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SOME RESULTS ON SINGLE VALUED NEUTROSOPHIC BI-MAGIC GRAPHS

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ABSTRACT

In this paper, we introduce the concepts of Bi-Magic labeling in single valued neutrosophic graphs. We investigate some properties of single valued neutrosophic bi-magic labeling on path, cycle and star graphs.

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Key words: single valued neutrosophic bi-magic labeling; path; cycle; star graphs.

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1. INTRODUCTION

Euler[6] introduced the concept of a graphs in 1736. Zadah[11] introduced the concept of fuzzy set in 1965. It is a mathematical structure to demonstrate the observable fact of uncertainty in real life problems. Rosenfeld [9] introduced the concept of fuzzy graph in 1975. Intuitionistic fuzzy set is an extension of fuzzy zet it was introduced by Atanassov [2]. In 1994, Sovan and Atanassov [3] introduced the concept of intutionistic fuzzy graph. Smarandache [10] introduced the concept of neutrosophic sets. Kotzig and Rosa [7] defined a magic labeling to be a total labeling in which the labels are the integers from 1 to $|V| + |E|$. The sum of labels on an edge and its two endpoints is constant. A. Nagoor Gani, Muhammad Akram and D. Subahashini [8] introduced the concept of fuzzy magic labeling graphs. In this paper, we introduce the concepts of Bi-Magic labeling in single valued neutrosophic graphs. We investigate some properties of single valued neutrosophic bi-magic labeling on path, cycle and star graphs.

2. PRELIMINARIES

Definition 2.1 [1] A Single Valued Neutrosophic (SVN) Graph with underlying set V is defined to be a pair G = (A, B) where

1. The function $T_A: V \to [0,1], I_A: V \to [0,1], F_A: V \to [0,1]$ denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element $v_i \in V$, respectively, and $0 \le T_A(v_i) + I_A(v_i) + F_A(v_i) \le 3$ for all $v_i \in V$.

2. The functions $T_B: E \subseteq V \times V \to [0,1], I_B: E \subseteq V \times V \to [0,1], F_B: E \subseteq V \times V \to [0,1]$ are defined by $T_B(v_i, v_j) \le min[T_A(v_i), T_A(v_j)], I_B(v_i, v_j) \le min[I_A(v_i), I_A(v_j)]$ and

$F_B(v_i, v_j) \le max[F_A(v_i), F_A(v_j)]$.

Definition 2.2 A Path P_n in a SVN graph is a sequence of distinct vertices $v_1, v_2, \dots, v_n$ such that $0 < T_B(v_i, v_{i+1}), I_B(v_i, v_{i+1}), F_B(v_i, v_{i+1}) \le 1$; $1 \le i \le n-1$; $n-1$ is called the length of the path P_n. A path P_n is called Cycle if $v_1 = v_n$ for $n \ge 3$.

Definition 2.3 A Star in a SVN graph consists of two vertex sets U and V with $|U| = 1$ and $|V| = n$ such that $0 < T_B(u, v_i), I_B(u, v_i), F_B(u, v_i) \le 1; 1 \le i \le n$.

3. SINGLE VALUED NEUTROSOPHIC BI-MAGIC LABELING GRAPHS

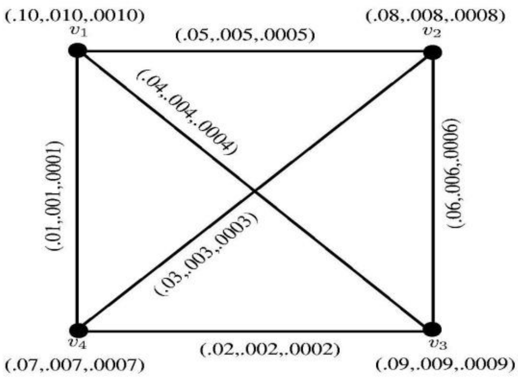
Definition 3.1 A SVN graph is said to be a single valued neutrosophic bi-magic graph if $\widetilde{Bm}_T(G) = T_A(u) + T_B(u,v) + T_A(v), \widetilde{Bm}_I(G) = I_A(u) + I_B(u,v) + I_A(v)$ and $\widetilde{Bm}_F(G) = F_A(u) + F_B(u,v) + F_A(v)$ has two different neutrosophic magic values $\widetilde{Bm}_1(G), \widetilde{Bm}_2(G)$ for all $u, v \in V$. Where $\widetilde{Bm}_1(G) = (\widetilde{Bm}_{T_1}(G), \widetilde{Bm}_{I_1}(G), \widetilde{Bm}_{F_1}(G))$ and $\widetilde{Bm}_2(G) = (\widetilde{Bm}_{T_2}(G), \widetilde{Bm}_{I_2}(G), \widetilde{Bm}_{F_2}(G))$. Bi-magic labeling of SVN graph G is $\widetilde{Bm}_0(G) = (\widetilde{Bm}_1(G), \widetilde{Bm}_2(G))$.

Example 3.1 Consider a SVN graph G=(A,B) such that $V = \{v_1, v_2, v_3, v_4\}$ and

$E = \{(v_1,v_2),(v_2,v_3),(v_3,v_4),(v_4,v_1),(v_1,v_3),(v_2,v_4)\}$.

The Bi-magic values of a graph G are $\widetilde{Bm}_1(G) = (0.23, 0.023, 0.0023)$ and $\widetilde{Bm}_2(G) = (0.18, 0.018, 0.0018)$.

Hence G is a SVN Bi-Magic labeling graph.



SVN Bi-Magic graph G

Theorem 3.1 For all $n \ge 3$, the path P_n is a single valued neutrosophic bi-magic labeling graph.

Proof. Let P_n be any path with $n \ge 3$. Then $v_1, v_2, v_3, \dots, v_n$ and $v_1v_2, v_2v_3, \dots, v_{n-1}v_n$ are vertices and edges of P_n. Let $l = min\{x: n < 3(10)^x, x = 0,1,2,\dots\}$, $\varepsilon_1 = 10^{-(l+1)}$, $\varepsilon_2 = 10^{-(l+2)}$ *and* $\varepsilon_3 = 10^{-(l+3)}$ where $\varepsilon_1, \varepsilon_2$ and ε_3 are the set of truth , indeterminacy and falsity membership degree in single valued neutrosophic labeling.

The single valued neutrosophic vertex and edge labeling is defined as follows:
Case - (1) When n is odd,

$$T_A(v_{2k}) = (2n-k)\varepsilon_1; 1 \leq k \leq \frac{n-1}{2},$$

$$I_A(v_{2k}) = (2n-k)\varepsilon_2; 1 \leq k \leq \frac{n-1}{2},$$

$$F_A(v_{2k}) = (2n-k)\varepsilon_3; 1 \leq k \leq \frac{n-1}{2}.$$

$$T_A(v_{2k-1}) = min\{T_A(v_{2i})|1 \leq i \leq \frac{n-1}{2}\} - k\varepsilon_1; 1 \leq k \leq \frac{n+1}{2},$$

$$I_A(v_{2k-1}) = min\{I_A(v_{2i})|1 \leq i \leq \frac{n-1}{2}\} - k\varepsilon_2; 1 \leq k \leq \frac{n+1}{2},$$

$$F_A(v_{2k-1}) = min\{F_A(v_{2i})|1 \leq i \leq \frac{n-1}{2}\} - k\varepsilon_3; 1 \leq k \leq \frac{n+1}{2}.$$

$$T_B(v_k, v_{k+1}) = (\frac{n-1}{2} + k)\varepsilon_1; 1 \leq k \leq \frac{n-1}{2},$$

$$I_B(v_k, v_{k+1}) = (\frac{n-1}{2} + k)\varepsilon_2; 1 \leq k \leq \frac{n-1}{2},$$

$$F_B(v_k, v_{k+1}) = (\frac{n-1}{2} + k)\varepsilon_3; 1 \leq k \leq \frac{n-1}{2}.$$

$$T_B(v_k, v_{k+1}) = (k - \frac{n-1}{2})\varepsilon_1; \frac{n+1}{2} \leq k \leq n-1,$$

$$I_B(v_k, v_{k+1}) = (k - \frac{n-1}{2})\varepsilon_2; \frac{n+1}{2} \leq k \leq n-1,$$

$$F_B(v_k, v_{k+1}) = (k - \frac{n-1}{2})\varepsilon_3; \frac{n+1}{2} \leq k \leq n-1.$$

For each edge (v_k, v_{k+1}) the SVN bi-magic labeling are,

$$\widetilde{Bm}_{T_1}(P_n) = T_A(v_k) + T_B(v_k, v_{k+1}) + T_A(v_{k+1}), 1 \leq k \leq \frac{n-1}{2},$$

$$\widetilde{Bm}_{T_1}(P_n) = (4n-1)\varepsilon_1.$$

$$\widetilde{Bm}_{T_2}(P_n) = T_A(v_k) + T_B(v_k, v_{k+1}) + T_A(v_{k+1}), \frac{n+1}{2} \leq k \leq n-1,$$

$$\widetilde{Bm}_{T_2}(P_n) = (3n)\varepsilon_1.$$

Similarly we can find,

$$\widetilde{Bm}_{I_1}(P_n) = (4n-1)\varepsilon_2,\ \widetilde{Bm}_{I_2}(P_n) = (3n)\varepsilon_2.$$

$$\widetilde{Bm}_{F_1}(P_n) = (4n-1)\varepsilon_3,\ \widetilde{Bm}_{F_2}(P_n) = (3n)\varepsilon_3.$$

Hence single valued neutrosophic bi-magic labeling of a even length path P_n are

$\widetilde{Bm}_1(P_n) = (\widetilde{Bm}_{T_1}(P_n), \widetilde{Bm}_{I_1}(P_n), \widetilde{Bm}_{F_1}(P_n))$ and
$\widetilde{Bm}_2(P_n) = \left(\widetilde{Bm}_{T_2}(P_n), \widetilde{Bm}_{I_2}(P_n), \widetilde{Bm}_{F_2}(P_n)\right)$.

Case - (2) When n is even,

$$T_A(v_{2k}) = (2n-k)\varepsilon_1; 1 \leq k \leq \frac{n}{2},$$

$$I_A(v_{2k}) = (2n-k)\varepsilon_2; 1 \leq k \leq \frac{n}{2},$$

$$F_A(v_{2k}) = (2n-k)\varepsilon_3; 1 \leq k \leq \frac{n}{2}.$$

$$T_A(v_{2k-1}) = min\{T_A(v_{2i}) | 1 \leq i \leq \frac{n}{2}\} - k\varepsilon_1; 1 \leq k \leq \frac{n}{2},$$

$$I_A(v_{2k-1}) = min\{I_A(v_{2i}) | 1 \leq i \leq \frac{n}{2}\} - k\varepsilon_2; 1 \leq k \leq \frac{n}{2},$$

$$F_A(v_{2k-1}) = min\{F_A(v_{2i}) | 1 \leq i \leq \frac{n}{2}\} - k\varepsilon_3; 1 \leq k \leq \frac{n}{2}.$$

$$T_B(v_k, v_{k+1}) = (\frac{n-2}{2} + k)\varepsilon_1; 1 \leq k \leq \frac{n}{2},$$

$$I_B(v_k, v_{k+1}) = (\frac{n-2}{2} + k)\varepsilon_2; 1 \leq k \leq \frac{n}{2},$$

$$F_B(v_k, v_{k+1}) = (\frac{n-2}{2} + k)\varepsilon_3; 1 \leq k \leq \frac{n}{2}.$$

$$T_B(v_k, v_{k+1}) = (k - \frac{\mathrm{n}}{2})\varepsilon_1; \frac{n}{2} + 1 \leq k \leq n-1,$$

$$I_B(v_k, v_{k+1}) = (k - \frac{n}{2})\varepsilon_2; \frac{n}{2} + 1 \leq k \leq n-1,$$

$$F_B(v_k, v_{k+1}) = (k - \frac{n}{2})\varepsilon_3; \frac{n}{2} + 1 \leq k \leq n-1.$$

For each edge (v_k, v_{k+1}) the SVN bi-magic labeling are,

$$\widetilde{Bm}_{T_1}(P_n) = T_A(v_k) + T_B(v_k, v_{k+1}) + T_A(v_{k+1}), 1 \leq k \leq \frac{n}{2},$$

$$\widetilde{Bm}_{T_1}(P_n) = (4n-2)\varepsilon_1.$$

$$\widetilde{Bm}_{T_2}(P_n) = T_A(v_k) + T_B(v_k, v_{k+1}) + T_A(v_{k+1}), \frac{n}{2} + 1 \leq k \leq n-1,$$

$$\widetilde{Bm}_{T_2}(P_n) = (3n-1)\varepsilon_1.$$

Similarly we can find,

$$\widetilde{Bm}_{I_1}(P_n) = (4n-2)\varepsilon_2, \widetilde{Bm}_{I_2}(P_n) = (3n-1)\varepsilon_2.$$

$$\widetilde{Bm}_{F_1}(P_n) = (4n-2)\varepsilon_3, \widetilde{Bm}_{F_2}(P_n) = (3n-1)\varepsilon_3.$$

Hence single valued neutrosophic bi-magic labeling of a odd length path P_n are

$$\widetilde{Bm}_1(P_n) = (\widetilde{Bm}_{T_1}(P_n), \widetilde{Bm}_{I_1}(P_n), \widetilde{Bm}_{F_1}(P_n))$$

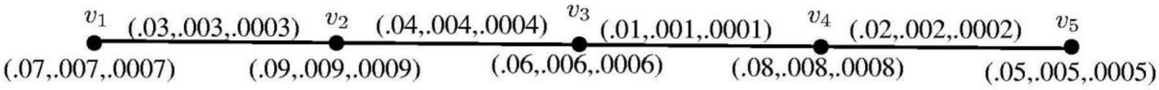
$$\text{and } \widetilde{Bm}_2(P_n) = (\widetilde{Bm}_{T_2}(P_n), \widetilde{Bm}_{I_2}(P_n), \widetilde{Bm}_{F_2}(P_n)).$$

Example 3.2 Consider a SVN Path graph P_5 such that $V = \{v_1, v_2, v_3, v_4, v_5\}$ and

$$E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_5)\}.$$

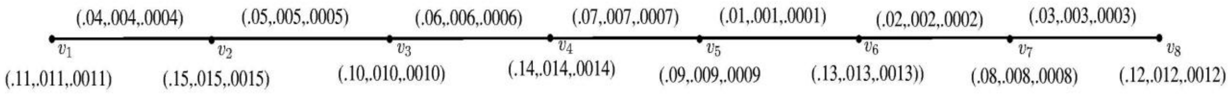
$$\widetilde{Bm}_1(P_5) = (0.19, 0.019, 0.0019) \text{ and } \widetilde{Bm}_2(P_5) = (0.15, 0.015, 0.0015).$$

Hence P_5 is a SVN Bi-Magic graph.



SVN Bi-Magic labeling of P_5

Example 3.3 Consider a SVN Path graph P_8 such that $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ and $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_5), (v_5, v_6), (v_6, v_7), (v_7, v_8)\}$.



SVN Bi-Magic graph P_8

$\widetilde{Bm}_1(P_8) = (0.30, 0.030, 0.0030)$ and $\widetilde{Bm}_2(P_8) = (0.23, 0.023, 0.0023)$.

Hence P_8 is a SVN Bi-Magic graph.

Theorem 3.2 For all $n \geq 3$, the cycle C_n is a single valued neutrosophic bi-magic labeling graph.

Proof. Let C_n be any cycle with $n \geq 3$. Then $v_1, v_2, v_3, \dots, v_n$ and $v_1v_2, v_2v_3, \dots, v_nv_1$ are vertices and edges of C_n. Let $l = min\{x : n < 3(10)^x, x = 0,1,2, \dots\}$, $\varepsilon_1 = 10^{-(l+1)}$, $\varepsilon_2 = 10^{-(l+2)}$ and $\varepsilon_3 = 10^{-(l+3)}$ where $\varepsilon_1, \varepsilon_2$ and ε_3 are the set of truth , indeterminacy and falsity membership degree in single valued neutrosophic labeling.

The single valued neutrosophic vertex and edge labeling is defined as follows:

Case - (1) When n is odd,

$$T_A(v_{2k}) = (2n - k + 1)\varepsilon_1; 1 \leq k \leq \frac{n-1}{2},$$

$$I_A(v_{2k}) = (2n - k + 1)\varepsilon_2; 1 \leq k \leq \frac{n-1}{2},$$

$$F_A(v_{2k}) = (2n - k + 1)\varepsilon_3; 1 \leq k \leq \frac{n-1}{2}.$$

$$T_A(v_{2k-1}) = min\{v_{2i} | 1 \leq i \leq \frac{n-1}{2}\} - k\varepsilon_1,$$

$$I_A(v_{2k-1}) = min\{v_{2i} | 1 \leq i \leq \frac{n-1}{2}\} - k\varepsilon_2,$$

$$F_A(v_{2k-1}) = min\{v_{2i}|1 \leq i \leq \frac{n-1}{2}\} - k\varepsilon_3.$$

$$T_B(v_k, v_{k+1}) = (\frac{n-1}{2} + k)\varepsilon_1; 1 \leq k \leq \frac{n+1}{2},$$

$$I_B(v_k, v_{k+1}) = (\frac{n-1}{2} + k)\varepsilon_2; 1 \leq k \leq \frac{n+1}{2},$$

$$F_B(v_k, v_{k+1}) = (\frac{n-1}{2} + k)\varepsilon_3; 1 \leq k \leq \frac{n+1}{2}.$$

$$T_B(v_k, v_{k+1}) = (k - \frac{n+1}{2})\varepsilon_1; \frac{n+3}{2} \leq k \leq n-1,$$

$$I_B(v_k, v_{k+1}) = (k - \frac{n+1}{2})\varepsilon_2; \frac{n+3}{2} \leq k \leq n-1,$$

$$F_B(v_k, v_{k+1}) = (k - \frac{n+1}{2})\varepsilon_3; \frac{n+3}{2} \leq k \leq n-1.$$

For each edge (v_k, v_{k+1}) the SVN bi-magic labeling are,

$$\widetilde{Bm}_{T_1}(C_n) = T_A(v_k) + T_B(v_k, v_{k+1}) + T_A(v_{k+1}), 1 \leq k \leq \frac{n+1}{2},$$

$$\widetilde{Bm}_{T_1}(C_n) = (4n+1)\varepsilon_1.$$

$$\widetilde{Bm}_{T_2}(C_n) = T_A(v_k) + T_B(v_k, v_{k+1}) + T_A(v_{k+1}), \frac{n+3}{2} \leq k \leq n-1,$$

$$\widetilde{Bm}_{T_2}(C_n) = (3n+1)\varepsilon_1.$$

Similarly we can find,

$$\widetilde{Bm}_{I_1}(C_n) = (4n+1)\varepsilon_2, \widetilde{Bm}_{I_2}(C_n) = (3n+1)\varepsilon_2.$$

$$\widetilde{Bm}_{F_1}(C_n) = (4n+1)\varepsilon_3, \widetilde{Bm}_{F_2}(C_n) = (3n+1)\varepsilon_3.$$

Hence single valued neutrosophic bi-magic labeling of a odd cycle C_n are

$$\widetilde{Bm}_1(C_n) = (\widetilde{Bm}_{T_1}(C_n), \widetilde{Bm}_{I_1}(C_n), \widetilde{Bm}_{F_1}(C_n))$$

and $\widetilde{Bm}_2(C_n) = (\widetilde{Bm}_{T_2}(C_n), \widetilde{Bm}_{I_2}(C_n), \widetilde{Bm}_{F_2}(C_n))$.

Case - (2) When n is even,

$$T_A(v_{2k-1}) = (2n - k + 1)\varepsilon_1; 1 \leq k \leq \frac{n}{2},$$

$$I_A(v_{2k-1}) = (2n - k + 1)\varepsilon_2; 1 \leq k \leq \frac{n}{2},$$

$$F_A(v_{2k-1}) = (2n - k + 1)\varepsilon_3; 1 \leq k \leq \frac{n}{2}.$$

$$T_A(v_{2k}) = min\{v_{2i-1}|1 \leq i \leq \frac{n}{2}\} - k\varepsilon_1,$$

$$I_A(v_{2k}) = min\{v_{2i-1}|1 \le i \le \frac{n}{2}\} - k\varepsilon_2,$$

$$F_A(v_{2k}) = min\{v_{2i-1}|1 \le i \le \frac{n}{2}\} - k\varepsilon_3.$$

$$T_B(v_1, v_n) = \varepsilon_1,$$

$$I_B(v_1, v_n) = \varepsilon_2,$$

$$F_B(v_1, v_n) = \varepsilon_3.$$

$$T_B(v_k, v_{k+1}) = (\frac{n}{2} + k)\varepsilon_1; 1 \le k \le \frac{n}{2},$$

$$I_B(v_k, v_{k+1}) = (\frac{n}{2} + k)\varepsilon_2; 1 \le k \le \frac{n}{2},$$

$$F_B(v_k, v_{k+1}) = (\frac{n}{2} + k)\varepsilon_3; 1 \le k \le \frac{n}{2}.$$

$$T_B(v_k, v_{k+1}) = (k + 1 - \frac{n}{2})\varepsilon_1; \frac{n+2}{2} \le k \le n - 1,$$

$$I_B(v_k, v_{k+1}) = (k + 1 - \frac{n}{2})\varepsilon_2; \frac{n+2}{2} \le k \le n - 1,$$

$$F_B(v_k, v_{k+1}) = (k + 1 - \frac{n}{2})\varepsilon_3; \frac{n+2}{2} \le k \le \mathrm{n} - 1.$$

For each edge (v_k, v_{k+1}) the SVN bi-magic labeling are,

$$\widetilde{Bm}_{T_1}(C_n) = T_A(v_k) + T_B(v_k, v_{k+1}) + T_A(v_{k+1}), 1 \le k \le \frac{n}{2},$$

$$\widetilde{Bm}_{T_1}(C_n) = (4n + 1)\varepsilon_1.$$

$$\widetilde{Bm}_{T_2}(C_n) = T_A(v_k) + T_B(v_k, v_{k+1}) + T_A(v_{k+1}), \frac{n+2}{2} \le k \le n - 1,$$

$$\widetilde{Bm}_{T_2}(C_n) = (3n + 2)\varepsilon_1.$$

Similarly we can find,

$$\widetilde{Bm}_{I_1}(C_n) = (4n + 1)\varepsilon_2, \widetilde{Bm}_{I_2}(C_n) = (3n + 2)\varepsilon_2.$$

$$\widetilde{Bm}_{F_1}(C_n) = (4n + 1)\varepsilon_3, \widetilde{Bm}_{F_2}(C_n) = (3n + 2)\varepsilon_3.$$

Hence single valued neutrosophic bi-magic labeling of a even cycle C_n are

$$\widetilde{Bm}_1(C_n) = (\widetilde{Bm}_{T_1}(C_n), \widetilde{Bm}_{I_1}(\mathrm{C}_n), \widetilde{Bm}_{F_1}(C_n))$$

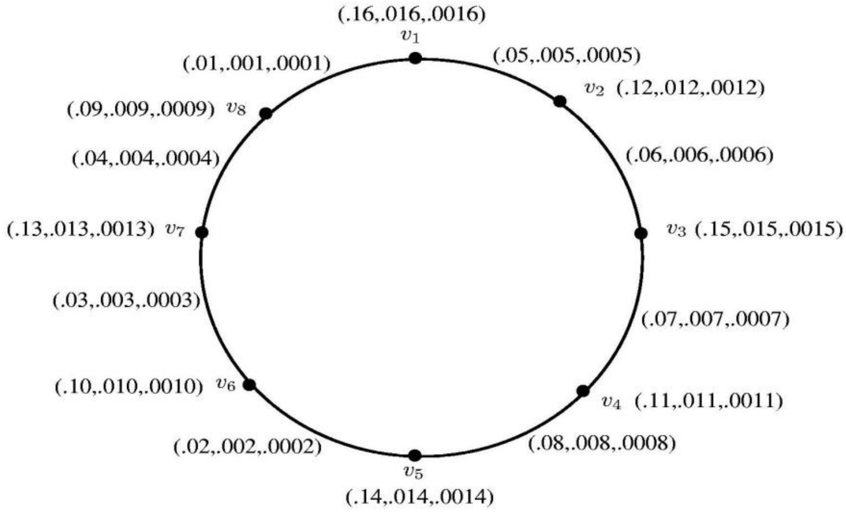
and $\widetilde{Bm}_2(C_n) = (\widetilde{Bm}_{T_2}(C_n), \widetilde{Bm}_{I_2}(C_n), \widetilde{Bm}_{F_2}(C_n))$.

Example 3.4 Consider a SVN cycle C_8 such that $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ and

$$E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_5), (v_5, v_6), (v_6, v_7), (v_7, v_8), (v_8, v_1)\}.$$

$$\widetilde{Bm}_1(C_8) = (0.33,0.033,0.0033) \text{ and } \widetilde{Bm}_2(C_8) = (0.26,0.026,0.0026).$$

Hence C_8 is a SVN Bi-Magic graph.



SVN Bi-Magic cycle graph C_8

Theorem 3.3 For any $n \geq 2$, Star graph $S_{1,n}$ is a single valued neutrosophic bi-magic graph.

Proof. Let $S_{1,n}$ be a star graph with $u, v_1, v_2, v_3, \ldots, v_n$ as vertices and $uv_1, uv_2, uv_3, \ldots, uv_n$ as edges. Let $l = min\{x: n < 3(10)^x, x = 0,1,2,\ldots\}$, $\varepsilon_1 = 10^{-(l+1)}$, $\varepsilon_2 = 10^{-(l+2)}$ *and* $\varepsilon_3 = 10^{-(l+3)}$ where $\varepsilon_1, \varepsilon_2$ and ε_3 are the set of truth , indeterminacy and falsity membership degree in single valued neutrosophic labeling.

The single valued neutrosophic vertex and edge labeling is defined as follows:

Case - (1) When n is odd,

$T_A(u) = (2n+1)\varepsilon_1,$

$$I_A(u) = (2n+1)\varepsilon_2,$$

$$F_A(u) = (2n+1)\varepsilon_3.$$

$$T_A(v_k) = T_A(u) - k\varepsilon_1; 1 \leq k \leq n,$$

$$I_A(v_k) = I_A(u) - k\varepsilon_2; 1 \leq k \leq n,$$

$$F_A(v_k) = F_A(u) - k\varepsilon_3; 1 \leq k \leq n.$$

$$T_B(u, v_k) = (\frac{n+1}{2} + k - 1)\varepsilon_1; 1 \leq k \leq \frac{n+1}{2},$$

$$I_B(u, v_k) = (\frac{n+1}{2} + k - 1)\varepsilon_2; 1 \leq k \leq \frac{n+1}{2},$$

$$F_B(u, v_k) = (\frac{n+1}{2} + k - 1)\varepsilon_3; 1 \leq k \leq \frac{n+1}{2}.$$

$$T_B(u, v_k) = (k - \frac{n+1}{2})\varepsilon_1; \frac{n+3}{2} \leq k \leq n,$$

$$I_B(u, v_k) = (k - \frac{n+1}{2})\varepsilon_2; \frac{n+3}{2} \leq k \leq n,$$

$$F_B(u, v_k) = (k - \frac{n+1}{2})\varepsilon_3; \frac{n+3}{2} \leq k \leq n.$$

For each edge (u, v_k) the SVN bi-magic labeling are,

$$\widetilde{Bm}_{T_1}(S_{1,n}) = T_A(u) + T_B(u.v_k) + T_A(v_k), 1 \leq k \leq \frac{n+1}{2},$$

$$\widetilde{Bm}_{T_1}(S_{1,n}) = (\frac{9n+3}{2})\varepsilon_1.$$

$$\widetilde{Bm}_{T_2}(S_{1,n}) = T_A(u) + T_B(u.v_k) + T_A(v_k), \frac{n+3}{2} \leq k \leq n,$$

$$\widetilde{Bm}_{T_2}(S_{1,n}) = (\frac{7n+3}{2})\varepsilon_1.$$

Similarly we can find,

$$\widetilde{Bm}_{I_1}(S_{1,n}) = (\frac{9n+3}{2})\varepsilon_2, \widetilde{Bm}_{I_2}(S_{1,n}) = (\frac{7n+3}{2})\varepsilon_2.$$

$$\widetilde{Bm}_{F_1}(S_{1,n}) = (\frac{9n+3}{2})\varepsilon_3, \widetilde{Bm}_{F_2}(S_{1,n}) = (\frac{7n+3}{2})\varepsilon_3.$$

Hence single valued neutrosophic bi-magic labeling of a star graph $S_{1,n}$ are

$$\widetilde{Bm}_1(S_{1,n}) = (\widetilde{Bm}_{T_1}(S_{1,n}), \widetilde{Bm}_{I_1}(S_{1,n}), \widetilde{Bm}_{F_1}(S_{1,n})) \text{ and}$$

$$\widetilde{Bm}_2(S_{1,n}) = (\widetilde{Bm}_{T_2}(S_{1,n}), \widetilde{Bm}_{I_2}(S_{1,n}), \widetilde{Bm}_{F_2}(S_{1,n})).$$

Case - (2) When n is even,

$$T_A(u) = (2n+1)\varepsilon_1,$$

$$I_A(u) = (2\text{n}+1)\varepsilon_2,$$

$$F_A(u) = (2n+1)\varepsilon_3.$$

$$T_A(v_k) = T_A(u) - k\varepsilon_1; 1 \leq k \leq n,$$

$$I_A(v_k) = I_A(u) - k\varepsilon_2; 1 \leq k \leq n,$$

$$F_A(v_k) = F_A(u) - k\varepsilon_3; 1 \leq k \leq n.$$

$$T_B(u, v_k) = (\frac{n}{2} + k)\varepsilon_1; 1 \leq k \leq \frac{n}{2},$$

$$I_B(u, v_k) = (\frac{n}{2} + k)\varepsilon_2; 1 \leq k \leq \frac{n}{2},$$

$$F_B(u, v_k) = (\frac{n}{2} + k)\varepsilon_3; 1 \leq k \leq \frac{n}{2}.$$

$$T_B(u, v_k) = (k - \frac{n}{2})\varepsilon_1; \frac{n+2}{2} \leq k \leq n,$$

$$I_B(u, v_k) = (k - \frac{n}{2})\varepsilon_2; \frac{n+2}{2} \leq k \leq n,$$

$$F_B(u, v_k) = (k - \frac{n}{2})\varepsilon_3; \frac{n+2}{2} \leq k \leq n.$$

For each edge (u, v_k) the SVN bi-magic labeling are,

$$\widetilde{Bm}_{T_1}(S_{1,n}) = T_A(u) + T_B(u.v_k) + T_A(v_k), 1 \leq k \leq \frac{n}{2},$$

$$\widetilde{Bm}_{T_1}(S_{1,n}) = (\frac{9n+4}{2})\varepsilon_1.$$

$$\widetilde{Bm}_{T_2}(S_{1,n}) = T_A(u) + T_B(u.v_k) + T_A(v_k), \frac{n+2}{2} \leq k \leq n,$$

$$\widetilde{Bm}_{T_2}(S_{1,n}) = (\frac{7n+4}{2})\varepsilon_1.$$

Similarly we can find,

$$\widetilde{Bm}_{I_1}(S_{1,n}) = (\frac{9n+4}{2})\varepsilon_2, \widetilde{Bm}_{I_2}(S_{1,n}) = (\frac{7n+4}{2})\varepsilon_2.$$

$$\widetilde{Bm}_{F_1}(S_{1,n}) = (\frac{9n+4}{2})\varepsilon_3, \widetilde{Bm}_{F_2}(S_{1,n}) = (\frac{7n+4}{2})\varepsilon_3.$$

Hence single valued neutrosophic bi-magic labeling of a star graph $S_{1,n}$ are

$$\widetilde{Bm}_1(S_{1,n}) = (\widetilde{Bm}_{T_1}(S_{1,n}), \widetilde{Bm}_{I_1}(S_{1,n}), \widetilde{Bm}_{F_1}(S_{1,n})) \text{ and}$$

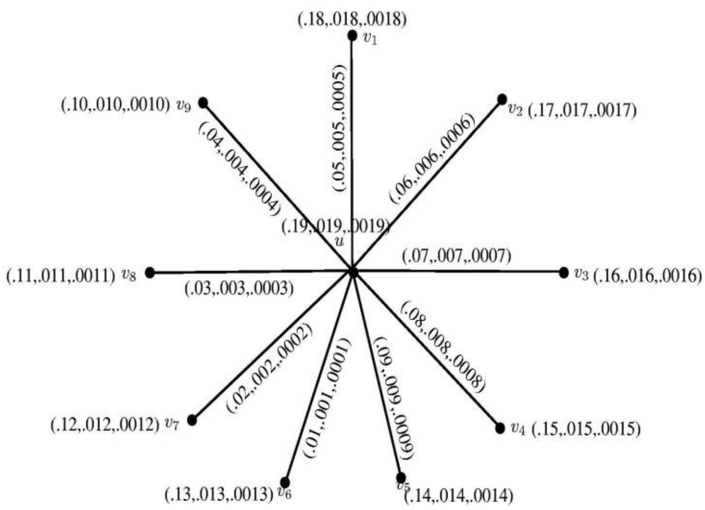
$$\widetilde{Bm}_2(S_{1,n}) = (\widetilde{Bm}_{T_2}(S_{1,n}), \widetilde{Bm}_{I_2}(S_{1,n}), \widetilde{Bm}_{F_2}(S_{1,n})).$$

Example 3.5 Consider a SVN star $S_{1,9}$ such that $V = \{u, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$ and

$$E = \{(u, v_1), (u, v_2), (u, v_3), (u, v_4), (u, v_5), (u, v_6), (u, v_7), (u, v_8), (u, v_9)\}.$$

$$\widetilde{Bm}_1(S_{1,9}) = (0.42, 0.042, 0.0042) \text{ and } \widetilde{Bm}_2(S_{1,9}) = (0.33, 0.033, 0.0033).$$

Hence $S_{1,9}$ is a SVN Bi-Magic graph.



SVN Bi-Magic star graph $S_{1,9}$

4. CONCLUSIONS

In this paper, the concepts of Bi-Magic labeling on single valued neutrosophic path, cycle and star graphs have been discussed. In future we can extend this Bi-Magic labeling on some single valued neutrosophic special graphs.

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