DERIVING THE LAFFER CURVE USING THE CONCEPT OF THE EXCESS BURDEN OF TAXATION

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Abstract

The Laffer Curve formula is derived from the concept of Excess Burden using a single parameter obtained from the theoretical relationship between the average Excess Burden ratio and the square of the average Tax Ratio. This is followed by a demonstration, using a chosen figure of excess burden, showing how to estimate the values of a Laffer Curve.

1. Introduction

History is full of examples of nations and empires collapsing, a major reason being given by historians is excess taxation. Taxes rise, the economy collapses, there is social unrest, there is invasion and defeat, finitum. Given the central role of excess taxation in the history of economic collapse, I am amazed at the minimal attention that has been paid to this subject. My suspicion is that all or most of the commentators want increased tax expenditure on their pet program(s), whether building palaces, increasing the military, or increasing welfare, and ignore the consequences.

The only relationship that appears to relate increased taxation to falling revenue is the Laffer Curve. This curve relates increasing tax rates to rising, then falling, tax revenues. See the diagram below. The Laffer Curve was proposed at a luncheon in the early 1970's by Arthur Laffer, and popularized by Jude Wanniski (Wanniski, 1978). Laffer proposed a diagram with two axes. The vertical axis is taxation revenue and the horizontal axis is the average tax rate. The curve is an inverted 'u' with one leg placed

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axiomatically at zero tax revenue and zero tax rate, and the other leg at an unspecified tax rate, where taxation revenue is again zero. At the center of the upturned ‘u’ there is a peak. To the left of the peak, revenue rises with the increasing tax rate, as most non-economists naturally expect. To the right of the peak, as the tax rate increases, revenue falls. Eventually at a certain tax rate, revenue falls to zero.

The Laffer Curve

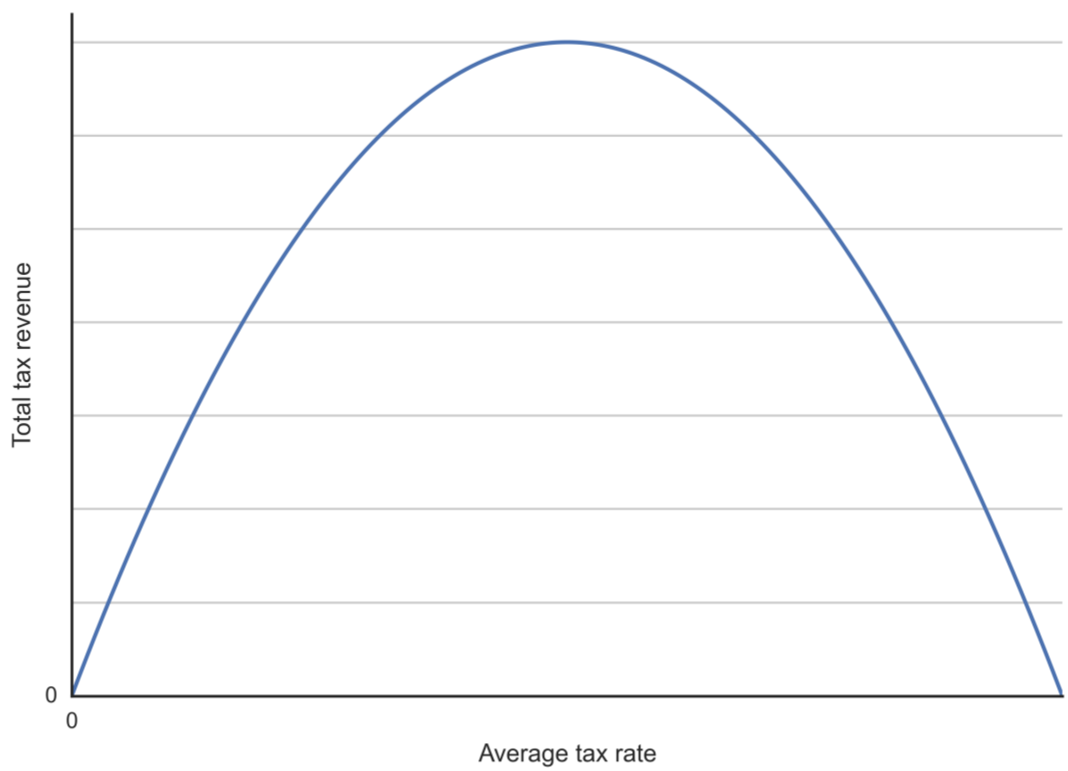

Figure 1. Diagram of the Laffer Curve

While there can be no objection among economists to the theoretical existence of the Laffer Curve, there have been many visceral objections that it exists in practice. Estimates have been made of the peak of the Laffer Curve using various methodologies. These range from Y. Hsing (1986) who gives a revenue maximizing personal income tax rate of between 32.67 percent and 35.21 percent, ranging to 100 percent (J. Malcomson (1986)).

Hitherto there has been no theoretical explanation connecting the Laffer Curve with other taxation concepts such as Excess Burden of Taxation. In this paper I attempt to do so.

In this article, I derive the Laffer Curve theoretically from the concept of the Excess Burden of Taxation. Estimated parameters for the USA can be derived from estimated values of Excess Burden in order to derive a Laffer Curve for the USA. This methodology is shown in this article. This is followed by a demonstration, using a figure for the excess burden estimated by Yorgenson and Yun in 1991, to show how a Laffer Curve can be estimated. The purpose of this approach is to avoid arguments over the chosen value of the excess burden and the final result of the peak of the Laffer Curve. At a later date, I shall provide updated values of the Laffer Curve using accepted current values of the estimated excess burden.

2. An explanation of Excess Burden

A. Definition of Excess Burden

"Loss of economic activity due to the imposition of a tax compared to a free market with no tax." Farlex Financial Dictionary, 2012, Farlex Inc.

B. Theory of Excess Burden

The theory of Excess Burden goes back to Hicks, but the two current major authorities are Auerbach and Hines (2001) Feldstein (2008), who have each written several papers on this subject.

So what is Excess Burden, otherwise known in the literature as the Deadweight Loss?

Excess burden is the efficiency cost, or deadweight loss, associated with taxation. Excess burden is usually measured by the area of a so-called Harberger Triangle, which is essentially a tax wedge inserted between the supply and demand functions of the commodity being taxed. Harberger (1971) measured the cost of tax distortions to labor supply, savings, capital allocation, and other economic decisions. More recent work by Auerbach and Hines (2001) and Feldstein (2008) estimated excess burdens based on more comprehensive measures of taxable income, using compensated demand and supply schedules. These studies reported sizeable excess burdens of existing taxes. As this paper is not about estimating the values of excess burden per se, I won't go into the detailed methodology of estimation, except to say that a hypothetical non-distorting tax system is used as a standard. In this model, all revenue is raised by means of a lump-sum levy that does not distort decisions and involves no loss in efficiency.

Excess burden may sound theoretical, but it has a serious practical effect. Its effect is just like a tax imposed by an enemy power, it is money and resources taken from the economy and not returned. Unlike most taxation, excess burden is a net loss to the economy. Thus, the effect of excess burden is far more serious than any other tax imposed on the economy, as most taxes are normally paid back into the economy.

As I wished only to provide a demonstration of how to estimate the values of the Laffer Curve, and at the same time eschewing any claims to where the peak of the Laffer Curve is currently, I took a value of the deadweight loss as estimated by Yorgenson and Yun in 1999. That is 18 per cent of…... When I have obtained a suitable up-to-date figure I shall use the methodology supplied to provide a current estimate of the peak of the Laffer Curve.

3. The theoretical derivation of the Laffer Curve

To do this we create a simple economic model. In this model, I make certain assumptions.

The basic assumption is that at a zero tax rate, zero tax revenue is raised. This ties one leg of the Laffer Curve to zero on both axes.

The next assumption is that total economic activity is related to the net profit less the "real" tax – defined as the actual tax plus the excess burden.

Thus where net profit is zero, total economic activity is zero, and the total revenue is zero. (It is noted in the economic literature, some economists oppose this salient point. Some say that the economy will continue to operate when there is zero revenue due to excess burden, as there will be tax avoidance, or the populace will resort to barter!).

Thus this concept ties the next leg of the Laffer Curve to the place where net profit is zero, the point where actual tax plus the excess burden takes 100 percent of profit.

Thus the Laffer Curve is tied to two points on the 'x' axis where tax revenue is zero.

The question is – is the curve in between an inverted 'u' with a maximum with positive revenue, and declining revenue past the maximum point?

We can first work out the theoretical structure of the Laffer Curve, and then with the parameter for the excess burden curve already we can we can put figures to the point where the maximum is.

Definitions

B Tax Base

P Gross Profit in dollars

N Net profit

p Profit rate

t Tax rate

E Excess Burden in Dollars

e Excess burden rate

T Tax revenue in dollars

Now the formula of the Laffer Curve relates the total tax revenue T in terms of the average tax rate t. Or:

T= fn(t) where the relationship is non-linear.

Now, the tax base B is a function of the rate of gross profit P less Tax T less Excess Burden E. Business activity, and thus the tax base, is assumed to expand as gross profit increases, declines as the tax burden increases, and declines as Excess Burden increases. Excess Burden, even though it is hidden, has a very real effect on total business activity. The relationship is non-linear.

(1) $B = \text{fn}\,(P - (T+E)$

or

(2) $B = n\,(P - (T+E))$ at any point in time.

Tax revenue T is equal to gross profits P times the tax rate t.

Thus

(3) $T = P \times t$

Divide (2) by B, the tax base, to normalize it.

Then

(4) $1 = n(p - (t + e))$ at any point in time.

as

(6) $t = T/B$

(8) $p = P/B$

(10) $e = E/B$

John Creedy (2004) and Ballard, Fullerton, Shoven and Whalley (1985) have shown that the rate of deadweight losses approximately increases with the *square* of the tax rate, and linearly with elasticities.

To be more precise I shall quote the function derived by Creedy (2004) page 17.

(12) $$EB_{cv} = (|\eta_0|/2)\,(X_0 P_0)\, t^2$$

where

EB_{cv} is rate of excess burden s

η_0 is the Hicksian point elasticity of demand

Creedy's methodology assumes a horizontal supply curve. However as point elasticities are assumed, a cross elasticity of supply and demand would be constant.

X_0 is the initial quantity of goods or activity

P_0 is the initial price

(the product of these is called initial income)

t is the rate of tax

Thus the level of excess burden is a function of the level of the rate of tax squared, keeping elasticity constant.

To keep things simple in this derivation I assume that the elasticity remains constant. I have taken the average tax rate to be the total tax revenue divided by the tax base.

It is more frequent to compare tax revenue to GDP. The proportion of tax to GDP in the US was 26.9 percent in 2014 (source Bureau of Economic Analysis (BEA)).

However in the formulation used here the tax is related to taxable income (the tax base). GDP includes government tax revenue. Let taxable income (the potential total tax base) be GDP less government tax revenue.

Thus t, the average rate of tax = government tax revenue/tax base

Given the relationship between the average tax rate and excess burden, excess burden losses increase with the square of the tax rate, the relation of the tax rate T to the excess burden can be described in a formula

(13) $e = \text{fn}\ t^2$

or $e = at^2$ assuming the relationship is linear, as the elasticity η_0 in the above function $EB_{cv} = (|\eta_0|/2)\ (X_0P_0)\ t^2$ is a point elasticity, and X_0P_0 is constant.

$e = EB_{cv}$ is the percentage of the Excess Burden to the tax base and t is the average tax rate percent.

Now

(14) $e = at^2$

Substitute for e in step (4)

(15) $1 = n(p - (t + at^2)$

Expanding

(16) $1 = np - nt + nat^2$

This is the formula of the Laffer Curve. It is a cubic curve, commencing at the intersection $r = 0$ and $t = 0$.

t, the tax rate, is a positive value, and lies between 0 and 1. The curve goes through (0,0). It also goes through a second point where $r = 0$ where $(pt - t^2 - at^3) = 0$.

As the curve is cubic it has a maximum and a minimum. As the maximum of the curve lies between the points t = 0 and $(pt - t^2 - at^3) = 0$ the curve is concave between those two points.

The maximum point of the Laffer Curve $2t + 3at^2 = p$ shows that t, the maximum tax ratio of the Laffer Curve, is dependent on the profit rate p. In periods when the profit rate is high, t increases, and thus maximum point of the Laffer Curve moves to the right. When the profit rate is lower, the maximum of the Laffer Curve moves to the left. Therefore it is incumbent on any government, if it desires to increase its maximum possible tax rate, to take steps to increase the pre-tax profit rate.

The amount of profit P in the economy is the conceptual remainder of the value of Labor to Output in the economy. There has been controversy that the ratio of the value of profits to labor has expanded over recent years. The increase in the proportion of P has pushed the peak of the Laffer Curve to the right. If this proportion reverses, the peak of the Laffer Curve will move to the left. If tax revenue reaches the Laffer Curve peak, any reversal of the Profit/Labor proportion is likely to lead to a fall in revenue as the tax ratio is now on the right side of the peak of the Laffer Curve.

4. Demonstration of the process of deriving the Laffer Curve

Step 1. Deriving the value of the parameter *a*.

For the sake of exposition, I have derived some results based on an assumed excess burden figure. This figure is based on the value of excess burden for the USA derived Yorgenson and Yun in 1991. (Yorgenson and Yun 1991). This derived value was 18 per cent of…….

I used the US economic statistics published by the Bureau of Economic Analysis (BEA) for the same year 1990.

So first of all, what is the Tax Ratio? The ratio of total government tax revenue to GDP. However Yorgenson and Yun did not use GDP in their estimate of the Excess Burden per cent. They used non-government income.

This figure can be estimated by deducting Government Tax Revenue from GDP. This gives the Government Revenue Tax Base. (Obviously the government does not tax itself).

So:

GDP of the USA in 1990 was $5979.6 billion in current dollars. (Source BEA GDP spreadsheet).

Total government tax revenue in 1990 = $1712.9 billion. (Source BEA Tax Revenue spreadsheet).

What is the tax base? Conceptually it is the total income available that potentially can be taxed. It can be taken to be the entire GDP. However, it can be argued that at any point in time a certain amount of tax is already being taken out of the economy, and as a tax base is an amount that can be potentially further be taxed, I have taken the value as GDP less the amount of existing tax. However, for the purpose of this exercise the precise value of the tax base B is not important. The tax base may indeed be taken as the entire GDP. The reason I have taken a different value of the tax base is to demonstrate that different values of the tax base B can be used to estimate the values of the Laffer Curve, and value of the tax base B need not taken to be the value of GDP. As will be seen later, the value of t can be normalized later to give a tax ratio to GDP.

Total tax base in 1990 = $5979.6 - $1712.9 billion

= $4263.7 billion

Tax ratio t = Tax Receipts/Tax Base

$= 1712.9/4263.7$

$t = 0.401740$ or 40.17 percent

Now $e = at^2$

$a = e/t^2$

$a = 18/(40.17)^2$

$a = 0.01115274$

This parameter is used throughout this successive work to find the Laffer Curve.

Step 2. Estimating the curve relating the value of the Excess Burden to the Tax Ratio

This step is optional, but the curve is interesting as it illustrates the rapid increase in excess burden with the increasing tax rate.

The values for e and t have been calculated in the following table using the formula $e = at^2$ and substituting for $a = 0.01115274$.

t Percent	*e* Percent
0	0
5	0.28
10	1.12
15	2.51
20	4.46
25	6.98

30	10.03
35	13.66
40	17.84
45	22.58
50	27.88
55	33.73
60	40.15
70	54.65
80	71.38

Table 2 - The value of the average tax ratio *t* to the excess burden ratio *e*

Chart of the above table

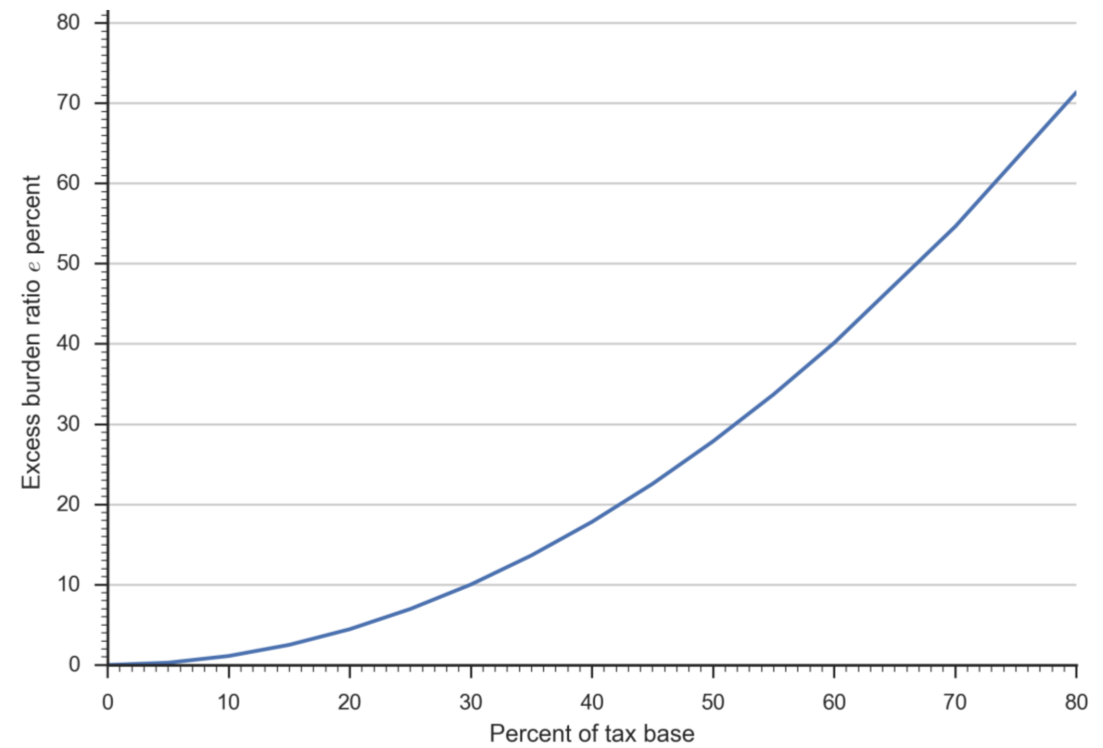

Figure 3. The relationship between the proportion of excess burden and the tax ratio

e is the rate of the Excess Burden percent of tax base

t is rate of taxation percent of tax base

Rate of taxation percent of tax base is a proxy for the average tax rate.

While these results may be rough and ready they demonstrate two essential points. First, the present tax to GDP ratio in the USA is around 27 percent. This gives the percentage tax to tax base ratio of around 36 percent, giving a current excess burden around 14 percent from the above table.

The second point is that this excess burden is increasing at a quadratic rate as the average tax rate rises, as can be seen in the above chart.

The question arises at what point does the total of the actual tax plus the corresponding excess burden equals 100 percent? That is, at what point does the actual tax plus the hidden tax of excess burden take everything? This is a question taxation economists have never really asked.

It can be seen from Table 2 that when the excess burden is 40.15 per cent, the average tax rate is 60 per cent, and adding these together, the total is over 100 hundred per cent. This is in effect a total confiscation of all income when the tax rate equals 60 per cent. The fact that the excess burden is “hidden” does not make it any less real.

5. Deriving the Laffer Curve values

Now we can put some figures on this curve.

We know that $a = 0.0115274$.

We assume that the initial profit rate P is constant and does not change as the tax rate rises.

Since r and t are measured in percentage terms, let p, the initial profit rate when tax is zero, be 100 percent.

$r = n\,(pt - t^2 - at^3)$

Let n be 1 for this exercise. To be precise the value of n = $(|\eta_0|/2)\,(X_0P_0)$, where η_0 is the point elasticity at the point X_0P_0 . Whatever its real value, its only effect would be to move the curve up and down in the vertical direction, not left and right. It will not affect the shape of the Laffer Curve or the position of the peak.

Thus $r = (1t - t^2 - 0.0011527\, t^3)$.

Estimating r for t from 0 to 1.0 (100 percent) we get the following results:-

Values of the Laffer Curve

Tax Rate t Percent	Tax Revenue r (measured as percent of maximum profit)
0	0
5	4.8
10	9.0
15	12.7
20	16.0
25	18.7
30	21.0
35	22.7
40	23.9
45	24.6
50	24.8
55	24.5
60	23.7
65	22.3
70	20.4
75	18.0
80	15.1

85	11.7
90	7.8
95	3.3
99	0.0

t

Table 4 – Tax revenue *r* in terms of average tax rate *t*

The calculated Laffer Curve is below.

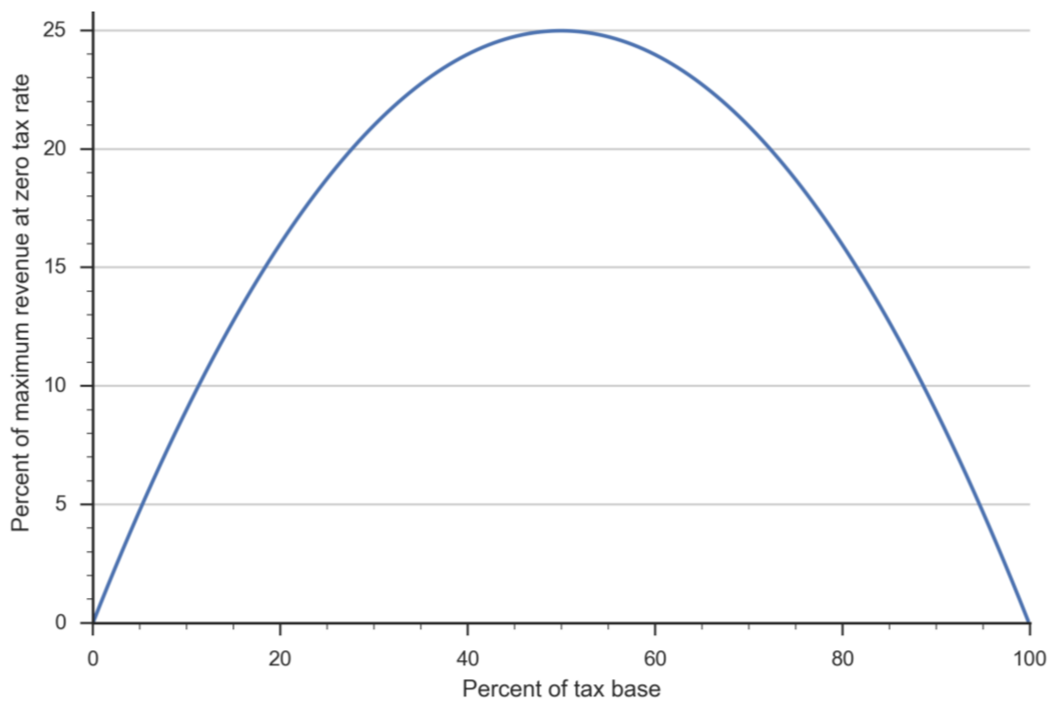

Figure 5. The values of the Laffer Curve derived by using the model and the value of parameter *a*

Note: The peak of this curve is slightly to the left of the 50 percent mark, and the curve reaches the "x" axis slightly left of the 100 percent mark. The position and shape of the Laffer Curve depends on the value of the parameter a. That the maximum of this particular curve is in the region of the 50 percent average tax ratio and the far end *reaches the "x" axis near the 100 percent average tax ratio mark is probably a coincidence, depending on economic conditions at the time.*

As can be seen, these calculations give the maximum point on the Laffer Curve in the USA to be around 50 percent average tax rate based on the tax revenue to potential tax base. This works out as 33 percent of the tax to GDP ratio.

Since the average tax rate in the USA is currently around 27 percent of GDP, these calculations indicate that the USA has nearly reached the peak of the Laffer Curve. Any further increases in the tax rate will lead to constant tax revenue and then revenue will start falling at a faster and faster rate. These calculations indicate when the average tax rate reaches 60 percent, (about 40 percent of tax to GDP ratio), the total tax take including excess burden is 100 percent.

These derived charts are for demonstration purposes only, as they are based on very outdated measure of excess burden, though the only ones currently available. There is no inference that the peak of the Laffer Curve is currently around 50 per cent of the tax base. In fact, the formula demonstrates that as the profit ratio rises, the peak of the Laffer Curve can shift to the right. This might have happened in the past decade.

6. Conclusion

The Laffer Curve was derived in this paper from the concept of Excess Burden. John Creedy (2004) and Ballard, Fullerton, Shoven and Whalley (1985) have shown that *rate of deadweight losses increase with the square of the tax rate* and linearly with elasticities. A parameter can be derived connecting this relationship. A formulation can then be derived of the Laffer Curve utilizing this parameter.

This formulation can be used to plot part of a cubic curve on a chart with the vertical axis as tax revenue and the horizontal axis as the average tax rate. The cubic curve goes through (0.0), reaches a peak, then falls back to the zero tax revenue level.

The inference from the above curve is that as tax rates increase tax revenue will stabilize and then start falling regardless how much the tax ratio is raised. Increasing the tax rate will causes a fall in tax revenue from the peak of the Laffer Curve onwards. The government cannot rely on growth to increase tax revenue, as part of the reason the Laffer Curve declines is that growth becomes negative after the tax ratio passes the peak of the curve.

Government assumptions and beliefs regarding the possibility of increasing expenditure will have to change when the peak of the Laffer Curve is approached. Increasing the rate of tax or imposing new taxes will not necessarily increase revenue.

As a consequence, near the peak of the Laffer Curve, if government expenditure increases in one area, there must be a compensating reduction in expenditure in other areas, unless this increased expenditure is met by increased borrowing. As tax revenue is capped by outside economic forces at the peak of the Laffer Curve, this will impose a cap on tax expenditure. Even increased growth cannot be depended on, because as you raise the tax past the peak of the Laffer Curve the economy will stagnate and then decline.

As growth will be zero at the peak of the Laffer Curve an inference of this result is that, as you move down the left side of the Laffer Curve as the ratio decreases, growth increases. Lower tax leads to higher growth.

As a derivation from this result, I suggest that an econometric model of the form $r = a + bt + ct^2 + dt^3$ be run, where r is tax revenue and t the tax ratio (not in percentage terms), and a, b, c and d are estimated parameters. I predict that a will be significantly no different from zero, and b and c are significantly no different from -1, whether t is the ratio of tax revenue to GDP or the tax base. The parameter d can be used to estimate the peak of the Laffer Curve. Furthermore d can be used to estimate the average Excess Burden in the economy, that will match the values estimated by alternative methods.

Refinements of this methodology may be needed. But what is also greatly needed is current excess burden data for every country. This would supply an updated value the current peak of the Laffer Curve for these countries.

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