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SINGLE-VALUED NEUTROSOPHIC HYPERGRAPHS

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Abstract. We introduce certain concepts, including single-valued neutrosophic hypergraph, line graph of single-valued neutrosophic hypergraph, dual single-valued neutrosophic hypergraph and transversal single-valued neutrosophic hypergraph.

Keywords: single-valued neutrosophic hypergraph, dual single-valued neutrosophic hypergraph.

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1. Introduction

Fuzzy set theory was introduced by Zadeh [17] to solve difficulties in dealing with uncertainties. Since then the theory of fuzzy sets and fuzzy logic have been examined by many researchers to solve many real life problems, involving ambiguous and uncertain environment. Atanassov [3] introduced the concept of intuitionistic fuzzy set as an extension of Zadeh's fuzzy set [17]. An intuitionistic fuzzy set can be viewed as an alternative approach when available information is not sufficient to define the impreciseness by the conventional fuzzy set. In fuzzy sets the degree of acceptance is considered only but intuitionistic fuzzy set is characterized by a membership(truth-membership) function and a non-membership(falsity-membership) function, the only requirement is that the sum of both values is less and equal to one. Intuitionistic fuzzy set can deal only with incomplete information but not the indeterminate information and inconsistent information which commonly exist in certainty system. In intuitionistic fuzzy sets, indeterminacy is its hesitation part by default. Smarandache [13] initiated the concept of neutrosophic set in 1998. "It is the branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interaction with different ideational spectra"[13]. A neutrosophic set is characterized by three components: truth-membership, indeterminacy-membership, and falsity-membership which are represented independently for dealing problems involving imprecise, indeterminacy and inconsistent data. In neutrosophic set, truth-membership, falsity-membership are independent, indeterminacy membership quantified explicitly, this assumption helps in a lot of situations such as information fusion when try to combine the data from different sensors. A neutrosophic set is a general framework which generalizes the concept of fuzzy set, interval valued fuzzy set, and intuitionistic fuzzy set. The single

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valued neutrosophic set was introduced for the first time by Smarandache in 1998 in his book: F. Smarandache, Neutrosophy / Neutrosophic probability, set, and logic, American Res. Press, see pages 7-8, 1998, which is also mentioned by Denis Howe, from England, in The Free Online Dictionary of Computing, 1999. Wang [15] discussed theoretic operators on single-valued neutrosophic set. Fuzzy hypergraph was introduced by the Kaufmann [7]. Chen [5] introduced interval-valued fuzzy hypergraphs. Lee-kwang *et al.* generalized the concept of fuzzy hypergraph and redefined it to be useful for fuzzy partition of a system. Akram and Dudek [1] investigated some properties of intuitionistic fuzzy hypergraph and gave applications of intuitionistic fuzzy hypergraph. In this paper, we introduce certain concepts, including single-valued neutrosophic hypergraph, line graph of single-valued neutrosophic hypergraph, dual single-valued neutrosophic hypergraph and transversal single-valued neutrosophic hypergraph.

2. Single-Valued Neutrosophic Hypergraphs

Definition 2.1. [15] *Let X be a space of points (objects), with a generic element in X denoted by x. A single-valued neutrosophic set (SVNS) A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each point x in X, $T_A(x), I_A(x), F_A(x) \in [0,1]$, i.e., $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.*

Definition 2.2. *The support of a single-valued neutrosophic set $A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$ is denoted by supp(A), defined by supp(A) $= \{x \mid T_A(x) \neq 0, I_A(x) \neq 0, F_A(x) \neq 0\}$. The support of a single-valued neutrosophic set is a crisp set.*

Definition 2.3. *The height of a single-valued neutrosophic set $A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$ is defined as $h(A) = (\sup_{x\in X} T_A(x), \sup_{x\in X} I_A(x), \inf_{x\in X} F_A(x))$. We call single-valued neutrosophic set A is normal if there exist at least one element $x \in X$ such that $T_A(x) = 1, I_A(x) = 1, F_A(x) = 0$.*

Definition 2.4. *Let $A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$ be a single-valued neutrosophic set on X and let $\alpha, \beta, \gamma \in [0,1]$ such that $\alpha + \beta + \gamma \le 3$. Then the set $A_{(\alpha,\beta,\gamma)} = \{x \mid T_A(x) \ge \alpha, I_A(x) \ge \beta, F_A(x) \le \gamma\}$ is called $(\alpha,\beta,\gamma)-$level subset of A. $(\alpha,\beta,\gamma)-$level set is a crisp set.*

Definition 2.5. *Let $V = \{v_1, v_2, \cdots, v_n\}$ be a finite set and $\mathbf{E} = \{E_1, E_2, \cdots, E_m\}$ be a finite family of non-trivial single-valued neutrosophic subsets of V such that*

$$V = \bigcup_i \text{supp}(E_i), i = 1, 2, 3, \cdots, m,$$

where the edges E_i are single-valued neutrosophic subsets of V, $E_i = \{(v_j, T_{E_i}(v_j), I_{E_i}(v_j), F_{E_i}(v_j))\}, E_i \neq \emptyset$, for $i = 1, 2, 3, \cdots, m$. Then the pair $H = (V, \mathbf{E})$ is a single-valued neutrosophic hypergraph on V, $\mathbf{E}$ is the family of single-valued neutrosophic hyperedges of H and V is the crisp vertex set of H.

In single-valued neutrosophic hypergraph two vertices u and v are adjacent if there exist an edge $E_i \in \mathbf{E}$ which contains the two vertices v and u, *i.e.,* $u, v \in \text{supp}(E_i)$. In single-valued neutrosophic hypergraphs H, if two vertices u and v are connected then there exists a sequence $u = u_0, u_1, u_2, \cdots, u_n = v$ of vertices of H such that u_{i-1} is adjacent u_i for $i = 1, 2, \cdots, n$. A connected single-valued neutrosophic hypergraph is a single-valued neutrosophic hypergraph in which every pair of vertices are connected. In a single-valued neutrosophic hypergraph two edges E_i and E_j are said to be adjacent if their intersection is non-empty, *i.e.*, $supp(E_i) \cap supp(E_j) \neq \emptyset, i \neq j$. The order of a single-valued neutrosophic hypergraph is denoted by $|V|$ and size (number of edges) is denoted by $|\mathbf{E}|$. If $|supp(E_i)| = k$ for each $E_i \in \mathbf{E}$, then single-valued neutrosophic hypergraph $H = (V, \mathbf{E})$ is k-uniform single-valued neutrosophic hypergraph. The element a_{ij} of the single-valued neutrosophic matrix represents the truth-membership (participation) degree, indeterminacy-membership degree and falsity-membership of v_i to E_j (that is $(T_{E_j}(v_i), I_{E_j}(v_i), F_{E_j}(v_i)))$. Since the diagram of single-valued neutrosophic hypergraph does not imply sufficiently the truth-membership degree, indeterminacy-membership degree and falsity-membership degree of vertex to edges, we use incidence matrix M_H for the description of single-valued neutrosophic hyperedes.

Definition 2.6. *The height of a single-valued neutrosophic hypergraph $H = (V, \mathbf{E})$, is denoted by $h(H)$, is defined by $h(H) = \bigvee_i \{h(E_i)|\ E_i \in \mathbf{E}\}$.*

Definition 2.7. *Let $H = (V, \mathbf{E})$ be a single-valued neutrosophic hypergraph, the cardinality of a single-valued neutrosophic hyperedge is the sum of truth-membership, indeterminacy-membership, and falsity-membership values of the vertices connected to an hyperedge, it is denoted by $|E_i|$. The degree of a single-valued neutrosophic hyperedge, $E_i \in \mathbf{E}$ is its cardinality, that is $d_H(E_i) = |E_i|$. The rank of a single-valued neutrosophic hypergraph is the maximum cardinality of any hyperedge in H, i.e., $\max_{E_i \in \mathbf{E}} d_H(E_i)$ and anti rank of a single-valued neutrosophic is the minimum cardinality of any hyperedge in H, i.e., $\min_{E_i \in \mathbf{E}} d_H(E_i)$.*

Definition 2.8. *A single-valued neutrosophic hypergraph is said to be linear single-valued neutrosophic hypergraph if every pair of distinct vertices of $H = (V, \mathbf{E})$ is in at most one edge of H, i.e., $|supp(E_i) \cap supp(E_j)| \leq 1$ for all $E_i, E_j \in \mathbf{E}$. A $2-$uniform linear single-valued neutrosophic hypergraph is a single-valued neutrosophic graph.*

Example 2.1. *Consider a single-valued neutrosophic hypergraph $H = (V, \mathbf{E})$ such that $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, $\mathbf{E} = \{E_1, E_2, E_3, E_4, E_5, E_6\}$, where $E_1 = \{(v_1, 0.3, 0.4, 0.6), (v_3, 0.7, 0.4, 0.4)\}, E_2 = \{(v_1, 0.3, 0.4, 0.6), (v_2, 0.5, 0.7, 0.6)\}, E_3 = \{(v_2, 0.5, 0.7, 0.6), (v_4, 0.6, 0.4, 0.8)\}$, $E_4 = \{(v_3, 0.7, 0.4, 0.4), (v_6, 0.4, 0.2, 0.7)\}$, $E_5 = \{(v_3, 0.7, 0.4, 0.4), (v_5, 0.6, 0.7, 0.5)\}$, $E_6 = \{(v_5, 0.6, 0.7, 0.5), (v_6, 0.4, 0.2, 0.7)\}$, and $E_7 = \{(v_4, 0.6, 0.4, 0.8), (v_6, 0.4, 0.2, 0.7)\}$.*

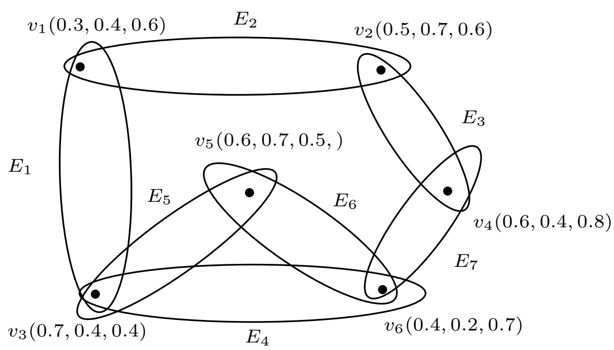

FIGURE 1. Single-valued neutrosophic hypergraph.

The single-valued neutrosophic hypergraph is shown in Figure. 1 and its incidence matrix M_H is given as follows:

M_H	E_1	E_2	E_3	E_4	E_5	E_6	E_7
v_1	$(0.3, 0.4, 0.6)$	$(0.3, 0.4, 0.6)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
v_2	$(0, 0, 0)$	$(0.5, 0.7, 0.6)$	$(0.5, 0.7, 0.6)$	$(0, 0., 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
v_3	$(0.7, 0.4, 0.4)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.7, 0.4, 0.4)$	$(0.7, 0.4, 0.4)$	$(0, 0, 0)$	$(0, 0, 0)$
v_4	$(0, 0, 0)$	$(0, 0, 0)$	$(0.6, 0.4, 0.8)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.6, 0.4, 0.8)$
v_5	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.6, 0.7, 0.5)$	$(0.6, 0.7, 0.5)$	$(0, 0, 0)$
v_6	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.4, 0.2, 0.7)$	$(0, 0, 0)$	$(0.4, 0.2, 0.7)$	$(0.4, 0.2, 0.7)$

Definition 2.9. *Let $H = (V, \mathbf{E})$ be a single-valued neutrosophic hypergraph, the degree $d_H(v)$ of a vertex v in H is $d_H(v) = \sum\limits_{v \in E_i} (T_{E_i}(v), I_{E_i}(v), F_{E_i}(v))$, where E_i are the edges that contain the vertex v. The maximum degree of a single-valued neutrosophic hypergraph is $\triangle(H) = \max\limits_{v \in V}(d_H(v))$. A single-valued neutrosophic hypergraph is said to be regular single-valued neutrosophic hypergraph in which all the vertices have same degree.*

Proposition 2.1. *Let $H = (V, \mathbf{E})$ be a single-valued neutrosophic hypergraph, then*

$$\sum_{v \in V} d_H(v) = \sum_{E_i \in \mathbf{E}} d_H(E_i).$$

Proof. Let M_H be the incidence matrix of single-valued neutrosophic hypergraph H, then the sum of the degrees of each vertex $v_i \in V$ and the sum of degrees of each edge $E_i \in \mathbf{E}$ are equal. We obtain $\sum\limits_{v \in V} d_H(v) = \sum\limits_{E_i \in \mathbf{E}} d_H(E_i)$. □

Definition 2.10. *The strength η of a single-valued neutrosophic hyperedge E_i is the minimum of truth-membership, indeterminacy-membership and maximum falsity-membership values in the edge E_i, i.e.,*

$$\eta(E_i) = \{\min_{v_j \in E_i} (T_{E_i}(v_j) \mid T_{E_i}(v_j) > 0), \min_{v_j \in E_i} (I_{E_i}(v_j) \mid I_{E_i}(v_j) > 0), \max_{v_j \in E_i} (F_{E_i}(v_j) \mid F_{E_i}(v_j) > 0)\}.$$

The strength of an edge in single-valued neutrosophic hypergraph interprets that the edge E_i group elements having participation degree at least $\eta(E_i)$.

Example 2.2. *Consider single-valued neutrosophic hypergraph as shown in Figure. 1, the height of H is $h(H) = (0.7, 0.7, 0.4)$, the strength of each edge is $\eta(E_1) = (0.3, 0.4, 0.6), \eta(E_2) = (0.3, 0.4, 0.6), \eta(E_3) = (0.5, 0.4, 0.8), \eta(E_4) = (0.4, 0.2, 0.7), \eta(E_5) = (0.6, 0.4, 0.5), \eta(E_6) = (0.4, 0.2, 0.7)$ and $\eta(E_7) = (0.4, 0.2, 0.8)$, respectively. The edges with high strength are called the strong edges because the interrelation (cohesion) in them is strong. Therefore, E_5 is stronger than each E_i, for $i = 1, 2, 3, 4, 6, 7$.*
If we assign $\eta(E_i) = (T_{\eta(E_i)}, I_{\eta(E_i)}, F_{\eta(E_i)})$ to each clique in single-valued neutrosophic graph mapped to an edge E_i in single-valued neutrosophic hypergraph, we obtain a single-valued neutrosophic graph which represents subset with grouping strength (interrelationship).

M_H	E_1	E_2	E_3	E_4	E_5	E_6	E_7
v_1	$(0.3, 0.4, 0.6)$	$(0.3, 0.4, 0.6)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
v_2	$(0, 0, 0)$	$(0.3, 0.4, 0.6)$	$(0.5, 0.4, 0.8)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
v_3	$(0.3, 0.4, 0.6)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.4, 0.2, 0.7)$	$(0.6, 0.4, 0.5)$	$(0, 0, 0)$	$(0, 0, 0)$
v_4	$(0, 0, 0)$	$(0, 0, 0)$	$(0.5, 0.4, 0.8)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.4, 0.2, 0.8)$
v_5	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.6, 0.4, 0.5)$	$(0.4, 0.2, 0.7)$	$(0, 0, 0)$
v_6	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.4, 0.2, 0.7)$	$(0, 0, 0)$	$(0.4, 0.2, 0.7)$	$(0.4, 0.2, 0.8)$

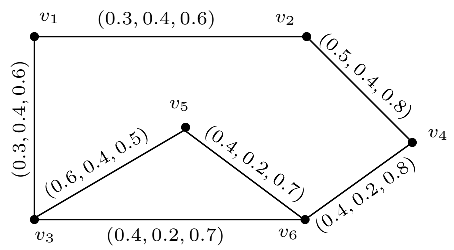

FIGURE 2. Corresponding single-valued neutrosophic graph.

We see that a single-valued neutrosophic graph can be associated with a single-valued neutrosophic hypergraph, a hyperedge with its strength η in the single-valued neutrosophic hypergraph is mapped to a clique in the single-valued neutrosophic graph, all edges in the clique have the same strength. Figure. 2 shows corresponding single-valued neutrosophic graph to the single-valued neutrosophic hypergraph H shown in Figure. 1. In corresponding single-valued neutrosophic graph, the numbers attached to the edges represents the truth-membership, indeterminacy-membership and falsity-membership of the edges.

Proposition 2.2. *Single-valued neutrosophic graphs and single-valued neutrosophic digraphs are special cases of the single-valued neutrosophic hypergraphs.*

Definition 2.11. *A single-valued neutrosophic set $A = \{(x, T_A(x)), I_A(x), F_A(x) \mid x \in X\}$ is an elementary single-valued neutrosophic set if A is single-valued on supp(A). An elementary single-valued neutrosophic hypergraph $H = (V, \mathbf{E})$ is a single-valued neutrosophic hypergraph in which each element of $\mathbf{E}$ is elementary.*

Definition 2.12. *A single-valued neutrosophic hypergraph $H = (V, \mathbf{E})$ is called simple single-valued neutrosophic hypergraph if $\mathbf{E}$ has no repeated single-valued neutrosophic hyperedges and whenever $E_i, E_j \in \mathbf{E}$ and $T_{E_i} \leq T_{E_j}, I_{E_i} \leq I_{E_j}, F_{E_i} \geq F_{E_j}$, then $T_{E_i} = T_{E_j}, I_{E_i} = I_{E_j}, F_{E_i} = F_{E_j}$. A single-valued neutrosophic hypergraph is called support simple, if whenever $E_i, E_j \in \mathbf{E}, E_i \subset E_j$ and supp(E_i) =supp(E_j), then $E_i = E_j$. A single-valued neutrosophic hypergraph is called strongly support simple if whenever $E_i, E_j \in \mathbf{E}$, and supp(E_i) =supp(E_j), then $E_i = Ej$.*

Definition 2.13. *Let $H = (V, \mathbf{E})$ be a single-valued neutrosophic hypergraph. Suppose that $\alpha, \beta, \gamma \in [0, 1]$. Let $E^{(\alpha,\beta,\gamma)} = \{E_i^{(\alpha,\beta,\gamma)} \mid E_i \in \mathbf{E}\}$ and $V^{(\alpha,\beta,\gamma)} = \bigcup_{E_i \in \mathbf{E}} E_i^{(\alpha,\beta,\gamma)}$. $H^{(\alpha,\beta,\gamma)} = (V^{(\alpha,\beta,\gamma)}, E^{(\alpha,\beta,\gamma)})$ is the (α, β, γ)-level hypergraph of $H = (V, \mathbf{E})$, where $E^{(\alpha,\beta,\gamma)} \neq \emptyset$. $H^{(\alpha,\beta,\gamma)}$ is a crisp hypergraph.*

Remark 2.1.
1. *A single-valued neutrosophic hypergraph $H = (V, \mathbf{E})$ is a single-valued neutrosophic graph (with loops) if and only if H is elementary, support simple and each edge has two (or one) element support.*
2. *For a simple single-valued neutrosophic hypergraph $H = (V, \mathbf{E})$, (α, β, γ)-level hypergraph $H^{(\alpha,\beta,\gamma)}$ may or may not be simple single-valued neutrosophic hypergraph. Clearly it is possible that $E_i^{(\alpha,\beta,\gamma)} = E_j^{(\alpha,\beta,\gamma)}$ for $E_i \neq E_j$.*
3. *$\mathcal{H}$ and $\mathcal{H}'$ are two families of crisp sets (hypergraphs) produced by the $(\alpha, \beta, \gamma)-$cuts of a single-valued neutrosophic hypergraph share an important relationship with each other such that for each set $H \in \mathcal{H}$ there is at least one set $H' \in \mathcal{H}'$ which contains H. We say that $\mathcal{H}'$ absorbs $\mathcal{H}$, i.e., $\mathcal{H} \sqsubseteq \mathcal{H}'$. Since it is possible $\mathcal{H}'$*

absorbs $\mathcal{H}$ while $\mathcal{H}' \cap \mathcal{H} = \emptyset$, we have that $\mathcal{H} \subseteq \mathcal{H}'$ implies $\mathcal{H} \sqsubseteq \mathcal{H}'$, but the converse is generally false, If $\mathcal{H} \sqsubseteq \mathcal{H}'$ and $\mathcal{H} \neq \mathcal{H}'$, then $\mathcal{H} \sqsubset \mathcal{H}'$.

Definition 2.14. *Let $H = (V, \mathbf{E})$ be a single-valued neutrosophic hypergraph, and let $h(H) = (r, s, t)$, $H^{(r_i,s_i,t_i)} = (V^{(r_i,s_i,t_i)}, E^{(r_i,s_i,t_i)})$ be the $(r_i, s_i, t_i)-$level hypergraphs of H. The sequence of real numbers $\{(r_1, s_1, t_1), (r_2, s_2, t_2), \cdots, (r_n, s_n, t_n)\}$, $0 < r_n < r_{n-1} < \cdots < r_1 = r$, $0 < s_n < s_{n-1} < \cdots < s_1 = s$, and $t_n > t_{n-1} > \cdots > t_1 = t > 0$, which satisfies the properties:*

1. *if $r_{i+1} < r' < r_i, s_{i+1} < s' < s_i, t_{i+1} > t' > t_i (t_i < t' < t_{i+1})$, then $E^{(r',s',t')} = E^{(r_i,s_i,t_i)}$,*
2. *$E^{(r_i,s_i,t_i)} \sqsubset E^{(r_i+1,s_i+1,t_i+1)}$,*

is called the fundamental sequence of H, and is denoted by $\mathrm{F}(H)$ and the set of $(r_i, s_i, t_i)-$level hypergraphs $\{H^{(r_1,s_1,t_1)}, H^{(r_2,s_2,t_2)}, \cdots, H^{(r_n,s_n,t_n)}\}$ is called the set of core hypergraphs of H, and is denoted by $\mathrm{C}(H)$.

If $r_1 < r \leq 1, s_1 < s \leq 1, 0 \leq t < t_1$, then $E^{(r,s,t)} = \{\emptyset\}$ and $H^{(r,s,t)}$ does not exist.

Definition 2.15. *Suppose $H = (V, \mathbf{E})$ is a single-valued neutrosophic hypergraph with $\mathrm{F}(H) = \{(r_1, s_1, t_1), (r_2, s_2, t_2), \cdots, (r_n, s_n, t_n)\}$ and $r_{n+1} = 0, s_{n+1} = 0, t_{n+1} = 0$. Then H is called sectionally elementary if for each $E_i \in E$ and each $(r_i, s_i, t_i) \in \mathrm{F}(H)$, $E_i^{(r_i,s_i,t_i)} = E_i^{(r,s,t)}$ for all $(r, s, t) \in \big((r_{i+1}, s_{i+1}, t_{i+1}), (r_i, s_i, t_i)\big]$.*

Definition 2.16. *Suppose that $H = (V, \mathbf{E})$ and $H' = (V', \mathbf{E}')$ are single-valued neutrosophic hypergraphs. H is called a partial single-valued neutrosophic hypergraph of H' if $\mathbf{E} \subseteq \mathbf{E}'$. If H is partial single-valued neutrosophic hypergraph of H', we write $H \subseteq H'$. If H is partial single-valued neutrosophic hypergraph of H' and $\mathbf{E} \subset \mathbf{E}'$, then we denote as $H \subset H'$.*

Example 2.3. *Consider the single-valued neutrosophic hypergraph $H = (V, \mathbf{E})$, where $V = \{v_1, v_2, v_3, v_4\}$ and $\mathbf{E} = \{E_1, E_2, E_3, E_4, E_5\}$, which is represented by the following incidence matrix:*

M_H	E_1	E_2	E_3	E_4	E_5
v_1	$(0.7, 0.6, 0.5)$	$(0.9, 0.8, 0.1)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.4, 0.3, 0.3)$
v_2	$(0.7, 0.6, 0.5)$	$(0.9, 0.8, 0.1)$	$(0.9, 0.8, 0.1)$	$(0.7, 0.6, 0.5)$	$(0, 0, 0)$
v_3	$(0, 0, 0)$	$(0, 0, 0)$	$(0.9, 0.8, 0.1)$	$(0.7, 0.6, 0.5)$	$(0.4, 0.3, 0.3)$
v_4	$(0, 0, 0)$	$(0.4, 0.3, 0.3)$	$(0, 0, 0)$	$(0.4, 0.3, 0.3)$	$(0.4, 0.3, 0.3)$

Clearly, $h(H) = (0.9, 0.8, 0.1)$, $E_1^* = E^{(0.9,0.8,0.1)} = \{\{v_2, v_3\}\}, E_2^* = E^{(0.7,0.6,0.5)} = \{\{v_1, v_2\}\}$ and $E_3^* = E^{(0.4,0.3,0.3)} = \{\{v_1, v_2, v_4\}, \{v_2, v_3\}, \{v_2, v_3, v_4\}\}$. Therefore, fundamental sequence is $\mathrm{F}(H) = \{(r_1, s_1, t_1) = (0.9, 0.8, 0.1), (r_2, s_2, t_2) = (0.7, 0.6, 0.5), (r_3, s_3, t_3) = (0.4, 0.3, 0.3)\}$ and the set of core hypergraph is $\mathrm{C}(H) = \{H^{(0.9,0.8,0.1)} = (V_1, E_1^*), H^{(0.7,0.6,0.5)} = (V_2, E_2^*), H^{(0.4,0.3,0.3)} = (V_3, E_3^*)\}$.
Note that, $E^{(0.9,0.8,0.1)} \sqsubseteq E^{(0.4,0.3,0.3)}$ and $E^{(0.9,0.8,0.1)} \neq E^{(0.4,0.3,0.3)}$. As $E_5 \subseteq E_2$, H is not simple single-valued neutrosophic hypergraph but H is support simple. In single-valued neutrosophic graph $H = (V, \mathbf{E})$, $E^{(r,s,t)} \neq E^{(0.9,0.8,0.1)}$ for $(r, s, t) = (0.7, 0.6, 0.5)$, H is not sectionally elementary.

The partial single-valued neutrosophic hypergraphs, $H' = (V', E')$, where $E' = \{E_2, E_3, E_4, E_1\}$ is simple, $H'' = (V'', E'')$, where $E'' = \{E_2, E_3, E_5\}$ is sectionally elementary, and $H''' = (V''', E''')$, where $E''' = \{E_1, E_3, E_5\}$ is elementary.

Definition 2.17. *A single-valued neutrosophic hypergraph H is said to be ordered if* $\mathrm{C}(H)$ *is ordered. That is, if* $\mathrm{C}(H) = \{H^{(r_1,s_1,t_1)}, H^{(r_2,s_2,t_2)}, \cdots, H^{(r_n,s_n,t_n)}\}$*, then* $H^{(r_1,s_1,t_1)} \subseteq H^{(r_2,s_2,t_2)} \subseteq \cdots \subseteq H^{(r_n,s_n,t_n)}$*. The single-valued neutrosophic hypergraph H is simply ordered if* $\mathrm{C}(H)$ *is ordered and if whenever* $E^* \in E^*_{i+1} \setminus E^*_i$*, then* $E^* \nsubseteq V_i$*.*

Proposition 2.3. *If $H = (V, \mathbf{E})$ is an elementary single-valued neutrosophic hypergraph, then H is ordered. Also, if $H = (V, \mathbf{E})$ is an ordered single-valued neutrosophic hypergraph with* $\mathrm{C}(H) = \{H^{(r_1,s_1,t_1)}, H^{(r_2,s_2,t_2)}, \cdots, H^{(r_n,s_n,t_n)}\}$ *and if* $H^{(r_n,s_n,t_n)}$ *is simple, then H is elementary.*

Definition 2.18. *A single-valued neutrosophic hypergraph $H = (V, \mathbf{E})$ is called a E^t tempered single-valued neutrosophic hypergraph of $H^* = (V, E^*)$ if there is a crisp hypergraph $H^* = (V, E^*)$ and single-valued neutrosophic set E^t is defined on V, where* $T_{E^t} : V \to (0, 1]$, $I_{E^t} : V \to (0, 1]$, *and* $F_{E^t} : V \to (0, 1]$ *such that* $\mathbf{E} = \{C_E \mid E \in E^*\}$*, where*

$$T_{C_E}(x) = \begin{cases} \wedge\{T_{E^t}(y) \mid y \in E\}, & \textit{if } x \in E; \\ 0, & \textit{otherwise.} \end{cases}$$

$$I_{C_E}(x) = \begin{cases} \wedge\{I_{E^t}(y) \mid y \in E\}, & \textit{if } x \in E; \\ 0, & \textit{otherwise.} \end{cases}$$

$$F_{C_E}(x) = \begin{cases} \vee\{F_{E^t}(y) \mid y \in E\}, & \textit{if } x \in E; \\ 0, & \textit{otherwise.} \end{cases}$$

We let $E^t \otimes H^*$ denotes the E^t tempered single-valued neutrosophic hypergraph of H^* determined by the crisp hypergraph $H^* = (V, E^*)$ and the single-valued neutrosophic set E^t.

Example 2.4. *Consider the single-valued neutrosophic hypergraph $H = (V, \mathbf{E})$, where $V = \{v_1, v_2, v_3, v_4\}$ and $\mathbf{E} = \{E_1, E_2, E_3, E_4\}$, which is represented by the following incidence matrix:*

M_H	E_1	E_2	E_3	E_4
v_1	$(0.3, 0.4, 0.6)$	$(0, 0, 0)$	$(0.1, 0.4, 0.5)$	$(0.3, 0.4, 0.5)$
v_2	$(0, 0, 0)$	$(0.1, 0.4, 0.3)$	$(0, 0, 0)$	$(0.3, 0.4, 0.5)$
v_3	$(0.3, 0.4, 0.6)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
v_4	$(0, 0, 0)$	$(0.1, 0.4, 0.3)$	$(0.1, 0.4, 0.5)$	$(0, 0, 0)$

Define $E^t = \{(v_1, 0.3, 0.4, 0.5), (v_2, 0.6, 0.5, 0.2), (v_3, 0.5, 0.4, 0.6), (v_4, 0.1, 0.4, 0.3)\}$*. Note that* $T_{\{v_1,v_3\}}(v_1) = T_{E^T}(v_1) \wedge T_{E^T}(v_3) = 0.3, I_{\{v_1,v_3\}}(v_1) = I_{E^t}(v_1) \wedge I_{E^t}(v_3) = 0.4, F_{\{v_1,v_3\}}(v_1) = F_{E^T}(v_1) \vee F_{E^T}(v_3) = 0.6$*, and* $T_{\{v_1,v_3\}}(v_3) = T_{E^T}(v_3) \wedge T_{E^T}(v_1) = 0.3, I_{\{v_1,v_3\}}(v_3) = I_{E^t}(v_3) \wedge I_{E^t}(v_1) = 0.4, F_{\{v_1,v_3\}}(v_3) = F_{E^T}(v_3) \vee F_{E^T}(v_1) = 0.6$*, then* $C_{\{v_1,v_3\}} = E_1$*. Also* $C_{\{v_2,v_4\}} = E_2$, $C_{\{v_1,v_4\}} = E_3$, $C_{\{v_1,v_2\}} = E_4$*. Thus H is E^t tempered.*

Theorem 2.1. *A single-valued neutrosophic hypergraph $H = (V, \mathbf{E})$ is a E^t tempered single-valued neutrosophic hypergraph of some crisp hypergraph H^* if and only if H is elementary, support simple and simple ordered.*

Proof. Suppose $H = (V, \mathbf{E})$ is a E^t tempered single-valued neutrosophic hypergraph of some crisp hypergraph H^*. Clearly, H is elementary and support simple. We show that H is simply ordered. Let $\mathrm{C}(H) = \{H^{(r_1,s_1,t_1)} = (V_1, E^*_1), H^{(r_2,s_2,t_2)} = (V_2, E^*_2), \cdots, H^{(r_n,s_n,t_n)} = (V_n, E^*_n)\}$. Since H is elementary, it follows from Proposition 2.3 H is ordered. To show that H is simply ordered, suppose there exist $E \in E^*_{i+1} \setminus E^*_i$. Then there exist $v \in E$ such that $T_E(v) = r_{i+1}, I_E(v) = s_{i+1}$, and $F_E(v) = t_{i+1}$. Since $T_E(v) = r_{i+1} < r_i, I_E(v) =$

$s_{i+1} < s_i$, and $F_E(v) = t_{i+1} > t_i$, it follows that $v \notin V_i$ and $E \nsubseteq V_i$, hence H is simply ordered.
Conversely, suppose $H = (V, \mathbf{E})$ is elementary, support simple and simply ordered. For $\mathrm{C}(H) = \{H^{(r_1,s_1,t_1)} = (V_1, E_1^*), H^{(r_2,s_2,t_2)} = (V_2, E_2^*), \cdots, H^{(r_n,s_n,t_n)} = (V_n, E_n^*)\}$, fundamental sequence is $\mathrm{F}(H) = \{(r_1,s_1,t_1), (r_2,s_2,t_2), \cdots, (r_n,s_n,t_n)\}$ with $0 < r_n < r_{n-1} < \cdots < r_1$, $0 < s_n < s_{n-1} < \cdots < s_1$, and $0 < t_1 < t_2 < \cdots < t_n$. $H^{(r_n,s_n,t_n)} = (V_n, E_n^*)$ and single-valued neutrosophic set E^t on V_n defined by

$$T_{E^t}(v) = \begin{cases} r_1, & \text{if } v \in V_1; \\ r_i, & \text{if } v \in V_i \setminus V_{i-1}, i = 2, 3, \cdots, n. \end{cases}$$

$$I_{E^t}(v) = \begin{cases} s_1, & \text{if } v \in V_1; \\ s_i, & \text{if } v \in V_i \setminus V_{i-1}, i = 2, 3, \cdots, n. \end{cases}$$

$$F_{E^t}(v) = \begin{cases} t_1, & \text{if } v \in V_1; \\ t_i, & \text{if } v \in V_i \setminus V_{i-1}, i = 2, 3, \cdots, n. \end{cases}$$

We show that $\mathbf{E} = \{C_E \mid E \in E_n^*\}$,where

$$T_{C_E}(x) = \begin{cases} \wedge\{T_{E^t}(y) \mid y \in E\}, & \text{if } x \in E; \\ 0, & \text{otherwise.} \end{cases}$$

$$I_{C_E}(x) = \begin{cases} \wedge\{I_{E^t}(y) \mid y \in E\}, & \text{if } x \in E; \\ 0, & \text{otherwise.} \end{cases}$$

$$F_{C_E}(x) = \begin{cases} \vee\{F_{E^t}(y) \mid y \in E\}, & \text{if } x \in E; \\ 0, & \text{otherwise.} \end{cases}$$

Let $E \in E_n^*$. Since H is elementary and support simple there is a unique single-valued neutrosophic hyperedge E_j in $\mathbf{E}$ having support $E \in E_n^*$. We have to show that E^t tempered single-valued neutrosophic hypergraph $H = (V, \mathbf{E})$ determined by the crisp graph $H_n^* = (V_n, E_n^*)$, i.e., $C_{E \in E_n^*} = E_i, i = 1, 2, \cdots, m$.
As all single-valued neutrosophic hyperedges are elementary and H is support simple, then different edges have different supports, that is $h(E_j)$ is equal to some member (r_i, s_i, t_i) of $\mathrm{F}(H)$. Consequently, $E \subseteq V_i$ and if $i > 1$, then $E \in E_i^* \setminus E_{i-1}^*$, $T_E(v) \geq r_i, I_E(v) \geq s_i$, and $F_E(v) \leq t_i$ for some $v \in E$.

Since $E \subseteq V_i$, we claim that $T_{E^t}(v) = r_i, I_{E^t}(v) = s_i, F_{E^t}(v) = t_i$ for some $v \in E$, if not then $T_{E^t}(v) \geq r_{i-1}, I_{E^t}(v) \geq s_{i-1}, F_{E^t}(v) \leq t_{i-1}$ for all $v \in E$ which implies $E \subseteq V_{i-1}$ and since H is simply ordered, $E \in E_i^* \setminus E_{i-1}^*$, then $E \nsubseteq V_{i-1}$, a contradiction. Thus $C_E = E_i, i = 1, 2, \cdots, m$, by the definition of C_E. □

Corollary 2.1. *Suppose $H = (V, \mathbf{E})$ is a simply ordered single-valued neutrosophic hypergraph and $\mathrm{F}(H) = \{(r_1,s_1,t_1), (r_2,s_2,t_2), \cdots, (r_n,s_n,t_n)\}$. If $H^{(r_n,s_n,t_n)}$ is a simple hypergraph, then there is a partial single-valued neutrosophic hypergraph $H' = (V, \mathbf{E}')$ of H such that following statements hold.*

(1) *H' is a E^t tempered single-valued neutrosophic hypergraph of $H^{(r_n,s_n,t_n)}$.*
(2) *$\mathrm{F}(H') = (H)$ and $\mathrm{C}(H') = \mathrm{C}(H)$.*

Proof. Since H is simple ordered, then H is an elementary single-valued neutrosophic hypergraph. We obtain the partial fuzzy hypergraph $H' = (V, \mathbf{E}')$ of $H = (V, \mathbf{E})$ by removing all edges from $\mathbf{E}$ that are properly contained in another edge of H, where $\mathbf{E}' = \{E_i \in E \mid \text{if } E_i \subseteq E_j \text{ and } E_j \in E, \text{ then } E_i = E_j\}$. Since $H^{(r_n,s_n,t_n)}$ is simple and all edges are elementary, any edge in H contain another edge then both have the same support. Hence $\mathrm{F}(H') = \mathrm{H}$ and $\mathrm{C}(H') = \mathrm{C}(H)$. By the definition of $\mathbf{E}'$, H' is elementary, support simple. Thus by the Theorem 2.1 H' is a E^t tempered single-valued neutrosophic graph. □

Definition 2.19. *Let $L(G^*) = (C, D)$ be a line graph of $G^* = (V, E)$, where $C = \{\{x\} \cup \{u_x, v_x\} \mid x \in E, u_x, v_x \in V, x = u_x v_x\}$ and $D = \{S_x S_y \mid S_x \cap S_y \neq \emptyset, x, y \in E, x \neq y\}$ and where $S_x = \{\{x\} \cup \{u_x, v_x\}\}, x \in E$. Let $G = (A_1, B_1)$ be a single-valued neutrosophic graph with underlying set V. Let A_2 be the single-valued neutrosophic vertex set on C, B_2 be the single-valued neutrosophic edge set on D. The single-valued neutrosophic line graph of G is a single-valued neutrosophic graph $L(G) = (A_2, B_2)$ such that*

(i) $T_{A_2}(S_x) = T_{B_1}(x) = T_{B_1}(u_x v_x)$,
$I_{A_2}(S_x) = I_{B_1}(x) = I_{B_1}(u_x v_x)$,
$F_{A_2}(S_x) = F_{B_1}(x) = F_{B_1}(u_x v_x)$,

(ii) $T_{B_2}(S_x S_y) = \min\{T_{B_1}(x), T_{B_1}(y)\}$,
$I_{B_2}(S_x S_y) = \min\{I_{B_1}(x), I_{B_1}(y)\}$,
$F_{B_2}(S_x S_y) = \max\{F_{B_1}(x), F_{B_1}(y)\}$ *for all* $S_x, S_y \in C, S_x S_y \in D$.

Proposition 2.4. *$L(G) = (A_2, B_2)$ is a single-valued neutrosophic line graph of some single-valued neutrosophic graph $G = (A_1, B_1)$ if and only if*

$$T_{B_2}(S_x S_y) = \min\{T_{A_2}(S_x), T_{A_2}(S_y)\},$$
$$I_{B_2}(S_x S_y) = \min\{I_{A_2}(S_x), T_{A_2}(S_y)\},$$
$$F_{B_2}(S_x S_y) = \max\{F_{A_2}(S_x), F_{A_2}(S_y)\},$$

for all $S_x S_y \in D$.

Definition 2.20. *Let $H = (V, \mathbf{E})$ be a single-valued neutrosophic hypergraph of a simple graph $H^* = (V, E)$, and $L(H^*) = (X, \varepsilon)$ be a line graph of H^*. The single-valued neutrosophic line graph $L(H)$ of a single-valued neutrosophic hypergraph H is defined to be a pair $L(H) = (A, B)$, where A is the vertex set of L(H) and B is the edge set of L(H) as follows:*

(i) *A is a single-valued neutrosophic set of X such that*
$T_A(E_i) = \max\limits_{v \in E_i}(T_{E_i}(v))$,
$I_A(E_i) = \max\limits_{v \in E_i}(I_{E_i}(v))$,
$F_A(E_i) = \min\limits_{v \in E_i}(F_{E_i}(v))$ *for all* $E_i \in \mathbf{E}$,

(ii) *B is a single-valued neutrosophic set of ε such that*
$T_B(E_j E_k) = \min\limits_i\{\min(T_{E_j}(v_i), T_{E_k}(v_i))\}$,
$T_B(E_j E_k) = \min\limits_i\{\min(T_{E_j}(v_i), T_{E_k}(v_i))\}$,
$T_B(E_j E_k) = \max\limits_i\{\max(T_{E_j}(v_i), T_{E_k}(v_i))\}$, *where* $v_i \in E_i \cap E_j$, $j, k = 1, 2, 3, \cdots, n$.

Example 2.5. *Consider $H^* = (V, E)$; $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $E = \{E_1, E_2, E_3, E_4, E_5, E_6\}$, where $E_1 = \{v_1, v_3\}, E_2 = \{v_1, v_2\}, E_3 = \{v_2, v_4\}$, $E_4 = \{v_3, v_6\}$, $E_5 = \{v_3, v_5\}$, $E_6 = \{v_5, v_6\}$, and $E_7 = \{(v_4, v_6\}$, $H = (V, \mathbf{E})$ as $\mathbf{E} = \{E_1, E_2, E_3, E_4, E_5, E_6, E_7\}$, such that*
$E_1 = \{(v_1, 0.3, 0.4, 0.6), (v_3, 0.7, 0.4, 0.4)\}, E_2 = \{(v_1, 0.3, 0.4, 0.6), (v_2, 0.5, 0.7, 0.6)\}$,
$E_3 = \{(v_2, 0.5, 0.7, 0.6), (v_4, 0.6, 0.4, 0.8)\}$, $E_4 = \{(v_3, 0.7, 0.4, 0.4), (v_6, 0.4, 0.2, 0.7)\}$,
$E_5 = \{(v_3, 0.7, 0.4, 0.4), (v_5, 0.6, 0.7, 0.5)\}$, $E_6 = \{(v_5, 0.6, 0.7, 0.5), (v_6, 0.4, 0.2, 0.7)\}$,
$E_7 = \{(v_4, 0.6, 0.4, 0.8), (v_6, 0.4, 0.2, 0.7)\}$.
The single-valued neutrosophic hypergraph $H = (V, \mathbf{E})$ is shown in Figure. 1.

The line graph $L(H)$ of single-valued neutrosophic hypergraph H is $L(H) = (A, B)$, where
$A = \{(E_1, 0.7, 0.4, 0.4), (E_2, 0.5, 0.7, 0.6), (E_3, 0.6, 0.7, 0.6), (E_4, 0.7, 0.4, 0.4), (E_5, 0.7, 0.7, 0.4), (E_6, 0.6, 0.7, 0.5), (E_7, 0.6, 0.4, 0.7)\}$ *is the vertex set and* $B = \{(E_1E_2, 0.3, 0.4, 0.6), (E_1E_5, 0.7, 0.4, 0.4), (E_1E_4, 0.7, 0.4, 0.4), (E_2E_3, 0.5, 0.7, 0.6), (E_3E_7, 0.6, 0.4, 0.8), (E_4E_5, 0.7, 0.4, 0.4), (E_4E_6, 0.4, 0.2, 0.7),$

$(E_4E_7, 0.4, 0.2, 0.7), (E_5E_6, 0.6, 0.7, 0.5), (E_6E_7, 0.4, 0.2, 0.7)\}$ *is the edge set of the single-valued neutrosophic line graph of* H.

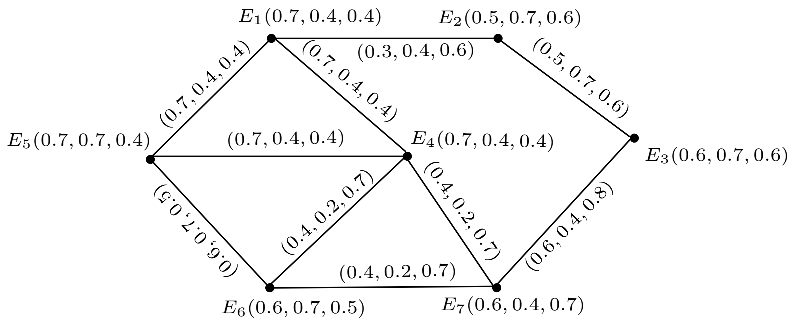

FIGURE 3. Single-valued neutrosophic line graph $L(H)$ of single-valued neutrosophic hypergraph H.

Proposition 2.5. *A single-valued neutrosophic hypergraph is connected if and only if line graph of a single-valued neutrosophic hypergraph is connected.*

Definition 2.21. *The* $2-$*section of a single-valued neutrosophic hypergraph* $H = (V, \mathbf{E})$, *denoted by* $[H]_2$, *is a single-valued neutrosophic graph* $G = (A, B)$, *where* A *is the single-valued neutrosophic vertex of* V, B *is the single-valued neutrosophic edge set in which any two vertices form an edge if they are in the same single-valued neutrosophic hyperedge such that*

$$T_B(e) = \min\{T_{E_k}(v_i), T_{E_k}(v_j)\},$$
$$I_B(e) = \min\{I_{E_k}(v_i), T_{E_k}(v_j)\},$$
$$F_B(e) = \max\{F_{E_k}(v_i), F_{E_k}(v_j)\},$$

for all $E_k \in \mathbf{E}, i \neq j, k = 1, 2, \cdots, m$.

We now introduce the concept of dual single-valued neutrosophic hypergraph for a single-valued neutrosophic hypergraph.

Definition 2.22. *The dual of a single-valued neutrosophic hypergraph* $H = (V, \mathbf{E})$ *is a single-valued neutrosophic hypergraph* $H^* = (E, \mathbf{V})$; $E = \{e_1, e_2, \cdots, e_n\}$ *set of vertices corresponding to* $E_1, E_2, , \cdots, E_n$, *respectively and* $\mathbf{V} = \{V_1, V_2, \cdots, V_n\}$ *set of hyperedges corresponding to* $v_1, v_2, \cdots, v_n$, *respectively.*

Example 2.6. *Consider a single-valued neutrosophic hypergraph* $H = (V, \mathbf{E})$ *such that* $V = \{v_1, v_2, v_3, v_4, v_5\}$ *and* $\mathbf{E} = \{E_1, E_2, E_3\}$, *where* $E_1 = \{(v_1, 0.5, 0.4, 0.6), (v_2, 0.4, 0.3, 0.8)\}, E_2 = \{(v_2, 0.4, 0.3, 0.8), (v_3, 0.6, 0.4, 0.8), (v_4, 0.7, 0.4, 0.5)\}$, *and* $E_3 = \{(v_4, 0.7, 0.4, 0.5), (v_5, 0.4, 0.2, 0.9)\}$.

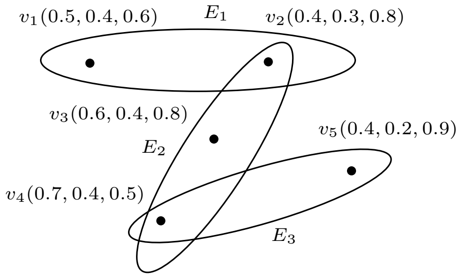

FIGURE 4. Single-valued neutrosophic hypergraph.

The single-valued neutrosophic hypergraph can be represented by the following incidence matrix:

M_H	E_1	E_2	E_3
v_1	$(0.5, 0.4, 0.6)$	$(0, 0, 0)$	$(0, 0, 0)$
v_2	$(0.4, 0.3, 0.8)$	$(0.4, 0.3, 0.8)$	$(0, 0, 0)$
v_3	$(0, 0, 0)$	$(0.6, 0.4, 0.8)$	$(0, 0, 0)$
v_4	$(0, 0, 0)$	$(0.7, 0.4, 0.5)$	$(0.7, 0.4, 0.5)$
v_5	$(0, 0, 0)$	$(0, 0, 0)$	$(0.4, 0.2, 0.9)$

Consider the dual single-valued neutrosophic hypergraph $H^* = (E, \mathbf{V})$ *of* H *such that* $E = \{e_1, e_2, e_3\}$, $\mathbf{V} = \{V_1, V_2, V_3, V_4, V_5\}$, *where* $V_1 = \{(e_1, 0.5, 0.4, 0.6), (e_2, 0, 0, 0), (e_3, 0, 0, 0)\}$, $V_2 = \{(e_1, 0.4, 0.3, 0.8), (e_2, 0.4, 0.3, 0.8), (e_3, 0, 0, 0)\}$, $V_3 = \{(e_1, 0, 0, 0), (e_2, 0.6, 0.4, 0.8), (e_3, 0, 0, 0)\}$, $V_4 = \{(e_1, 0, 0, 0), (e_2, 0.7, 0.4, 0.5), (e_3, 0.7, 0.4, 0.5)\}$ *and* $V_5 = \{(e_1, 0, 0, 0), (e_2, 0, 0, 0), (e_3, 0.4, 0.2, 0.9)\}$.

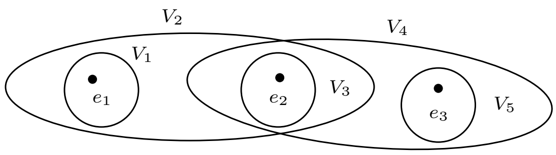

FIGURE 5. Dual single-valued neutrosophic hypergraph H^*.

The dual single-valued neutrosophic hypergraph is shown in Figure. 5 and its incidence matrix M_{H^*} *is defined as follows:*

M_{H^*}	V_1	V_2	V_3	V_4	V_5
e_1	$(0.5, 0.4, 0.6)$	$(0.4, 0.3, 0.8)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
e_2	$(0, 0, 0)$	$(0.4, 0.3, 0.8)$	$(0.6, 0.4, 0.8)$	$(0.7, 0.4, 0.5)$	$(0, 0, 0)$
e_3	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0.7, 0.4, 0.5)$	$(0.4, 0.2, 0.9)$

Remark 2.2. H^* *is a single-valued neutrosophic hypergraph whose incidence matrix is the transpose of the incidence matrix of* H *and* $\triangle(H) = rank(H^*)$. *The dual single-valued neutrosophic hypergraph* H^* *of a simple single-valued neutrosophic hypergraph* H *may or may not be simple.*

Proposition 2.6. *The dual* H^* *of a linear single-valued neutrosophic hypergraph without isolated vertex is also linear single-valued neutrosophic hypergraph.*

Proof. Let H be a linear hypergraph. Assume that H^* is not linear. There is two distinct hyperedges V_i and V_j of H^* which intersect with at least two vertices e_1 and e_2. The definition of duality implies that v_i and v_j belong to E_1 and E_2 (the single-valued neutrosophic hyperedges of H standing for the vertices e_1, e_2 of H^* respectively) so H is not linear. Contradiction since H is linear. Hence dual H^* of a linear single-valued neutrosophic hypergraph without isolated vertex is also linear single-valued neutrosophic hypergraph. □

Definition 2.23. *Let $H = (V, \mathbf{E})$ be a single-valued neutrosophic hypergraph. A single-valued neutrosophic transversal τ of H is a single-valued neutrosophic subset of V with the property that $\tau^{h(E)} \cap E^{h(E)} \neq \emptyset$ for each $E \in \mathbf{E}$, where $h(E)$ is the height of hyperedge E. A minimal single-valued neutrosophic transversal τ for H is a transversal of H with the property that if $\tau' \subset \tau$, then τ' is not a single-valued neutrosophic transversal of H.*

Proposition 2.7. *If τ is a single-valued neutrosophic transversal of a single-valued neutrosophic hypergraph $H = (V, \mathbf{E})$, then $h(\tau) > h(E)$ for each $E \in \mathbf{E}$. Moreover, if τ is a minimal single-valued neutrosophic transversal of H, then $h(\tau) = \vee\{h(E) \mid E \in \mathbf{E}\} = h(H)$.*

Theorem 2.2. *For a single-valued neutrosophic hypergraph H, $Tr(H) \neq \emptyset$, where $Tr(H)$ is the family of minimal single-valued neutrosophic transversal of H.*

Proposition 2.8. *Let $H = (V, E)$ be a single-valued neutrosophic hypergraph. The following statements are equivalent:*

(i) *τ is a single-valued neutrosophic transversal of H*
(ii) *For each $E \in \mathbf{E}$, $h(E) = (r', s', t')$, and each $0 < r \leq r', 0 < s \leq s', t \geq t'$, $\tau^{(r,s,t)} \cap E^{(r,s,t)} \neq \emptyset$*

If the $(r, s, t)-$cut $\tau^{(r,s,t)}$ is a subset of the vertex set of $H^{(r,s,t)}$ for each (r, s, t), $0 < r \leq r', 0 < s \leq s', t \geq t'$,then

(iii) *For each (r, s, t), $0 < r \leq r', 0 < s \leq s', t \geq t'$, $\tau^{(r,s,t)}$ is a transversal of $H^{(r,s,t)}$*
(iv) *Every single-valued neutrosophic transversal τ of H contains a single-valued neutrosophic transversal τ' for each (r, s, t), $0 < r \leq r', 0 < s \leq s', t \geq t'$, ${\tau'}^{(r,s,t)}$ is a transversal of $H^{(r,s,t)}$*

Observation: If τ is a minimal transversal of single valued neutrosophic graph H, then $\tau^{(r,s,t)}$ not necessarily belongs to $Tr(H^{(r,s,t)})$ for each (r, s, t), satisfying $0 < r \leq r', 0 < s \leq s', t \geq t'$. Let $Tr^*(H)$ represents the collection of those minimal single valued neutrosophic transversal, τ of H, where $\tau^{(r,s,t)}$ is a minimal transversal of $H^{(r,s,t)}$, for each (r, s, t), $0 < r \leq r', 0 < s \leq s', t \geq t'$, i.e., $Tr^* = \{\tau \in Tr(H) \mid h(\tau) = h(H) \text{ and } \tau^{(r,s,t)} \in Tr(H^{(r,s,t)})\}$.

Example 2.7. *Consider the single-valued neutrosophic hypergraph $H = (V, \mathbf{E})$, where $V = \{v_1, v_2, v_3\}$ and $\mathbf{E} = \{E_1, E_2, E_3\}$, which is represented by the following incidence matrix:*

M_H	E_1	E_2	E_3
v_1	$(0.9, 0.6, 0.1)$	$(0, 0, 0)$	$(0.4, 0.3, 0.2)$
v_2	$(0.4, 0.3, 0.2)$	$(0.4, 0.3, 0.2)$	$(0.4, 0.3, 0.2)$
v_3	$(0, 0, 0)$	$(0, 0, 0)$	$(0.4, 0.3, 0.2)$

Clearly, $h(H) = (0.9, 0.6, 0.1)$, the only minimal transversal τ of single-valued neutrosophic hypergraph H is $\tau(H) = \{(v_1, 0.9, 0.6, 0.1), (v_2, 0.4, 0.3, 0.2)\}$. The fundamental sequence of H is $\mathrm{F}(H) = \{(0.9, 0.6, 0.1), (0.4, 0.3, 0.2)\}$, $\tau^{(0.9,0.6,0.1)} = \{v_1\}$ and $\tau^{(0.4,0.3,0.2)} = \{v_1, v_2\}$. Since $\{v_2\}$ is the only minimal transversal of the $H^{(0.4,0.3,0.2)}$, $E^{(0.4,0.3,0.2)} =$

$\{\{v_1, v_2\}, \{v_2\}, \{v_1, v_2, v_3\}\}$, *it follows that the only minimal transversal* τ *of* H *is not a member of* $Tr^*(H)$. *Hence* $Tr^*(H) = \emptyset$.

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