The First Cosmic Velocity and The Coriolis Force

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Abstract

It was shown that the value of the first cosmic velocity of a satellite moving in a circular orbit in the equatorial plane of the rotating Earth is smaller in the easterly direction than in the westerly direction. This effect is caused by inertial forces – centrifugal and Coriolis.

Keywords: first cosmic velocity, centrifugal inertial force, Coriolis force

1. Introduction

In the following, we will show that the value of the first cosmic velocity of a satellite moving in a circular orbit in the equatorial plane of the rotating Earth is smaller in the easterly direction than in the westerly direction. This effect is caused by inertial forces – centrifugal and Coriolis.

2. The first cosmic velocity for a circular orbit in the case of a non-rotating planet

The first cosmic velocity (**v**$_I$) is the velocity that should be given to the particle of mass (m), to move in a uniform motion along a circle with a radius $r \geq R$ in the field of a point source or a homogeneous sphere with mass (M) and radius (R). The first cosmic velocity is perpendicular to the position vector with the beginning in the center of the source.

In the case of a non-rotating planet, gravitational force acts as a centripetal force that maintains a satellite moving in a circular orbit.

$$F_{gravit} = F_{centripetal}$$

$$F_{gravit} = -\frac{GMm}{r^2} \quad \text{– radial vector coordinate of gravitational force}$$

$$F_{centripetal} = -\frac{mv_I^2}{|r|} \quad \text{– radial vector coordinate of centripetal force}$$

$$\boxed{v_I = \sqrt{\frac{GM}{|r|}}}$$

3. Non-inertial reference systems

By definition, in the inertial system the particle either remains at rest or continues to move at a constant velocity if and only if the sum of external forces acting on it is equal to zero. Systems that do not meet this condition are called non-inertial systems. They are systems moving relative to the inertial system with the translatory (accelerated or retarded), vibrational, rotary, curvilinear motion, etc. In such systems appear apparent forces, called the inertial forces. In order to be able to apply the second principle of dynamics also in non-inertial systems, among the forces acting on a particle, one should also take into account the inertial forces.

According to the observer associated with the non-inertial system, which is a rotating planet, the centripetal force that maintains the satellite moving in a circular orbit in the equatorial plane is the sum of gravitational force and inertial forces – centrifugal and Coriolis.

4. Centrifugal inertial force

In the equatorial plane of a rotating planet with angular velocity ($\boldsymbol{\omega}$) the value of centrifugal inertial force ($|\mathbf{F}_{centrifugal}|$) acting on a particle of mass (m) moving on a circular orbit with radius (r) is the largest and amounts to:

$$\left|\mathbf{F}_{centrifugal}\right| = m\omega^2 |r|$$

This force is then directed radially from the center of the field source regardless of the direction of the angular velocity. Radial coordinate of centrifugal inertial force is positive.

$$F_{centrifugal} = +|\mathbf{F}_{centrifugal}|$$

5. Coriolis force

On a particle of mass (m), moving at a linear velocity ($\mathbf{v}$) in a rotating system at angular velocity ($\boldsymbol{\omega}$) acts the Coriolis force ($\mathbf{F}_C$).

$$\mathbf{F}_C = 2m\mathbf{v} \times \boldsymbol{\omega}$$

For simplicity, we will limit ourselves to the situation when a particle moves on a circular orbit in the equatorial plane of the rotating planet.

When the particle moves in the direction of the planet's rotation, the angle between the vectors of the linear velocity of the particle ($\mathbf{v}$) and the angular velocity of the planet ($\boldsymbol{\omega}$) is $+90°$. Coriolis force ($\mathbf{F}_C$) is then directed radially from the center of the field source. The radial coordinate of vector ($\mathbf{F}_C$) is positive in this case.

$$F_C = +2m|v||\omega|$$

When the particle moves in the opposite direction to the planet's rotation, the angle between the vectors of the linear velocity of the particle ($\mathbf{v}$) and the angular velocity of the planet ($\boldsymbol{\omega}$) is $-90°$. Coriolis force ($\mathbf{F}_C$) is then directed radially to the center of the field source. The radial coordinate of vector ($\mathbf{F}_C$) is negative in this case.

$$F_C = -2m|v||\omega|$$

6. The first cosmic velocity for a circular orbit in the case of the rotating planet

In the case of the rotating planet, the centripetal force, which maintains the satellite moving in a circular orbit in the equatorial plane, is the sum of gravitational force and inertial forces – centrifugal and Coriolis.

$$\mathbf{F}_{\text{centripetal}} = \mathbf{F}_{\text{gravit}} + \mathbf{F}_{\text{centrifugal}} + \mathbf{F}_{\text{C}}$$

$F_{\text{centripetal}} = -\dfrac{mv^2}{|r|}$ – radial vector coordinate of centripetal force

$F_{\text{gravit}} = -\dfrac{GMm}{r^2}$ – radial vector coordinate of gravtational force

$F_{\text{cetrifugal}} = m\omega^2|r|$ – radial vector coordinate of centrifugal inertial force

$F_{\text{C}} = \pm 2m|v||\omega|$ – radial vector coordinates of Coriolis force

$\mathbf{F}_{\text{C}} = 2m(\mathbf{v}\times\boldsymbol{\omega})$

$$\boxed{\begin{aligned} &\frac{1}{r}|v|^2 + 2|\omega||v| + \omega^2|r| - \frac{GM}{r^2} = 0, \qquad v = +|v| > 0 \\ &\frac{1}{r}|v|^2 - 2|\omega||v| + \omega^2|r| - \frac{GM}{r^2} = 0, \qquad v = -|v| < 0 \end{aligned}}$$

Among the four solutions of the above quadratic equations for | v |, only two solutions are physical.

$$|v_1| = -|\omega||r| + \sqrt{\frac{GM}{|r|}}, \qquad v_1 = +|v_1| > 0$$

$$|v_2| = +|\omega||r| + \sqrt{\frac{GM}{|r|}}, \qquad v_2 = -|v_2| < 0$$

EXAMPLE

Sample calculations will be made for a satellite moving just above the surface of the Earth.

$$\boxed{\begin{aligned} &\omega = 2\pi\frac{\text{rad}}{24\,\text{hours}} = 7.3\cdot 10^{-5}\,\frac{\text{rad}}{\text{s}} \\ &r = 6.4\cdot 10^{6}\,\text{m} \\ &M = 6\cdot 10^{24}\,\text{kg} \\ &G = 6.67\cdot 10^{-11}\,\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2} \end{aligned}}$$

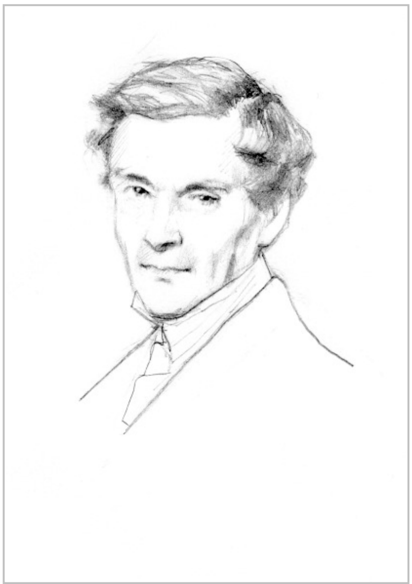
$$\omega r = 467.2\,\frac{\text{m}}{\text{s}} \approx 0.5\cdot 10^3\,\frac{\text{m}}{\text{s}}, \qquad \sqrt{\frac{GM}{r}} = 7.9\cdot 10^3\,\frac{\text{m}}{\text{s}}$$

$$v_1 = +7.4\cdot 10^3\,\frac{\text{m}}{\text{s}}$$

$$v_2 = -8.4\cdot 10^3\,\frac{\text{m}}{\text{s}}$$

The velocity $\mathbf{v}_1$ should be given to the satellite in an easterly direction, Coriolis force is then directed radially from the center of the field source. The velocity $\mathbf{v}_2$ should be given to the satellite in a westerly direction, the Coriolis force is then directed radially towards center of the field source.
The time of returning to the same point of the Earth, when the satellite moves in the direction of the earth's rotation, is greater than in the opposite direction.

7. Coriolis



Gaspard Gustave de Coriolis (1792-1843)
French physicist and engineer

1792 – He was born on May 21 in Paris.
1808 – He began his studies at the École Polytechnique, which he continued in the École des Ponts et Chaussées.
1816 – He became a professor at the École Polytechnique.
1836 – He was elected a member of the Paris Academy of Sciences.
1843 – He died on September 19 in Paris.

Coriolis is known primarily from the fact that:
• Introduced (1829) definition of work and kinetic energy (force vive) [1].
• Discovered (1835) a new inertial force, which allowed him to formulate equations of motion in a rotating system [2].

8. Final remarks

Two different values of the first cosmic velocity can also be obtained [3], analyzing, as part of general relativity, the free fall of a test particle on the rotating planet. If we assume that the radius of the planet is much larger than the Schwarzschild radius, and the speed of the satellite and the product of the angular speed and the radius of the planet are much smaller than the speed of light, it achieved the same result as in the classical calculus [4].

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References

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