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Improved Symmetry Measures of Simplified Neutrosophic Sets and Their Decision-Making Method Based on a Sine Entropy Weight Model

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Received: 31 May 2018; Accepted: 13 June 2018; Published: 15 June 2018

Abstract: This work indicates the insufficiency of existing symmetry measures (SMs) between asymmetry measures of simplified neutrosophic sets (SNSs) and proposes the improved normalized SMs of SNSs, including the improved SMs and weighted SMs in single-valued and interval neutrosophic settings. Then, the sine entropy measures of SNSs are presented to establish a sine entropy weight model for solving the criteria weights in decision-making. Based on the improved weighted SMs of SNSs and the sine entropy weight model, a multi-criteria decision-making (MCDM) method with unknown criteria weights (an improved MCDM method) is established in the SNS setting. In the MCDM process, corresponding to the criteria weights obtained by the sine entropy model, the ranking order of all alternatives and the best one are given by means of the improved weighted SMs between the ideal solution and each alternative. Lastly, the improved MCDM method is applied to an actual decision example in single-valued and interval neutrosophic settings to indicate the feasibility of the improved MCDM method. By comparative analysis with existing MCDM methods, the improved SMs and the sine entropy weight model not only provide a simpler and more effective method for MCDM problems with unknown criteria weights in the SNS setting, but can also overcome the insufficiency of the existing SMs and MCDM method.

Keywords: simplified neutrosophic symmetry measure; sine entropy; interval neutrosophic set; single-valued neutrosophic set; decision making

1. Introduction

Since a neutrosophic set (NS) [1] provides an effective way to express inconsistent, incomplete, and indeterminate information in the real world, which cannot be expressed by the fuzzy set and (interval-valued) intuitionistic fuzzy set [2–5], it has been widely applied in various fields, such as image processing [6–9], object tracking [10–12], and decision-making [13]. As a subclass of NS, a simplified neutrosophic set (SNS) [14], implying single-valued neutrosophic set (SVNS) and interval neutrosophic set (INS) concepts, is composed of the truth, indeterminacy, and falsity components, where their membership degrees are constrained in the real standard interval [0, 1]. A large number of studies of SNSs/SVNSs/INSs have been applied to decision-making problems with known/given criteria weights [15–24] and unknown criteria weights [25]. However, various measures between SNSs/SVNSs/INSs are important mathematical tools in multi-criteria decision-making (MCDM) problems. For instance, three vector similarity measures (the cosine, Dice, jaccard measures) of SNSs [16], similarity measures of INSs [26], hybrid vector similarity measures of SNSs [27], and the generalized Dice measures of SNSs (containing the Dice measures and asymmetry measures as their special cases) [28] were presented for MCDM problems. Then, the cross-entropy measures of

SVNSs and INSs [15,29] were used for MCDM problems. After that, the projection and bidirectional projection measures of SVNSs [30] and the harmonic averaging projection measures of SNSs [31] were developed for MCDM problems.

Especially in more recent research, Tu et al. [32] firstly proposed the normalized SMs based on asymmetry measures of SNSs and their MCDM method with known criteria weights and indicated its main advantage of the strengthened resolution/discrimination in the decision-making process. However, the SMs of SNSs presented in [32] may produce undefined/unmeaningful situations in some cases, which will indicate their insufficiency in the following section. Furthermore, the SM-based MCDM method introduced in [32] cannot deal with decision-making problems with unknown criteria weights. To solve these issues, this work proposes the improved normalized SMs and weighted SMs based on the asymmetry measures of SNSs, the sine entropy of SNS, and their MCDM method with unknown criteria weights in SVNS and INS (SNS) settings.

This study is constructed by the following framework: Section 2 describes the existing SMs based on asymmetry measures of SNSs and indicates their insufficiency in some cases. In Section 3, the improved normalized SMs and improved weighted SMs of SNSs based on the asymmetry measures of SNSs are proposed in SVNS and INS settings. Section 4 presents the sine entropy of SNS based on sine function and its proof. In Section 5, a MCDM method with unknown criteria weights (an improved MCDM method) is developed based on the improved weighted SMs and the sine entropy weight model. Section 6 presents an actual decision example in SVNS and INS setting to show the application of the improved MCDM method and compares the improved MCDM method with an existing MCDM method by considering the given criteria weights and sine entropy weights to demonstrate the feasibility and effectiveness of the improved MCDM method. Finally, conclusions and future research are contained in Section 7.

2. Existing SMs between Simplified Neutrosophic Asymmetry Measures and Insufficiency

This section introduces the normalized SMs between simplified neutrosophic asymmetry measures presented in [32] and indicates their insufficiency.

The SNS introduced by Ye [14] can be expressed as $Y = \{\langle z, \alpha_Y(z), \beta_Y(z), \gamma_Y(z)\rangle | z \in Z\}$ in the universe of discourse Z, such that $\alpha_Y(z)$: $Z \to [0, 1]$, $\beta_Y(z)$: $Z \to [0, 1]$, and $\gamma_Y(z)$: $Z \to [0, 1]$, which are depicted by the truth, indeterminacy, and falsity membership degrees, with either $0 \le \sup \alpha_Y(z) + \sup \beta_Y(z) + \sup \gamma_Y(z) \le 3$ for INS or $0 \le \alpha_Y(z) + \beta_Y(z) + \gamma_Y(z) \le 3$ for SVNS and $z \in Z$. Then an element $\langle z, \alpha_Y(z), \beta_Y(z), \gamma_Y(z)\rangle$ in the SNS Y is denoted by the simplified neutrosophic number (SNN) $y = \langle \alpha_y, \beta_y, \gamma_y \rangle$ for short.

Assume that $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ are two SNSs, where $x_j = \langle \alpha_{xj}, \beta_{xj}, \gamma_{xj} \rangle$ and $y_j = \langle \alpha_{yj}, \beta_{yj}, \gamma_{yj} \rangle$ is the j-th single-valued neutrosophic numbers (SVNNs) (j = 1, 2, ..., n) and $x_j = \langle [\alpha^-_{xj}, \alpha^+_{xj}], [\beta^-_{xj}, \beta^+_{xj}], [\gamma^-_{xj}, \gamma^+_{xj}] \rangle$ and $y_j = \langle [\alpha^-_{yj}, \alpha^+_{yj}], [\beta^-_{yj}, \beta^+_{yj}], [\gamma^-_{yj}, \gamma^+_{yj}] \rangle$ is the j-th interval neutrosophic numbers (INNs) (j = 1, 2, ..., n). Then asymmetry measures between X and Y are defined as follows [32]:

$$P_Y(X) = \frac{X \cdot Y}{\|Y\|^2} = \frac{\sum_{j=1}^{n} (\alpha_{xj}\alpha_{yj} + \beta_{xj}\beta_{yj} + \gamma_{xj}\gamma_{yj})}{\sum_{j=1}^{n} (\alpha^2_{yj} + \beta^2_{yj} + \gamma^2_{yj})} \text{ for SVNSs,} \tag{1}$$

$$P_X(Y) = \frac{X \cdot Y}{\|X\|^2} = \frac{\sum_{j=1}^{n} (\alpha_{xj}\alpha_{yj} + \beta_{xj}\beta_{yj} + \gamma_{xj}\gamma_{yj})}{\sum_{j=1}^{n} (\alpha^2_{xj} + \beta^2_{xj} + \gamma^2_{xj})} \text{ for SVNSs,} \tag{2}$$

$$P_Y(X)=\frac{X\cdot Y}{\|Y\|^2}=\frac{\sum_{j=1}^{n}\left(\alpha_{xj}^{-}\alpha_{yj}^{-}+\alpha_{xj}^{+}\alpha_{yj}^{+}+\beta_{xj}^{-}\beta_{yj}^{-}+\beta_{xj}^{+}\beta_{yj}^{+}+\gamma_{xj}^{-}\gamma_{yj}^{-}+\gamma_{xj}^{+}\gamma_{yj}^{+}\right)}{\sum_{j=1}^{n}\left[(\alpha_{yj}^{-})^2+(\alpha_{yj}^{+})^2+(\beta_{yj}^{-})^2+(\beta_{yj}^{+})^2+(\gamma_{yj}^{-})^2+(\gamma_{yj}^{+})^2\right]}\text{ for INSs,} \tag{3}$$

$$P_X(Y)=\frac{X\cdot Y}{\|X\|^2}=\frac{\sum_{j=1}^{n}\left(\alpha_{xj}^{-}\alpha_{yj}^{-}+\alpha_{xj}^{+}\alpha_{yj}^{+}+\beta_{xj}^{-}\beta_{yj}^{-}+\beta_{xj}^{+}\beta_{yj}^{+}+\gamma_{xj}^{-}\gamma_{yj}^{-}+\gamma_{xj}^{+}\gamma_{yj}^{+}\right)}{\sum_{j=1}^{n}\left[(\alpha_{xj}^{-})^2+(\alpha_{xj}^{+})^2+(\beta_{xj}^{-})^2+(\beta_{xj}^{+})^2+(\gamma_{xj}^{-})^2+(\gamma_{xj}^{+})^2\right]}\text{ for INSs.} \tag{4}$$

Thus, the normalized SM between SNSs (SVNSs and INSs) X and Y introduced in [32] is presented as follows:

$$M(X,Y)=\frac{1}{1+\left|\frac{X\cdot Y}{\|X\|^2}-\frac{X\cdot Y}{\|Y\|^2}\right|}=\frac{\|X\|^2\|Y\|^2}{\|X\|^2\|Y\|^2+\left|\|X\|^2-\|Y\|^2\right|X\cdot Y}, \tag{5}$$

which contains the following normalized SMs of single-valued and interval neutrosophic asymmetry measures:

$$\begin{aligned}M_1(X,Y)&=\frac{\|X\|^2\|Y\|^2}{\|X\|^2\|Y\|^2+\left|\|X\|^2-\|Y\|^2\right|X\cdot Y}\\&=\frac{\sum_{j=1}^{n}\left[(\alpha_{xj})^2+(\beta_{xj})^2+(\gamma_{xj})^2\right]\times\sum_{j=1}^{n}\left[(\alpha_{yj})^2+(\beta_{yj})^2+(\gamma_{yj})^2\right]}{\left\{\begin{array}{l}\sum_{j=1}^{n}\left[(\alpha_{xj})^2+(\beta_{xj})^2+(\gamma_{xj})^2\right]\times\sum_{j=1}^{n}\left[(\alpha_{yj})^2+(\beta_{yj})^2+(\gamma_{yj})^2\right]\\+\left|\sum_{j=1}^{n}\left[(\alpha_{xj})^2+(\beta_{xj})^2+(\gamma_{xj})^2\right]-\sum_{j=1}^{n}\left[(\alpha_{yj})^2+(\beta_{yj})^2+(\gamma_{yj})^2\right]\right|\\\times\sum_{j=1}^{n}\left[\alpha_{xj}\alpha_{yj}+\beta_{xj}\beta_{yj}+\gamma_{xj}\gamma_{yj}\right]\end{array}\right\}}\text{ for SVNSs,}\end{aligned} \tag{6}$$

$$\begin{aligned}M_2(X,Y)&=\frac{\|X\|^2\|Y\|^2}{\|X\|^2\|Y\|^2+\left|\|X\|^2-\|Y\|^2\right|X\cdot Y}\\&=\frac{\sum_{j=1}^{n}\left[(\alpha_{xj}^{-})^2+(\alpha_{xj}^{+})^2+(\beta_{xj}^{-})^2+(\beta_{xj}^{+})^2+(\gamma_{xj}^{-})^2+(\gamma_{xj}^{+})^2\right]\times\sum_{j=1}^{n}\left[(\alpha_{yj}^{-})^2+(\alpha_{yj}^{+})^2+(\beta_{yj}^{-})^2+(\beta_{yj}^{+})^2+(\gamma_{yj}^{-})^2+(\gamma_{yj}^{+})^2\right]}{\left\{\begin{array}{l}\sum_{j=1}^{n}\left[(\alpha_{xj}^{-})^2+(\alpha_{xj}^{+})^2+(\beta_{xj}^{-})^2+(\beta_{xj}^{+})^2+(\gamma_{xj}^{-})^2+(\gamma_{xj}^{+})^2\right]\times\sum_{j=1}^{n}\left[(\alpha_{yj}^{-})^2+(\alpha_{yj}^{+})^2+(\beta_{yj}^{-})^2+(\beta_{yj}^{+})^2+(\gamma_{yj}^{-})^2+(\gamma_{yj}^{+})^2\right]\\+\left|\sum_{j=1}^{n}\left[(\alpha_{xj}^{-})^2+(\alpha_{xj}^{+})^2+(\beta_{xj}^{-})^2+(\beta_{xj}^{+})^2+(\gamma_{xj}^{-})^2+(\gamma_{xj}^{+})^2\right]-\sum_{j=1}^{n}\left[(\alpha_{yj}^{-})^2+(\alpha_{yj}^{+})^2+(\beta_{yj}^{-})^2+(\beta_{yj}^{+})^2+(\gamma_{yj}^{-})^2+(\gamma_{yj}^{+})^2\right]\right|\\\times\sum_{j=1}^{n}\left(\alpha_{xj}^{-}\alpha_{yj}^{-}+\alpha_{xj}^{+}\alpha_{yj}^{+}+\beta_{xj}^{-}\beta_{yj}^{-}+\beta_{xj}^{+}\beta_{yj}^{+}+\gamma_{xj}^{-}\gamma_{yj}^{-}+\gamma_{xj}^{+}\gamma_{yj}^{+}\right)\end{array}\right\}}\text{ for INSs.}\end{aligned} \tag{7}$$

However, when $x_j=\langle\alpha_{xj},\beta_{xj},\gamma_{xj}\rangle=<0,0,0>$ or $y_j=\langle\alpha_{yj},\beta_{yj},\gamma_{yj}\rangle=<0,0,0>$ and $x_j=<[\alpha_{xj}^{-},\alpha_{xj}^{+}],[\beta_{xj}^{-},\beta_{xj}^{+}],[\gamma_{xj}^{-},\gamma_{xj}^{+}]>=<[0,0],[0,0],[0,0]>$ or $y_j=<[\alpha_{yj}^{-},\alpha_{yj}^{+}],[\beta_{yj}^{-},\beta_{yj}^{+}],[\gamma_{yj}^{-},\gamma_{yj}^{+}]>=<[0,0],[0,0],[0,0]>$ (j = 1, 2, ... , n) in X or Y, Equations (6) and (7) are undefined/unmeaningful. In these cases, existing SMs of SNSs (SVNSs and INSs) cannot be suitable for the decision-making and pattern recognition problems with the neutrosophic information. Hence, it is necessary to improve the algorithm of the existing normalized SMs.

3. Improved Normalized SMs of SNSs

To overcome the aforementioned insufficiency of the existing SMs [32], an improved normalized SM based on simplified neutrosophic asymmetry measures between SNSs X and Y is proposed as follows:

$$H(X,Y)=1-\frac{|P_X(Y)-P_Y(X)|}{P_X(Y)+P_Y(X)}=1-\frac{\left|\|Y\|^2-\|X\|^2\right|}{\|Y\|^2+\|X\|^2}, \tag{8}$$

which contains the following normalized SMs of SVNSs and INSs:

$$H_1(X,Y) = 1 - \frac{\left| \|Y\|^2 - \|X\|^2 \right|}{\|Y\|^2 + \|X\|^2} = 1 - \frac{\left| \sum_{j=1}^{n} (\alpha_{yj}^2 + \beta_{yj}^2 + \gamma_{yj}^2) - \sum_{j=1}^{n} (\alpha_{xj}^2 + \beta_{xj}^2 + \gamma_{xj}^2) \right|}{\sum_{j=1}^{n} (\alpha_{yj}^2 + \beta_{yj}^2 + \gamma_{yj}^2) + \sum_{j=1}^{n} (\alpha_{xj}^2 + \beta_{xj}^2 + \gamma_{xj}^2)} \text{ for SVNSs,} \quad (9)$$

$$H_2(X,Y) = 1 - \frac{\left| \|Y\|^2 - \|X\|^2 \right|}{\|Y\|^2 + \|X\|^2} = 1 - \frac{\left| \begin{array}{l} \sum_{j=1}^{n} [(\alpha_{yj}^-)^2 + (\alpha_{yj}^+)^2 + (\beta_{yj}^-)^2 + (\beta_{yj}^+)^2 + (\gamma_{yj}^-)^2 + (\gamma_{yj}^+)^2] \\ - \sum_{j=1}^{n} [(\alpha_{xj}^-)^2 + (\alpha_{xj}^+)^2 + (\beta_{xj}^-)^2 + (\beta_{xj}^+)^2 + (\gamma_{xj}^-)^2 + (\gamma_{xj}^+)^2] \end{array} \right|}{\left(\begin{array}{l} \sum_{j=1}^{n} [(\alpha_{yj}^-)^2 + (\alpha_{yj}^+)^2 + (\beta_{yj}^-)^2 + (\beta_{yj}^+)^2 + (\gamma_{yj}^-)^2 + (\gamma_{yj}^+)^2] \\ + \sum_{j=1}^{n} [(\alpha_{xj}^-)^2 + (\alpha_{xj}^+)^2 + (\beta_{xj}^-)^2 + (\beta_{xj}^+)^2 + (\gamma_{xj}^-)^2 + (\gamma_{xj}^+)^2] \end{array} \right)} \text{ for INSs,} \quad (10)$$

where $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ are the two SNSs, including the SVNNs $x_j = \langle \alpha_{xj}, \beta_{xj}, \gamma_{xj} \rangle$ and $y_j = \langle \alpha_{yj}, \beta_{yj}, \gamma_{yj} \rangle$ (j = 1, 2, ..., n) and the INNs $x_j = \langle [\alpha_{xj}^-, \alpha_{xj}^+], [\beta_{xj}^-, \beta_{xj}^+], [\gamma_{xj}^-, \gamma_{xj}^+] \rangle$ and $y_j = \langle [\alpha_{yj}^-, \alpha_{yj}^+], [\beta_{yj}^-, \beta_{yj}^+], [\gamma_{yj}^-, \gamma_{yj}^+] \rangle$ (j = 1, 2, ..., n) in X and Y.

Since Equations (9) and (10) are the normalized SMs of SVNSs and INSs, they satisfy the conditions: $H_k(X, Y) = H_k(Y, X)$ and $0 \leq H_k(X, Y) \leq 1$ for k = 1, 2. Then the improved SMs of SVNSs and INSs can overcome the insufficiency of the existing SMs of SVNSs and INSs because the improved SMs do not imply the aforementioned undefined/unmeaningful situation, and also show simpler algorithms in comparison to Equations (6) and (7).

If the importance of each element x_j or y_j (j = 1, 2, ..., n) is considered in X and Y by w_j, with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, the improved weighted SM between asymmetry measures of SNSs can be presented by:

$$W(X,Y) = 1 - \frac{|P_{Xw}(Y) - P_{Yw}(X)|}{P_{Xw}(Y) + P_{Yw}(X)} = 1 - \frac{\left| \|Y\|_w^2 - \|X\|_w^2 \right|}{\|Y\|_w^2 + \|X\|_w^2}, \quad (11)$$

which contains the following improved weighted SMs of SVNSs and INSs:

$$W_1(X,Y) = 1 - \frac{\left| \sum_{j=1}^{n} w_j^2 (\alpha_{yj}^2 + \beta_{yj}^2 + \gamma_{yj}^2) - \sum_{j=1}^{n} w_j^2 (\alpha_{xj}^2 + \beta_{xj}^2 + \gamma_{xj}^2) \right|}{\sum_{j=1}^{n} w_j^2 (\alpha_{yj}^2 + \beta_{yj}^2 + \gamma_{yj}^2) + \sum_{j=1}^{n} w_j^2 (\alpha_{xj}^2 + \beta_{xj}^2 + \gamma_{xj}^2)} \text{ for SVNSs,} \quad (12)$$

$$W_2(X,Y) = 1 - \frac{\left| \begin{array}{l} \sum_{j=1}^{n} w_j^2 [(\alpha_{yj}^-)^2 + (\alpha_{yj}^+)^2 + (\beta_{yj}^-)^2 + (\beta_{yj}^+)^2 + (\gamma_{yj}^-)^2 + (\gamma_{yj}^+)^2] \\ - \sum_{j=1}^{n} w_j^2 [(\alpha_{xj}^-)^2 + (\alpha_{xj}^+)^2 + (\beta_{xj}^-)^2 + (\beta_{xj}^+)^2 + (\gamma_{xj}^-)^2 + (\gamma_{xj}^+)^2] \end{array} \right|}{\left\{ \begin{array}{l} \sum_{j=1}^{n} w_j^2 [(\alpha_{yj}^-)^2 + (\alpha_{yj}^+)^2 + (\beta_{yj}^-)^2 + (\beta_{yj}^+)^2 + (\gamma_{yj}^-)^2 + (\gamma_{yj}^+)^2] \\ + \sum_{j=1}^{n} w_j^2 [(\alpha_{xj}^-)^2 + (\alpha_{xj}^+)^2 + (\beta_{xj}^-)^2 + (\beta_{xj}^+)^2 + (\gamma_{xj}^-)^2 + (\gamma_{xj}^+)^2] \end{array} \right\}} \text{ for INSs.} \quad (13)$$

Since Equations (12) and (13) are the weighted normalized SMs of SVNSs and INSs, they also satisfy these conditions: $W_k(X, Y) = W_k(Y, X)$ and $0 \leq W_k(X, Y) \leq 1$ for k = 1, 2.

4. Simplified Neutrosophic Sine Entropy

In this section, we propose the sine entropy of SNS based on sine function to determine unknown criteria weights in the following MCDM problems.

Definition 1. *Let* $Y = \{y_1, y_2, \ldots, y_n\}$ *be an SNS, where* $y_j = \langle \alpha_{yj}, \beta_{yj}, \gamma_{yj} \rangle$ *is the j-th SVNN and* $y_j = \langle [\alpha_{yj}^-, \alpha_{yj}^+], [\beta_{yj}^-, \beta_{yj}^+], [\gamma_{yj}^-, \gamma_{yj}^+] \rangle$ *is the j-th INN (j = 1, 2, ..., n). Then the sine entropy measures of Y are defined as follows:*

$$S_{E1}(Y) = \frac{1}{3n}\sum_{j=1}^{n}[\sin(\alpha_j\pi) + \sin(\beta_j\pi) + \sin(\gamma_j\pi)] \text{ for the SVNS } Y, \tag{14}$$

$$S_{E2}(Y) = \frac{1}{6n}\sum_{j=1}^{n}[\sin(\alpha_j^-\pi) + \sin(\beta_j^-\pi) + \sin(\gamma_j^-\pi) + \sin(\alpha_j^+\pi) + \sin(\beta_j^+\pi) + \sin(\gamma_j^+\pi)] \text{ for the INS } Y. \tag{15}$$

Following an axiomatic definition of the entropy measures of SNSs [33,34], the sine entropy measures of SVNS and INS have the following theorem.

Theorem 1. *Let the fuzziest SVNN be* a_j *= <0.5, 0.5, 0.5> or the fuzziest INN be* a_j *= <[0.5, 0.5], [0.5, 0.5], [0.5, 0.5]> (j = 1, 2, ..., n) in the fuzziest SNS* $A = \{a_1, a_2, \ldots, a_n\}$*. Then, the sine entropy measure* $S_{Ek}(Y)$ *(k = 1, 2) satisfies the following properties:*

(E1) $S_{Ek}(Y) = 0$ if $Y = \{y_1, y_2, \ldots, y_n\}$ is a crisp set, i.e., y_j = <1, 0, 0> or y_j = <0, 0, 1> for SVNN and y_j = <[1, 1], [0, 0], [0, 0]> or y_j = <[0, 0], [0, 0], [1, 1]> (j = 1, 2, ..., n) for INS;
(E2) $S_{Ek}(Y) = 1$ if and only if $y_j = a_j$ (j = 1, 2, ..., n);
(E3) If the closer an SNS Y is to the fuzziest SNS A than an SNS X, the fuzzier Y is than X, then $S_{Ek}(X) \leq S_{Ek}(Y)$;
(E4) $S_{Ek}(Y) = S_{Ek}(Y^c)$ if Y^c is the complement of Y.

Proof:

(E1) For a crisp set $Y = \{y_1, y_2, \ldots, y_n\}$, i.e., y_j = <1, 0, 0> or y_j = <0, 0, 1> for SVNN and y_j = <[1, 1], [0, 0], [0, 0]> or y_j = <[0, 0], [0, 0], [1, 1]> (j = 1, 2, ..., n) for INN, by use of Equation (14) we obtain the following result:

$$S_{E1}(Y) = \frac{1}{3n}\sum_{j=1}^{n}[\sin(\alpha_j\pi) + \sin(\beta_j\pi) + \sin(\gamma_j\pi)] = \frac{n}{3n}[\sin(1\times\pi) + \sin(0\times\pi) + \sin(0\times\pi)] = 0,$$

or

$$S_{E1}(Y) = \frac{1}{3n}\sum_{j=1}^{n}[\sin(\alpha_j\pi) + \sin(\beta_j\pi) + \sin(\gamma_j\pi)] = \frac{n}{3n}[\sin(0\times\pi) + \sin(0\times\pi) + \sin(1\times\pi)] = 0,$$

and by use of Equation (15) we also obtain the following result:

$$\begin{aligned} S_{E2}(Y) &= \tfrac{1}{6n}\sum_{j=1}^{n}[\sin(\alpha_j^-\pi) + \sin(\beta_j^-\pi) + \sin(\gamma_j^-\pi) + \sin(\alpha_j^+\pi) + \sin(\beta_j^+\pi) + \sin(\gamma_j^+\pi)] \\ &= \tfrac{n}{6n}[\sin(1\times\pi) + \sin(0\times\pi) + \sin(0\times\pi) + \sin(1\times\pi) + \sin(0\times\pi) + \sin(0\times\pi)] = 0, \end{aligned}$$

or:

$$\begin{aligned} S_{E2}(Y) &= \tfrac{1}{6n}\sum_{j=1}^{n}[\sin(\alpha_j^-\pi) + \sin(\beta_j^-\pi) + \sin(\gamma_j^-\pi) + \sin(\alpha_j^+\pi) + \sin(\beta_j^+\pi) + \sin(\gamma_j^+\pi)] \\ &= \tfrac{n}{6n}[\sin(0\times\pi) + \sin(0\times\pi) + \sin(1\times\pi) + \sin(0\times\pi) + \sin(0\times\pi) + \sin(1\times\pi)] = 0. \end{aligned}$$

(E2) Let the sine function be $f(z_j) = \sin(z_j\pi)$ for $z_j \in [0, 1]$ ($j = 1, 2, \ldots, n$). By differentiating $f(z_j)$ with respect to z_j and equating to zero, there exist the following results:

$$\frac{\partial f(z_j)}{\partial z_j} = \pi\cos(z_j\pi), \tag{16}$$

$$\frac{\partial f(z_j)}{\partial z_j} = \pi\cos(z_j\pi) = 0.$$

Thus, the critical point of z_j is $z_j = 0.5$.
By differentiating Equation (16) with respect to z_j, we obtain:

$$\frac{\partial^2 f(z_j)}{\partial(z_j)^2} = -\pi^2\sin(z_j\pi). \tag{17}$$

Hence, Equation (17) can indicate the following inequality:

$$\frac{\partial^2 f(z_j)}{\partial(z_j)^2} < 0 \text{ for } z_j = 0.5.$$

Obviously, $f(z_j)$ for $z_j \in [0, 1]$ implies a concave function with the global maximum $f(z_j) = 1$ at $z_j = 0.5$. Then, the sine entropy measures of an SNS Y can be expressed as the following two forms:

$$S_{E1}(Y) = \frac{1}{3n}\sum_{j=1}^{n}\left[f(\alpha_{yj}) + f(\beta_{yj}) + f(\gamma_{yj})\right] \text{ for the SVNS } Y,$$

$$S_{E2}(Y) = \frac{1}{6n}\sum_{j=1}^{n}\begin{bmatrix} f\left(\alpha_{yj}^{-}\right) + f\left(\beta_{yj}^{-}\right) + f\left(\gamma_{yj}^{-}\right) \\ +f\left(\alpha_{yj}^{+}\right) + f\left(\beta_{yj}^{+}\right) + f\left(\gamma_{yj}^{+}\right) \end{bmatrix} \text{ for the INS } Y.$$

Clearly, $S_{Ek}(Y) = 1$ ($k = 1, 2$) $\Leftrightarrow$ $y_j = a_j = <0.5, 0.5, 0.5>$ or $y_j = a_j = <[0.5, 0.5], [0.5, 0.5], [0.5, 0.5]>$ ($j = 1, 2, \ldots, n$).

(E3) According to Equation (16), $f(z_j)$ for $z_j \in [0, 1]$ ($j = 1, 2, \ldots, n$) is an increasing function if $z_j < 0.5$ and then it is a decreasing function if $z_j > 0.5$.

Therefore, the closer an SNS Y is to the fuzziest SNS A than an SNS X, and then $S_{Ek}(X) \leq S_{Ek}(Y)$ ($k = 1, 2$).

(E4) Since the complement of the SVNN $y_j = \langle\alpha_{yj}, \beta_{yj}, \gamma_{yj}\rangle$ in Y is $y_j^c = \langle\gamma_{yj}, 1 - \beta_{yj}, \alpha_{yj}\rangle$, i.e., $(\alpha_{yj})^c = \gamma_{yj}$ and $(\beta_{yj})^c = 1 - \beta_{yj}$ ($j = 1, 2, \ldots, n$) and the complement of the INN $y_j = \langle[\alpha_{yj}^{-}, \alpha_{yj}^{+}], [\beta_{yj}^{-}, \beta_{yj}^{+}], [\gamma_{yj}^{-}, \gamma_{yj}^{+}]\rangle$ in Y is $y_j^c = \langle[\gamma_{yj}^{-}, \gamma_{yj}^{+}], [1 - \beta_{yj}^{+}, 1 - \beta_{yj}^{-}], [\alpha_{yj}^{-}, \alpha_{yj}^{+}]\rangle$, i.e., $[\alpha_{yj}^{-}, \alpha_{yj}^{+}]^c = [\gamma_{yj}^{-}, \gamma_{yj}^{+}]$ and $[\beta_{yj}^{-}, \beta_{yj}^{+}]^c = [1 - \beta_{yj}^{+}, 1 - \beta_{yj}^{-}]$ ($j = 1, 2, \ldots, n$). Then, there is $S_{Ek}(Y^c) = S_{Ek}(Y)$ ($k = 1, 2$) by using Equations (14) and (15).

This completes the proof of the theorem. □

It is worth noting that the SVNS Y is a special case of the INS Y if $\alpha_{yj}^{-} = \alpha_{yj}^{+} = \alpha_{yj}$, $\beta_{yj}^{-} = \beta_{yj}^{+} = \beta_{yj}$, and $\gamma_{yj}^{-} = \gamma_{yj}^{+} = \gamma_{yj}$ in the INN $y_j = \langle[\alpha_{yj}^{-}, \alpha_{yj}^{+}], [\beta_{yj}^{-}, \beta_{yj}^{+}], [\gamma_{yj}^{-}, \gamma_{yj}^{+}]\rangle$ ($j = 1, 2, \ldots, n$). In this case, Equation (15) is reduced to Equation (14).

5. Decision-Making Method Using the Improved Weighted SMs of SNSs

In this section, the improved weighted SMs of SNSs (SVNSs and INSs) and the simplified neutrosophic sine entropy are utilized for MCDM problems with unknown criteria weights.

In a MCDM problem with unknown criteria weights, suppose that a set of alternatives is $Y = \{Y_1, Y_2, \ldots, Y_m\}$ and a set of criteria is $R = \{R_1, R_2, \ldots, R_n\}$. Thus, we propose the MCDM method based on the improved weighted SMs of SNSs and the sine entropy weights of SNSs in SVNS and INS setting, which is called the improved MCDM method in the following.

In SNS (SVNS and INS) setting, the suitable evaluations of each alternative Y_j ($j = 1, 2, \ldots, n$) over criteria R_i ($i = 1, 2, \ldots, m$) are represented by an SNS $Y_i = \{y_{i1}, y_{i2}, \ldots, y_{in}\}$, where $y_{ij} = <\alpha_{ij}, \beta_{ij}, \gamma_{ij}>$ is an SVNN for $\alpha_{ij}, \beta_{ij}, \gamma_{ij} \in [0, 1]$ and $0 \le \alpha_{ij} + \beta_{ij} + \gamma_{ij} \le 3$ or $y_{ij} = \langle [\alpha_{ij}^-, \alpha_{ij}^+], [\beta_{ij}^-, \beta_{ij}^+], [\gamma_{ij}^-, \gamma_{ij}^+] \rangle$ is an INN for $\alpha_{ij}, \beta_{ij}, \gamma_{ij} \subseteq [0, 1]$ and $0 \le \alpha_{ij}^+ + \beta_{ij}^+ + \gamma_{ij}^+ \le 3$. Thus, the decision matrix of SNSs $M = (y_{ij})_{m\times n}$ can be established in SVNS or INS setting. Thus, the improved MCDM method is indicated by the following steps:

Step 1. Based on the concept of an ideal solution (alternative), we can determine the ideal solution $Y^* = \{y_1^*, y_2^*, \ldots, y_n^*\}$ where $y_j^* =< \alpha_j^*, \beta_j^*, \gamma_j^* >=< \max_i(\alpha_{ij}), \min_i(\beta_{ij}), \min_i(\gamma_{ij}) >$ is an ideal SVNN or $y_j^* =< \alpha_j^*, \beta_j^*, \gamma_j^* >=< [\max_i(\alpha_{ij}^-), \max_i(\alpha_{ij}^+)], [\min_i(\beta_{ij}^-), \min_i(\beta_{ij}^+)], [\min_i(\gamma_{ij}^-), \min_i(\gamma_{ij}^+) >$ is an ideal INN ($j = 1, 2, \ldots, n$; $i = 1, 2, \ldots, m$).

Step 2. Since the fuzziness/uncertainty of a criterion evaluation increases, the criterion weight should decrease. So, based on the sine entropy measure formula Equation (14) or (15) we can calculate unknown weights of each criterion by the following sine entropy weight model:

$$w_j = \frac{1 - S_{Ek}(y_{ij})}{n - \sum_{j=1}^{n} S_{Ek}(y_{ij})}, \tag{18}$$

where $S_{E1}(y_{ij}) = \frac{1}{3m}\sum_{i=1}^{m}[\sin(\alpha_{ij}\pi) + \sin(\beta_{ij}\pi) + \sin(\gamma_{ij}\pi)]$ for $k = 1$ is the sine entropy of SVNNs or $S_{E2}(y_{ij}) = \frac{1}{6m}\sum_{i=1}^{m}[\sin(\alpha_{ij}^-\pi) + \sin(\beta_{ij}^-\pi) + \sin(\gamma_{ij}^-\pi) + \sin(\alpha_{ij}^+\pi) + \sin(\beta_{ij}^+\pi) + \sin(\gamma_{ij}^+\pi)]$ for $k = 2$ is the sine entropy of INNs, and $\sum_{j=1}^{n} w_j = 1$.

Step 3. By use of Equation (12) for SVNSs or Equation (13) for INSs, the improved weighted SM between Y_i ($i = 1, 2, \ldots, m$) and Y^* is given by:

$$W_1(Y_i, Y^*) = 1 - \frac{\left|\sum_{j=1}^{n} w_j^2(\alpha_{ij}^2 + \beta_{ij}^2 + \gamma_{ij}^2) - \sum_{j=1}^{n} w_j^2[(\alpha_j^*)^2 + (\beta_j^*)^2 + (\gamma_j^*)^2]\right|}{\sum_{j=1}^{n} w_j^2(\alpha_{ij}^2 + \beta_{ij}^2 + \gamma_{ij}^2) + \sum_{j=1}^{n} w_j^2[(\alpha_j^*)^2 + (\beta_j^*)^2 + (\gamma_j^*)^2]}, \tag{19}$$

or

$$W_2(Y_i, Y^*) = 1 - \frac{\left|\begin{array}{c} \sum_{j=1}^{n} w_j^2[(\alpha_{ij}^-)^2 + (\alpha_{ij}^+)^2 + (\beta_{ij}^-)^2 + (\beta_{ij}^+)^2 + (\gamma_{ij}^-)^2 + (\gamma_{ij}^+)^2] \\ -\sum_{j=1}^{n} w_j^2[(\alpha_j^{*-})^2 + (\alpha_j^{*+})^2 + (\beta_j^{*-})^2 + (\beta_j^{*+})^2 + (\gamma_j^{*-})^2 + (\gamma_j^{*+})^2] \end{array}\right|}{\left\{\begin{array}{c} \sum_{j=1}^{n} w_j^2[(\alpha_{ij}^-)^2 + (\alpha_{ij}^+)^2 + (\beta_{ij}^-)^2 + (\beta_{ij}^+)^2 + (\gamma_{ij}^-)^2 + (\gamma_{ij}^+)^2] \\ +\sum_{j=1}^{n} w_j^2[(\alpha_j^{*-})^2 + (\alpha_j^{*+})^2 + (\beta_j^{*-})^2 + (\beta_j^{*+})^2 + (\gamma_j^{*-})^2 + (\gamma_j^{*+})^2] \end{array}\right\}}. \tag{20}$$

Step 4. According to the improved weighted SM values of $W_1(Y_i, Y^*)$ or $W_2(Y_i, Y^*)$, we can rank alternatives and choose the best one.

Step 5. End.

6. Actual Decision Examples and Comparative Analysis

For convenient comparison, we adapt an actual decision example of manufacturing schemes (alternatives) from the literature [32] in the SNS (SVNS and INS) setting to indicate the applicability of the improved MCDM method, and then present the comparative analysis with existing MCDM method [32] in SVNS and INN setting to indicate the effectiveness and merits of the improved MCDM method.

6.1. Actual Decision Example

A MCDM problem of manufacturing schemes (alternatives) in the flexible manufacturing system is adapted from the literature [32]. A set of four potential alternatives for the flexible manufacturing system is provided by $Y = \{Y_1, Y_2, Y_3, Y_4\}$. Then, decision-makers should select the best one, which must satisfy the requirements of the three criteria: the improvement of quality (R_1), the market response (R_2), and the manufacturing cost (R_3). However, the criteria weights are unknown in this decision-making situation.

In the environment of SVNSs, the decision-makers are required to make the suitable evaluation of each alternative Y_i (i = 1, 2, 3, 4) over the criteria R_j (j = 1, 2, 3) by the evaluation information of SVNSs, which can be established as the following decision matrix of SVNSs:

$$M_1 = \begin{bmatrix} <0.75, 0.2, 0.2> & <0.7, 0.24, 0.26> & <0.6, 0.2, 0.25> \\ <0.8, 0.1, 0.1> & <0.75, 0.2, 0.3> & <0.7, 0.3, 0.1> \\ <0.7, 0.2, 0.15> & <0.8, 0.2, 0.1> & <0.75, 0.25, 0.2> \\ <0.8, 0.1, 0.2> & <0.7, 0.15, 0.2> & <0.7, 0.2, 0.3> \end{bmatrix}.$$

Thus, the improved MCDM method for the MCDM problem is described by the following decision steps:

Step 1. By $y_j^* = <\alpha_j^*, \beta_j^*, \gamma_j^*> = <\max_i(\alpha_{ij}), \min_i(\beta_{ij}), \min_i(\gamma_{ij})>$ (j = 1, 2, 3; i = 1, 2, 3, 4), the ideal solution (ideal alternative) of SVNSs is given as:

$$Y^* = \{y_1^*, y_2^*, y_3^*\} = \{<0.8, 0.1, 0.1>, <0.8, 0.15, 0.1>, <0.75, 0.2, 0.1>\}.$$

Step 2. By Equation (18), the criteria weight vector is obtained as follows:

$$W = (w_1, w_2, w_3) = (0.5682, 0.2952, 0.1366).$$

Step 3. By Equation (19), the improved weighted SM values between Y_i (i = 1, 2, 3, 4) and Y^* can be yielded as follows:

$$W_1(Y_1, Y^*) = 0.9757,\ W_1(Y_2, Y^*) = 0.9977,\ W_1(Y_3, Y^*) = 0.\ 9397, \text{ and } W_1(Y_4, Y^*) = 0.9989.$$

Step 4. The four alternatives are ranked by $Y_4 > Y_2 > Y_1 > Y_3$ since the SM values are $W_1(Y_4, Y^*) > W_1(S_2, S^*) > W_1(S_1, S^*) > W_1(Y_3, Y^*)$. It is obvious that Y_4 is the best scheme.

In the environment of INSs, on the other hand, the suitable evaluations of the four alternatives over the three criteria are given by INS information, which can be established as the following decision matrix of INSs:

$$M_2 = \begin{bmatrix} <[0.7, 0.8], [0.1, 0.2], [0.15, 0.3]> & <[0.7, 0.8], [0.2, 0.3], [0.1, 0.3]> & <[0.6, 0.7], [0, 0.2], [0.1, 0.4]> \\ <[0.75, 0.9], [0.1, 0.2], [0.1, 0.2]> & <[0.7, 0.8], [0.1, 0.2], [0.1, 0.3]> & <[0.6, 0.7], [0.2, 0.3], [0.1, 0.3]> \\ <[0.6, 0.8], [0.1, 0.3], [0.1, 0.2]> & <[0.7, 0.8], [0.1, 0.3], [0.1, 0.2]> & <[0.7, 0.8], [0.2, 0.4], [0.1, 0.3]> \\ <[0.8, 0.9], [0.1, 0.2], [0.1, 0.2]> & <[0.7, 0.8], [0.1, 0.2], [0.1, 0.3]> & <[0.6, 0.8], [0.2, 0.3], [0.2, 0.4]> \end{bmatrix}$$

Step 1. By $y_j^* = < \alpha_j^*, \beta_j^*, \gamma_j^* > = < [\max_i(\alpha_{ij}^-), \max_i(\alpha_{ij}^+)], [\min_i(\beta_{ij}^-), \min_i(\beta_{ij}^+)], [\min_i(\gamma_{ij}^-), \min_i(\gamma_{ij}^+) >$ for j = 1, 2, 3 and i = 1, 2, 3, 4, we can give the ideal solution of INSs (ideal alternative):

$$Y^* = \{y_1^*, y_2^*, \ldots, y_n^*\} = \{< [0.8, 0.9], [0.1, 0.2], [0.1, 0.2] >, < [0.7, 0.8], [0.1, 0.2], [0.1, 0.2] >, < [0.7, 0.8], [0, 0.2], [0.1, 0.3] >\}$$

Step 2. By Equation (18), the criteria weight vector is obtained as follows:

$$W = (w_1, w_2, w_3) = (0.4976, 0.3557, 0.1467).$$

Step 3. By Equation (20), the improved weighted SM values between Y_i (i = 1, 2, 3, 4) and Y^* can be yielded as the following results:

$$W_2(Y_1, Y^*) = 0.9521,\ W_2(Y_2, Y^*) = 0.9848,\ W_2(Y_3, Y^*) = 0.\ 9145, \text{ and } W_2(Y_4, Y^*) = 0.9933.$$

Step 4. Since the SM values are $W_2(Y_4, Y^*) > W_2(Y_2, Y^*) > W_2(Y_1, Y^*) > W_2(Y_3, Y^*)$, the four alternatives are ranked by $Y_4 > Y_2 > Y_1 > Y_3$. Hence, the alternative Y_4 is the best one.

For the above decision results in SVNS and INS setting, two kinds of ranking orders of the four alternatives and the best scheme are identical.

6.2. Comparative Analysis

This section compares the improved MCDM method with existing MCDM method in [32] to show the effectiveness and rationality of the improved MCDM method in SVNS and INS setting.

For the convenient comparison, we also give the decision results of existing MCDM method in [32] and the improved MCDM method by considering the same given/known criteria weight vector W = (0.36, 0.3, 0.34) [32] and the improved MCDM method regarding the sine entropy weights, which are indicated in Table 1.

Table 1. Decision results of existing multi-criteria decision-making (MCDM) method [32] and the improved MCDM method.

MCDM Method	SM Value between Y_i and Y^* in SVNS Setting	SM Value between Y_i and Y^* in INS Setting	Ranking Order in SVNS Setting	Ranking Order in INS Setting
Existing MCDM with the given weights [32]	0.8945, 0.9964, 0.9717, 0.9730	0.9053, 0.9423, 0.9401, 0.9762	$Y_2 > Y_4 > Y_3 > Y_1$	$Y_4 > Y_2 > Y_3 > Y_1$
Improved MCDM method with the given weights	0.9394, 0.9981, 0.9853, 0.9859	0.9472, 0.9691, 0.9675, 0.9877	$Y_2 > Y_4 > Y_3 > Y_1$	$Y_4 > Y_2 > Y_3 > Y_1$
Improved MCDM method with the sine entropy weights	0.9757, 0.9977, 0.9397, 0.9989	0.9521, 0.9848, 0.9145, 0.9933	$Y_4 > Y_2 > Y_1 > Y_3$	$Y_4 > Y_2 > Y_1 > Y_3$

Firstly, by the comparison between the existing MCDM method [32] and the improved MCDM method for considering the same given/known criteria weight vector W = (0.36, 0.3, 0.34) adopted from [32], we can see from Table 1 that they provide the same ranking orders in either SVNS settings or INS settings. In this case, it is obvious that the improved SMs are effective and feasible. Then, by the comparison between existing MCDM method with the given/known criteria weights [32] and the improved MCDM with the sine entropy weights, both demonstrate the different ranking orders because of the difference between the given/known weights and the sine entropy weights. Clearly, the criteria weights given by decision-makers' preference imply their subjectivity, while the sine entropy weight method implies its objectivity. Hence, the entropy weight method is more reasonable and more practicable than the given criteria weight method in actual MCDM problems.

However, the main highlights of the improved SMs of SNSs and the improved MCDM method are summarized as follows:

(1) The improved SMs of SNSs not only indicate simpler algorithms than the existing SMs of SNSs [32], but also can overcome the insufficiency of the existing SMs of SNSs.

(2) The improved MCDM method based on the sine entropy weight model can handle MCDM problems with unknown criteria weights, while the existing MCDM method [32] can only handle MCDM problems with known criteria weights. Hence, the former is superior to the latter in the MCDM problems.
(3) The objective criteria weights obtained by the sine entropy weight model are more reasonable and more practicable than the known criteria weights/subjective criteria weights given by decision-makers' preference.
(4) The improved MCDM method based on the sine entropy weight model is simple and effective in simplified neutrosophic MCDM problems with unknown criteria weights.

7. Conclusions

This work indicated the insufficiency of existing SMs of SNSs introduced in [32] and proposed the improved normalized SMs of SNSs, including the improved normalized SMs between asymmetry measures of SVNSs and INSs, to overcome the insufficiency of the existing SMs, and then the novel sine entropy of SNS was presented to establish a sine entropy weight model in MCDM problems with unknown criteria weights. Based on the improved SMs of SNSs and the sine entropy weight model, an improved MCDM method for MCDM problems with unknown criteria weights was developed in SVNS and INS settings. By means of the improved weighted SM values between each alternative and the ideal solution, all alternatives can be ranked and the best one can be easily chosen as well. Lastly, an actual decision example demonstrated the applicability of the improved MCDM method, and then its effectiveness and merits are indicated by a comparative analysis with the existing MCDM method in SVNS and INS settings.

The main advantages of the improved SMs of SNSs are that it not only has simpler algorithms than the existing SMs of SNSs, but also can overcome the insufficiency of the existing SMs of SNSs, and then the objective criteria weights obtained by the sine entropy weight model is more suitable for actual MCDM problems with unknown criteria weights than the known criteria weights/subjective criteria weights given by the decision-makers' preferences. In the future research, we shall extend the improved SMs of SNSs to other application areas, such as pattern recognition, image processing, and medical diagnosis.

Author Contributions: J.Y. proposed the improved SMs of SNSs and the sine entropy of SNSs; W.C. established the MCDM method based on the improved SMs and the sine entropy weight model in SVNS and INS setting; and we presented the actual decision example and comparative analysis and wrote this paper together.

Acknowledgments: This paper was supported by the National Natural Science Foundation of China (No. 61703280).

Conflicts of Interest: The authors declare no conflict of interest.

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