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Neutrosophic Baire Spaces

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Abstract: In this paper, we introduce the concept of neutrosophic Baire space and present some of its characterizations.

Keywords:neutrosophic first category ; neutrosophic second category; neutrosophic residual set; neutrosophic Baire space.

1 Introduction and Preliminaries

The fuzzy idea has invaded all branches of science as far back as the presentation of fuzzy sets by L. A. Zadeh [17]. The important concept of fuzzy topological space was offered by C. L. Chang [6] and from that point forward different ideas in topology have been reached out to fuzzy topological space. The concept of "intuitionistic fuzzy set" was first presented by Atanassov [1]. He and his associates studied this useful concept [2, 3, 4]. Afterward, this idea was generalized to "intuitionistic L - fuzzy sets" by Atanassov and Stoeva [5]. The idea of somewhat fuzzy continuous functions and somewhat fuzzy open hereditarily irresolvable were introduced and investigated by by G. Thangaraj and G. Balasubramanian in [15]. The idea of intuitionistic fuzzy nowhere dense set in intuitionistic fuzzy topological space presented and studied by by Dhavaseelan and et al. in [16]. The concepts of neutrosophy and neutrosophic set were introduced by F. Smarandache [[13], [14]]. Afterwards, the works of Smarandache inspired A. A. Salama and S. A. Alblowi[12] to introduce and study the concepts of neutrosophic crisp set and neutrosophic crisp topological spaces. The Basic definitions and Proposition related to neutrosophic topological spaces was introduced and discussed by Dhavaseelan et al. [9]. In this paper the concepts of neutrosophic Baire spaces are introduced and characterizations of neutrosophic baire spaces are studied.

Definition 1.1. [13, 14] Let T,I,F be real standard or non standard subsets of $]0^-, 1^+[$, with $sup_T = t_{sup}, inf_T = t_{inf}$
$sup_I = i_{sup}, inf_I = i_{inf}$
$sup_F = f_{sup}, inf_F = f_{inf}$
$n-sup = t_{sup} + i_{sup} + f_{sup}$
$n-inf = t_{inf}+i_{inf}+f_{inf}$. T,I,F are neutrosophic components.

Definition 1.2. [13, 14] Let X be a nonempty fixed set. A neutrosophic set [briefly NS] A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set A.

Remark 1.1. [13, 14]

(1) A neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ can be identified to an ordered triple $\langle \mu_A, \sigma_A, \gamma_A\rangle$ in $]0^-, 1^+[$ on X.

(2) For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \sigma_A, \gamma_A\rangle$ for the neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 1.3. [13, 14] Let X be a nonempty set and the neutrosophic sets A and B in the form
$A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$. Then

(a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$;

(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;

(c) $\bar{A} = \{\langle x, \gamma_A(x), \sigma_A(x), \mu_A(x)\rangle : x \in X\}$; [Complement of A]

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$;

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$;

(f) $[\,]A = \{\langle x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x)\rangle : x \in X\}$;

(g) $\langle\rangle A = \{\langle x, 1 - \gamma_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 1.4. [13, 14] Let $\{A_i : i \in J\}$ be an arbitrary family of neutrosophic sets in X. Then

(a) $\bigcap A_i = \{\langle x, \wedge\mu_{A_i}(x), \wedge\sigma_{A_i}(x), \vee\gamma_{A_i}(x)\rangle : x \in X\}$;

(b) $\bigcup A_i = \{\langle x, \vee\mu_{A_i}(x), \vee\sigma_{A_i}(x), \wedge\gamma_{A_i}(x)\rangle : x \in X\}$.

Since our main purpose is to construct the tools for developing neutrosophic topological spaces, we introduce the neutrosophic sets 0_N and 1_N in X as follows:

Definition 1.5. [13, 14] $0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$ and $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$.

Definition 1.6. [9] A neutrosophic topology (NT) on a nonempty set X is a family T of neutrosophic sets in X satisfying the following axioms:

(i) $0_N, 1_N \in T$,

(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T$,

(iii) $\cup G_i \in T$ for arbitrary family $\{G_i \mid i \in \Lambda\} \subseteq T$.

In this case the ordered pair (X, T) or simply X is called a neutrosophic topological space (briefly NTS) and each neutrosophic set in T is called a neutrosophic open set (briefly NOS). The complement $\overline{A}$ of a NOS A in X is called a neutrosophic closed set (briefly NCS) in X.

Definition 1.7. [9] Let A be a neutrosophic set in a neutrosophic topological space X. Then

$Nint(A) = \bigcup\{G \mid G$ is a neutrosophic open set in X and $G \subseteq A\}$ is called the neutrosophic interior of A;

$Ncl(A) = \bigcap\{G \mid G$ is a neutrosophic closed set in X and $G \supseteq A\}$ is called the neutrosophic closure of A.

Definition 1.8. [9] Let X be a nonempty set. If r, t, s be real standard or non standard subsets of $]0^-, 1^+[$ then the neutrosophic set $x_{r,t,s}$ is called a neutrosophic point(in short NP)in X given by

$$x_{r,t,s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p \\ (0, 0, 1), & \text{if } x \neq x_p \end{cases}$$

for $x_p \in X$ is called the support of $x_{r,t,s}$.where r denotes the degree of membership value, t denotes the degree of indeterminacy and s is the degree of non-membership value of $x_{r,t,s}$.

Definition 1.9. [11] A neutrosophic set A in neutrosophic topological space (X, T) is called neutrosophic dense if there exists no neutrosophic closed set B in (X, T) such that $A \subset B \subset 1_N$

Proposition 1.1. [11] If A is a neutrosophic nowhere dense set in (X, T), then $\overline{A}$ is a neutrosophic dense set in (X, T) .

Proposition 1.2. [11] Let A be a neutrosophic set. If A is a neutrosophic closed set in (X, T) with $Nint(A) = 0_N$, then A is a neutrosophic nowhere dense set in (X, T).

2 Neutrosophic Baire Spaces

Definition 2.1. Let (X, T) be a neutrosophic topological space. A neutrosophic set A in (X, T) is called neutrosophic first category if $A = \bigcup_{i=1}^{\infty} B_i$, where B_i's are neutrosophic nowhere dense sets in (X, T). Any other neutrosophic set in (X, T) is said to be of neutrosophic second category.

Definition 2.2. A neutrosophic topological space (X, T) is called neutrosophic first category space if the neutrosophic set 1_N is a neutrosophic first category set in (X, T).That is, $1_N = \bigcup_{i=1}^{\infty} A_i$ where A_i's are neutrosophic nowhere dense sets in (X, T). Otherwise (X, T) will be called a neutrosophic second category space.

Proposition 2.1. If A be a neutrosophic first category set in (X, T),then $\overline{A} = \bigcap_{i=1}^{\infty} B_i$ where $Ncl(B_i) = 1_N$.

Proof. Let A be a neutrosophic first category set in (X, T). Then $A = \bigcup_{i=1}^{\infty} A_i$, where A_i's are neutrosophic nowhere dense sets in (X, T). Now $\overline{A} = \overline{\bigcup_{i=1}^{\infty} A_i} = \bigcap_{i=1}^{\infty}(\overline{A_i})$. Now A_i is a neutrosophic nowhere dense set in (X, T). Then, by Proposition 1.1, we have $\overline{A_i}$ is a neutrosophic dense set in (X, T). Let us put $B_i = \overline{A_i}$. Then $\overline{A} = \bigcap_{i=1}^{\infty} B_i$ where $Ncl(B_i) = 1_N$. □

Definition 2.3. Let A be a neutrosophic first category set in (X, T). Then $\overline{A}$ is called a neutrosophic residual set in (X, T).

Definition 2.4. Let (X, T) be a neutrosophic topological space. Then (X, T) is said to neutrosophic Baire space if $Nint(\bigcup_{i=1}^{\infty} A_i) = 0_N$, where A_i's are neutrosophic nowhere dense sets in (X, T).

Example 2.1. Let $X = \{a, b, c\}$. Define the neutrosophic sets A, B,C and D as follows :
$A = \langle x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.5})\rangle$,
$B = \langle x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3})\rangle$,
$C = \langle x, (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4}), (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4}), (\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.4})\rangle$ and
$D = \langle x, (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}), (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}), (\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.7})\rangle$. Then the family $T = \{0_N, 1_N, A\}$ is a neutrosophic topologies on X. Thus, (X, T) is a neutrosophic topological spaces. Now $\overline{A}, \overline{B}, C$ and D are neutrosophic nowhere dense sets in (X, T). Also $Nint(\overline{A} \cup \overline{B} \cup C \cup D) = 0_N$. Hence (X, T) is a neutrosophic Baire space.

Proposition 2.2. If $Nint(\bigcup_{i=1}^{\infty} A_i) = 0_N$ where $Nint(A_i) = 0_N$ and $A_i \in T$, then (X, T) is a neutrosophic Baire space.

Proof. Now $A_i \in T$ implies that A_i is a neutrosophic open set in (X, T). Since $Nint(A_i) = 0_N$. By Proposition 1.2, A_i is a neutrosophic nowhere dense set in (X, T). Therefore $Nint(\bigcup_{i=1}^{\infty} A_i) = 0_N$. where A_i's are neutrosophic nowhere dense set in (X, T). Hence (X, T) is a neutrosophic Baire space. □

Proposition 2.3. If $Ncl(\bigcap_{i=1}^{\infty} A_i) = 1_N$ where A_i's are neutrosophic dense and neutrosophic open sets in (X, T), then (X, T) is a neutrosophic Baire Space.

Proof. Now $Ncl(\bigcap_{i=1}^{\infty} A_i) = 1_N$ implies that $\overline{Ncl(\bigcap_{i=1}^{\infty} A_i)} = 0_N$. Then we have $Nint(\overline{\bigcap_{i=1}^{\infty} A_i}) = 0_N$. Which implies that $Nint(\bigcup_{i=1}^{\infty} \overline{A_i}) = 0_N$. Let $B_i = \overline{A_i}$. Then $Nint(\bigcup_{i=1}^{\infty} B_i) =$

0_N. Now $A_i \in T$ implies that $\overline{A_i}$ is a neutrosophic closed set in (X,T) and hence B_i is a neutrosophic closed and $Nint(B_i) = Nint(\overline{A_i}) = \overline{Ncl(A_i)} = 0_N$. Hence By Proposition 1.2, B_i is a neutrosophic nowhere dense set in (X,T). Hence $Nint(\bigcup_{i=1}^{\infty} B_i) = 0_N$ where B_i's are neutrosophic nowhere dense sets, implies that (X,T) is a neutrosophic Baire space. □

Proposition 2.4. Let (X,T) be a neutrosophic topological space. Then the following are equivalent

(i) (X,T) is a neutrosophic Baire space.

(ii) $Nint(A) = 0_N$, for every neutrosophic first category set A in (X,T).

(iii) $Ncl(B) = 1_N$, for every neutrosophic residual set B in (X,T).

Proof. $(i) \Rightarrow (ii)$ Let A be a neutrosophic first category set in (X,T). Then $A = (\bigcup_{i=1}^{\infty} A_i)$ where A_i's are neutrosophic nowhere dense sets in (X,T). Now $Nint(A) = Nint(\bigcup_{i=1}^{\infty} A_i) = 0_N$. Since (X,T) is a neutrosophic Baire space. Therefore $Nint(A) = 0_N$.

$(ii) \Rightarrow (iii)$ Let B be a neutrosophic residual set in (X,T). Then $\overline{B}$ is a neutrosophic first category set in (X,T). By hypothesis $Nint(\overline{B}) = 0_N$ which implies that $\overline{Ncl(A)} = 0_N$. Hence $Ncl(A) = 1_N$.

$(iii) \Rightarrow (i)$ Let A be a neutrosophic first category set in (X,T). Then $A = (\bigcup_{i=1}^{\infty} A_i)$ where A_i's are neutrosophic nowhere dense sets in (X,T). Now A is a neutrosophic first category set implies that $\overline{A}$ is a neutrosophic residual set in (X,T). By hypothesis, we have $Ncl(\overline{A}) = 1_N$, which implies that $\overline{Nint(A)} = 1_N$. Hence $Nint(A) = 0_N$. That is, $Nint(\bigcup_{i=1}^{\infty} A_i) = 0_N$, where A_i's are neutrosophic nowhere dense sets in (X,T). Hence (X,T) is a neutrosophic Baire space. □

Proposition 2.5. A neutrosophic topological space (X,T) is a neutrosophic Baire space if and only if $(\bigcup_{i=1}^{\infty} A_i) = 1_N$, where A_i's is a neutrosophic closed set in (X,T) with $Nint(A_i) = 0_N$, implies that $Nint(\bigcup_{i=1}^{\infty} A_i) = 0_N$.

Proof. Let (X,T) be a neutrosophic Baire space. Now A_i is a neutrosophic closed in (X,T) and $Nint(A_i) = 0_N$, implies that A_i is a neutrosophic nowhere dense set in (X,T). Now $\bigcup_{i=1}^{\infty} A_i = 1_N$ implies that 1_N is a neutrosophic first category set in (X,T). Since (X,T) is a neutrosophic Baire space space, by Proposition 2.4, $Nint(1_N) = 0_N$. That is, $Nint(\bigcup_{i=1}^{\infty} A_i) = 0_N$.

Conversely suppose that $Nint(\bigcup_{i=1}^{\infty} A_i) = 0_N$ where A_i. By Proposition 1.2, A_i is a neutrosophic nowhere dense set in (X,T). Hence $Nint(\bigcup_{i=1}^{\infty} A_i) = 0_N$ implies that (X,T) is a neutrosophic Baire space. □

Definition 2.5. Let (X,T) and (Y,S) be any two neutrosophic topological spaces. A map $f : (X,T) \to (Y,S)$ is said to be a neutrosophic open if the image of every neutrosophic open set A in (X,T) is neutrosophic open $f(A)$ in (Y,S).

Definition 2.6. [10] Let (X,T) and (Y,S) be any two neutrosophic topological spaces. A map $f : (X,T) \to (Y,S)$ is called neutrosophic contra continuous if the inverse image of every neutrosophic open set in (Y,S) is neutrosophic closed in (X,T).

Proposition 2.6. Let (X,T) and (Y,S) be any two neutrosophic topological spaces. If $f : (X,T) \to (Y,S)$ is an onto neutrosophic contra continuous and neutrosophic open then (Y,S) is a neutrosophic Baire space.

Proof. Let A be a neutrosophic first category set in (Y,S). Then $A = (\bigcup_{i=1}^{\infty} A_i)$ where A_i are neutrosophic nowhere dense sets in (Y,S). Suppose $Nint(A) \neq 0_N$. Then there exists a neutrosophic open set $B \neq 0_N$ in (Y,S), such that $B \subseteq A$. Then $f^{-1}(B) \subseteq f^{-1}(A) = f^{-1}(\bigcup_{i=1}^{\infty} A_i) = \bigcup_{i=1}^{\infty} f^{-1}(A_i)$. Hence

$$f^{-1}(B) \subseteq \bigcup_{i=1}^{\infty} f^{-1}(Ncl(A_i)). \tag{2.1}$$

Since f is neutrosophic contra continuous and $Ncl(A_i)$ is a neutrosophic closed set in (Y,S), $f^{-1}(Ncl(A_i))$ is a neutrosophic open in (X,T). From (2.1) we have

$$f^{-1}(B) \subseteq \bigcup_{i=1}^{\infty} f^{-1}(Ncl(A_i)) = \bigcup_{i=1}^{\infty} Nint(f^{-1}(Ncl(A_i)). \tag{2.2}$$

Since f is intuitionsitic fuzzy open and onto, $Nint(f^{-1}(A_i)) \subseteq f^{-1}(Nint(A_i))$. From 2.2, we have $f^{-1}(B) \subseteq \bigcup_{i=1}^{\infty} f^{-1}(NintNcl(A_i)) \subseteq \bigcup_{i=1}^{\infty} f^{-1}(0_N) = 0_N$. Since A_i is a neutrosophic nowhere dense. That is, $f^{-1}(B) \subseteq 0_N$ and hence $f^{-1}(B) = 0_N$ which implies that $B = 0_N$, which is a contradiction to $B \neq 0_N$. Hence $Nint(A) = 0_N$ where A is a neutrosophic first category set in (Y,S). Hence by Proposition 2.4, (Y,S) is a neutrosophic Baire space.

□

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