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Multi-Criteria Decision-Making Method Based on Prioritized Muirhead Mean Aggregation Operator under Neutrosophic Set Environment

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Abstract: The aim of this paper is to introduce some new operators for aggregating single-valued neutrosophic (SVN) information and to apply them to solve the multi-criteria decision-making (MCDM) problems. Single-valued neutrosophic set, as an extension and generalization of an intuitionistic fuzzy set, is a powerful tool to describe the fuzziness and uncertainty, and Muirhead mean (MM) is a well-known aggregation operator which can consider interrelationships among any number of arguments assigned by a variable vector. In order to make full use of the advantages of both, we introduce two new prioritized MM aggregation operators, such as the SVN prioritized MM (SVNPMM) and SVN prioritized dual MM (SVNPDMM) under SVN set environment. In addition, some properties of these new aggregation operators are investigated and some special cases are discussed. Furthermore, we propose a new method based on these operators for solving the MCDM problems. Finally, an illustrative example is presented to testify the efficiency and superiority of the proposed method by comparing it with the existing method.

Keywords: neutrosophic set; prioritized operator; Muirhead mean; multicriteria decision-making; aggregation operators; dual aggregation operators

1. Introduction

Multicriteria decision-making (MCDM) is one of the hot topics in the decision-making field to choose the best alternative to the set of the feasible one. In this process, the rating values of each alternative include both precise data and experts' subjective information [1,2]. However, traditionally, it is assumed that the information provided by them are crisp in nature. However, due to the complexity of the system day-by-day, the real-life contains many MCDM problems where the information is either vague, imprecise or uncertain in nature [3]. To deal with it, the theory of fuzzy set (FS) [4] or extended fuzzy sets such as intuitionistic fuzzy set (IFS) [5], interval-valued IFS (IVIFS) [6] are the most successful ones, which characterize the criterion values in terms of membership degrees. Since their existence, numerous researchers were paying more attention to these theories and developed several approaches using different aggregation operators [7–10] and ranking methods [11–13] in the processing of the information values.

It is remarked that neither the FS nor the IFS theory are able to deal with indeterminate and inconsistent data. For instance, consider an expert which gives their opinion about a certain object in such a way that 0.5 being the possibility that the statement is true, 0.7 being the possibility that the statement is false and 0.2 being the possibility that he or she is not sure. Such type of data is not handled with FS, IFS or IVIFS. To resolve this, Smarandache [14] introduced the concept neutrosophic sets (NSs). In NS, each element in the universe of discourse set has degrees of truth membership, indeterminacy-membership and falsity membership, which takes values in the non-standard unit interval $(0^-, 1^+)$. Due to this non-standard unit interval, NS theory is hard to implement on the practical problems. So in order to use NSs in engineering problems more easily, some classes of NSs and their theories were proposed [15,16]. Wang et al. [16] presented the class of NS named as interval NS while in Wang et al. [15], a class of single-valued NS (SVNS) is presented. Due to its importance, several researchers have made their efforts to enrich the concept of NSs in the decision-making process and some theories such as distance measures [17], score functions [18], aggregation operators [19–23] and so on.

Generally, aggregation operators (AOs) play an important role in the process of MCDM problems whose main target is to aggregate a collection of the input to a single number. In that direction, Ye [21] presented the operational laws of SVNSs and proposed the single-valued neutrosophic (SVN) weighted averaging (SVNWA) and SVN weighted geometric average (SVNWGA) operators. Peng et al. [22] defined the improved operations of SVN numbers (SVNNs) and developed their corresponding ordered weighted average/geometric aggregation operator. Nancy and Garg [24] developed the weighted average and geometric average operators by using the Frank norm operations. Liu et al. [25] developed some generalized neutrosophic aggregation operators based on Hamacher operations. Zhang et al. [26] presented the aggregation operators under interval neutrosophic set (INS) environment and Aiwu et al. [27] proposed some of its generalized operators. Garg and Nancy [19] developed a nonlinear optimization model to solve the MCDM problem under the INS environment.

From the above mentioned AOs, it is analyzed that all these studies assume that all the input arguments used during aggregation are independent of each other and hence there is no interrelationship between the argument values. However, in real-world problems, there always occurs a proper relationship between them. For instance, if a person wants to purchase a house then there is a certain relationship between its cost and the locality. Clearly, both the factors are mutually dependent and interacting. In order to consider the interrelationship of the input arguments, Bonferroni mean (BM)[28], Maclaurin symmetric mean (MSM) [29], Heronian mean (HM) [30] etc., are the useful aggregation functions. Yager [31] proposed the concept of BM whose main characteristic is its capability to capture the interrelationship between the input arguments. Garg and Arora [32] presented BM aggregation operators under the intuitionistic fuzzy soft set environment. In these functions, BM can capture the interrelationship between two arguments while others can capture more than two relationships. Taking the advantages of these functions in a neutrosophic domain, Liu and Wang [33] applied the BM to a neutrosophic environment and introduce the SVN normalized weighted Bonferroni mean (SVNNWBM) operator. Wang et al. [34] proposed the MSM aggregation operators to capture the correlation between the aggregated arguments. Li et al. [20] presented HM operators to solve the MCDM problems under SVNS environment. Garg and Nancy [35] presented prioritized AOs under the linguistic SVNS environment to solve the decision-making problems. Wu et al. [36] developed some prioritized weighted averaging and geometric aggregation operators for SVNNs. Ji et al. [37] established the single-valued prioritized BM operator by using the Frank operations. An alternative to these aggregations, the Muirhead mean (MM) [38] is a powerful and useful aggregation technique. The prominent advantage of the MM is that it can consider the interrelationships among all arguments, which makes it more powerful and comprehensive than BM, MSM and HM. In addition, MM has a parameter vector which can make the aggregation process more flexible.

Based on the above analysis, we know the decision-making problems are becoming more and more complex in the real world. In order to select the best alternative(s) for the MCDM problems, it is necessary to express the uncertain information in a more profitable way. In addition, it is important to deal with how to consider the relationship between input arguments. Keeping all these features in mind, and by taking the advantages of the SVNS, we combine the prioritized aggregation and MM and propose prioritized MM (PMM) operator by considering the advantages of both. These considerations have led us to consider the following main objectives for this paper:

1. to handle the impact of the some unduly high or unduly low values provided by the decision makers on to the final ranking;
2. to present some new aggregation operators to aggregate the preferences of experts element;
3. to develop an algorithm to solve the decision-making problems based on proposed operators;
4. to present some example in which relevance of the preferences in SVN decision problems is made explicit.

Since in our real decision-making problems, we always encounter a problem of some attributes' values, provided by the decision makers, whose impact on the decision-making process are unduly high or unduly low; this consequently results in a bad impression on the final results. To handle it, in the first objective we utilize prioritized averaging (PA) as an aggregation function which can handle such a problem very well. To achieve the second objective, we develop two new AOs, named as SVN prioritized MM (SVNPMM) and SVN prioritized dual MM (SVNPDMM) operators, by extending the operations of SVNNs by using MM and PA operators. MM operator is a powerful and useful aggregation technique with the feature that it considers the interrelationships among all arguments which makes it more powerful and comprehensive than BM [28], MSM [29] and HM [30]. Moreover, the MM has a parameter vector which can make the aggregation process more flexible. Several properties and some special cases from the proposed operators are investigated. To achieve the third objective, we establish an MCDM method based on these proposed operators under the SVNS environment where preferences related to each alternative is expressed in terms of SVNNs. An illustrative example is presented to testify the efficiency and superiority of the proposed method by comparative analysis with the other existing methods for fulfilling the fourth objective. Further, apart from these, we verify that the methods proposed in this paper have advantages with respect to existing operators as follows: (1) some of the existing AOs can be taken as a special case of the proposed operators under NSs environment, (2) they consider the interrelationship among all arguments, (3) they are more adaptable and feasible than the existing AOs based on the parameter vector, (4) the presented approach considers the preferences of the decision maker in terms of risk preference as well as risk aversion.

The rest of the manuscript is organized as follows. In Section 2, we briefly review the concepts of SVNS and the aggregation operators. In Section 3, two new AOs based on PA and MM operations are developed under SVNS environment and their desirable properties are investigated. In addition, some special cases of the operators by varying the parametric value are discussed. In Section 4, we explore the applications of SVNN to MCDM problems with the aid of the proposed decision-making method and demonstrate with a numerical example. Finally, Section 5 gives the concluding remarks.

2. Preliminaries

In this section, some basic concepts related to SVNSs have been defined over the universal set X with a generic element $x \in X$.

Definition 1 ([14])**.** *A neutrosophic set (NS) α comprises of three independent degrees in particular truth (μ_α), indeterminacy (ρ_α), and falsity (ν_α) which are characterized as*

$$\alpha = \{\langle x, \mu_\alpha(x), \rho_\alpha(x), \nu_\alpha(x) \mid x \in X\rangle\}, \tag{1}$$

where $\mu_\alpha(x)$, $\rho_\alpha(x)$, $\nu_\alpha(x)$ is the subset of the non-standard unit interval $(0^-, 1^+)$ such that $0^- \leq \mu_\alpha(x) + \rho_\alpha(x) + \nu_\alpha(x) \leq 3^+$.

Definition 2 ([16])**.** *A single-valued neutrosophic set (SVNS) α in X is defined as*

$$\alpha = \{\langle x, \mu_\alpha(x), \rho_\alpha(x), \nu_\alpha(x) \mid x \in X\rangle\}, \tag{2}$$

where $\mu_\alpha(x)$, $\rho_\alpha(x)$, $\nu_\alpha(x) \in [0,1]$ such that $0 \le \mu_\alpha(x) + \rho_\alpha(x) + \nu_\alpha(x) \le 3$ for all $x \in X$. A SVNS is an instance of an NS.

For convenience, we denote this pair as $\alpha = (\mu_\alpha, \rho_\alpha, \nu_\alpha)$, throughout this article, and called as SVNN with the conditions $\mu_\alpha, \rho_\alpha, \nu_\alpha \in [0,1]$ and $\mu_\alpha + \rho_\alpha + \nu_\alpha \le 3$.

Definition 3 ([18])**.** *Let $\alpha = (\mu_\alpha, \rho_\alpha, \nu_\alpha)$ be a SVNN. A score function s of α is defined as*

$$s(\alpha) = \frac{1 + (\mu_\alpha - 2\rho_\alpha - \nu_\alpha)(2 - \mu_\alpha - \nu_\alpha)}{2}. \tag{3}$$

Based on this function, an ordered relation between two SVNNs α and β is stated as, if $s(\alpha) > s(\beta)$ then $\alpha > \beta$.

Definition 4 ([16,22])**.** *Let $\alpha = (\mu, \rho, \nu)$, $\alpha_1 = (\mu_1, \rho_1, \nu_1)$ and $\alpha_2 = (\mu_2, \rho_2, \nu_2)$ be three SVNNs and $\lambda > 0$ be real number. Then, we have*

1. $\alpha^c = (\nu, \rho, \mu)$;
2. $\alpha_1 \le \alpha_2$ *if* $\mu_1 \le \mu_2, \rho_1 \ge \rho_2$ *and* $\nu_1 \ge \nu_2$;
3. $\alpha_1 = \alpha_2$ *if and only if* $\alpha_1 \le \alpha_2$ *and* $\alpha_2 \le \alpha_1$;
4. $\alpha_1 \cap \alpha_2 = (\min(\mu_1, \mu_2), \max(\rho_1, \rho_2), \max(\nu_1, \nu_2))$;
5. $\alpha_1 \cup \alpha_2 = (\max(\mu_1, \mu_2), \min(\rho_1, \rho_2), \min(\nu_1, \nu_2))$;
6. $\alpha_1 \oplus \alpha_2 = (\mu_1 + \mu_2 - \mu_1\mu_2, \rho_1\rho_2, \nu_1\nu_2)$;
7. $\alpha_1 \otimes \alpha_2 = (\mu_1\mu_2, \rho_1 + \rho_2 - \rho_1\rho_2, \nu_1 + \nu_2 - \nu_1\nu_2)$;
8. $\lambda\alpha_1 = (1 - (1 - \mu_1)^\lambda, \rho_1^\lambda, \nu_1^\lambda)$;
9. $\alpha_1^\lambda = (\mu_1^\lambda, 1 - (1 - \rho_1)^\lambda, 1 - (1 - \nu_1)^\lambda)$.

Definition 5 ([36])**.** *For a collection of SVNNs $\alpha_j = (\mu_j, \rho_j, \nu_j)(j = 1, 2, \ldots, n)$, the prioritized weighted aggregation operators are defined as*

1. *SVN prioritized weighted average (SVNPWA) operator*

$$SVNPWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left(1 - \prod_{j=1}^{n}(1 - \mu_j)^{\frac{H_j}{\sum_{j=1}^{n} H_j}}, \prod_{j=1}^{n}(\rho_j)^{\frac{H_j}{\sum_{j=1}^{n} H_j}}, \prod_{j=1}^{n}(\nu_j)^{\frac{H_j}{\sum_{j=1}^{n} H_j}}\right), \tag{4}$$

2. *SVN prioritized geometric average (SVNPGA) operator*

$$SVNPGA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left(\prod_{j=1}^{n}(\mu_j)^{\frac{H_j}{\sum_{j=1}^{n} H_j}}, 1 - \prod_{j=1}^{n}(1 - \rho_j)^{\frac{H_j}{\sum_{j=1}^{n} H_j}}, 1 - \prod_{j=1}^{n}(1 - \nu_j)^{\frac{H_j}{\sum_{j=1}^{n} H_j}}\right), \tag{5}$$

where $H_1 = 1$ and $H_j = \prod_{k=1}^{j-1} s(\alpha_k); (j = 2, \ldots, n)$.

Definition 6 ([38])**.** *For a non-negative real numbers $h_j (j = 1, 2, \ldots, n)$, (MM) operator over the parameter $P = (p_1, p_2, \ldots, p_n) \in R^n$ is defined as*

$$MM^P(h_1, h_2, \ldots, h_n) = \left(\frac{1}{n!} \sum_{\sigma \in S_n} \prod_{j=1}^{n} h_{\sigma(j)}^{p_j}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \tag{6}$$

where σ is the permutation of $(1, 2, \ldots, n)$ and S_n is set of all permutations of $(1, 2, \ldots, n)$.

By assigning some special vectors to P, we can obtain some special cases of the MM:

1. If $P = (1, 0, \ldots, 0)$, the MM is reduced to

$$\mathrm{MM}^{(1,0,\ldots,0)}(h_1, h_2, \ldots, h_n) = \frac{1}{n}\sum_{j=1}^{n} h_j, \tag{7}$$

which is the arithmetic averaging operator.

2. If $P = (1/n, 1/n, \ldots, 1/n)$, the MM is reduced to

$$\mathrm{MM}^{(1/n,1/n,\ldots,1/n)}(h_1, h_2, \ldots, h_n) = \prod_{j=1}^{n} h_j^{1/n}, \tag{8}$$

which is the geometric averaging operator.

3. If $P = (1, 1, 0, 0, \ldots, 0)$, then the MM is reduced to

$$\mathrm{MM}^{(1,1,0,0,\ldots,0)}(h_1, h_2, \ldots, h_n) = \left(\frac{1}{n(n+1)}\sum_{\substack{i,j=1\\ i\neq j}}^{n} h_i h_j\right)^{1/2}, \tag{9}$$

which is the BM operator [28].

4. If $P = (\overbrace{1, 1, \ldots, 1}^{k}, \overbrace{0, 0, \ldots, 0}^{n-k})$, then the MM is reduced to

$$\mathrm{MM}^{(\overbrace{1,1,\ldots,1}^{k}, \overbrace{0,0,\ldots,0}^{n-k})}(h_1, h_2, \ldots, h_n) = \left(\frac{1}{C_k^n}\sum_{\substack{1\leq i_1<\\ \ldots<i_k\leq n}} \prod_{j=1}^{k} h_{i_j}\right)^{1/k}, \tag{10}$$

which is the MSM operator [29].

3. Neutrosophic Prioritized Muirhead Mean Operators

In this section, by considering the overall interrelationships among the multiple input arguments, we develop some new prioritized based MM aggregation operators for a collection of SVNNs α_j; $(j = 1, 2, \ldots, n)$, denoted by Ω. Assume that σ is the permutation of $(1, 2, \ldots, n)$ such that $\alpha_{\sigma(j-1)} \leq \alpha_{\sigma(j)}$ for $j = 2, 3, \ldots, n$.

3.1. Single-Valued Neutrosophic Prioritized Muirhead Mean (SVNPMM) Operator

Definition 7. *For a collection of SVNNs $\alpha_j (j = 1, 2, \ldots, n)$, a SVNPMM operator is a mapping SVNPMM : $\Omega \to \Omega$ defined as*

$$SVNPMM(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left(\frac{1}{n!}\bigoplus_{\sigma\in S_n}\prod_{j=1}^{n}\left(n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}\alpha_{\sigma(j)}\right)^{p_j}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \tag{11}$$

where $H_1 = 1$, $H_j = \prod_{k=1}^{j-1} s(\alpha_k);(j = 2, \ldots, n)$, S_n is collection of all permutations of $(1, 2, \ldots, n)$ and $P = (p_1, p_1, \ldots, p_n) \in R^n$ be a vector of parameters.

Theorem 1. *For a collection of SVNNs $\alpha_j = (\mu_j, \rho_j, \nu_j)(j = 1, 2, \ldots, n)$, the aggregated value by Equation (11) is again a SVNN and given by*

$$SVNPMM(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left(\begin{array}{c} \left(1 - \left(\prod_{\sigma \in S_n}\left(1 - \prod_{j=1}^{n}\left(1 - (1-\mu_{\sigma(j)})^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \\ 1 - \left(1 - \left(\prod_{\sigma \in S_n}\left(1 - \prod_{j=1}^{n}\left(1 - \rho_{\sigma(j)}^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \\ 1 - \left(1 - \left(\prod_{\sigma \in S_n}\left(1 - \prod_{j=1}^{n}\left(1 - \nu_{\sigma(j)}^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}} \end{array} \right). \quad (12)$$

Proof. For SVNN $\alpha_j (j = 1, 2, \ldots, n)$ and by Definition 4, we have

$$n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}\alpha_{\sigma}(j) = \left(1 - (1-\mu_{\sigma(j)})^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}, \rho_{\sigma(j)}^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}, \nu_{\sigma(j)}^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)$$

and

$$\left(n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}\alpha_{\sigma(j)}\right)^{p_j} = \left(\begin{array}{c} \left(1 - (1-\mu_{\sigma(j)})^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j}, 1 - \left(1 - \rho_{\sigma(j)}^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j}, \\ 1 - \left(1 - \nu_{\sigma(j)}^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j} \end{array} \right).$$

Thus,

$$\bigoplus_{\sigma \in S_n}\prod_{j=1}^{n}\left(n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}\alpha_{\sigma(j)}\right)^{p_j} = \left(\begin{array}{c} 1 - \prod_{\sigma \in S_n}\left(1 - \prod_{j=1}^{n}\left(1 - (1-\mu_{\sigma(j)})^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j}\right), \\ \prod_{\sigma \in S_n}\left(1 - \prod_{j=1}^{n}\left(1 - \rho_{\sigma(j)}^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j}\right), \\ \prod_{\sigma \in S_n}\left(1 - \prod_{j=1}^{n}\left(1 - \nu_{\sigma(j)}^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j}\right) \end{array} \right).$$

Now,

$$\mathrm{SVNPMM}(\alpha_1,\alpha_2,\ldots,\alpha_n)=\left(\frac{1}{n!}\bigoplus_{\sigma\in S_n}\prod_{j=1}^{n}\left(n\frac{H_{\sigma(j)}}{\sum\limits_{j=1}^{n}H_j}\alpha_{\sigma(j)}\right)^{p_j}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}$$

$$=\left(\begin{array}{c}\left(1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-(1-\mu_{\sigma(j)})^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}},\\ 1-\left(1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\rho_{\sigma(j)}^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}},\\ 1-\left(1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\nu_{\sigma(j)}^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}\end{array}\right).$$

Thus Equation (12) holds. Furthermore, $0\leq\mu_{\sigma(j)},\rho_{\sigma(j)},\nu_{\sigma(j)}\leq 1$ so we have

$$1-\left(1-\mu_{\sigma(j)}\right)^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\in[0,1]$$

and

$$\prod_{j=1}^{n}\left(1-(1-\mu_{\sigma(j)})^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{p_j}\in[0,1],$$

which implies that

$$1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-(1-\mu_{\sigma(j)})^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{p_j}\right)\right)\in[0,1].$$

Hence,

$$0\leq\left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-(1-\mu_{\sigma(j)})^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}\leq 1.$$

Similarly, we have

$$0 \leq 1 - \left(1 - \left(\prod_{\sigma \in S_n}\left(1 - \prod_{j=1}^{n}\left(1 - \rho_{\sigma(j)}^{\,n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}} \leq 1$$

and

$$0 \leq 1 - \left(1 - \left(\prod_{\sigma \in S_n}\left(1 - \prod_{j=1}^{n}\left(1 - \nu_{\sigma(j)}^{\,n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}} \leq 1.$$

which complete the proof. □

The working of the proposed operator is demonstrated through a numerical example, which is illustrated as follow.

Example 1. *Let* $\alpha_1 = (0.5,\ 0.2,\ 0.3)$, $\alpha_2 = (0.3,\ 0.5,\ 0.4)$ *and* $\alpha_3 = (0.6,\ 0.5,\ 0.2)$ *be three SVNNs and* $P = (1,\ 0.5,\ 0.3)$ *be the given parameter vector. By utilizing the given information and* $H_j = \prod_{k=1}^{j-1} s(\alpha_k)$; $(j = 2,3)$, *we get* $H_1 = 1$, $H_2 = 0.74$ *and* $H_3 = 0.2257$. *Therefore,*

$$\begin{aligned}
& \prod_{\sigma \in S_3}\left(1 - \prod_{j=1}^{3}\left(1 - (1 - \mu_{\sigma(j)})^{3\frac{H_{\sigma(j)}}{\sum_{j=1}^{3} H_j}}\right)^{p_j}\right) \\
= \ & \left\{1 - \left(1 - (1-0.5)^{3\times 0.5087}\right)^{1} \times \left(1 - (1-0.3)^{3\times 0.3765}\right)^{0.5} \times \left(1 - (1-0.6)^{3\times 0.1148}\right)^{0.3}\right\} \\
\times \ & \left\{1 - \left(1 - (1-0.3)^{3\times 0.3765}\right)^{1} \times \left(1 - (1-0.5)^{3\times 0.5087}\right)^{0.5} \times \left(1 - (1-0.6)^{3\times 0.1148}\right)^{0.3}\right\} \\
\times \ & \left\{1 - \left(1 - (1-0.6)^{3\times 0.1148}\right)^{1} \times \left(1 - (1-0.3)^{3\times 0.3765}\right)^{0.5} \times \left(1 - (1-0.5)^{3\times 0.5087}\right)^{0.3}\right\} \\
\times \ & \left\{1 - \left(1 - (1-0.3)^{3\times 0.3765}\right)^{1} \times \left(1 - (1-0.6)^{3\times 0.1148}\right)^{0.5} \times \left(1 - (1-0.5)^{3\times 0.5087}\right)^{0.3}\right\} \\
\times \ & \left\{1 - \left(1 - (1-0.5)^{3\times 0.5087}\right)^{1} \times \left(1 - (1-0.6)^{3\times 0.1148}\right)^{0.5} \times \left(1 - (1-0.3)^{3\times 0.3765}\right)^{0.3}\right\} \\
\times \ & \left\{1 - \left(1 - (1-0.6)^{3\times 0.1148}\right)^{1} \times \left(1 - (1-0.5)^{3\times 0.5087}\right)^{0.5} \times \left(1 - (1-0.3)^{3\times 0.3765}\right)^{0.3}\right\} \\
= \ & 0.0052.
\end{aligned}$$

Similarly, we have

$$
\begin{aligned}
&\prod_{\sigma\in S_3}\left(1-\prod_{j=1}^{3}\left(1-\rho_{\sigma(j)}^{3\frac{H_{\sigma(j)}}{\sum_{j=1}^{3}H_j}}\right)^{p_j}\right)\\
=\;&\left\{1-\left(1-(0.2)^{3\times 0.5087}\right)^{1}\times\left(1-(0.5)^{3\times 0.3765}\right)^{0.5}\times\left(1-(0.5)^{3\times 0.1148}\right)^{0.3}\right\}\\
\times\;&\left\{1-\left(1-(0.5)^{3\times 0.3765}\right)^{1}\times\left(1-(0.2)^{3\times 0.5087}\right)^{0.5}\times\left(1-(0.5)^{3\times 0.1148}\right)^{0.3}\right\}\\
\times\;&\left\{1-\left(1-(0.5)^{3\times 0.1148}\right)^{1}\times\left(1-(0.5)^{3\times 0.3765}\right)^{0.5}\times\left(1-(0.2)^{3\times 0.5087}\right)^{0.3}\right\}\\
\times\;&\left\{1-\left(1-(0.5)^{3\times 0.3765}\right)^{1}\times\left(1-(0.5)^{3\times 0.1148}\right)^{0.5}\times\left(1-(0.2)^{3\times 0.5087}\right)^{0.3}\right\}\\
\times\;&\left\{1-\left(1-(0.2)^{3\times 0.5087}\right)^{1}\times\left(1-(0.5)^{3\times 0.1148}\right)^{0.5}\times\left(1-(0.5)^{3\times 0.3765}\right)^{0.3}\right\}\\
\times\;&\left\{1-\left(1-(0.5)^{3\times 0.1148}\right)^{1}\times\left(1-(0.2)^{3\times 0.5087}\right)^{0.5}\times\left(1-(0.5)^{3\times 0.3765}\right)^{0.3}\right\}\\
=\;&0.000093196
\end{aligned}
$$

and

$$
\begin{aligned}
&\prod_{\sigma\in S_3}\left(1-\prod_{j=1}^{3}\left(1-\nu_{\sigma(j)}^{3\frac{H_{\sigma(j)}}{\sum_{j=1}^{3}H_j}}\right)^{p_j}\right)\\
=\;&\left\{1-\left(1-(0.3)^{3\times 0.5087}\right)^{1}\times\left(1-(0.4)^{3\times 0.3765}\right)^{0.5}\times\left(1-(0.2)^{3\times 0.1148}\right)^{0.3}\right\}\\
\times\;&\left\{1-\left(1-(0.4)^{3\times 0.3765}\right)^{1}\times\left(1-(0.3)^{3\times 0.5087}\right)^{0.5}\times\left(1-(0.2)^{3\times 0.1148}\right)^{0.3}\right\}\\
\times\;&\left\{1-\left(1-(0.2)^{3\times 0.1148}\right)^{1}\times\left(1-(0.4)^{3\times 0.3765}\right)^{0.5}\times\left(1-(0.3)^{3\times 0.5087}\right)^{0.3}\right\}\\
\times\;&\left\{1-\left(1-(0.4)^{3\times 0.3765}\right)^{1}\times\left(1-(0.2)^{3\times 0.1148}\right)^{0.5}\times\left(1-(0.3)^{3\times 0.5087}\right)^{0.3}\right\}\\
\times\;&\left\{1-\left(1-(0.3)^{3\times 0.5087}\right)^{1}\times\left(1-(0.2)^{3\times 0.1148}\right)^{0.5}\times\left(1-(0.4)^{3\times 0.3765}\right)^{0.3}\right\}\\
\times\;&\left\{1-\left(1-(0.2)^{3\times 0.1148}\right)^{1}\times\left(1-(0.3)^{3\times 0.5087}\right)^{0.5}\times\left(1-(0.4)^{3\times 0.3765}\right)^{0.3}\right\}\\
=\;&0.00000093195.
\end{aligned}
$$

Hence, by using Equation (12), we get the aggregated value by SVNPMM is

$$\begin{aligned}
&SVNPMM(\alpha_1, \alpha_2, \alpha_3) \\
= &\left(\begin{array}{c} \left(1-(0.0052)^{1/6}\right)^{1/1.8}, 1-\left(1-(0.000093196)^{1/6}\right)^{1/1.8}, \\ 1-\left(1-(0.00000093195)^{1/6}\right)^{1/1.8} \end{array} \right) \\
= &(0.7415, 0.1246, 0.0562).
\end{aligned}$$

It is observed from the proposed operator that it satisfies the certain properties which are stated as follows.

Theorem 2. *If $\alpha_j = (\mu_j, \rho_j, \nu_j)$ and $\alpha'_j = (\mu'_j, \rho'_j, \nu'_j)$ are two SVNNs such that $\mu_j \leq \mu'_j$, $\rho_j \geq \rho'_j$ and $\nu_j \geq \nu'_j$ for all j, then*

$$SVNPMM(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq SVNPMM(\alpha'_1, \alpha'_2, \ldots, \alpha'_n).$$

This property is called monotonicity.

Proof. For two SVNNs α_j and α'_j, we have $\alpha_{\sigma(j)} \leq \alpha'_{\sigma(j)}$, for all j which implies that $\mu_{\sigma(j)} \leq \mu'_{\sigma(j)}$ and $(1-\mu_{\sigma(j)})^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}} \geq (1-\mu'_{\sigma(j)})^{n\frac{H'_{\sigma(j)}}{\sum_{j=1}^{n} H'_j}}$, where $H_1 = 1$, $H_j = \prod_{k=1}^{j-1} s(\alpha_k)$ and $H'_1 = 1$, $H'_j = \prod_{k=1}^{j-1} s(\alpha'_k)$ for $(j = 2, 3, \ldots, n)$. Thus,

$$\left(1-(1-\mu_{\sigma(j)})^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j} \leq \left(1-(1-\mu'_{\sigma(j)})^{n\frac{H'_{\sigma(j)}}{\sum_{j=1}^{n} H'_j}}\right)^{p_j}$$

$$\text{and} \quad \prod_{j=1}^{n}\left(1-(1-\mu_{\sigma(j)})^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j} \leq \prod_{j=1}^{n}\left(1-(1-\mu'_{\sigma(j)})^{n\frac{H'_{\sigma(j)}}{\sum_{j=1}^{n} H'_j}}\right)^{p_j}.$$

Further, we have

$$\begin{aligned}
&\prod_{\sigma \in S_n}\left(1-\prod_{j=1}^{n}\left(1-(1-\mu_{\sigma(j)})^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j}\right) \\
&\qquad \geq \prod_{\sigma \in S_n}\left(1-\prod_{j=1}^{n}\left(1-(1-\mu'_{\sigma(j)})^{n\frac{H'_{\sigma(j)}}{\sum_{j=1}^{n} H'_j}}\right)^{p_j}\right)
\end{aligned}$$

and

$$\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-\mu_{\sigma(j)}\right)^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}$$
$$\geq\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-\mu'_{\sigma(j)}\right)^{n\frac{H'_{\sigma(j)}}{\sum_{j=1}^{n}H'_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}.$$

Hence, we get

$$\left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-\mu_{\sigma(j)}\right)^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}$$
$$\leq\left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-\mu'_{\sigma(j)}\right)^{n\frac{H'_{\sigma(j)}}{\sum_{j=1}^{n}H'_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}.$$

Similarly, we have

$$1-\left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\rho_{\sigma(j)}^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}$$
$$\geq 1-\left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\rho'^{\,n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}_{\sigma(j)}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}$$

and

$$1-\left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\nu_{\sigma(j)}^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}$$
$$\geq 1-\left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\nu'^{\,n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}_{\sigma(j)}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}.$$

Therefore, by Definition 4, we have

$$\mathrm{SVNPMM}(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq \mathrm{SVNPMM}(\alpha'_1, \alpha'_2, \ldots, \alpha'_n).$$

□

Theorem 3. *For a collection of SVNNs $\alpha_j = (\mu_j, \rho_j, \nu_j)(j = 1, 2, \ldots, n)$. Let $\alpha^- = (\mu^-, \rho^-, \nu^-)$ and $\alpha^+ = (\mu^+, \rho^+, \nu^+)$ be the lower and upper bound, respectively, of the SVNNs where $\mu^- = \min_j\{\mu_j\}$, $\rho^- = \max_j\{\rho_j\}$, $\nu^- = \max_j\{\nu_j\}$, $\mu^+ = \max_j\{\mu_j\}$, $\rho^+ = \min_j\{\rho_j\}$ and $\nu^+ = \min_j\{\nu_j\}$, then*

$$\alpha^- \leq SVNPMM(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq \alpha^+.$$

This property is called boundedness.

Proof. Since $\min_j\{\mu_j\} \leq \mu_j$, therefore $\min_j\{\mu_j\} \leq \mu_{\sigma(j)}$, which implies

$$\left(1 - \min_j \mu_j\right)^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}} \geq \left(1 - \mu_{\sigma(j)}\right)^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}$$

and

$$\left(1 - \left(1 - \min_j \mu_j\right)^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j} \leq \left(1 - \left(1 - \mu_{\sigma(j)}\right)^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j}.$$

Then,

$$\prod_{j=1}^{n}\left(1 - \left(1 - \min_j \mu_j\right)^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j} \leq \prod_{j=1}^{n}\left(1 - \left(1 - \mu_{\sigma(j)}\right)^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j}.$$

Further,

$$\prod_{\sigma \in S_n}\left(1 - \prod_{j=1}^{n}\left(1 - \left(1 - \min_j \mu_j\right)^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j}\right)$$
$$\geq \prod_{\sigma \in S_n}\left(1 - \prod_{j=1}^{n}\left(1 - \left(1 - \mu_{\sigma(j)}\right)^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n} H_j}}\right)^{p_j}\right),$$

which implies that

$$\left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-\min_j \mu_j\right)^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}$$

$$\leq\left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-\mu_{\sigma(j)}\right)^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}},$$

i.e.,

$$\mu^-\leq\left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-(1-\mu_{\sigma(j)})^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}.$$

In the same manner, we get

$$\rho^-\geq 1-\left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\rho_{\sigma(j)}{}^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}$$

and

$$\nu^-\geq 1-\left(1-\left(\prod_{\sigma\in S_n}\left(1-\prod_{j=1}^{n}\left(1-\nu_{\sigma(j)}{}^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}.$$

Hence, $(\mu^-,\rho^-,\nu^-)\leq \mathrm{SVNPMM}(\alpha_1,\alpha_2,\ldots,\alpha_n)$. Similarly, we have

$$\mathrm{SVNPMM}(\alpha_1,\alpha_2,\ldots,\alpha_n)\leq(\mu^+,\rho^+,\nu^+),$$

which completes the proof. □

Theorem 4. *Let $\tilde{\alpha}_j$ be any permutation of α_j then we have*

$$SVNPMM(\alpha_1,\alpha_2,\ldots,\alpha_n)=SVNPMM(\tilde{\alpha}_1,\tilde{\alpha}_2,\ldots,\tilde{\alpha}_n).$$

This property is called commutativity.

Proof. The proof of this theorem can be easily followed from Equation (12), so we omit it here. □

Theorem 5. *If the priority level of all the SVNNs is taken to be the same then SVNPMM operator reduces to single-valued neutrosophic Muirhead mean (SVNMM) operator. This property is called reducibility.*

Proof. Take $\xi_j = \frac{H_j}{\sum\limits_{j=1}^{n} H_j} = \frac{1}{n}$ for all j denotes the prioritized level. As ξ_j is same for all j, so, we have $(n\xi_j)\alpha_{\sigma(j)} = \alpha_{\sigma(j)}$, which implies

$$\begin{aligned}\text{SVNPMM}(\alpha_1, \alpha_2, \ldots, \alpha_n) &= \left(\frac{1}{n!} \bigoplus_{\sigma \in S_n} \prod_{j=1}^{n} \alpha_{\sigma(j)}^{p_j}\right)^{\frac{1}{\sum\limits_{j=1}^{n} p_j}} \\ &= \text{SVNMM}(\alpha_1, \alpha_2, \ldots, \alpha_n).\end{aligned}$$

□

However, apart from these, the following particular cases are observed from the proposed SVNPMM operator by assigning different values to $P = (p_1, p_2, \ldots, p_n)$.

1. If $P = (1, 0, \ldots, 0)$, then SVNPMM operator becomes the SVN prioritized weighted average (SVNPWA) operator which is given as

$$\begin{aligned}\text{SVNPMM}(\alpha_1, \alpha_2, \ldots, \alpha_n) &= \left(\frac{1}{n!} \bigoplus_{\sigma \in S_n} \left(n \frac{H_{\sigma(1)}}{\sum\limits_{j=1}^{n} H_j} \alpha_{\sigma(1)}\right)\right)^{\frac{1}{\sum\limits_{j=1}^{n} p_j}} \\ &= \bigoplus_{j=1}^{n} \frac{H_j}{\sum\limits_{j=1}^{n} H_j} \alpha_j \\ &= \text{SVNPWA}(\alpha_1, \alpha_2, \ldots, \alpha_n).\end{aligned}$$

2. When $P = (\lambda, 0, \ldots, 0)$, then SVNPMM operator yields to SVN generalized hybrid prioritized weighted average (SVNGHPWA) operator as shown below

$$\begin{aligned}\text{SVNPMM}(\alpha_1, \alpha_2, \ldots, \alpha_n) &= \left(\frac{1}{n!} \bigoplus_{\sigma \in S_n} \left(n \frac{H_{\sigma(1)}}{\sum\limits_{j=1}^{n} H_j} \alpha_{\sigma(1)}\right)^{\lambda}\right)^{\frac{1}{\lambda}} \\ &= \left(\frac{1}{n} \bigoplus_{j=1}^{n} \left(n \frac{H_j}{\sum\limits_{j=1}^{n} H_j} \alpha_j\right)^{\lambda}\right)^{\frac{1}{\lambda}} \\ &= \text{SVNGHPWA}(\alpha_1, \alpha_2, \ldots, \alpha_n).\end{aligned}$$

3. If $P = (1, 1, 0, \ldots, 0)$, then Equation (11) reduces to SVN prioritized bonferroni mean (SVNPBM) operator as below

$$\begin{aligned}\text{SVNPMM}(\alpha_1,\alpha_2,\ldots,\alpha_n) &= \left(\frac{1}{n!}\bigoplus_{\sigma\in S_n}\left(n\frac{H_{\sigma(1)}}{\sum\limits_{j=1}^{n}H_j}\alpha_{\sigma(1)}\right)\left(n\frac{H_{\sigma(2)}}{\sum\limits_{j=1}^{n}H_j}\alpha_{\sigma(2)}\right)\right)^{\frac{1}{2}}\\ &= \left(\frac{n^2}{n!}\bigoplus_{\substack{r,s=1\\ r\neq s}}^{n}\left(\frac{H_r}{\sum\limits_{r=1}^{n}H_r}\alpha_r\right)\left(\frac{H_s}{\sum\limits_{s=1}^{n}H_s}\alpha_s\right)\right)^{\frac{1}{2}}\\ &= \text{SVNPBM}(\alpha_1,\alpha_2,\ldots,\alpha_n).\end{aligned}$$

4. If $P = (\overbrace{1,1\ldots,1}^{t\text{ terms}}, \overbrace{0,0\ldots,0}^{n-t\text{ terms}})$, then SVNPMM operator yields to SVN prioritized Maclaurin symmetric mean (SVNPMSM) operator as follows

$$\text{SVNPMM}(\alpha_1,\alpha_2,\ldots,\alpha_n) = \left(\frac{2n^t t}{n!}\bigoplus_{\substack{1<j_1<\cdots\\ <j_t<n}}\bigotimes_{q=1}^{t}\left(\frac{H_{j_q}}{\sum\limits_{r=1}^{n}H_r}\alpha_{j_q}\right)\right)^{\frac{1}{t}}.$$

3.2. Single-Valued Neutrosophic Prioritized Dual Muirhead Mean Operator

In this section, we propose prioritized dual aggregation operator based on the MM under the SVNS environment.

Definition 8. *A SVNPDMM operator is a mapping SVNPDMM : $\Omega^n \to \Omega$ given by*

$$SVNPDMM(\alpha_1,\alpha_2,\ldots,\alpha_n) = \frac{1}{\sum\limits_{j=1}^{n}p_j}\left(\prod_{\sigma\in S_n}\bigoplus_{j=1}^{n}\left(p_j\alpha_{\sigma(j)}\right)^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{\frac{1}{n!}}. \tag{13}$$

Theorem 6. *The collective value by using Equation (13) is still a SVNN and is given as*

$$SVNPDMM(\alpha_1,\alpha_2,\ldots,\alpha_n)$$
$$= \left(\begin{array}{c} 1-\left(1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-\mu_{\sigma(j)}^{\,n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}, \\ \left(1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-(1-\rho_{\sigma(j)})^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}}, \\ \left(1-\left(\prod\limits_{\sigma\in S_n}\left(1-\prod\limits_{j=1}^{n}\left(1-(1-\nu_{\sigma(j)})^{n\frac{H_{\sigma(j)}}{\sum_{j=1}^{n}H_j}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n}p_j}} \end{array}\right). \tag{14}$$

Proof. The proof follows from Theorem 1. □

In order to illustrate the working of this operator, we demonstrate it through an illustrative example as follows.

Example 2. *If we have taken the data as considered in Example 1 to illustrate the aggregation operator as defined in Theorem 6 then, we have*

$$
\begin{aligned}
&\prod_{\sigma\in S_3}\left(1-\prod_{j=1}^{3}\left(1-\mu_{\sigma(j)}^{3\frac{H_{\sigma(j)}}{\sum_{j=1}^{3}H_j}}\right)^{p_j}\right)\\
=&\left\{1-\left(1-(0.5)^{3\times0.5087}\right)^{1}\times\left(1-(0.3)^{3\times0.3765}\right)^{0.5}\times\left(1-(0.6)^{3\times0.1148}\right)^{0.3}\right\}\\
\times&\left\{1-\left(1-(0.3)^{3\times0.3765}\right)^{1}\times\left(1-(0.5)^{3\times0.5087}\right)^{0.5}\times\left(1-(0.6)^{3\times0.1148}\right)^{0.3}\right\}\\
\times&\left\{1-\left(1-(0.6)^{3\times0.1148}\right)^{1}\times\left(1-(0.3)^{3\times0.3765}\right)^{0.5}\times\left(1-(0.5)^{3\times0.5087}\right)^{0.3}\right\}\\
\times&\left\{1-\left(1-(0.3)^{3\times0.3765}\right)^{1}\times\left(1-(0.6)^{3\times0.1148}\right)^{0.5}\times\left(1-(0.5)^{3\times0.5087}\right)^{0.3}\right\}\\
\times&\left\{1-\left(1-(0.5)^{3\times0.5087}\right)^{1}\times\left(1-(0.6)^{3\times0.1148}\right)^{0.5}\times\left(1-(0.3)^{3\times0.3765}\right)^{0.3}\right\}\\
\times&\left\{1-\left(1-(0.6)^{3\times0.1148}\right)^{1}\times\left(1-(0.5)^{3\times0.5087}\right)^{0.5}\times\left(1-(0.3)^{3\times0.3765}\right)^{0.3}\right\}\\
=&\ 0.00042495.
\end{aligned}
$$

Similarly, we have

$$
\begin{aligned}
&\prod_{\sigma\in S_3}\left(1-\prod_{j=1}^{3}\left(1-(1-\rho_{\sigma(j)})^{3\frac{H_{\sigma(j)}}{\sum_{j=1}^{3}H_j}}\right)^{p_j}\right)\\
=&\left\{1-\left(1-(1-0.2)^{3\times0.5087}\right)^{1}\times\left(1-(1-0.5)^{3\times0.3765}\right)^{0.5}\times\left(1-(1-0.5)^{3\times0.1148}\right)^{0.3}\right\}\\
\times&\left\{1-\left(1-(1-0.5)^{3\times0.3765}\right)^{1}\times\left(1-(1-0.2)^{3\times0.5087}\right)^{0.5}\times\left(1-(1-0.5)^{3\times0.1148}\right)^{0.3}\right\}\\
\times&\left\{1-\left(1-(1-0.5)^{3\times0.1148}\right)^{1}\times\left(1-(1-0.5)^{3\times0.3765}\right)^{0.5}\times\left(1-(1-0.2)^{3\times0.5087}\right)^{0.3}\right\}\\
\times&\left\{1-\left(1-(1-0.5)^{3\times0.3765}\right)^{1}\times\left(1-(1-0.5)^{3\times0.1148}\right)^{0.5}\times\left(1-(1-0.2)^{3\times0.5087}\right)^{0.3}\right\}\\
\times&\left\{1-\left(1-(1-0.2)^{3\times0.5087}\right)^{1}\times\left(1-(1-0.5)^{3\times0.1148}\right)^{0.5}\times\left(1-(1-0.5)^{3\times0.3765}\right)^{0.3}\right\}\\
\times&\left\{1-\left(1-(1-0.5)^{3\times0.1148}\right)^{1}\times\left(1-(1-0.2)^{3\times0.5087}\right)^{0.5}\times\left(1-(1-0.5)^{3\times0.3765}\right)^{0.3}\right\}\\
=&\ 0.0268
\end{aligned}
$$

and

$$
\begin{aligned}
& \prod_{\sigma \in S_3}\left(1-\prod_{j=1}^{3}\left(1-\left(1-\nu_{\sigma(j)}\right)^{3\frac{H_{\sigma(j)}}{\sum_{j=1}^{3} H_j}}\right)^{p_j}\right) \\
= & \left\{1-\left(1-(1-0.3)^{3\times 0.5087}\right)^{1}\times\left(1-(1-0.4)^{3\times 0.3765}\right)^{0.5}\times\left(1-(1-0.2)^{3\times 0.1148}\right)^{0.3}\right\} \\
\times & \left\{1-\left(1-(1-0.4)^{3\times 0.3765}\right)^{1}\times\left(1-(1-0.3)^{3\times 0.5087}\right)^{0.5}\times\left(1-(1-0.2)^{3\times 0.1148}\right)^{0.3}\right\} \\
\times & \left\{1-\left(1-(1-0.2)^{3\times 0.1148}\right)^{1}\times\left(1-(1-0.4)^{3\times 0.3765}\right)^{0.5}\times\left(1-(1-0.3)^{3\times 0.5087}\right)^{0.3}\right\} \\
\times & \left\{1-\left(1-(1-0.4)^{3\times 0.3765}\right)^{1}\times\left(1-(1-0.2)^{3\times 0.1148}\right)^{0.5}\times\left(1-(1-0.3)^{3\times 0.5087}\right)^{0.3}\right\} \\
\times & \left\{1-\left(1-(1-0.3)^{3\times 0.5087}\right)^{1}\times\left(1-(1-0.2)^{3\times 0.1148}\right)^{0.5}\times\left(1-(1-0.4)^{3\times 0.3765}\right)^{0.3}\right\} \\
\times & \left\{1-\left(1-(1-0.2)^{3\times 0.1148}\right)^{1}\times\left(1-(1-0.3)^{3\times 0.5087}\right)^{0.5}\times\left(1-(1-0.4)^{3\times 0.3765}\right)^{0.3}\right\} \\
= & \ 0.0791.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
SVNPDMM(\alpha_1,\alpha_2,\alpha_3) &= \begin{pmatrix} 1-\left(1-(0.00042495)^{\frac{1}{6}}\right)^{\frac{1}{1.8}}, \left(1-(0.0268)^{\frac{1}{6}}\right)^{\frac{1}{1.8}}, \\ \left(1-(0.0791)^{\frac{1}{6}}\right)^{\frac{1}{1.8}} \end{pmatrix} \\
&= (0.1631, 0.6441, 0.5535).
\end{aligned}
$$

Similar to SVNPMM operator, it is observed that this SVNPDMM operator also satisfies same properties for a collection of SVNNs $\alpha_j (j = 1, 2, \ldots, n)$ which are stated without proof as below.

(P1) Monotonicity: If $\alpha_j \leq \alpha'_j$ for all j, then

$$\text{SVNPDMM}(\alpha_1,\alpha_2,\ldots,\alpha_n) \leq \text{SVNPDMM}(\alpha'_1,\alpha'_2,\ldots,\alpha'_n).$$

(P2) Boundedness: If α^-, and α^+ are lower and upper bound of SVNNs then

$$\alpha^- \leq \text{SVNPDMM}(\alpha_1,\alpha_2,\ldots,\alpha_n) \leq \alpha^+.$$

(P3) Commutativity: For any permutation $(\tilde{\alpha_1},\tilde{\alpha_2},\ldots,\tilde{\alpha_n})$ of the $(\alpha_1,\alpha_2,\ldots,\alpha_n)$, we have

$$\text{SVNPDMM}(\alpha_1,\alpha_2,\ldots,\alpha_n) = \text{SVNPDMM}(\tilde{\alpha_1},\tilde{\alpha_2},\ldots,\tilde{\alpha_n}).$$

4. Multi-Criteria Decision-Making Approach Based on Proposed Operators

In this section, we present an MCDM approach for solving the decision-making problem under the SVNS environment by using the proposed operators. A practical example from a field of decision-making has been taken to illustrate it.

4.1. Proposed Decision-Making Approach

Consider an MCDM problem which consists of m alternatives $A_1, A_2, \ldots, A_m$ which are evaluated under the n criteria $C_1, C_2, \ldots, C_n$. For this, an expert was invited to evaluate these alternatives under the SVN environment such that their rating values were given in the form of SVNNs. For instance, corresponding to alternative A_i under criterion C_j, when we ask the opinion of an expert about the alternative A_i with respect to the criterion C_j, he or she may observe that the possibility degree in which the statement is good is μ_{ij}, the statement is false is ν_{ij} and the degree in which he or she is unsure is ρ_{ij}. In this case, the evaluation of these alternatives are represented as SVNN $\alpha_{ij} = (\mu_{ij}, \rho_{ij}, \nu_{ij})$ such that $0 \leq \mu_{ij}, \rho_{ij}, \nu_{ij} \leq 1$ and $\mu_{ij} + \rho_{ij} + \nu_{ij} \leq 3$. This collective information is represented in the form of the neutrosophic decision-matrix D which is represented as

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{pmatrix} \alpha_{11} & \alpha_{12} & \ldots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \ldots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \ldots & \alpha_{mn} \end{pmatrix} \end{array}.$$

Based on this information, the procedure to find the best alternative(s) is summarized as follows:

Step 1: If in the considered decision-making problem, there exist two kinds of criteria, namely the benefit and the cost types, then all the cost type criteria should be normalized into the benefit type by using the following equation

$$r_{ij} = \begin{cases} (\nu_{ij}, \rho_{ij}, \mu_{ij}) & ; \quad \text{for cost type criteria,} \\ (\mu_{ij}, \rho_{ij}, \nu_{ij}) & ; \quad \text{for benefit type criteria.} \end{cases} \tag{15}$$

Step 2: Compute $H_{ij} (i = 1, 2, \ldots, m)$ as

$$H_{ij} = \begin{cases} 1 & ; \quad j = 1, \\ \prod\limits_{k=1}^{j-1} s(r_{ik}) & ; \quad j = 2, \ldots, n. \end{cases} \tag{16}$$

Step 3: For a given parameter $P = (p_1, p_2, \ldots, p_n)$, utilize either SVNPMM or SVNPDMM operator to get the collective values $r_i = (\mu_i, \rho_i, \nu_i) (i = 1, 2, \ldots, m)$ for each alternative as

$$\begin{aligned} r_i &= \text{SVNPMM}(r_{i1}, r_{i2}, \ldots, r_{in}) \\ &= \left(\begin{array}{c} \left(1 - \left(\prod\limits_{\sigma \in S_n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - (1 - \mu_{i\sigma(j)})^{n \frac{H_{i\sigma(j)}}{\sum_{j=1}^{n} H_{ij}}} \right)^{p_j} \right) \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \\ 1 - \left(1 - \left(\prod\limits_{\sigma \in S_n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \rho_{i\sigma(j)}^{n \frac{H_{i\sigma(j)}}{\sum_{j=1}^{n} H_{ij}}} \right)^{p_j} \right) \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \\ 1 - \left(1 - \left(\prod\limits_{\sigma \in S_n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \nu_{i\sigma(j)}^{n \frac{H_{i\sigma(j)}}{\sum_{j=1}^{n} H_{ij}}} \right)^{p_j} \right) \right)^{\frac{1}{n!}} \right)^{\frac{1}{\sum_{j=1}^{n} p_j}} \end{array} \right) \end{aligned} \tag{17}$$

or

$$
\begin{aligned}
r_i &= \text{SVNPDMM}(r_{i1}, r_{i2}, \ldots, r_{in}) \\
&= \left(\begin{array}{c}
1 - \left(1 - \left(\prod\limits_{\sigma \in S_n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \mu_{i\sigma(j)}^{n\frac{H_{i\sigma(j)}}{\sum_{j=1}^{n} H_{ij}}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \\
\left(1 - \left(\prod\limits_{\sigma \in S_n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - (1 - \rho_{i\sigma(j)})^{n\frac{H_{i\sigma(j)}}{\sum_{j=1}^{n} H_{ij}}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}, \\
\left(1 - \left(\prod\limits_{\sigma \in S_n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - (1 - \nu_{i\sigma(j)})^{n\frac{H_{i\sigma(j)}}{\sum_{j=1}^{n} H_{ij}}}\right)^{p_j}\right)\right)^{\frac{1}{n!}}\right)^{\frac{1}{\sum_{j=1}^{n} p_j}}
\end{array} \right). \qquad (18)
\end{aligned}
$$

Step 4: Calculate score values of the overall aggregated values $r_i = (\mu_i, \rho_i, \nu_i)$ $(i = 1, 2, \ldots, m)$ by using equation

$$s(r_i) = \frac{1 + (\mu_i - 2\rho_i - \nu_i)(2 - \mu_i - \nu_i)}{2}. \qquad (19)$$

Step 5: Rank all the feasible alternatives $A_i (i = 1, 2, \ldots, m)$ according to Definition 3 and hence select the most desirable alternative(s).

The above mentioned approach has been illustrated with a numerical example discussed in Section 4.2.

4.2. Illustrative Example

A travel agency named Marricot Tripmate has excelled in providing travel related services to domestic and inbound tourists. The agency wants to provide more facilities like detailed information, online booking capabilities, the ability to book and sell airline tickets, and other travel related services to their customers. For this purpose, the agency intends to find an appropriate information technology (IT) software company that delivers affordable solutions through software development. To complete this motive, the agency forms a set of five companies (alternatives), namely, Zensar Tech (A_1), NIIT Tech (A_2), HCL Tech(A_3), Hexaware Tech(A_4), and Tech Mahindra (A_5) and the selection is held on the basis of the different criteria, namely, technology expertise (C_1), service quality (C_2), project management (C_3) and industry experience (C_4). The prioritization relationship for the criterion is $C_1 \succ C_2 \succ C_3 \succ C_4$. In order to access these alternatives, an expert was invited and he gives their preferences toward each alternative in the form of SVNN. Their complete preferences of the expert are summarized in Table 1.

Table 1. Single-valued neutrosophic decision making matrix.

	C_1	C_2	C_3	C_4
A_1	(0.5, 0.3, 0.4)	(0.5, 0.2, 0.3)	(0.2, 0.2, 0.6)	(0.3, 0.2, 0.4)
A_2	(0.7, 0.1, 0.3)	(0.7, 0.2, 0.3)	(0.6, 0.3, 0.2)	(0.6, 0.4, 0.2)
A_3	(0.5, 0.3, 0.4)	(0.6, 0.2, 0.4)	(0.6, 0.1, 0.2)	(0.5, 0.1, 0.3)
A_4	(0.7, 0.3, 0.2)	(0.7, 0.2, 0.2)	(0.4, 0.5, 0.2)	(0.5, 0.2, 0.2)
A_5	(0.4, 0.1, 0.3)	(0.5, 0.1, 0.2)	(0.4, 0.1, 0.5)	(0.4, 0.3, 0.6)

Then, the following steps of the proposed approach have been executed as below

Step 1: As all the criteria values are of the same types, the original decision matrix need not be normalized.

Step 2: Compute $H_{ij}(j = 1, 2, 3, 4)$ by using Equation (16), we get

$$H = \begin{bmatrix} 1 & 0.6650 & 0.4921 & 0.3642 \\ 1 & 0.9000 & 0.7200 & 0.4464 \\ 1 & 0.6650 & 0.5320 & 0.4575 \\ 1 & 0.6650 & 0.5154 & 0.1134 \\ 1 & 0.8250 & 0.6806 & 0.6024 \end{bmatrix}.$$

Step 3: Without loss of generality, we take $P = (0.25, 0.25, 0.25, 0.25)$ and use SVNPMM operator given in Equation (17) to aggregate $r_{ij}(j = 1, 2, 3, 4)$ and hence we get $r_1 = (0.9026, 0.0004, 0.0118)$; $r_2 = (0.9963, 0.0008, 0.0007)$; $r_3 = (0.9858, 0.0001, 0.0029)$; $r_4 = (0.9877, 0.0021, 0.0002)$ and $r_5 = (0.9474, 0.0000, 0.0093)$.

Step 4: By Equation (19), we get $s(r_1) = 0.9959$, $s(r_2) = 0.9992$, $s(r_3) = 0.9998$, $s(r_4) = 0.9978$ and $s(r_5) = 0.9990$.

Step 5: Since $s(r_3) > s(r_2) > s(r_5) > s(r_4) > s(r_1)$ and thus ranking order of their corresponding alternatives is $A_3 \succ A_2 \succ A_5 \succ A_4 \succ A_1$. Here $\succ$ refers "preferred to". Therefore, A_3 is the best one according to the requirement of the travel agency.

Contrary to this, if we utilize SVNPDMM operator then the following steps are executed as:

Step 1: Similar to above Step 1.

Step 2: Similar to above Step 2.

Step 3: For a parameter $P = (0.25, 0.25, 0.25, 0.25)$, use SVNPDMM operator given in Equation (18) we get $r_1 = (0.0069, 0.7379, 0.9413)$; $r_2 = (0.1034, 0.7423, 0.7782)$; $r_3 = (0.0428, 0.6021, 0.8672)$; $r_4 = (0.0625, 0.8271, 0.6966)$ and $r_5 = (0.0109, 0.5340, 0.9125)$.

Step 4: The evaluated score values by using Equation (19) are $s(r_1) = 0.2226$, $s(r_2) = 0.1628$, $s(r_3) = 0.3396$, $s(r_4) = -0.0554$ and $s(r_5) = 0.4222$.

Step 5: The ranking order of the alternatives, based on the score values, is $A_5 \succ A_3 \succ A_1 \succ A_2 \succ A_4$ and hence A_5 as the best alternative among the others.

4.3. Comparison Study

If we apply the existing prioritized aggregation operator named as SVN prioritized operator [36] on the considered problem, then the following steps of the Wu et al. [36] approach have been executed as follows:

Step 1: Use SVNPWA operator as given in Equation (4) to calculate the aggregated values $\beta_i(i = 1, 2, 3, 4, 5)$ of each alternative A_i are $\beta_1 = (0.4392, 0.2407, 0.3981)$, $\beta_2 = (0.6681, 0.1864, 0.2602)$, $\beta_3 = (0.5461, 0.1929, 0.3414)$, $\beta_4 = (0.6294, 0.2844, 0.2000)$ and $\beta_5 = (0.4291, 0.1141, 0.3232)$.

Step 2: Compute the cross entropy E for each β_i from $A^+ = (1, 0, 0)$ and $A^- = (0, 0, 1)$ based on the equation $E(\alpha_1, \alpha_2) = (\sin\mu_1 - \sin\mu_2) \times (\sin(\mu_1 - \mu_2)) + (\sin\rho_1 - \sin\rho_2) \times (\sin(\rho_1 - \rho_2)) + (\sin\nu_1 - \sin\nu_2) \times (\sin(\nu_1 - \nu_2))$ and then evaluate S_{β_i} by using equation $S_{\beta_i} = \frac{E(\beta_i, A^+)}{E(\beta_i, A^+) + E(\beta_i, A^-)}$. The values corresponding to it are: $S_{\beta_1} = 0.4642$, $S_{\beta_2} = 0.1755$, $S_{\beta_3} = 0.3199$, $S_{\beta_4} = 0.1914$ and $S_{\beta_5} = 0.4007$.

Step 3: The final ranking of alternative, according to the values of S_{β_i}, is $A_2 \succ A_4 \succ A_3 \succ A_5 \succ A_1$.

From above, we have concluded that the A_2 is the best alternative and A_1 is the worst one. However, from their approach [36], it has been concluded that they have completely ignored the interrelationships among the multi-input arguments and hence the ranking order are quite different. Thus, from it, we can see the influence of the interrelationships among all the criteria on the decision-making process.

4.4. Influence of Parameter P on the Decision-Making Process

The proposed aggregation operators have two prominent advantages. First, it can reduce the bad effects of the unduly high and low assessments on the final results. Second, it can capture the interrelationship between SVN attributes values. Moreover, both of the two aggregation operators have a parameter vector P, which leads to a more flexibility during the aggregation process. Further, the parameter vector P plays a significant role in the final ranking results. In order to illustrate the influence of the parameter vector $P = (p_1, p_2, \ldots, p_n)$ on the score functions and the ranking results, we set different values to P in the SVNPMM and SVNPDMM operators and their corresponding results are summarized in Table 2. From this table, it is concluded that the score value of each alternative decreases by SVNPMM operator while it increases by SVNPDMM operator. Therefore, based on the decision maker behavior, either A_3 or A_5 are the best alternatives to be chosen for their desired goals. Thus, the parameter vector P can be viewed as decision makers' risk preference.

4.5. Further Discussion

The prominent advantage of the proposed aggregation operators is that the interrelationship among all SVNNs can be taken into consideration. Moreover, it has a parameter vector that leads to flexible aggregation operators. To show the validity and superiorities of the proposed operators, we conduct a comparative analysis whose characteristics are presented in Table 3.

Table 2. Ranking results of alternatives using proposed operators for different values of P.

Parameter Vector P	Operator	Score Values of Alternatives					Ranking Results
		A_1	A_2	A_3	A_4	A_5	
(1, 0, 0, 0)	SVNPMM	0.9975	0.9997	0.9999	0.9989	0.9990	$A_3 \succ A_2 \succ A_5 \succ A_4 \succ A_1$
	SVNPDMM	0.2184	0.0876	0.2942	-0.1233	0.3632	$A_5 \succ A_3 \succ A_1 \succ A_2 \succ A_4$
(1, 1, 0, 0)	SVNPMM	0.9844	0.9969	0.9988	0.9920	0.9940	$A_3 \succ A_2 \succ A_5 \succ A_4 \succ A_1$
	SVNPDMM	0.3638	0.2891	0.4851	0.0162	0.5597	$A_5 \succ A_3 \succ A_1 \succ A_2 \succ A_4$
(1, 1, 1, 0)	SVNPMM	0.9723	0.9926	0.9968	0.9809	0.9887	$A_3 \succ A_2 \succ A_5 \succ A_4 \succ A_1$
	SVNPDMM	0.4268	0.3846	0.5529	0.1219	0.6053	$A_5 \succ A_3 \succ A_1 \succ A_2 \succ A_4$
(1, 1, 1, 1)	SVNPMM	0.9624	0.9868	0.9942	0.9659	0.9851	$A_3 \succ A_2 \succ A_5 \succ A_4 \succ A_1$
	SVNPDMM	0.4617	0.4507	0.5955	0.2079	0.6341	$A_5 \succ A_3 \succ A_1 \succ A_2 \succ A_4$
(2, 2, 2, 2)	SVNPMM	0.9443	0.9633	0.9836	0.9189	0.9767	$A_3 \succ A_5 \succ A_2 \succ A_1 \succ A_4$
	SVNPDMM	0.5165	0.5024	0.640	0.3016	0.6698	$A_5 \succ A_3 \succ A_1 \succ A_2 \succ A_4$
(3, 3, 3, 3)	SVNPMM	0.9322	0.9440	0.9744	0.8896	0.9715	$A_3 \succ A_5 \succ A_2 \succ A_1 \succ A_4$
	SVNPDMM	0.5369	0.5018	0.6490	0.3142	0.6853	$A_5 \succ A_3 \succ A_1 \succ A_2 \succ A_4$
$\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$	SVNPMM	0.9824	0.9965	0.9987	0.9903	0.9943	$A_3 \succ A_2 \succ A_5 \succ A_4 \succ A_1$
	SVNPDMM	0.3652	0.3217	0.4982	0.0490	0.5661	$A_5 \succ A_3 \succ A_1 \succ A_2 \succ A_4$
$\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$	SVNPMM	0.9959	0.9992	0.9998	0.9978	0.9990	$A_3 \succ A_2 \succ A_5 \succ A_4 \succ A_1$
	SVNPDMM	0.2226	0.1628	0.3396	-0.0554	0.4222	$A_5 \succ A_3 \succ A_1 \succ A_2 \succ A_4$
(2, 0, 0, 0)	SVNPMM	0.9890	0.9984	0.9990	0.9953	0.9931	$A_3 \succ A_2 \succ A_4 \succ A_5 \succ A_1$
	SVNPDMM	0.3571	0.1886	0.4228	-0.1009	0.4781	$A_5 \succ A_3 \succ A_1 \succ A_2 \succ A_4$
(3, 0, 0, 0)	SVNPMM	0.9814	0.9964	0.9974	0.9898	0.9860	$A_3 \succ A_2 \succ A_4 \succ A_5 \succ A_1$
	SVNPDMM	0.4139	0.2426	0.4645	-0.0595	0.5008	$A_5 \succ A_3 \succ A_1 \succ A_2 \succ A_4$

SVNPMM: single-valued neutrosophic prioritized Muirhead mean, SVNPDMM: single-valued neutrosophic prioritized dual Muirhead mean.

Table 3. Comparison of different approaches and aggregation operators.

Approaches	Whether the Interrelationship of Two Attributes Is Captured	Whether the Interrelationship of Three Attributes Is Captured	Whether the Relationship of Multiple Attributes Is Captured	Whether the Bad Effects of the Unduly High Unduly Low Arguments Can Be Reduced	Whether It Makes the Method Flexible by the Parameter Vector
NWA [21]	×	×	×	×	×
SVNWA [22]	×	×	×	×	×
SVNOWA [22]	×	×	×	×	×
SVNWG [22]	×	×	×	×	×
SVNOWG [22]	×	×	×	×	×
SVNHWA [25]	×	×	×	×	×
SVNHWG [25]	×	×	×	×	×
NWG [21]	×	×	×	×	×
SVNFWG [24]	×	×	×	×	✓
SVNFWA [24]	×	×	×	×	✓
SVNFNPBM [37]	✓	×	×	×	✓
WSVNLMSM [34]	✓	✓	✓	×	✓
SVNNWBM [33]	✓	×	×	×	✓
SVNIGWHM [20]	✓	✓	✓	×	✓
GNNHWA [25]	×	×	×	×	✓
The proposed method	✓	✓	✓	✓	✓

NWA: neutrosophic weighted averaging, SVNWA: single-valued neutrosophic weighted averaging, SVNOWA: single-valued neutrosophic ordered weighted averaging, SVNWG: single-valued neutrosophic weighted geometric, SVNOWG: single-valued neutrosophic ordered weighted geometric, SVNHWA: single-valued neutrosophic hybrid weighted averaging, SVNHWG: single-valued neutrosophic hybrid weighted geometric, NWG: neutrosophic weighted geometric, SVNFWG: single-valued neutrosophic Frank weighted geometric, SVNFWA: single-valued neutrosophic Frank weighted averaging, SVNFNPBM: single-valued neutrosophic Frank normalized prioritized Bonferroni mean, WSVNLMSM: weighted single-valued neutrosophic linguistic Maclaurin symmetric mean, SVNNWBM: single-valued neutrosophic normalized weighted Bonferroni mean, IGWHM: single-valued neutrosophic improved generalized weighted Heronian mean, GNNHWA: generalized neutrosophic number Hamacher weighted averaging.

The approaches in [21,22,25] are based on a simple weighted averaging operator. However, in these approaches, some of the weakness are (1) they assume that all the input arguments are independent, which is somewhat inconsistent with reality; (2) they cannot consider the interrelationship among input arguments. However, on the contrary, the proposed method can capture the interrelationship among input arguments. In addition to that, the proposed operator has an additional parameter P which provide a feasible aggregation process. In addition, some of the existing operators are deduced from the proposed operators. Thus, the proposed method is more powerful and flexible than the methods in [21,22,25].

In [33,37], authors presented an approach based on the BM aggregation operator where they considered the interrelationship between the arguments. However, the main flaws of these approaches are that they consider only two arguments during the interrelationship. On the other hand, in [34] authors have presented an aggregation operator based on MSM by considering two or more arguments during the interrelationship; however, these methods [33,34,37] fail to reflect the interrelationship among all input arguments. Finally, in [20] authors used the Heronian mean AOs without considering any interrelationship between the arguments.

As compared with these existing approaches, the merits of the proposed approach are that it can reflect the interrelationships among all the input arguments. In addition, the proposed operators have an additional parameter P which makes the proposed approach more flexible and feasible.

5. Conclusions

Muirhead mean aggregation operator is more flexible by using a variable and considering the multiple interrelationships between the pairs of the input arguments. On the other hand, SVNS is more of a generalization of the fuzzy set, intuitionistic fuzzy set to describe the uncertainties in the data. In order to combine their advantages, in the present paper, we develop some new MM aggregation operators for the SVNSs including the SVNPMM and the SVNPDMM. The desirable properties of these proposed operators and some special cases are discussed in detail. Moreover, we presented two new methods to solve the MCDM problem based on the proposed operators. The proposed method is more general and flexible, not only by considering the parametric vector P but also by taking into account the multiple interrelationships between the input argument. Apart from this, the remarkable characteristic of the proposed operator is to reflect the correlations of the aggregated arguments by considering the fact that those different criteria having different priority levels. The mentioned approach has been demonstrated through a numerical example and compares their corresponding proposed results with some of the results of existing approaches. From the computed results, it has been observed that the proposed approach can be efficiently utilized to solve decision-making problems where uncertainties and vagueness in the data occur concurrently. Moreover, by changing the values of the parameter P, an analysis has been done which concludes that the proposed operators provide more choices to the decision makers according to their preferences. In addition, it is also regarded as considering the risk preference of decision makers by the parameter P. So, the proposed approach is more suitable and flexible to solve the practical and complex MCDM problems.

In future works, we will apply our proposed method for more practical decision-making problems. In addition, considering the superiority of MM operator, we can extend it to some new fuzzy sets, such as Pythagorean fuzzy sets [39–41], applications to MCDM [42–44], multiplicative sets [45,46] and so on.

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