing Eq. (B20) with the Faraday's law,

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t},$$

and assuming that the term $\frac{\partial\mathbf{B}}{\partial\mathbf{t}}$ will generate the same axial field, we identify the axial vector field **Z** as the axial electric field **E**. Eq. (B20) is identical to the Faraday's law.

We re-derive the Faraday's law mathematically.

5. Interpretation of axial B and Axial E Fields

Consider a charge Q and an observer. When the charge is at rest relative to the observer, it generates a vector **E** field; when the charge is moving with a constant velocity relative to the observer, beside the vector **E** field, the observer also experiences a axial **B** and axial **E** fields generated by the velocity of the charge, $\mathbf{B} = \mathbf{v}\times\mathbf{E}$, and $\mathbf{E} = -\mathbf{v}\times\mathbf{B}.$ Namely, the constant velocity of a charge generates two axial fields (Table B1).

Table B1: Fields generated by stationary charge and the motion of charges

	Generated Field
Stationary Charge: $\mathbf{v} = \mathbf{0}$	$\mathbf{E}$
Uniform motion of charge: $\mathbf{v} \neq \mathbf{0},\ \dot{\mathbf{v}} = \mathbf{0}$	$\mathbf{B} = \mathbf{v}\times\mathbf{E}$ $\mathbf{E} = -\mathbf{v}\times\mathbf{B}$

6. Ampere-Maxwell law

Now let's study the term, $(\mathbf{v}\cdot\nabla)\mathbf{E}$, of Eq. (B13). Applying the vector analysis formula, Eq. (B6). Let $\mathbf{T} = \mathbf{E}$, Eq. (B6) gives

$$\boldsymbol{\nabla}(\mathbf{v}\cdot\mathbf{E}) = (\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{E} + \mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{E}).$$

Or rewrite as

$$(\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{E} = \boldsymbol{\nabla}(\mathbf{v}\cdot\mathbf{E}) - \mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{E}). \tag{B21}$$

Substituting Eq. (B21) into Eq. (B10), we obtain

$$\nabla\times\mathbf{Y} = 4\pi Q\mathbf{v} - \boldsymbol{\nabla}(\mathbf{v}\cdot\mathbf{E}) + \mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{E}). \tag{B22}$$

Next let's find $\mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{E})$. For this aim, utilizing Eq. (B20) and (B16) with $\mathbf{Z} = \mathbf{E}$, we obtain

$$\mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{E}) = -\mathbf{v}\times\frac{\partial\mathbf{B}}{\partial t} = -\frac{\partial\mathbf{v}\times\mathbf{B}}{\partial t} = \frac{\partial\mathbf{E}}{\partial t}. \tag{B23}$$

Now calculate the term $\boldsymbol{\nabla}(\mathbf{v}\cdot\mathbf{E})$. From Eq. (B7) and (B16) with $\mathbf{Z} = \mathbf{E}$, we have

$$\mathbf{v}\cdot\mathbf{E} = -\mathbf{v}\cdot(\mathbf{v}\times\mathbf{B}) = 0. \tag{B24}$$

Then, substituting Eq. (B23 and B24) into Eq. (B22), we obtain,

$$\nabla\times\mathbf{Y} = 4\pi Q\mathbf{v} + \frac{\partial\mathbf{E}}{\partial t}. \tag{B25}$$

Comparing Eq. (B25) with the Ampere-Maxwell's equation,

$$\nabla\times\mathbf{B} = 4\pi Q\mathbf{v} + \frac{\partial\mathbf{E}}{\partial t}$$

and assuming that the term, $4\pi Q\mathbf{v} + \frac{\partial\mathbf{E}}{\partial t}$, will generate the same axial vector field, thus we identify the axial field $\mathbf{Y}$ as the magnetic field $\mathbf{B}$ again. We derive mathematically the Ampere-Maxwell's law.

7. Summary

Based on the Gauss/Coulomb's law, for a uniformly moving charge, we derive mathematically the rest of Maxwell equations,

Gauss/Coulomb' law: $\nabla\cdot\mathbf{E} = 4\pi Q$

Ampere-Maxwell's law: $\nabla\times\mathbf{B} = 4\pi Q\mathbf{v} + \frac{\partial\mathbf{E}}{\partial t}$. (B25)

Faraday's law: $\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial\mathbf{t}}$. (B20)

Gauss's law for B field: $\nabla\cdot\mathbf{B} = 0$. (B11)

The conclusion is that the uniform motion of a charge must generate an axial vector magnetic field and an axial vector electric field. Eq. (B1) and (B2) predetermine this fact mathematically. This fact demonstrates once more the famous sentence that "The world is built mathematically". The experiments-based Maxwell equations have their mathematical origin.

Therefore, the approach is valid for derive Maxwell-type equations for non-uniformly moving electric shrges