ISSN: 2277-9655
[Banerjee *et al.*, Vol. 7(5): May, 2018] Impact Factor: 5.164
IC™ Value: 3.00 CODEN: IJESS7

IJESRT

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

SINGLE-OBJECTIVE LINEAR GOAL PROGRAMMING PROBLEM WITH NEUTROSOPHIC NUMBERS

Durga Banerjee[2], Surapati Pramanik [*2]
[1]Ranaghat Yusuf Institution, Rathtala, P.O.- Ranaghat, District- Nadia, Pin Code -741201, West Bengal, India
[*] Department of Mathematics, Nandalal Ghosh B. T. College, Panpur, P.O. Narayanpur, District. North 24 Parganas, West Bengal, India, Pin 743126.

DOI: 10.5281/zenodo.1252834

ABSTRACT

This paper deals with single-objective linear goal programming problem with neutrosophic numbers. The coefficients of objective function and the constraints are considered as neutrosophic numbers of the form (p + qI), where p, q are real numbers and I denotes indeterminacy. In the solution process, the neutrosophic numbers are transformed into interval numbers. Then, the problem reduces to single-objective linear interval programming problem. Employing interval programming technique, the target interval of the objective function is determined. For the sake of achieving the target goals, the goal achievement function is constructed. Three new goal programming models are developed to solve the reduced problem. Two numerical examples are solved to illustrate the proposed method. The obtained results are also compared with the existing methods.

KEYWORDS: : neutrosophic goal programming, goal programming, fuzzy goal programming, neutrosophic number, Smarandache neutrosophic number

1. INTRODUCTION

Goal programming is a branch of multi-objective optimization. GP can be viewed as an extension or a generalization of linear programming to handle multiple, normally conflicting objective measures. The basic idea of goal programming is found in the work of Charnes, Cooper and Ferguson [1]. Charnes and Cooper [2] first coined the term goal programming to deal with infeasible linear programming. Charnes and Cooper [2]), Ijiri [3], Lee [4]), Ignizio [5], Romero [6], Schniederjans [7], Chang [8], Dey and Pramanik [9] and many pioneer researchers established different approaches to goal programming in crisp environment. Inuguchi and Kume [10] investigated interval goal programming.

Narasimhan [11] proposed goal programming in fuzzy environment, which is called Fuzzy Goal Programming (FGP). FGP has been developed by several authors such as Hannan (12), Ignizio [13], Tiwari, Dharma, and Rao [14, 15], Mohamed [16], Pramanik [17, 18], Pramanik and Roy [19-24], Pramanik and Dey [25], Tabrizi, Shahanaghi, and Jabalameli [26], and other researchers.

Pramanik and Roy [27] proposed goal programming in intuitionistic fuzzy environment called intuitionistic FGP (IFGP). Pramanik and Roy [28-29] presented some applications of IFGP in transportation and quality control problem. Pramanik, Dey, and Roy [30] presented IFGP approach to bi-level programming problem. Razmi, Jafarian, and Amin [31] presented Pareto-optimal solutions to multi-objective programming problems under intuitionistic fuzzy environment.

Smarandache [32] introduced neutrosophic set based on neutrosophy, a new branch of philosophy. Wang, Smarandache, Zhang, and Sunderraman [33] proposed Single Valued Neutrosophic Set (SVNS) to deal realistic problems. SVNS has been applied in different areas such as multi-attribute decision making (MADM) [34-49] conflict resolution [50], educational problem [51-52], data mining [53], social problem [54- 55], etc.

MADM has been further studied in different neutrosophic extension and hybrid neutrosophic environment such as interval neutrosophic set environment [56-59], neutrosophic soft set environment [60-65], rough neutrosophic environment [66-77], neutrosophic bipolar set environment [78-84], neutrosophic hesitant fuzzy set environment [85-92] neutrosophic refine set environment [93-98], linguistic refine set [99], neutrosophic cubic set environment [100-105], complex rough neutrosophic set environment [106-107], etc. In 2018, Pramanik, Mallick & Dasgupta [110] presented a brief survey of contribution of Indian researchers in MADM. Some applications of neutrosophic sets in MADM can be found in [109-111].

Optimization technique in neutrosophic environment has been recently introduced in the literature. Optimization technique in neutrosophic hybrid environment is yet to appear in the literature. Roy and Das [112] proposed neutrosophic multi-objective linear programming problem (MOLPP). Das and Roy [113] presented multi-objective non-linear programming problem using neutrosophic optimization technique. Hezam, Abdel-Baset, and Smarandache [114] proposed neutrosophic multi-objective programming problem using Taylor series approximation. Abdel-Baset et al. [115] proposed neutrosophic goal programming. In the studies [112-115], indeterminacy membership function was maximized. Following the studies [112-115] in the literature, Pramanik [116] contended that in real decision making situation, maximizing indeterminacy is not acceptable and the technique with minimizing indeterminacy and falsity is more realistic model. Pramanik [116] presented framework of neutrosophic goal programming by introducing three neutrosophic goal programming models. Pramanik [117] also presented framework for neutrosophic multi-objective linear programming based on the same philosophy [116] of minimizing indeterminacy and falsity simultaneously. Sarkar and Roy [118] recently presented a single objective neutrosophic optimization algorithm where indeterminacy is maximized in one model and in another model indeterminacy is minimized but their difference and impact were not studied. The neutrosophic optimization models [112-125] need further modifications to reflect the real implication in optimization technique.

Smarandache [126-127] developed Neutrosophic Number (NN) and established its basic properties. The NN is expressed in the form (p + qI), where p, q are real numbers and *I* represents indeterminacy. Smarandache [128] defined neutrosophic interval function (thick function). Some theoretical development and application of NNs have been reported in the literature [129-133]. Ye [134] presented some basic operations of NNs and NN function. In the same study, Ye [134] developed a linear programming method with NNs and discussed production planning problem. In 2018, Ye et al. [135] formulated NN nonlinear programming.

Goal programming with neutrosophic coefficient is yet to appear in the literature. To fill the gap, we initiate the single-objective linear programming problem based on goal programming approach. The coefficients of objective function and constraints are considered as neutrosophic number of the form (p + q I), where p, q are real number and I represents indeterminacy. The NNs are converted into interval numbers. The entire programming problem reduces to linear interval programming problem. The target interval of the number functions is constructed using the technique of interval programming. Three new neutrosophic goal programming models are developed to solve the revised problem. Three numerical examples are solved to demonstrate the feasibility, applicability and effectiveness of the proposed models.

The remainder of the paper is organized as follows: Section 2 presents some basic discussions regarding NNs, interval numbers. Section 3 recalls interval linear programming. Section 4 devotes to formulate of single-objective linear goal programming with NNs. Section 5 presents numerical example. Section 6 presents conclusion and future scope of research.

II. SOME BASIC DISCUSSIONS

Here we recall some basic definitions and properties of neutrosophic sets, single valued, neutrosophic sets, neutrosophic numbers, interval numbers.

2.1 Neutrosophic numbers

A neutrosophic number [126, 127] α = m + nI consists of its determinate part m and its indeterminate part nI. Here, m, and n are real numbers and I is indeterminacy.

α = p+ qI, where $I \in [I^L, I^U]$ $\therefore$ $\alpha = [p + q I^U, p + q I^U] = [\alpha^L, \alpha^U]$ (say).

Example 1:

Assume that α = 2+3I, where 2 is the determinate part and 3I is the indeterminate part. Suppose $I \in [0.1, 0.2]$, then α becomes an interval α = [2.3, 2.6]. Thus for a given interval of the part I, NNs are converted into interval number.

2.6 Some basic properties of interval number

Here some basic properties of interval analysis are stated below.

An interval is defined by $\alpha = [\alpha^L, \alpha^U] = \{x \in \Re \mid \alpha^L \le x \le \alpha^U\}$, where α^L and α^U are left and right limit of the interval α on the real line R.

Let m (α) and w (α) be the midpoint and the width respectively of an interval α .

Then, $m(\alpha) = (1/2)(\alpha^L + \alpha^U)$ and $w(\alpha) = (\alpha^U - \alpha^L)$. (1)

The different operations [136] are defined as follows:

The scalar multiplication of α is defined as:

$$\lambda\alpha = \begin{cases}[\lambda\alpha^L, \lambda\alpha^U], \lambda \geq 0 \\ [\lambda\alpha^U, \lambda\alpha^L], \lambda \leq 0\end{cases} \quad (2)$$

Absolute value of α is defined as $|\alpha| = \begin{cases}[\alpha^L, \alpha^U], & \alpha^L \geq 0 \\ [0, \max(-\alpha^L, \alpha^U)], & \alpha^L < 0 < \alpha^U \\ [-\alpha^U, -\alpha^L], & \alpha^U \leq 0\end{cases}$ (3)

(iii) The binary operation '*' are defined between two interval numbers $\alpha = [\alpha^L, \alpha^U]$ and $\beta = [\beta^L, \beta^U]$ as: $\alpha * \beta = \{x * y : x \in \alpha, y \in \beta\}$ where $\alpha^L \leq x \leq \alpha^U$, $\beta^L \leq y \leq \beta^U$.
'*' is designated as any of the operation of four conventional arithmetic operations.

2.7 Some basic properties of NNs

Here we define some properties of NNs [126, 127].

Let $\alpha_1 = p_1 + q_1 I_1$ and $\alpha_2 = p_2 + q_2 I_2$ where $I_1 \in [I_1^L, I_1^U], I_2 \in [I_2^L, I_2^U]$ then

$\therefore \alpha_1 = [p_1 + q_1 I_1^L, p_1 + q_1 I_1^U] = [\alpha_1^L, \alpha_1^U]$(say) and $\alpha_2 = [p_2 + q_2 I_2^L, p_2 + q_2 I_2^U] = [\alpha_2^L, \alpha_2^U]$(say).

$$\alpha_1 + \alpha_2 = [\alpha_1^L + \alpha_2^L, \alpha_1^U + \alpha_2^U]$$

$$\alpha_1 - \alpha_2 = [\alpha_1^L - \alpha_2^U, \alpha_1^U - \alpha_2^L]$$

$$\alpha_1 * \alpha_2 = [\min(\alpha_1^L * \alpha_2^L, \alpha_1^L * \alpha_2^U, \alpha_1^U * \alpha_2^L, \alpha_1^U * \alpha_2^U), \max(\alpha_1^L * \alpha_2^L, \alpha_1^L * \alpha_2^U, \alpha_1^U * \alpha_2^L, \alpha_1^U * \alpha_2^U)]$$

$$\text{(iv)}\ \alpha_1 \div \alpha_2 = \begin{cases}[\alpha_1^L, \alpha_1^U] * [\frac{1}{\alpha_2^U}, \frac{1}{\alpha_2^L}] \text{ or} \\ [\min(\alpha_1^L/\alpha_2^L, \alpha_1^L/\alpha_2^U, \alpha_1^U/\alpha_2^L, \alpha_1^U/\alpha_2^U), \max(\alpha_1^L/\alpha_2^L, \alpha_1^L/\alpha_2^U, \alpha_1^U/\alpha_2^L, \alpha_1^U/\alpha_2^U)] \text{ if } 0 \notin \alpha_2 \\ \text{Undefined if } 0 \in \alpha_2\end{cases}.$$

III. INTERVAL VALUED LINEAR PROGRAMMING

In this section, first we recall the general model of interval linear programming.

$$\text{Optimize } Z_p(\bar{Y}) = \sum_{j=1}^{n} [c_{pj}^L, c_{pj}^U] y_j, \quad p = 1, 2, ..., P \quad (4)$$

subject to

$$\bar{A}\bar{Y} \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} \bar{b}, \quad (5)$$

$$\bar{Y} = (y_1, y_2, ..., y_n) \geq \bar{0} \quad (6)$$

where $\bar{Y}$ is a decision vector of order n x 1, $[c_{pj}^L, c_{pj}^U]$ (j=1,2,...,n; p = 1,2,...,P) is interval coefficient of p-th objective function, $\bar{A}$ is q x n matrix, $\bar{b}$ is q x 1 vector and c_{pj}^L and c_{pj}^U represent lower and upper bounds of the coefficients respectively.

Again, the multi objective linear programming with interval coefficients in objective functions as well as constraints can be presented as:

$$\text{Optimize } Z_p(\bar{Y}) = \sum_{j=1}^{n} [c_{pj}^L, c_{pj}^U] y_j, \quad p = 1, 2, ..., P \quad (7)$$

subject to $\sum_{j=1}^{n} [a_{kj}^L, a_{kj}^U] y_j \leq [b_k^L, b_k^U], \quad k = 1, 2, ..., q$

$y_j \geq 0$, j = 1, 2, …, n

where $\bar{Y}$ is a decision vector of order n x 1, $[c_{Pj}^L, c_{Pj}^U]$, $[b_k^L, b_k^U]$ (j = 1, 2, ..., n; k =1,2,...,q; p = 1, 2,..., P) are closed intervals.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

According to Shaocheng [137] and Ramadan [138] the interval inequality of the form $\sum_{j=1}^{n}[a_j^L y_j, a_j^U y_j] \geq [b^L, b^U]\ \forall y_j \geq 0$ can be transformed into two inequalities

$$\sum_{j=1}^{n} a_j^L y_j \geq b^U, \sum_{j=1}^{n} a_j^U y_j \geq b^L, \forall y_j \geq 0 \tag{8}$$

Minimization problem [136]is stated as follows:

$$\text{Minimize } Z_p(\bar{Y}) = \sum_{j=1}^{n}[c_{pj}^L, c_{pj}^U]y_j, \quad p = 1, 2, \ldots, P \tag{9}$$

subject to $\sum_{j=1}^{n}[a_{kj}^L, a_{kj}^U]y_j \geq [b_k^L, b_k^U], \quad k = 1, 2, \ldots, q$

$y_j \geq 0$, j = 1, 2, …, n.

For the best optimal solution, we solve the problem

$$\text{Minimize } Z_p(\bar{Y}) = \sum_{j=1}^{n} c_{pj}^L y_j, \quad p = 1, 2, \ldots, P \tag{10}$$

subject to

$\sum_{j=1}^{n} a_{kj}^U y_j \geq b_k^L, \quad k = 1, 2, \ldots, q$

$y_j \geq 0$, j = 1, 2, …, n.

For the worst optimal solution, we solve the problem

$$\text{Minimize } Z_p(\bar{Y}) = \sum_{j=1}^{n} c_{pj}^U y_j, \quad p = 1, 2, \ldots, P \tag{11}$$

subject to

$\sum_{j=1}^{n} a_{kj}^L y_j \geq b_k^U, \quad k = 1, 2, \ldots, q$

$y_j \geq 0$, j = 1, 2, …, n.

Suppose, the best solution point by solving (10) is $\bar{Y}^B = (y_1^B, y_2^B, \ldots, y_n^B) \geq 0$ (12)

with the best objective value $Z_p^B(\bar{Y}^B) = \sum_{j=1}^{n} c_{pj}^L y_j^B, \quad p = 1, 2, \ldots, P$ (13)

Suppose, the worst solution point by solving (11) is $\bar{Y}^W = (y^W{}_1, y^W{}_2, \ldots, y^W{}_n) \geq 0$ (14)

with the worst objective value $Z_p^W(\bar{Y}^W) = \sum_{j=1}^{n} c_{pj}^L y_j^W, \quad p = 1, 2, \ldots, P$ (15)

Then the optimal value of the p-th objective function is $[Z_p^B(\bar{Y}^B), Z_p^W(\bar{Y}^W)]$. (16)

Now using the technique of goal programming we get the optimal solution of the problem.

IV. FORMULATIO OF SINGLE –OBJECTIVE LINEAR GOAL PROGRAMMING WITH NNS

Let us consider the minimization problem as follows:

$$\text{Minimize } Z(\bar{Y}) = \sum_{j=1}^{n}(a_j + I_j b_j)y_j \tag{17}$$

subject to $\sum_{j=1}^{n}(c_{kj} + I_{kj} d_{kj})y_j \leq \alpha_K + I_k \beta_k$,

$y_j \geq 0, j = 1, 2, \ldots, n,$

where $I_j \in [I_j^L, I_j^U]$ and $I_{kj} \in [I_{kj}^L, I_{kj}^U]$ $I_k \in [I_k^L, I_k^U]$ $j = 1, 2, \ldots, n$ and $k = 1, 2, \ldots, q$.

Now,

$$Z(\bar{Y}) = \sum_{j=1}^{n} (a_j + I_j b_j) y_j = \sum_{j=1}^{n} [(a_j + I_j^L b_j) y_j, (a_j + I_j^U b_j) y_j]$$

$$= [\sum_{j=1}^{n} (a_j + I_j^L b_j) y_j, \sum_{j=1}^{n} (a_j + I_j^U b_j) y_j] = [Z^L, Z^U] (\text{say})$$

where, $\sum_{j=1}^{n} (a_j + I_j^L b_j) y_j = Z^L(\bar{Y})$ and $\sum_{j=1}^{n} (a_j + I_j^U b_j) y_j = Z^U(\bar{Y})$ (18)

The constraints reduce to

$$\sum_{j=1}^{n} (c_{kj} + I_{kj} d_{kj}) y_j \leq \alpha_k + I_k \beta_k$$

$$\Rightarrow [\sum_{j=1}^{n} (c_{kj} + I_{kj}^L d_{kj}) y_j, \sum_{j=1}^{n} (c_{kj} + I_{kj}^U d_{kj}) y_j] \leq [\alpha_k + I_k^L \beta_k, \alpha_k + I_k^U \beta_k]$$

Let $\alpha_k + I_k^L \beta_k = b_k^L$, $\alpha_k + I_k^U \beta_k = b_k^U$

Then $[\sum_{j=1}^{n} (c_{kj} + I_{kj}^L d_{kj}) y_j, \sum_{j=1}^{n} (c_{kj} + I_{kj}^U d_{kj}) y_j] \leq [b_k^L, b_k^U], k = 1, 2, \ldots, q.$ (19)

Assume that the decision maker fixes $[Z^{*L}, Z^{*U}]$ as the target interval of the objective function Z.
Applying the procedure discussed in the Section 3, we find out the target level of the objective function Z.
Thus we have,

$Z^U \geq Z^{*L}$ and $Z^L \leq Z^{*U}$. (20)

The goal achievement functions are written as:

$-Z^U + d^U = -Z^{*L}$ and $Z^L + d^L = Z^{*U}$. (21)

Here $d^L \geq 0$, and $d^U \geq 0$ are negative deviational variables.

GOAL PROGRAMMING MODEL I (22)

Min $(d^U + d^L)$

subject to

$-Z^U(\bar{Y}) + d^U = -Z^{*L}$,

$Z^L(\bar{Y}) + d^L = Z^{*U}$,

$\sum_{j=1}^{n} (c_{kj} + I_{kj}^L d_{kj}) y_j \leq b_k^U$,

$\sum_{j=1}^{n} (c_{kj} + I_{kj}^U d_{kj}) y_j \leq b_k^L$,

$d^L \geq 0, d^U \geq 0, y_j \geq 0, j = 1, 2, \ldots, n,$ and $k = 1, 2, \ldots, q.$

GOAL PROGRAMMING MODEL II (23)

Min $(w^U d^U + w^L d^L)$

subject to

$-Z^U(\bar{Y}) + d^U = -Z^{*L}$,

$Z^L(\bar{Y}) + d^L = Z^{*U}$,

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

$$\sum_{j=1}^{n}(c_{kj}+I_{kj}^{L}d_{kj})y_j \le b_k^U,$$
$$\sum_{j=1}^{n}(c_{kj}+I_{kj}^{U}d_{kj})y_j \le b_k^L,$$
$$d^L \ge 0, d^U \ge 0,\ w^L, w^U, y_j \ge 0, \text{and } j=1,2,...,n; k=1,2,...,q.$$

Here, w^U, w^L are the numerical weight of corresponding negative deviational variables d^L and d^U respectively prescribed by decision makers.

GOAL PROGRAMMING MODEL III (24)

$$\text{Min}\,\lambda$$

subject to

$$-Z^U(\bar{Y})+d^U=-Z^{*L},$$
$$Z^L(\bar{Y})+d^L=Z^{*U},$$
$$\lambda \ge d^L,$$
$$\lambda \ge d^U,$$
$$\sum_{j=1}^{n}(c_{kj}+I_{kj}^{L}d_{kj})y_j \le b_k^U,$$
$$\sum_{j=1}^{n}(c_{kj}+I_{kj}^{U}d_{kj})y_j \le b_k^L,$$
$$d^L \ge 0, d^U \ge 0,\ y_j \ge 0, \text{and } j=1,2,...,n; k=1,2,...,q.$$

V. NUMERICAL EXAMPLES

Example I

We consider an application in production planning studied by Ye [134].

"*A company manufactures two types of products: Types A and B. To manufacture Type A, its each product needs 9-kgmaterial, 3 + 0.3I working hours, and 4 + 0.4I kW power on the machine 1, and then, each product in Type A can obtain 60 + 6I$ profits, where the indeterminacy I may be Considered as a possible range within the interval [0,1] under some specified situation. In Type B,each product needs 4-kg material, 10 working hours, and 5kW power on the machine 2, and then, each product in Types B can obtain 120$ profits. If the company can provide 360-kg material, 300 working hours, and 200kW power per day, then the company needs how much products in Types A and B must be manufactured, respectively, for each day so as to obtain the maximum profit*".

Let the two types A and B manufacture per day be x_1 and x_2 pieces, respectively. For this case, the NN linear programming model is presented as follows:

$$\text{Max } Z(\bar{X}, I)=(60+6I)x_1+120x_2),$$

Subject to

$$9x_1+4x_2 \le 360,$$
$$(3+0.3I)\,x_1+10\,x_2 \le 300,$$
$$(4+0.4I)\,x_1+5x_2 \le 200,$$
$$x_1 \ge 0,\ x_2 \ge 0,\ I \in [0,1].$$

The problem can be presented as follows:

$$\text{Max } Z(\bar{X})=[60x_1+120x_2, 66x_1+120x_2],$$

Subject to

$$9x_1+4x_2 \le 360,$$
$$[3x_1+10x_2,\ 3.3x_1+10x_2] \le 300,$$
$$[4x_1+5x_2,\ 4.4x_1+5x_2] \le 200,$$
$$x_1 \ge 0,\ x_2 \ge 0,\ I \in [0,1].$$

The problem can be transformed into minimization type as follows:

$-Z(\bar{X}) = [-66x_1 - 120x_2, -60x_1 - 120x_2] = [Z^L, Z^U]$

Subject to

$-9x_1 - 4x_2 \geq -360$,

$[-3.3x_1 - 10x_2, -3x_1 - 10x_2] \geq -300$,

$[-4.4x_1 - 5x_2, -4x_1 - 5x_2] \geq -200$,

$x_1 \geq 0,\ x_2 \geq 0$.

For best solution:

Min $Z^L(\bar{X}) = (-66x_1 - 120x_2)$

$-9x_1 - 4x_2 \geq -360$,

$-3x_1 - 10x_2 \geq -300$,

$-4x_1 - 5x_2 \geq -200$,

$x_1 \geq 0,\ x_2 \geq 0$,

Solving the above model, the obtained solution is Min $Z^U = -4200$ at (20, 24).

For worst solution:

Min $Z^U(\bar{X}) = (-60x_1 - 120x_2)$

$-9x_1 - 4x_2 \geq -360$,

$-3.3x_1 - 10x_2 \geq -300$,

$-4.4x_1 - 5x_2 \geq -200$, $x_1 \geq 0,\ x_2 \geq 0$,

Solving the above model, the obtained solution is Min $Z^U = -3970.91$ at (18.18, 24).

Then the optimal value would be between [-4200, -3970.91]. The optimal value of the original maximization problem would be between [3970.91, 4200].

The objective functions with targets can be written as:

$-66x_1 - 120x_2 \leq -4000$, $-60x_1 - 120x_2 \geq -4200$,

The goal functions with targets can be written as:

$-66x_1 - 120x_2 + d^L = -4000$,

$60x_1 + 120x_2 + d^U = 4200$,

$d^L \geq 0, d^U \geq 0$.

Using the goal programming model (22) for single objective, the GP Model I is presented as follows:

GP Model I

$\text{Min}\,(d^L + d^U)$

$-66x_1 - 120x_2 + d^L = -4000$,

$60x_1 + 120x_2 + d^U = 4200$,

$-9x_1 - 4x_2 \geq -360$,

$-3.3x_1 - 10x_2 \geq -300$,

$-4.4x_1 - 5x_2 \geq -200$,

$-3.3x_1 - 10x_2 \geq -300$,

$-4.4x_1 - 5x_2 \geq -200$,

$x_1 \geq 0,\ x_2 \geq 0$,

$d^L \geq 0, d^U \geq 0$

Using the goal programming model (23) for single objective, the GP Model II is presented as follows:

GP Model II

$\text{Min}\,(w^L d^L + w^U d^U)$

$-66x_1 - 120x_2 + d^L = -4000$,

$60x_1 + 120x_2 + d^U = 4200$,

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

$-9x_1 - 4x_2 \geq -360$,
$-3.3x_1 - 10x_2 \geq -300$,
$-4.4x_1 - 5x_2 \geq -200$,
$-3.3x_1 - 10x_2 \geq -300$,
$-4.4x_1 - 5x_2 \geq -200$,
$x_1 \geq 0,\ x_2 \geq 0$,
$d^L \geq 0, d^U \geq 0,\ w^L \geq 0,\ w^U \geq 0.$

Using the goal programming model (24) for single objective, the GP Model III is presented as follows:

GP Model III

$\text{Min}\,\lambda$

$-66x_1 - 120x_2 + d^L = -4000,$

$60x_1 + 120x_2 + d^U = 4200,$

$-9x_1 - 4x_2 \geq -360$,
$-3.3x_1 - 10x_2 \geq -300$,
$-4.4x_1 - 5x_2 \geq -200$,
$-3.3x_1 - 10x_2 \geq -300$,
$-4.4x_1 - 5x_2 \geq -200$,
$x_1 \geq 0,\ x_2 \geq 0$,
$d^L \geq 0, d^U \geq 0\ \ \lambda \geq d^L, \lambda \geq d^U.$

The obtained optimal solutions from the proposed three GP Models are shown in Table 1.

Table 1: Optimal solution for $I \in [0,1]$

Goal Programming model	Z	$\overline{X}^*$
Goal programming Model I	[3909.12, 4000.032]	(15.152, 25)
Goal programming Model II	[3909.12, 4000.032]	(15.152, 25)
Goal programming Model III	[3970.92, 4080.012]	(18.182, 24)

For I = 0, the problem reduces to

$\text{Max}\ \ Z(\overline{X}) = (60x_1 + 120x_2)$

$9x_1 + 4x_2 \leq 360$,
$3x_1 + 10x_2 \leq 300$,
$4x_1 + 5x_2 \leq 200$, $\ x_1 \geq 0,\ x_2 \geq 0$,

Solving the above model, the obtained solution is Max $Z = 4080$ at (20, 24).

The comparison between proposed model and the existing model of Ye [134] is shown in the Table 2.

Table 2: Comparison between the proposed models and Ye [134] $I \in [0,1]$

Method	Max Z
Proposed model I	[3909.12, 4000.032]
Proposed model II	[3909.12, 4000.032]
Proposed model III	[3970.92, 4080.012]
Ye [132]	[3971,4080]

Example III

Consider a NNLP problem studied by Ye [134] with two variables (unknowns) x_1 and x_2 which is stated as follows:

$\text{Max}\ Z(\overline{X}) = 5x_1 + (4+I)x_2$,

Subject to
$x_1 + 3x_2 \leq 90$,
$2x_1 + (1+I)\,x_2 \leq 80$,
$x_1 + x_2 \leq 45$,

$x_1 \geq 0$, $x_2 \geq 0$, $I \in [0, 0.1]$.

$Z(\bar{X}) = [5x_1 + 4x_2, 5x_1 + 4.1x_2]$,

Subject to

$x_1 + 3x_2 \leq 90$,

$[2x_1 + x_2, 2x_1 + 1.1x_2] \leq 80$,

$x_1 + x_2 \leq 45$,

$x_1 \geq 0$, $x_2 \geq 0$, $I \in [0, 0.1]$.

$-Z(\bar{X}) = [-5x_1 - 4.1x_2, -5x_1 - 4x_2] = [Z_1^L, Z_1^U]$,

Subject to

$-x_1 - 3x_2 \geq -90$,

$[-2x_1 - 1.1x_2, -2x_1 - x_2] \geq -80$,

$-x_1 - x_2 \geq -45$, $x_1 \geq 0$, $x_2 \geq 0$.

For worst solution:

$\text{Min } Z^U(\bar{X}) = (-5x_1 - 4x_2)$

$-x_1 - 3x_2 \geq -90$,

$-2x_1 - 1.1x_2 \geq -80$,

$-x_1 - x_2 \geq -45$, $x_1 \geq 0$, $x_2 \geq 0$.

Solving the above model, the obtained solution is Min $Z^U = -213$ at (33.89, 11.11).

For best solution:

$\text{Min } Z^L(\bar{X}) = (-5x_1 - 4.1x_2)$

$-x_1 - 3x_2 \geq -90$,

$-2x_1 - x_2 \geq -80$,

$-x_1 - x_2 \geq -45$, $x_1 \geq 0$, $x_2 \geq 0$.

Solving the above model, the obtained solution is Min $Z^L = -216$ at (35, 10).

Then the optimal value would be between [-216, -213]. The optimal value of the original maximization problem would be between [213, 216].

The objective functions with targets can be written as:

$-5x_1 - 4.1x_2 \leq -213, -5x_1 - 4x_2 \geq -216,$

The goal functions with targets can be written as:

$-5x_1 - 4.1x_2 + d^L = -213,$

$5x_1 + 4x_2 + d^U = 216,$

$d^L \geq 0, d^U \geq 0.$

Using the goal programming model (22) for single objective, the GP Model I has been described as follows:

GP Model I

$\text{Min}(d^L + d^U)$

$-5x_1 - 4.1x_2 + d^L = -213,$

$5x_1 + 4x_2 + d^U = 216,$

$-x_1 - 3x_2 \geq -90$,

$-2x_1 - x_2 \geq -80$,

$-2x_1 - 1.1x_2 \geq -80$

$-x_1 - x_2 \geq -45$,

$x_1 \geq 0$, $x_2 \geq 0$.

$d^L \geq 0, d^U \geq 0$

Using the goal programming model (23) for single objective, the GP Model II is presented as follows:

GP Model II

$\text{Min}(w^L d^L + w^U d^U)$

$-5x_1 - 4.1x_2 + d^L = -213,$

$5x_1 + 4x_2 + d^U = 216,$

$-x_1 - 3x_2 \geq -90,$

$-2x_1 - x_2 \geq -80,$

$-2x_1 - 1.1x_2 \geq -80$

$-x_1 - x_2 \geq -45,$

$d^L \geq 0, d^U \geq 0,\ w^L \geq 0,\ w^U \geq 0,\ x_1 \geq 0,\ x_2 \geq 0.$

Using the goal programming model (24) for single objective, the GP Model III is presented as follows:

GP Model III

$\text{Min}\,\lambda$

$-5x_1 - 4.1x_2 + d^L = -213,$

$5x_1 + 4x_2 + d^U = 216,$

$-x_1 - 3x_2 \geq -90,$

$-2x_1 - x_2 \geq -80,$

$-2x_1 - 1.1x_2 \geq -80$

$-x_1 - x_2 \geq -45,$

$\lambda \geq d^L, \lambda \geq d^U,$

$d^L \geq 0, d^U \geq 0,\ x_1 \geq 0,\ x_2 \geq 0.$

The obtained optimal solutions from the proposed three GP Models are shown in Table 3.

Table 3: Optimal solution for $I \in [0, 0.1]$

Goal programming model	Z	$\overline{X}^*$
Goal programming Model I	[212.02, 212.983]	(34.70, 9.63)
Goal programming Model II	[212.02, 212.983]	(34.70, 9.63)
Goal programming Model III	[213.89, 215.001]	(33.89, 11.11)

The comparison between proposed model and the existing model of Ye [134] is shown in the Table 4.

Table 4: Comparison between the proposed GP Models and Ye [134].

Method	Max Z
Proposed model I	[212.02, 212.983]
Proposed model II	[212.02, 212.983]
Proposed model III	[213.89, 215.001]
Ye [134]	[170, 270]

VI. CONCLUSION

This paper has presented single-objective linear goal programming problem with neutrosophic numbers as coefficients of both objective functions and constraints. The neutrosophic coefficient of the form p+ qI is converted into interval coefficient with the prescribed range of I. Adopting the concept of solving linear interval programming problem, three new neutrosophic goal programming models have been developed and solved by considering two numerical examples. Comparative analysis with the existing models has been provided.

We hope that the proposed method for solving single-objective linear goal programming with neutrosophic coefficients will open up a new way for the future research work on neutrosophic optimization technique. Using this approach many areas involving neutrosophic number of the form p + qI can be explored. The proposed model can be extended to multi-objective programming problem with neutrosophic numbers.

ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

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ISSN: 2277-9655
Impact Factor: 5.164
CODEN: IJESS7

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