Electrodynamics for Non-uniformly Moving Electric Charge

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Abstract

In this article, we postulate and demonstrate mathematically that the acceleration of an electric charge generates new **N** and **M** fields. The equations of the **N** and **M** fields obtained are derived from pure mathematic formulas and Maxwell equations. Therefore, the new fields should exist and need to be tested experimentally. The new field equations have the similar forms as that of Maxwell equations, and are supplement to Maxwell equations. The effects of new fields are to vary the Lorentz force with time. The new fields propagate as waves. We suggest that those new fields are worth to be systematically studied experimentally. The study of those new fields might open a new window.

Key words: Maxwell equations, Lorentz force, electrodynamics, non-uniform moving electric charge, wave, electromagnetic experiment,

1. Introduction

Physics laws describing phenomena that relate with uniform motion include:

(1) Maxwell equations govern electromagnetic fields generated by uniform motion of charges.

(2) Special Relativity postulates that physics laws are invariant in all inertial frames of reference.

(3) de Broglie wavelength $\lambda = {}^{h}/_{p}$ of uniformly moving particles.

In the 1998, scientists reported a revolutionary discovery that the expansion of the universe is accelerating [1], which implies that everything in the universe is accelerating.

A task is: modify physics laws to describe accelerating motion related phenomena, namely, to include the terms of acceleration in physics laws. Recently Hubble's law and Doppler's law have been extended to contain terms of acceleration for describing phenomena caused by acceleration [2]. Indeed the acceleration does cause significant differences in those phenomena.

Galileo transformation and Lorentz transformation are between inertial frames with low and high velocity respectively. For accelerating frames, both transformations no long hold. There is no appropriate transformation either between an inertial frame and a non-inertial frame or between non-inertial systems.

At quantum level, the phenomenon of wavefunction collapse is explained as due to the acceleration of particle [3].

In this article, we study what effect/field will the acceleration of an electric charge induce. We demonstrate mathematically: (1) the acceleration of a charge generates new fields; (2) those new fields affect stationary, uniformly moving, and accelerating test charges, differently; (3) new fields generated by the acceleration and jerk of charge propagate as wave.

2. Review of Electrodynamics

First let's review Maxwell equations and the Lorentz force. Maxwell equations are invariant under Lorentz transformation and describe the fields generated by stationary charge and uniform motion of charges. There is an interesting phenomenon: When a charge is at rest, it generates an electric field, and doesn't generate a magnetic field. However, when the same charge moves uniformly, the velocity of the charge generates a magneto-static field. The magnetic field is completely different from electric field in the following senses: (1) the way the fields generated; (2) the nature of the fields; (3) effects of the fields on charges.

Many physics students have a question: Why the uniform velocity of a charge generates such different magnetic field? A teacher will explain to them that magnetism is the combination of electric field with special relativity.

Table 1: Fields generated by stationary charge and the motion of charges

	Generated Field	Force
Stationary Charge: $\mathbf{v} = \mathbf{0}$	Electro-static field $\mathbf{E}$	$\mathbf{F} = q\mathbf{E}$
Constant velocity of charge: $\mathbf{v} \neq \mathbf{0},\ \dot{\mathbf{v}} = \mathbf{0}$	Magneto-static field $\mathbf{B}$	$\mathbf{F} = q\mathbf{u}\times\mathbf{B}$
Acceleration of charge: $\mathbf{v} \neq \mathbf{0},\ \dot{\mathbf{v}} \neq \mathbf{0}$	?	?

Now we ask further questions: What new fields will the acceleration, instead of the velocity, of a charge generate? What are the effects of new fields on stationary, uniformly moving, and accelerating test charges (Table 1)?

The non-uniform motion of a charge is characterized by velocity and acceleration. Thus, at any given instant, the non-uniform motion can be considered as uniform, and the instant velocity will generate magnetic field. The question is: are there new fields generated by the acceleration? Further more, for an accelerating charge, one needs to explain why the non-uniform motion of charge generates the magnetic field and new fields. The term "acceleration" means either acceleration or deceleration.

3. New Fields Generated by Non-uniform Motion of Charge

Maxwell equations, except the displacement current, were established based on series experiments. Up to now, as I know of, there is no experiment (s) to study systematically the effects of the acceleration of charges.

To answer those questions, we propose a mathematic approach.

We postulate that the acceleration of a charge generates new effects on testing charges. There are new fields mediating those effects. Moreover, we postulate that the acceleration of a charge generates new fields in the same way as that either the velocity of a charge generates magnetic field or the time-varying electric/magnetic field generates the induced magnetic/electric field.

The basic concept is that, since the uniform velocity of a charge generates magnetic field that is curl field, the acceleration of a charge should generates curl fields like magnetic field. The new field equations should be supplement to Maxwell equations and can be utilized to investigate electromagnetic phenomena caused by the acceleration of a charge at the level of the field strengths. The issue is that those new field equations don't obey Lorentz transformation.

3.1. Magnetic-type N Field

Thus let's start with a mathematic approach.

There is a formula in vector analysis,

$$\nabla\times(\mathbf{S}\times\mathbf{T}) = \mathbf{S}(\nabla\cdot\mathbf{T}) - \mathbf{T}(\nabla\cdot\mathbf{S}) + (\mathbf{T}\cdot\nabla)\mathbf{S} - (\mathbf{S}\cdot\nabla)\mathbf{T}. \quad (1)$$

What is significant of this formula is that the combinations of the divergence and the gradient of two either non-curl or curl vectors, **S** and **T**, induce inevitable a curl vector, $\mathbf{S}\times\mathbf{T}$. Namely a curl vector is inevitable born of any two vectors.

Combining the vector analysis formula with the postulations, let's $\mathbf{S} = \dot{\mathbf{v}},\ \mathbf{T} = \mathbf{E},$ where $\dot{\mathbf{v}}$ is acceleration; $\mathbf{E}$ is the electric field. The "dot" on the top of velocity **v** represents the time derivative of the velocity. Eq. (1) gives

$$\nabla\times(\dot{\mathbf{v}}\times\mathbf{E}) = \dot{\mathbf{v}}(\nabla\cdot\mathbf{E}) - \mathbf{E}(\nabla\cdot\dot{\mathbf{v}}) + (\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} - (\dot{\mathbf{v}}\cdot\nabla)\mathbf{E}. \quad (2)$$

Define a new field, $\mathbf{N},$

$$\mathbf{N} \equiv \dot{\mathbf{v}}\times\mathbf{E}, \quad (3)$$

Note the **N** field is an acceleration correspondence of magnetic field **B**,

$$\mathbf{B} \equiv \mathbf{v}\times\mathbf{E}. \quad (4)$$

Therefore we call the **N** field the magnetic-type field.

Combining Eq. (2 and 3), we obtain,

$$\nabla\times\mathbf{N} = 4\pi Q\dot{\mathbf{v}} - \mathbf{E}(\nabla\cdot\dot{\mathbf{v}}) + (\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} - (\dot{\mathbf{v}}\cdot\nabla)\mathbf{E}, \quad (5)$$

where Maxwell equation, $\nabla\cdot\mathbf{E} = 4\pi Q$, has been applied.

For non-spatially-varying acceleration, $\mathbf{E}(\nabla\cdot\dot{\mathbf{v}}) = (\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} = \mathbf{0},$ Eq. (5) can be simplified as

$$\nabla\times\mathbf{N} = 4\pi Q\dot{\mathbf{v}} - (\dot{\mathbf{v}}\cdot\nabla)\mathbf{E}. \quad (6)$$

The acceleration of a charge generates inevitablely a curl field **N** like the velocity of a charge generates a curl field **B**. Eq. (6) show that the **N** field, acceleration $\dot{\mathbf{v}}$, and electric field **E** correlate.

3.2. Electric-type M field

Based on symmetry, it is nature to expect that the acceleration of charges generate an electric-type field as well. For this aim, let's apply the formula, Eq. (1), again. Set $\mathbf{S} = \dot{\mathbf{v}},\ \mathbf{T} = \mathbf{B},$ where $\mathbf{B}$ is magnetic field, we obtain,

$$\nabla\times(\dot{\mathbf{v}}\times\mathbf{B}) = -\mathbf{B}(\nabla\cdot\dot{\mathbf{v}}) + (\mathbf{B}\cdot\nabla)\dot{\mathbf{v}} - (\dot{\mathbf{v}}\cdot\nabla)\mathbf{B}, \quad (7)$$

where Maxwell equation, $\nabla\cdot\mathbf{B} = 0$, has been utilized.

Define a new curl field, **M,**

$$\mathbf{M} \equiv -\dot{\mathbf{v}}\times\mathbf{B}. \quad (8)$$

Note the **M** field is an acceleration correspondence of electric field **E**,

$$\mathbf{E} \equiv -\mathbf{v}\times\mathbf{B}. \quad (9)$$

Thus call the **M** field the electric-type field. Substituting the **M** field into Eq. (7), we obtain,

$$\nabla\times\mathbf{M} = \mathbf{B}(\nabla\cdot\dot{\mathbf{v}}) - (\mathbf{B}\cdot\nabla)\dot{\mathbf{v}} + (\dot{\mathbf{v}}\cdot\nabla)\mathbf{B}. \quad (10)$$

For non-spatially-varying acceleration, $\mathbf{E}(\nabla\cdot\dot{\mathbf{v}}) = (\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} = \mathbf{0},$ we obtain the **M** field equation,

$$\nabla\times\mathbf{M} = (\dot{\mathbf{v}}\cdot\nabla)\mathbf{B}. \quad (11)$$

The acceleration $\dot{\mathbf{v}}$ of a charge indeed generates a curl field **M**. Eq. (11) relates the **M** field, acceleration $\dot{\mathbf{v}}$, and magnetic field **B.**

3.3. Interpretation of the N and M Fields

The physical meaning of the **N** and **M** fields can be interpreted as the following.

Consider a charge Q and an observer O. when the charge is at rest relative to the observer, it generates an electro-static field; when the charge is moving with a constant velocity relative to the observer, beside the electric field, the observer also observes a magnetic field generated by the uniform motion, $\mathbf{B} = \mathbf{v}\times\mathbf{E}$; when the charge is moving with acceleration $\dot{\mathbf{v}}$, beside the electric and magnetic field, the observer will observe both a curl **N** field, $\mathbf{N} = \dot{\mathbf{v}}\times\mathbf{E}$, and a curl **M** field, $\mathbf{M} = -\dot{\mathbf{v}}\times\mathbf{B}.$ Namely, the constant velocity of a charge generates one curl field; the acceleration of a charge generates two additional curl fields (Table 2).

Table 2: Fields generated by stationary charge and the motion of charges

	Generated Field
Stationary Charge: $\mathbf{v} = \mathbf{0}$	Electro-static field $\mathbf{E}$
Constant velocity of charge: $\mathbf{v} \neq \mathbf{0},\ \dot{\mathbf{v}} = \mathbf{0}$	$\mathbf{B} = \mathbf{v}\times\mathbf{E}$
Non-uniform motion of charge: $\mathbf{v}(t) \neq \mathbf{0},\ \dot{\mathbf{v}} \neq 0$, or $\dot{\mathbf{v}}(t) \neq \mathbf{0}$	$\mathbf{N} = \dot{\mathbf{v}}\times\mathbf{E}$ $\mathbf{M} = -\dot{\mathbf{v}}\times\mathbf{B}$

3.3. Correlation between the N and M Fields

3.4.1. Ampere-law-type Correlation

Eq. (6) and (11) shows the correlations between the **N** field, acceleration $\dot{\mathbf{v}}$, and electric field **E** and between the **M** field, acceleration $\dot{\mathbf{v}}$, and magnetic field **B**, respectively. Now we need to find the correlation between the **N** and **M** fields. For this aim, we need to re-express the term of $(\dot{\mathbf{v}}\cdot\nabla)\mathbf{E}$ in Eq. (6) and $(\dot{\mathbf{v}}\cdot\nabla)\mathbf{B}$ in Eq. (11).

Applying another vector analysis formula,

$$\boldsymbol{\nabla}(\mathbf{S}\cdot\mathbf{T}) = (\mathbf{S}\cdot\boldsymbol{\nabla})\mathbf{T} + (\mathbf{T}\cdot\boldsymbol{\nabla})\mathbf{S} + \mathbf{S}\times(\boldsymbol{\nabla}\times\mathbf{T}) + \mathbf{T}\times(\boldsymbol{\nabla}\times\mathbf{S}). \tag{12}$$

Let's $\mathbf{S} = \dot{\mathbf{v}}$ and $\mathbf{T} = \mathbf{E}$, where **E** is electric field, Eq. (12) gives

$$\boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) = (\dot{\mathbf{v}}\cdot\boldsymbol{\nabla})\mathbf{E} + (\mathbf{E}\cdot\boldsymbol{\nabla})\dot{\mathbf{v}} + \dot{\mathbf{v}}\times(\boldsymbol{\nabla}\times\mathbf{E}) + \mathbf{E}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}). \tag{13}$$

Or rewrite as

$$(\dot{\mathbf{v}}\cdot\boldsymbol{\nabla})\mathbf{E} = \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) - (\mathbf{E}\cdot\boldsymbol{\nabla})\dot{\mathbf{v}} - \dot{\mathbf{v}}\times(\boldsymbol{\nabla}\times\mathbf{E}) - \mathbf{E}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}). \tag{14}$$

Substituting Eq. (14) into Eq. (5) and Eq. (6) respectively, we obtain

$$\nabla\times\mathbf{N} = 4\pi Q\dot{\mathbf{v}} - \mathbf{E}(\nabla\cdot\dot{\mathbf{v}}) + 2(\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} - \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) + \dot{\mathbf{v}}\times(\boldsymbol{\nabla}\times\mathbf{E}) + \mathbf{E}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}). \tag{15}$$

For non-spatially-varying acceleration, $(\mathbf{E}\cdot\boldsymbol{\nabla})\dot{\mathbf{v}} = (\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} = \mathbf{E}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}) = \mathbf{0}$, Eq. (15) becomes,

$$\nabla\times\mathbf{N} = 4\pi Q\dot{\mathbf{v}} - \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) + \dot{\mathbf{v}}\times(\boldsymbol{\nabla}\times\mathbf{E}). \tag{16}$$

Next let's find $\dot{\mathbf{v}}\times(\boldsymbol{\nabla}\times\mathbf{E})$ in term of the **M** field. For this aim, taking time derivative of the **M** field, we obtain,

$$\frac{\partial\mathbf{M}}{\partial t} = -\frac{\partial}{\partial t}(\dot{\mathbf{v}}\times\mathbf{B}) = -\ddot{\mathbf{v}}\times\mathbf{B} - \dot{\mathbf{v}}\times\frac{\partial\mathbf{B}}{\partial t} = -\ddot{\mathbf{v}}\times\mathbf{B} + \dot{\mathbf{v}}\times[\nabla\times\mathbf{E}]. \tag{17}$$

Where Maxwell equation, $\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$, is used; " $\ddot{\mathbf{v}}$ " is jerk and has four cases: (1) accelerating the acceleration; (2) decelerating the acceleration; (3) accelerating the deceleration; and (4) decelerating the deceleration. In this article we just use letter " $\ddot{\mathbf{v}}$ " to represent all of four cases.

Substituting Eq. (17) into Eq. (15) and (16), respectively, we obtain Ampere-law-type correlation,

$$\nabla\times\mathbf{N} = 4\pi Q\dot{\mathbf{v}} + \frac{\partial\mathbf{M}}{\partial t} + \ddot{\mathbf{v}}\times\mathbf{B} - \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) - \mathbf{E}(\nabla\cdot\dot{\mathbf{v}}) + 2(\mathbf{E}\cdot\nabla)\dot{\mathbf{v}} + \mathbf{E}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}), \tag{18}$$

and

$$\nabla\times\mathbf{N} = 4\pi Q\dot{\mathbf{v}} + \frac{\partial\mathbf{M}}{\partial t} + \ddot{\mathbf{v}}\times\mathbf{B} - \nabla(\dot{\mathbf{v}}\cdot\mathbf{E}), \tag{19}$$

which is the acceleration correspondence of the Ampere's law,

$$\nabla\times\mathbf{B} = 4\pi Q\mathbf{v} + \frac{\partial\mathbf{E}}{\partial t}. \tag{20}$$

Eq. (18) and (19) are the Ampere-law-type correlation of the **N** and **M** fields.

3.4.2. Faraday-law-type Correlation

Based on symmetry, we postulate another correlation between the **N** and **M** fields.

Let's $\mathbf{S} = \dot{\mathbf{v}}$ and $\mathbf{T} = \mathbf{B}$, where **B** is magnetic field. Substituting into Eq. (12), gives

$$\boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B}) = (\dot{\mathbf{v}}\cdot\boldsymbol{\nabla})\mathbf{B} + (\mathbf{B}\cdot\boldsymbol{\nabla})\dot{\mathbf{v}} + \dot{\mathbf{v}}\times(\boldsymbol{\nabla}\times\mathbf{B}) + \mathbf{B}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}). \tag{21}$$

Or rewrite as

$$(\dot{\mathbf{v}}\cdot\boldsymbol{\nabla})\mathbf{B} = \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B}) - (\mathbf{B}\cdot\boldsymbol{\nabla})\dot{\mathbf{v}} - \dot{\mathbf{v}}\times(\boldsymbol{\nabla}\times\mathbf{B}) - \mathbf{B}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}). \tag{22}$$

Substituting Eq. (22) into Eq. (10), we obtain,

$$\nabla\times\mathbf{M} = \mathbf{B}(\nabla\cdot\dot{\mathbf{v}}) - 2(\mathbf{B}\cdot\nabla)\dot{\mathbf{v}} + \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B}) - \dot{\mathbf{v}}\times(\boldsymbol{\nabla}\times\mathbf{B}) - \mathbf{B}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}). \tag{23}$$

For non-spatially-varying acceleration, $\mathbf{B}(\nabla\cdot\dot{\mathbf{v}}) = (\mathbf{B}\cdot\boldsymbol{\nabla})\dot{\mathbf{v}} = \mathbf{B}\times(\boldsymbol{\nabla}\times\dot{\mathbf{v}}) = \mathbf{0}$, Eq. (23) becomes,

$$\nabla\times\mathbf{M} = \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B}) - \dot{\mathbf{v}}\times(\boldsymbol{\nabla}\times\mathbf{B}). \tag{24}$$

Next let's find $\dot{\mathbf{v}}\times(\nabla\times\mathbf{B})$ in term of **N** field. Taking time derivative of the **N** field, we obtain,

$$\frac{\partial\mathbf{N}}{\partial t}=\frac{\partial}{\partial t}(\dot{\mathbf{v}}\times\mathbf{E})=\ddot{\mathbf{v}}\times\mathbf{E}+\dot{\mathbf{v}}\times(\nabla\times\mathbf{B})-4\pi Q\dot{\mathbf{v}}\times\mathbf{v}, \tag{25}$$

Substituting Eq. (25) into Eq. (23) and (24), respectively, we obtain the correlation equation of the **N** and **M** fields,

$$\nabla\times\mathbf{M}=-\frac{\partial\mathbf{N}}{\partial t}+\mathbf{B}(\nabla\cdot\dot{\mathbf{v}})-2(\mathbf{B}\cdot\nabla)\dot{\mathbf{v}}+\nabla(\dot{\mathbf{v}}\cdot\mathbf{B})+\ddot{\mathbf{v}}\times\mathbf{E}-4\pi Q\dot{\mathbf{v}}\times\mathbf{v}-\mathbf{B}\times(\nabla\times\dot{\mathbf{v}}). \tag{26}$$

$$\nabla\times\mathbf{M}=-\frac{\partial\mathbf{N}}{\partial t}+\ddot{\mathbf{v}}\times\mathbf{E}+\nabla(\dot{\mathbf{v}}\cdot\mathbf{B})-4\pi Q\dot{\mathbf{v}}\times\mathbf{v}. \tag{27}$$

For the case of $\dot{\mathbf{v}}//\mathbf{v}$, E. (26) and (27) give, respectively,

$$\nabla\times\mathbf{M}=-\frac{\partial\mathbf{N}}{\partial t}+\mathbf{B}(\nabla\cdot\dot{\mathbf{v}})-2(\mathbf{B}\cdot\nabla)\dot{\mathbf{v}}+\nabla(\dot{\mathbf{v}}\cdot\mathbf{B})+\ddot{\mathbf{v}}\times\mathbf{E}-\mathbf{B}\times(\nabla\times\dot{\mathbf{v}}). \tag{28}$$

$$\nabla\times\mathbf{M}=-\frac{\partial\mathbf{N}}{\partial t}+\ddot{\mathbf{v}}\times\mathbf{E}+\nabla(\dot{\mathbf{v}}\cdot\mathbf{B}). \tag{29}$$

which is the acceleration correspondence of the Faraday's law,

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}. \tag{30}$$

Eq. (26)-(29) are the Faraday-law-type Correlation of the **N** and **M** fields.

The equations of the N and M fields obtained above are derived from pure mathematic formulas and Maxwell equations. Therefore, the new fields should exist and need to be tested experimentally.

3.4. Comparison Between Maxwell Equation and the N and M field Equation

Now let's summarize the **N** and **M** field equations and compare with Maxwell equations (Table 3).

Table 3: Comparision between Maxwell equation and **N** and **M** Field equation

Maxwell Equation $\mathbf{v}\neq 0,\ \dot{\mathbf{v}}=0$	**N** Field $\mathbf{v}\neq 0,\ \dot{\mathbf{v}}\neq 0,\ \ddot{\mathbf{v}}\neq 0$	**M** Field $\mathbf{v}\neq 0,\ \dot{\mathbf{v}}\neq 0,\ \ddot{\mathbf{v}}\neq 0$
$\nabla\times\mathbf{B}=4\pi Q\mathbf{v}+\frac{\partial\mathbf{E}}{\partial t}$	$\nabla\times\mathbf{N}=4\pi Q\dot{\mathbf{v}}+\frac{\partial\mathbf{M}}{\partial t}-$ $-\nabla(\dot{\mathbf{v}}\cdot\mathbf{E})+\ddot{\mathbf{v}}\times\mathbf{B}$	
$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$		$\nabla\times\mathbf{M}=-\frac{\partial\mathbf{N}}{\partial t}+$ $+\nabla(\dot{\mathbf{v}}\cdot\mathbf{B})+\ddot{\mathbf{v}}\times\mathbf{E}$

The combination of Maxwell equations and the **N** and **M** field equations is a complete set of equations to describe fields generated by non-uniform motion of charges.

4. Equation of Motion

In this section, we study effects of the **N** and **M** fields. We postulate that the effects of the **N** and **M** fields are to change the Lorentz force with the time. For demonstration, let's start with Lorentz force,

$$\mathbf{F}=\mathrm{q}\mathbf{E}+\mathrm{q}\mathbf{u}\times\mathbf{B}. \tag{31}$$

Considering two frames of reference, S and S'. The sources of electric field $\mathbf{E}'$ and magnetic field $\mathbf{B}'$ are in S' frame. An observer and a test charge q are in S frame. The S' frame moves relative to the S frame with velocity **v** and acceleration $\dot{\mathbf{v}}$. We assume: (1) speed is slow; (2) at a given instant, the S' frame is approximately an inertial frame. Thus we have transformation,

$$\mathbf{E}\approx\mathbf{E}'-\mathbf{v}\times\mathbf{B}', \tag{32a}$$

$$\mathbf{B} \approx \mathbf{B}' + \mathbf{v}\times\mathbf{E}'. \tag{32b}$$

The $\mathbf{E}$ and $\mathbf{B}$ are measured in the S frame. The $\mathbf{u}$ is the velocity of the test charge q. The force varies with time as

$$\dot{\mathbf{F}} = \mathrm{q}\dot{\mathbf{E}} + \mathrm{q}\frac{\mathrm{d}(\mathbf{u}\times\mathbf{B})}{\mathrm{dt}} = \mathrm{q}\dot{\mathbf{E}} + \mathrm{q}\dot{\mathbf{u}}\times\mathbf{B} + \mathrm{q}\mathbf{u}\times\dot{\mathbf{B}}. \tag{33}$$

Next let's calculate $\frac{\mathrm{d}\mathbf{E}}{\mathrm{dt}}$ and $\frac{\mathrm{d}(\mathbf{u}\times\mathbf{B})}{\mathrm{dt}}$, respectively. Taking time derivative of Eq. (32a) and (32b) respectively, we have,

$$\dot{\mathbf{E}} = \dot{\mathbf{E}}' - \dot{\mathbf{v}}\times\mathbf{B}' - \mathbf{v}\times\dot{\mathbf{B}}', \tag{34a}$$

$$\dot{\mathbf{B}} = \dot{\mathbf{B}}' + \dot{\mathbf{v}}\times\mathbf{E}' + \mathbf{v}\times\dot{\mathbf{E}}'. \tag{34b}$$

Without losing generality, assume that in the S' frame, the electric and magnetic fields are static, $\dot{\mathbf{E}}' = \dot{\mathbf{B}}' = 0$. Eq. (34a) and (34b) become,

$$\dot{\mathbf{E}} = -\dot{\mathbf{v}}\times\mathbf{B}', \tag{35}$$

$$\dot{\mathbf{B}} = \dot{\mathbf{v}}\times\mathbf{E}'. \tag{36}$$

Substituting Eq. (32a) and (32b) into Eq. (35a) and (35b), we obtain

$$\dot{\mathbf{E}} = -\dot{\mathbf{v}}\times(\mathbf{B} - \mathbf{E}'\times\mathbf{v}) = -\dot{\mathbf{v}}\times\mathbf{B} + \dot{\mathbf{v}}\times(\mathbf{E}'\times\mathbf{v}) = \mathbf{M} + \dot{\mathbf{v}}\times(\mathbf{E}'\times\mathbf{v}), \tag{37}$$

$$\dot{\mathbf{B}} = \dot{\mathbf{v}}\times(\mathbf{E} + \mathbf{B}'\times\mathbf{v}) = \dot{\mathbf{v}}\times\mathrm{E} + \dot{\mathbf{v}}\times(\mathbf{B}'\times\mathbf{v}) = \mathbf{N} + \dot{\mathbf{v}}\times(\mathbf{B}'\times\mathbf{v}). \tag{38}$$

Substituting Eq. (36a and 36b) into Eq. (33), we obtain

$$\dot{\mathbf{F}} = \mathrm{q}\mathbf{M} + \mathrm{q}\dot{\mathbf{v}}\times(\mathbf{E}'\times\mathbf{v}) + \mathrm{q}\dot{\mathbf{u}}\times\mathbf{B} + \mathrm{q}\mathbf{u}\times\mathbf{N} + \mathrm{q}\mathbf{u}\times[\dot{\mathbf{v}}\times(\mathbf{B}'\times\mathbf{v})], \tag{39}$$

or

$$\dot{\mathbf{F}} = \mathrm{q}\mathbf{M} + \mathrm{q}\mathbf{u}\times\mathbf{N} + \mathrm{q}\dot{\mathbf{u}}\times\mathbf{B} + \mathrm{q}\mathbf{u}\times[\dot{\mathbf{v}}\times(\mathbf{B}'\times\mathbf{v})] + \mathrm{q}\dot{\mathbf{v}}\times(\mathbf{E}'\times\mathbf{v}), \tag{40}$$

The acceleration of the source charge generates new **N** and **M** fields. The effect of the **N** and **M** fields is to change the Lorentz force with time on a test charge. Eq. (40) shows that the **N** and **M** fields affect stationary test charge via $[\mathrm{q}\mathbf{M} + \mathrm{q}\dot{\mathbf{v}}\times(\mathbf{E}'\times\mathbf{v})]$, uniformly moving test charge via $\mathrm{q}\mathbf{M} + \mathrm{q}\mathbf{u}\times\mathbf{N} + \mathrm{q}\mathbf{u}\times[\dot{\mathbf{v}}\times(\mathbf{B}'\times\mathbf{v})] + \mathrm{q}\dot{\mathbf{v}}\times(\mathbf{E}'\times\mathbf{v})$, and accelerating test charge via all terms on the right hand side of Eq. (40), differently.

At a given instant, the non-uniform motions of a source charge can be considered as uniform motions, and generate regular electromagnetic fields. At the same instant, a non-uniformly moving test charge can be considered as a uniformly moving charge and under the influence of the electromagnetic fields, which is govern by Lorentz force law. Eq. (40) is a supplement formula to the Lorentz force Eq. (31). We need both Eq. (31) and Eq. (40) to describe the motion of an accelerating test charge under the influence of the **E, B, N,** and **M** fields generated by accelerating source charges.

There is correspondence between Eq. (40) and Eq. (31) (Table 4):

$$\mathbf{F} \rightarrow \frac{\mathrm{d}\mathbf{F}}{\mathrm{dt}},\ \mathbf{E} \rightarrow \mathbf{M},\ \mathbf{B} \rightarrow \mathbf{N}.$$

Table 4: Comparison of Effects of **E, B, M,** and **N** Fields

	Electromagnetic fields: **E, B**	**N** and **M** fields
Effects	Force on test charge $\mathbf{F} = \mathrm{q}\mathbf{E} + \mathrm{q}\mathbf{u}\times\mathbf{B}$	Time-varying of force on test charge $\frac{\mathrm{d}\mathbf{F}}{\mathrm{dt}} \approx \mathrm{q}\mathbf{M} + \mathrm{q}\mathbf{u}\times\mathbf{N} + \mathrm{q}\dot{\mathbf{u}}\times\mathbf{B} + \cdots$

5. Wave Equations of N and M Fields

We postulate that the **N** and **M** fields propagate as wave. Let's derive wave equations of the new fields **N** and **M**.

5.1. Wave Equation of N field

For simplicity and without loss generality, we start with Eq. (20 and 30) instead Eq. (19 and 29). Taking curl of Eq. (20),

$$\nabla\times\nabla\times\mathbf{N} = 4\pi Q\boldsymbol{\nabla}\times\dot{\mathbf{v}} + \frac{\partial\nabla\times\mathbf{M}}{\partial t} - \boldsymbol{\nabla}\times\boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) + \nabla\times(\ddot{\mathbf{v}}\times\mathbf{B}). \quad (41)$$

and substituting Eq. (30) into Eq. (41), we obtain

$$\nabla^2\mathbf{N} = -4\pi Q\boldsymbol{\nabla}\times\dot{\mathbf{v}} - \frac{\partial}{\partial t}\left(-\frac{\partial\mathbf{N}}{\partial t} + \ddot{\mathbf{v}}\times\mathbf{E} + \boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B})\right) - \nabla\times(\ddot{\mathbf{v}}\times\mathbf{B}). \quad (42)$$

Where $\boldsymbol{\nabla}\times\boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) = \mathbf{0}.$ For the case of acceleration having the same direction as velocity, but velocity is perpendicular to B field, so $\dot{\mathbf{v}}\cdot\mathbf{B} = \mathbf{0}$. We have

$$\nabla^2\mathbf{N} - \frac{\partial^2\mathbf{N}}{\partial t^2} = -4\pi Q\boldsymbol{\nabla}\times\dot{\mathbf{v}} - \frac{\partial}{\partial t}[\ddot{\mathbf{v}}\times\mathbf{E}] - \nabla\times(\ddot{\mathbf{v}}\times\mathbf{B}). \quad (43)$$

which implies that the $\dot{\mathbf{v}}$, $\ddot{\mathbf{v}}$ and $\dddot{\mathbf{v}}$ generate the **N** wave. As a comparison, the acceleration of charges is the source of **E** wave,

$$\nabla^2\mathbf{E} - \frac{\partial^2\mathbf{E}}{\partial t^2} = 4\pi\rho\dot{\mathbf{v}}.$$

For constant and non-spatially-varying acceleration, Eq. (43) becomes

$$\frac{\partial^2\mathbf{N}}{\partial t^2} - \nabla^2\mathbf{N} = \mathbf{0}, \quad (44)$$

Eq. (43) and (44) are the wave equations of the **N** field with and without a source.

5.2. Wave Equation of M field

For simplicity and without loss generality, we start with Eq. (30). Let's take curl of Eq. (30),

$$\nabla\times\nabla\times\mathbf{M} = -\frac{\partial\nabla\times\mathbf{N}}{\partial t} + \nabla\times(\ddot{\mathbf{v}}\times\mathbf{E}). \quad (45)$$

Where $\nabla\times\boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{B}) = 0$ and $\dot{\mathbf{v}}\cdot\mathbf{B} = 0$ have be used. Substituting Eq. (20) into Eq. (45), we obtain,

$$\nabla^2\mathbf{M} - \frac{\partial^2\mathbf{M}}{\partial t^2} = 4\pi Q\ddot{\mathbf{v}} - \frac{\partial}{\partial t}[\boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) - \ddot{\mathbf{v}}\times\mathbf{B}] - \nabla\times(\ddot{\mathbf{v}}\times\mathbf{E}), \quad (46)$$

which is the **M** wave equation and implies that the $\dot{\mathbf{v}}$, $\ddot{\mathbf{v}}$ and $\dddot{\mathbf{v}}$ are the sour of the **M** wave.

Without the source terms, Eq. (46) becomes

$$\frac{\partial^2\mathbf{M}}{\partial t^2} - \nabla^2\mathbf{M} = \mathbf{0}. \quad (47)$$

Table 5: Comparision between Wave Equations of **E, B, N, M** fields

E Wave $\mathbf{v}\neq 0,\ \dot{\mathbf{v}}\neq 0$	$\nabla^2\mathbf{E} - \frac{\partial^2\mathbf{E}}{\partial t^2} = 4\pi Q\dot{\mathbf{v}}$
M Wave $\mathbf{v}\neq 0,\ \dot{\mathbf{v}}\neq 0,\ \ddot{\mathbf{v}}\neq 0$	$\nabla^2\mathbf{M} - \frac{\partial^2\mathbf{M}}{\partial t^2} = 4\pi Q\ddot{\mathbf{v}} - \frac{\partial}{\partial t}[\boldsymbol{\nabla}(\dot{\mathbf{v}}\cdot\mathbf{E}) - \ddot{\mathbf{v}}\times\mathbf{B}] - \nabla\times(\ddot{\mathbf{v}}\times\mathbf{E})$
B Wave $\mathbf{v}\neq 0,\ \dot{\mathbf{v}} = 0$	$\nabla^2\mathbf{B} - \frac{\partial^2\mathbf{B}}{\partial t^2} = 4\pi Q\nabla\times\mathbf{v}$
N Wave $\mathbf{v}\neq 0,\ \dot{\mathbf{v}}\neq 0,\ \ddot{\mathbf{v}}\neq 0$	$\nabla^2\mathbf{N} - \frac{\partial^2\mathbf{N}}{\partial t^2} = -4\pi Q\boldsymbol{\nabla}\times\dot{\mathbf{v}} - \frac{\partial}{\partial t}[\ddot{\mathbf{v}}\times\mathbf{E}] - \nabla\times(\ddot{\mathbf{v}}\times\mathbf{B})$

6. Summary and Discussion

We postulate and demonstrate mathematically that the acceleration of an electric charge generates new fields. The new field equations have the similar forms as that of Maxwell equations, and are supplement to Maxwell equations. The combination of Maxwell equations and the **N** and **M** field equations is a complete set of field equations to describe fields generated by accelerating charges.

The new fields vary the Lorentz force with time.

The new fields generated by acceleration and jerk of charges propagate as waves.

We suggest that those new fields are worth to be systematically studied experimentally.

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