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Abstract

Purpose – Recently, F. Smarandache generalized the Atanassov's intuitionistic fuzzy sets (IFSs) and other kinds of sets to neutrosophic sets (NSs) and also defined various notions of neutrosophic topologies on the non-standard interval. One can expect some relation between the intuitionistic fuzzy topology (IFT) on an IFS and neutrosophic topologies on the non-standard interval. The purpose of this paper is to show that this is false.

Design/methodology/approach – The possible relations between the intuitionistic fuzzzy topology and neutrosophic topologies are studied.

Findings – Relations on IFT and neutrosophic topologies.

Research limitations/implications – Clearly, the paper is confined to IFSs and NSs.

Practical implications – The main applications are in the mathematical field.

Originality/value – The paper shows original results on fuzzy sets and topology.

Keywords Logic, Fuzzy logic, Topology, Set theory

Paper type Research paper

1. Introduction

In various recent papers, Smarandache (1998, 2002, 2003, 2005a, b) generalizes intuitionistic fuzzy sets (IFSs) and other kinds of sets to neutrosophic sets (NSs). In Smarandache (2005a, b), some distinctions between NSs and IFSs are underlined.

The notion of IFS defined by Atanassov (1983) has been applied by Çoker (1997) for study intuitionistic fuzzy topological spaces (IFTSs). This concept has been developed by many authors (Bayhan and Çoker, 2003; Çoker, 1996, 1997; Çoker and Eş, 1995; Eş and Çoker, 1996; Gürçay *et al.*, 1997; Hanafy, 2003; Hur *et al.*, 2004); Lee and Lee, 2000; Lupiáñez, 2004a, b, 2006a, b, 2007, 2008; Turanh and Çoker, 2000).

Smarandache (2002, 2005b) also defined various notions of neutrosophic topologies on the non-standard interval.

One can expect some relation between the intuitionistic fuzzy topology (IFT) on an IFS and the neutrosophic topology. We show in this paper that this is false. Indeed, the union and the intersection of IFSs do not coincide with the corresponding operations for NSs, and an IFT is not necessarily a neutrosophic topology on the non-standard interval, in the various senses defined by Smarandache.

2. Basic definitions

First, we present some basic definitions.

Definition 1. Let X be a non-empty set. An IFS A, is an object having the form $A = \{ < x, \mu_A, \gamma_A > /x \in X\}$ where the functions $\mu_A : X \rightarrow I$ and $\gamma_A : X \rightarrow I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$ (Atanassov, 1983).

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Definition 2. Let X be a non-empty set, and the IFSs $A = \{ < x, \mu_A, \gamma_A > |x \in X\}$, $B = \{ < x, \mu_B, \gamma_B > |x \in X\}$. Let (Atanassov, 1988):

$$\bar{A} = \{ < x, \gamma_A, \mu_A > |x \in X\}$$

$$A \cap B = \{ < x, \mu_A \wedge \mu_B, \gamma_A \vee \gamma_B > |x \in X\}$$

$$A \cup B = \{ < x, \mu_A \vee \mu_B, \gamma_A \wedge \gamma_B > |x \in X\}.$$

Definition 3. Let X be a non-empty set. Let $0_\sim = \{ < x, 0, 1 > |x \in X\}$ and $1_\sim = \{ < x, 1, 0 > |x \in X\}$ (Çoker, 1997).

Definition 4. An IFT on a non-empty set X is a family τ of IFSs in X satisfying:

- $0_\sim, 1_\sim \in \tau$;
- $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$; and
- $\cup G_j \in \tau$ for any family $\{G_j | j \in J\} \subset \tau$.

In this case, the pair (X, τ) is called an IFTS and any IFS in τ is called an intuitionistic fuzzy open set in X (Çoker, 1997).

Definition 5. Let T, I, F be real standard or non-standard subsets of the non-standard unit interval $]^-0, 1^+[$, with:

$$\sup T = t_{\sup}, \quad \inf T = t_{\inf}$$

$$\sup I = i_{\sup}, \quad \inf \mathrm{I} = i_{\inf}$$

$$\sup F = f_{\sup}, \quad \inf F = f_{\inf} \quad and \quad n_{\sup} = t_{\sup} + i_{\sup} + f_{\sup} \quad n_{\inf} = t_{\inf} + i_{\inf} + f_{\inf},$$

T, I, F are called neutrosophic components. Let U be an universe of discourse, and M a set included in U. An element x from U is noted with respect to the set M as $x\,(T, I, F)$ and belongs to M in the following way: it is $t\%$ true in the set, $i\%$ indeterminate (unknown if it is) in the set, and $f\,\%$ false, where t varies in T, i varies in I, f varies in F. The set M is called a NS (Smarandache, 2005a).

Remark. All IFS is a NS.

Definition 6. Let $J \in \{T, I, F\}$ be a component. Most known N-norms are:

- The algebraic product N-norm: $N_{n-a\lg\mathrm{ebraic}}\, J(x, y) = x \cdot y$.
- The bounded N-norm: $N_{n-bounded}\, J(x, y) = \max\{0, x + y - 1\}$.
- The default (min) N-norm: $N_{n-\min} J(x, y) = \min\{x, y\}$.
- N_n represent the intersection operator in NS theory. Indeed, $x \wedge y = (T_\wedge, I_\wedge, F_\wedge)$ (Smarandache, 2005b).

Definition 7. Let $J \in \{T, I, F\}$ be a component. Most known N-conorms are:

- The algebraic product N-conorm: $N_{c-a\lg\mathrm{ebraic}}\, J(x, y) = x + y - x \cdot y$.
- The bounded N-conorm: $N_{c-bounded}\, J(x, y) = \min\{1, x + y\}$.
- The default (max) N-conorm: $N_{c-\max} J(x, y) = \max\{x, y\}$.
- N_c represent the union operator in NS theory. Indeed, $x \vee y = (T_\vee, I_V, F_\vee)$ (Smarandache, 2005b).

3. Results

Proposition 1. Let A and B be two IFSs in X, and $j(A)$ and $j(B)$ be the corresponding NSs. We have that $j(A) \cup j(B)$ is not necessarily $j(A \cup B)$, and $j(A) \cap j(B)$ is not necessarily $j(A \cap B)$, for any of three definitions of intersection of NSs.

Proof. Let $A =< x, 1/2, 1/3 >$ and $B =< x, 1/2, 1/2 >$ (i.e. $\mu_A,\ \nu_A,\ \mu_B,\ \nu_B$ are constant maps).

Then, $A \cup B =< x, \mu_A \vee \mu_B, \gamma_A \wedge \gamma_B >=< x, 1/2, 1/3 >$ and $x(1/2, 1/6, 1/3) \in j(A \cup B)$. On the other hand, $x(1/2, 1/6, 1/3) \in j(A)$, $x(1/2, 0, 1/2) \in j(B)$.

Then, we have that:

(1) for the union operator defined by the algebraic product N-conorm $x(3/4, 1/6, 2/3) \in j(A) \cup j(B)$;

(2) for the union operator defined by the bounded N-conorm $x(1, 1/6, 5/6) \in j(A) \cup j(B)$; and

(3) for the union operator defined by the default (max) N-conorm $x(1/2, 1/6, 1/2) \in j(A) \cup j(B)$.

Thus, $j(A \cup B) \neq j(A) \cup j(B)$, with the three definitions.

Analogously, $A \cap B =< x, \mu_A \wedge \mu_B, \gamma_A \vee \gamma_B >=< x, 1/2, 1/2 >$ and $x(1/2, 0, 1/2) \in j(A \cap B)$.

And, we have that:

(1) for the intersection operator defined by the algebraic product N-norm $x(1/4, 0, 1/6) \in j(A) \cap j(B)$;

(2) for the intersection operator defined by the bounded N-norm $x(0, 0, 0) \in j(A) \cap j(B)$; and

(3) for the intersection operator defined by the default (min) N-norm $x(1/2, 0, 1/3) \in j(A) \cap j(B)$.

Thus, $j(A \cap B) \neq j(A) \cap j(B)$, with the three definitions. □

Definition 8. Let us construct a neutrosophic topology on $NT =]^-0, 1^+[$, considering the associated family of standard or non-standard subsets included in NT, and the empty set which is closed under set union and finite intersection neutrosophic. The interval NT endowed with this topology forms a neutrosophic topological space. There exist various notions of neutrosophic topologies on NT, defined by using various N-norm/N-conorm operators (Smarandache, 2002, 2005b).

Proposition 2. Let (X, τ) be an IFTS. Then, the family $\{j(U) | U \in \tau\}$ is not necessarily a neutrosophic topology on NT (in the three defined senses).

Proof. Let $\tau = \{1_\sim, 0_\sim, A\}$ where $A =< x, 1/2, 1/2 >$ then $x(1, 0, 0) \in j(1_\sim)$, $x \in (0, 0, 1) \in j(0_\sim)$ and $x(1/2, 0, 1/2) \in j(A)$. Thus, $\tau^* = \{j(1_\sim), j(0_\sim), j(A)\}$ is not a neutrosophic topology, because this family is not closed by finite intersections, for any neutrosophic topology on NT. Indeed:

- For the intersection defined by the algebraic product N-norm, we have that $x(1/2, 0, 0) \in j(1_\sim) \cap j(A)$, and this NS is not in the family τ^*.
- For the intersection defined by the bounded N-norm, we have also that $x(1/2, 0, 0) \in j(1_\sim) \cap j(A)$, and this NS is not in the family τ^*.

- For the intersection defined by the default (min) N-norm, we have also that $x(1/2, 0, 0) \in j(1_{\sim}) \cap j(A)$, and this NS is not in the family τ^*.

□

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