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An Extended Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) with Maximizing Deviation Method Based on Integrated Weight Measure for Single-Valued Neutrosophic Sets

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Abstract: A single-valued neutrosophic set (SVNS) is a special case of a neutrosophic set which is characterized by a truth, indeterminacy, and falsity membership function, each of which lies in the standard interval of $[0, 1]$. This paper presents a modified Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) with maximizing deviation method based on the single-valued neutrosophic set (SVNS) model. An integrated weight measure approach that takes into consideration both the objective and subjective weights of the attributes is used. The maximizing deviation method is used to compute the objective weight of the attributes, and the non-linear weighted comprehensive method is used to determine the combined weights for each attributes. The use of the maximizing deviation method allows our proposed method to handle situations in which information pertaining to the weight coefficients of the attributes are completely unknown or only partially known. The proposed method is then applied to a multi-attribute decision-making (MADM) problem. Lastly, a comprehensive comparative studies is presented, in which the performance of our proposed algorithm is compared and contrasted with other recent approaches involving SVNSs in literature.

Keywords: Single-valued neutrosophic set; Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS); integrated weight; maximizing deviation; multi-attribute decision-making (MADM)

1. Introduction

The study of fuzzy set theory proposed by Zadeh [1] was an important milestone in the study of uncertainty and vagueness. The widespread success of this theory has led to the introduction of many extensions of fuzzy sets such as the intuitionistic fuzzy set (IFS) [2], interval-valued fuzzy set (IV-FS) [3], vague set [4], and hesitant fuzzy set [5]. The most widely used among these models is the IFS model which has also spawned other extensions such as the interval-valued intuitionistic fuzzy set [6] and bipolar intuitionistic fuzzy set [7]. Smarandache [8] then introduced an improvement to IFS theory called neutrosophic set theory which loosely refers to neutral knowledge. The study of the

neutrality aspect of knowledge is the main distinguishing criteria between the theory of fuzzy sets, IFSs, and neutrosophic sets. The classical neutrosophic set (NS) is characterized by three membership functions which describe the degree of truth (T), the degree of indeterminacy (I), and the degree of falsity (F), whereby all of these functions assume values in the non-standard interval of $]0^-, 1^+[$. The truth and falsity membership functions in a NS are analogous to the membership and non-membership functions in an IFS, and expresses the degree of belongingness and non-belongingness of the elements, whereas the indeterminacy membership function expresses the degree of neutrality in the information. This additional indeterminacy membership function gives NSs the ability to handle the neutrality aspects of the information, which fuzzy sets and its extensions are unable to handle. Another distinguishing factor between NSs and other fuzzy-based models is the fact that all the three membership functions in a NS are entirely independent of one another, unlike the membership and non-membership functions in an IFS or other fuzzy-based models in which values of the membership and non-membership functions are dependent on one another. This gives NSs the ability to handle uncertain, imprecise, inconsistent, and indeterminate information, particularly in situations whereby the factors affecting these aspects of the information are independent of one another. This also makes the NS more versatile compared to IFSs and other fuzzy- or IF-based models in literature.

Smarandache [8] and Wang et al. [9] pointed out that the non-standard interval of $]0^-, 1^+[$ in which the NS is defined in, makes it impractical to be used in real-life problems. Furthermore, values in this non-standard interval are less intuitive and the significance of values in this interval can be difficult to be interpreted. This led to the conceptualization of the single-valued neutrosophic set (SVNS). The SVNS is a straightforward extension of NS which is defined in the standard unit interval of $[0, 1]$. As values in $[0, 1]$ are compatible with the range of acceptable values in conventional fuzzy set theory and IFS theory, it is better able to capture the intuitiveness of the process of assigning membership values. This makes the SVNS model easier to be applied in modelling real-life problems as the results obtained are a lot easier to be interpreted compared to values in the interval $]0^-, 1^+[$.

The SVNS model has garnered a lot of attention since its introduction in [9], and has been actively applied in various multi-attribute decision-making (MADM) problems using a myriad of different approaches. Wang et al. [9] introduced some set theoretic operators for SVNSs, and studied some additional properties of the SVNS model. Ye [10,11] introduced a decision-making algorithm based on the correlation coefficients for SVNSs, and applied this algorithm in solving some MADM problems. Ye [12,13] introduced a clustering method and also some decision-making methods that are based on the similarity measures of SVNSs, whereas Huang [14] introduced a new decision-making method for SVNSs and applied this method in clustering analysis and MADM problems. Peng and Liu [15] on the other hand proposed three decision-making methods based on a new similarity measure, the EDAS method and level soft sets for neutrosophic soft sets, and applied this new measure to MADM problems set in a neutrosophic environment. The relations between SVNSs and its properties were first studied by Yang et al. [16], whereas the graph theory of SVNSs and bipolar SVNSs were introduced by Broumi et al. in [17–19] and [20–22], respectively. The aggregation operators of simplified neutrosophic sets (SNSs) were studied by Tian et al. [23] and Wu et al. [24]. Tian et al. [23] introduced a generalized prioritized aggregation operator for SNSs and applied this operator in a MADM problem set in an uncertain linguistic environment, whereas Wu et al. [24] introduced a cross-entropy measure and a prioritized aggregation operator for SNSs and applied these in a MADM problem. Sahin and Kucuk [25] proposed a subsethood measure for SVNSs and applied these to MADM problems.

The fuzzy Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) method for SVNSs were studied by Ye [26] and Biswas et al. [27]. Ye [26] introduced the TOPSIS method for group decision-making (MAGDM) that is based on single-valued neutrosophic linguistic numbers, to deal with linguistic decision-making. This TOPSIS method uses subjective weighting method whereby attribute weights are randomly assigned by the users. Maximizing deviation method or any other objective weighting methods are not used. Biswas et al. [27] proposed a TOPSIS method for group decision-making (MAGDM) based on the SVNS model. This TOPSIS method is based on the

original fuzzy TOPSIS method and does not use the maximizing deviation method to calculate the objective weights for each attribute. The subjective weight of each attribute is determined by using the single-valued neutrosophic weighted averaging aggregation operator to calculate the aggregated weights of the attributes using the subjective weights that are assigned by each decision maker.

The process of assigning weights to the attributes is an important phase of decision making. Most research in this area usually use either objective or subjective weights. However, considering the fact that different values for the weights of the attributes has a significant influence on the ranking of the alternatives, it is imperative that both the objective and subjective weights of the attributes are taken into account in the decision-making process. In view of this, we consider the attributes' subjective weights which are assigned by the decision makers, and the objective weights which are computed using the maximizing deviation method. These weights are then combined using the non-linear weighted comprehensive method to obtain the integrated weight of the attributes.

The advantages and drawbacks of the methods that were introduced in the works described above served as the main motivation for the work proposed in this paper, as we seek to introduce an effective SVN-based decision-making method that is free of all the problems that are inherent in the other existing methods in literature. In addition to these advantages and drawbacks, the works described above have the added disadvantage of not being able to function (i.e., provide reasonable solutions) under all circumstances. In view of this, the objective of this paper is to introduce a novel TOPSIS with maximizing deviation method for SVNSs that is able to provide effective solutions under any circumstances. Our proposed TOPSIS method is designed to handle MADM problems, and uses the maximizing deviation method to calculate the objective weights of attributes, utilizing an integrated weight measure that takes into consideration both the subjective and objective weights of the attributes. The robustness of our TOPSIS method is verified through a comprehensive series of tests which proves that our proposed method is the only method that shows compliance to all the tests, and is able to provide effective solutions under all different types of situations, thus out-performing all of the other considered methods.

The remainder of this paper is organized as follows. In Section 2, we recapitulate some of the fundamental concepts related to neutrosophic sets and SVNSs. In Section 3, we define an SVN-based TOPSIS and maximizing deviation methods and an accompanying decision-making algorithm. The proposed decision-making method is applied to a supplier selection problem in Section 4. In Section 5, a comprehensive comparative analysis of the results obtained via our proposed method and other recent approaches is presented. The similarities and differences in the performance of the existing algorithms and our algorithm is discussed, and it is proved that our algorithm is effective and provides reliable results in every type of situation. Concluding remarks are given in Section 6, followed by the acknowledgements and list of references.

2. Preliminaries

In this section, we recapitulate some important concepts pertaining to the theory of neutrosophic sets and SVNSs. We refer the readers to [8,9] for further details pertaining to these models.

The neutrosophic set model [8] is a relatively new tool for representing and measuring uncertainty and vagueness of information. It is fast becoming a preferred general framework for the analysis of uncertainty in data sets due to its capability in the handling big data sets, as well as its ability in representing all the different types of uncertainties that exists in data, in an effective and concise manner via a triple membership structure. This triple membership structure captures not only the degree of belongingness and non-belongingness of the objects in a data set, but also the degree of neutrality and indeterminacy that exists in the data set, thereby making it superior to ordinary fuzzy sets [1] and its extensions such as IFSs [2], vague sets [4], and interval-valued fuzzy sets [3]. The formal definition of a neutrosophic set is as given below.

Let U be a universe of discourse, with a class of elements in U denoted by x .

Definition 1. [8] A neutrosophic set A is an object having the form $A = \{x, T_A(x), I_A(x), F_A(x) : x \in U\}$, where the functions $T, I, F : U \rightarrow]-0, 1+[$ denote the truth, indeterminacy, and falsity membership functions, respectively, of the element $x \in U$ with respect to A . The membership functions must satisfy the condition $-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Definition 2. [8] A neutrosophic set A is contained in another neutrosophic set B , if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, and $F_A(x) \geq F_B(x)$, for all $x \in U$. This relationship is denoted as $A \subseteq B$.

Wang et al. [9] then introduced a special case of the NS model called the single-valued neutrosophic set (SVNS) model, which is as defined below. This SVNS model is better suited to applied in real-life problems compared to NSs due to the structure of its membership functions which are defined in the standard unit interval of $[0, 1]$.

Definition 3. [9] A SVNS A is a neutrosophic set that is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, where $T_A(x), I_A(x), F_A(x) \in [0, 1]$. This set A can thus be written as

$$A = \{\langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in U\}. \quad (1)$$

The sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ must fulfill the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. For a SVNS A in U , the triplet $(T_A(x), I_A(x), F_A(x))$ is called a single-valued neutrosophic number (SVNN). For the sake of convenience, we simply let $x = (T_x, I_x, F_x)$ to represent a SVNN as an element in the SVNS A .

Next, we present some important results pertaining to the concepts and operations of SVNSs. The subset, equality, complement, union, and intersection of SVNSs, and some additional operations between SVNSs were all defined by Wang et al. [9], and these are presented in Definitions 4 and 5, respectively.

Definition 4. [9] Let A and B be two SVNSs over a universe U .

- (i) A is contained in B , if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, and $F_A(x) \geq F_B(x)$, for all $x \in U$. This relationship is denoted as $A \subseteq B$.
- (ii) A and B are said to be equal if $A \subseteq B$ and $B \subseteq A$.
- (iii) $A^c = (x, (F_A(x), 1 - I_A(x), T_A(x)))$, for all $x \in U$.
- (iv) $A \cup B = (x, (\max(T_A, T_B), \min(I_A, I_B), \min(F_A, F_B)))$, for all $x \in U$.
- (v) $A \cap B = (x, (\min(T_A, T_B), \max(I_A, I_B), \max(F_A, F_B)))$, for all $x \in U$.

Definition 5. [9] Let $x = (T_x, I_x, F_x)$ and $y = (T_y, I_y, F_y)$ be two SVNNs. The operations for SVNNs can be defined as follows:

- (i) $x \oplus y = (T_x + T_y - T_x * T_y, I_x * I_y, F_x * F_y)$
- (ii) $x \otimes y = (T_x * T_y, I_x + I_y - I_x * I_y, F_x + F_y - F_x * F_y)$
- (iii) $\lambda x = (1 - (1 - T_x)^\lambda, (I_x)^\lambda, (F_x)^\lambda)$, where $\lambda > 0$
- (iv) $x^\lambda = ((T_x)^\lambda, 1 - (1 - I_x)^\lambda, 1 - (1 - F_x)^\lambda)$, where $\lambda > 0$.

Majumdar and Samanta [28] introduced the information measures of distance, similarity, and entropy for SVNSs. Here we only present the definition of the distance measures between SVNSs as it is the only component that is relevant to this paper.

Definition 6. [28] Let A and B be two SVNSs over a finite universe $U = \{x_1, x_2, \dots, x_n\}$. Then the various distance measures between A and B are defined as follows:

(i) The Hamming distance between A and B are defined as:

$$d_H(A, B) = \sum_{i=1}^n \{|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|\} \quad (2)$$

(ii) The normalized Hamming distance between A and B are defined as:

$$d_H^N(A, B) = \frac{1}{3n} \sum_{i=1}^n \{|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|\} \quad (3)$$

(ii) The Euclidean distance between A and B are defined as:

$$d_E(A, B) = \sqrt{\sum_{i=1}^n \{(T_A(x_i) - T_B(x_i))^2 + (I_A(x_i) - I_B(x_i))^2 + (F_A(x_i) - F_B(x_i))^2\}} \quad (4)$$

(iv) The normalized Euclidean distance between A and B are defined as:

$$d_E^N(A, B) = \sqrt{\frac{1}{3n} \sum_{i=1}^n \{(T_A(x_i) - T_B(x_i))^2 + (I_A(x_i) - I_B(x_i))^2 + (F_A(x_i) - F_B(x_i))^2\}} \quad (5)$$

3. A TOPSIS Method for Single-Valued Neutrosophic Sets

In this section, we present the description of the problem that is being studied followed by our proposed TOPSIS method for SVNNS. The accompanying decision-making algorithm which is based on the proposed TOPSIS method is presented. This algorithm uses the maximizing deviation method to systematically determine the objective weight coefficients for the attributes.

3.1. Description of Problem

Let $U = \{u_1, u_2, \dots, u_m\}$ denote a finite set of m alternatives, $A = \{e_1, e_2, \dots, e_n\}$ be a set of n parameters, with the weight parameter w_j of each e_j completely unknown or only partially known, $w_j \in [0, 1]$, and $\sum_{j=1}^n w_j = 1$.

Let A be an SVNNS in which $x_{ij} = (T_{ij}, I_{ij}, F_{ij})$ represents the SVNNS that represents the information pertaining to the i th alternative x_i that satisfies the corresponding j th parameter e_j . The tabular representation of A is as given in Table 1.

Table 1. Tabular representation of the Single Valued Neutrosophic Set (SVNS) A .

| u | e_1 | e_2 | \dots | e_n |
|----------|----------------------------|----------------------------|----------|----------------------------|
| x_1 | (T_{11}, I_{11}, F_{11}) | (T_{12}, I_{12}, F_{12}) | \dots | (T_{1n}, I_{1n}, F_{1n}) |
| x_2 | (T_{21}, I_{21}, F_{21}) | (T_{22}, I_{22}, F_{22}) | \dots | (T_{2n}, I_{2n}, F_{2n}) |
| \vdots | \vdots | \vdots | \ddots | \vdots |
| x_m | (T_{m1}, I_{m1}, F_{m1}) | (T_{m2}, I_{m2}, F_{m2}) | \dots | (T_{mn}, I_{mn}, F_{mn}) |

3.2. The Maximizing Deviation Method for Computing Incomplete or Completely Unknown Attribute Weights

The maximizing deviation method was proposed by Wang [29] with the aim of applying it in MADM problems in which the weights of the attributes are completely unknown or only partially known. This method uses the law of input arguments i.e., it takes into account the magnitude of the membership functions of each alternative for each attribute, and uses this information to obtain exact and reliable evaluation results pertaining to the weight coefficients for each attribute. As such,

this method is able to compute the weight coefficients of the attributes without any subjectivity, in a fair and objective manner.

The maximizing deviation method used in this paper is a modification of the original version introduced in Wang [29] that has been made compatible with the structure of the SVNS model. The definitions of the important concepts involved in this method are as given below.

Definition 7. For the parameter $e_j \in A$, the deviation of the alternative x_i to all the other alternatives is defined as:

$$D_{ij}(w_j) = \sum_{k=1}^m w_j d(x_{ij}, x_{kj}), \quad (6)$$

where x_{ij}, x_{kj} are the elements of the SVNS A , $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$ and $d(x_{ij}, x_{kj})$ denotes the distance between elements x_{ij} and x_{kj} .

The other deviation values include the deviation value of all alternatives to other alternatives, and the total deviation value of all parameters to all alternatives, both of which are as defined below:

- (i) The deviation value of all alternatives to other alternatives for the parameter $e_j \in A$, denoted by $D_j(w_j)$, is defined as:

$$D_j(w_j) = \sum_{i=1}^m D_{ij}(w_j) = \sum_{i=1}^m \sum_{k=1}^m w_j d(x_{ij}, x_{kj}), \quad (7)$$

where $j = 1, 2, \dots, n$.

- (ii) The total deviation value of all parameters to all alternatives, denoted by $D(w_j)$, is defined as:

$$(w_j) = \sum_{j=1}^n D_j(w_j) = \sum_{j=1}^n \sum_{i=1}^m \sum_{k=1}^m w_j d(x_{ij}, x_{kj}), \quad (8)$$

where w_j represents the weight of the parameter $e_j \in A$.

- (iii) The individual objective weight of each parameter $e_j \in A$, denoted by θ_j , is defined as:

$$\theta_j = \frac{\sum_{i=1}^m \sum_{k=1}^m d(x_{ij}, x_{kj})}{\sum_{j=1}^n \sum_{i=1}^m \sum_{k=1}^m d(x_{ij}, x_{kj})} \quad (9)$$

It should be noted that any valid distance measure between SVNSs can be used in Equations (6)–(9). However, to improve the effective resolution of the decision-making process, in this paper, we use the normalized Euclidean distance measure given in Equation (5) in the computation of Equations (6)–(9).

3.3. TOPSIS Method for MADM Problems with Incomplete Weight Information

The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) was originally introduced by Hwang and Yoon [30], and has since been extended to fuzzy sets, IFSs, and other fuzzy-based models. The TOPSIS method works by ranking the alternatives based on their distance from the positive ideal solution and the negative ideal solution. The basic guiding principle is that the most preferred alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution (Hwang and Yoon [30], Chen and Tzeng [31]). In this section, we present a decision-making algorithm for solving MADM problems in single-valued neutrosophic environments, with incomplete or completely unknown weight information.

3.3.1. The Proposed TOPSIS Method for SVNSs

After obtaining information pertaining to the weight values for each parameter based on the maximizing deviation method, we develop a modified TOPSIS method for the SVNS model. To achieve our goal, we introduce several definitions that are the important components of our proposed TOPSIS method.

Let the relative neutrosophic positive ideal solution (RNPIS) and relative neutrosophic negative ideal solution (RNNIS) be denoted by b^+ and b^- , respectively, where these solutions are as defined below:

$$b^+ = \left\{ \left(\max_i T_{ij}, \min_i I_{ij}, \min_i F_{ij} \right) \mid j = 1, 2, \dots, n \right\}, \quad (10)$$

and

$$b^- = \left\{ \left(\min_i T_{ij}, \max_i I_{ij}, \max_i F_{ij} \right) \mid j = 1, 2, \dots, n \right\} \quad (11)$$

The difference between each object and the RNPIS, denoted by D_i^+ , and the difference between each object and the RNNIS, denoted by D_i^- , can then be calculated using the normalized Euclidean distance given in Equation (5) and by the formula given in Equations (12) and (13).

$$D_i^+ = \sum_{j=1}^n w_j d_{NE}(b_{ij}, b_j^+), \quad i = 1, 2, \dots, m \quad (12)$$

and

$$D_i^- = \sum_{j=1}^n w_j d_{NE}(b_{ij}, b_j^-), \quad i = 1, 2, \dots, m \quad (13)$$

Here, w_j denotes the integrated weight for each of the attributes.

The optimal alternative can then be found using the measure of the relative closeness coefficient of each alternative, denoted by C_i , which is as defined below:

$$C_i = \frac{D_i^-}{\max_j D_j^-} - \frac{D_i^+}{\min_j D_j^+}, \quad i, j = 1, 2, \dots, m \quad (14)$$

From the structure of the closeness coefficient in Equation (14), it is obvious that the larger the difference between an alternative and the fuzzy negative ideal object, the larger the value of the closeness coefficient of the said alternative. Therefore, by the principal of maximum similarity between an alternative and the fuzzy positive ideal object, the objective of the algorithm is to determine the alternative with the maximum closeness coefficient. This alternative would then be chosen as the optimal alternative.

3.3.2. Attribute Weight Determination Method: An Integrated WEIGHT MEASure

In any decision-making process, there are two main types of weight coefficients, namely the subjective and objective weights that need to be taken into consideration. Subjective weight refers to the values assigned to each attribute by the decision makers based on their individual preferences and experience, and is very much dependent on the risk attitude of the decision makers. Objective weight refers to the weights of the attributes that are computed mathematically using any appropriate computation method. Objective weighting methods uses the law of input arguments (i.e., the input values of the data) as it determines the attribute weights based on the magnitude of the membership functions that are assigned to each alternative for each attribute.

Therefore, using only subjective weighting in the decision-making process would be inaccurate as it only reflects the opinions of the decision makers while ignoring the importance of each attribute that are reflected by the input values. Using only objective weighting would also be inaccurate as it only

reflects the relative importance of the attributes based on the law of input arguments, but fails to take into consideration the preferences and risk attitude of the decision makers.

To overcome this drawback and improve the accuracy and reliability of the decision-making process, we use an integrated weight measure which combines the subjective and objective weights of the attributes. This factor makes our decision-making algorithm more accurate compared to most of the other existing methods in literature that only take into consideration either the objective or subjective weights.

Based on the formula and weighting method given above, we develop a practical and effective decision-making algorithm based on the TOPSIS approach for the SVNS model with incomplete weight information. The proposed Algorithm 1 is as given below.

Algorithm 1. (based on a modified TOPSIS approach).

Step 1. Input the SVNS A which represents the information pertaining to the problem.

Step 2. Input the subjective weight h_j for each of the attributes $e_j \in A$ as given by the decision makers.

Step 3. Compute the objective weight θ_j for each of the attributes $e_j \in A$, using Equation (9).

Step 4. The integrated weight coefficient w_j for each of the attributes $e_j \in A$, is computed using Equation as follow:

$$w_j = \frac{h_j \theta_j}{\sum_{j=1}^n h_j \theta_j}$$

Step 5. The values of RNPIS b^+ and RNNIS b^- are computed using Equations (10) and (11).

Step 6. The difference between each alternative and the RNPIS, D^+ and the RNNIS D^- are computed using Equations (12) and (13), respectively.

Step 7. The relative closeness coefficient C_i for each alternative is calculated using Equation (14).

Step 8. Choose the optimal alternative based on the principal of maximum closeness coefficient.

4. Application of the Topsis Method in a Made Problem

The implementation process and utility of our proposed decision-making algorithm is illustrated via an example related to a supplier selection problem.

4.1. Illustrative Example

In today's extremely competitive business environment, firms must be able to produce good quality products at reasonable prices in order to be successful. Since the quality of the products is directly dependent on the effectiveness and performance of its suppliers, the importance of supplier selection has become increasingly recognized. In recent years, this problem has been handled using various mathematical tools. Some of the recent research in this area can be found in [32–38].

Example 1. A manufacturing company is looking to select a supplier for one of the products manufactured by the company. The company has shortlisted ten suppliers from an initial list of suppliers. These ten suppliers form the set of alternatives U that are under consideration,

$$U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}.$$

The procurement manager and his team of buyers evaluate the suppliers based on a set of evaluation attributes E which is defined as:

$$E = \{e_1 = \text{service quality}, e_2 = \text{pricing and cost structure}, e_3 = \text{financial stability}, \\ e_4 = \text{environmental regulation compliance}, e_5 = \text{reliability}, \\ e_6 = \text{relevant experience}\}.$$

The firm then evaluates each of the alternatives x_i ($i = 1, 2, \dots, 10$), with respect to the attributes e_j ($j = 1, 2, \dots, 6$). The evaluation done by the procurement team is expressed in the form of SVNNS in a SVNS A .

Now suppose that the company would like to select one of the five shortlisted suppliers to be their supplier. We apply the proposed Algorithm 1 outlined in Section 3.3 to this problem with the aim of selecting a supplier that best satisfies the specific needs and requirements of the company. The steps involved in the implementation process of this algorithm are outlined below (Algorithm 2).

Algorithm 2. (based on the modified TOPSIS approach).

Step 1. The SVNS A constructed for this problem is given in tabular form in Table 2

Step 2. The subjective weight h_j for each attribute $e_j \in A$ as given by the procurement team (the decision makers) are $h = \{h_1 = 0.15, h_2 = 0.15, h_3 = 0.22, h_4 = 0.25, h_5 = 0.14, h_6 = 0.09\}$.

Step 3. The objective weight θ_j for each attribute $e_j \in A$ is computed using Equation (9) are as given below:
 $\theta = \{\theta_1 = 0.139072, \theta_2 = 0.170256, \theta_3 = 0.198570, \theta_4 = 0.169934, \theta_5 = 0.142685,$
 $\theta_6 = 0.179484\}$.

Step 4. The integrated weight w_j for each attribute $e_j \in A$ is computed using Equation (15). The integrated weight coefficient obtained for each attribute is:

$w = \{w_1 = 0.123658, w_2 = 0.151386, w_3 = 0.258957, w_4 = 0.251833, w_5 = 0.118412,$
 $w_6 = 0.0957547\}$.

Step 5. Use Equations (10) and (11) to compute the values of b^+ and b^- from the neutrosophic numbers given in Table 2. The values are as given below:

$b^+ = \{b_1^+ = [0.7, 0.2, 0.1], b_2^+ = [0.9, 0, 0.1], b_3^+ = [0.8, 0, 0], b_4^+ = [0.9, 0.3, 0],$
 $b_5^+ = [0.7, 0.2, 0.2], b_6^+ = [0.8, 0.2, 0.1]\}$

and

$b^- = \{b_1^- = [0.5, 0.8, 0.5], b_2^- = [0.6, 0.8, 0.5], b_3^- = [0.1, 0.7, 0.5], b_4^- = [0.3, 0.8, 0.7],$
 $b_5^- = [0.5, 0.8, 0.7], b_6^- = [0.5, 0.8, 0.9]\}$.

Step 6. Use Equations (12) and (13) to compute the difference between each alternative and the RNPIS and the RNNIS, respectively. The values of D^+ and D^- are as given below:

$D^+ = \{D_1^+ = 0.262072, D_2^+ = 0.306496, D_3^+ = 0.340921, D_4^+ = 0.276215, D_5^+ = 0.292443,$
 $D_6^+ = 0.345226, D_7^+ = 0.303001, D_8^+ = 0.346428, D_9^+ = 0.271012, D_{10}^+ = 0.339093\}$.

and

$D^- = \{D_1^- = 0.374468, D_2^- = 0.307641, D_3^- = 0.294889, D_4^- = 0.355857, D_5^- = 0.323740$
 $D_6^- = 0.348903, D_7^- = 0.360103, D_8^- = 0.338725, D_9^- = 0.379516, D_{10}^- = 0.349703\}$.

Step 7. Using Equation (14), the closeness coefficient C_i for each alternative is:

$C_1 = -0.0133, C_2 = -0.3589, C_3 = -0.5239, C_4 = -0.1163, C_5 = -0.2629,$
 $C_6 = -0.3980, C_7 = -0.2073, C_8 = -0.4294, C_9 = -0.0341, C_{10} = -0.3725.$

Step 8. The ranking of the alternatives obtained from the closeness coefficient is as given below:

$$x_1 > x_9 > x_4 > x_7 > x_5 > x_2 > x_{10} > x_6 > x_8 > x_3.$$

Therefore the optimal decision is to select supplier x_1 .

Table 2. Tabular representation of SVNS A .

| U | e ₁ | e ₂ | e ₃ |
|----------|-----------------|-----------------|-----------------|
| x_1 | (0.7, 0.5, 0.1) | (0.7, 0.5, 0.3) | (0.8, 0.6, 0.2) |
| x_2 | (0.6, 0.5, 0.2) | (0.7, 0.5, 0.1) | (0.6, 0.3, 0.5) |
| x_3 | (0.6, 0.2, 0.3) | (0.6, 0.6, 0.4) | (0.7, 0.7, 0.2) |
| x_4 | (0.5, 0.5, 0.4) | (0.6, 0.4, 0.4) | (0.7, 0.7, 0.3) |
| x_5 | (0.7, 0.5, 0.5) | (0.8, 0.3, 0.1) | (0.7, 0.6, 0.2) |
| U | e ₁ | e ₂ | e ₃ |
| x_6 | (0.5, 0.5, 0.5) | (0.7, 0.8, 0.1) | (0.7, 0.3, 0.5) |
| x_7 | (0.6, 0.8, 0.1) | (0.7, 0.2, 0.1) | (0.6, 0.3, 0.4) |
| x_8 | (0.7, 0.8, 0.3) | (0.6, 0.6, 0.5) | (0.8, 0, 0.5) |
| x_9 | (0.6, 0.7, 0.1) | (0.7, 0, 0.1) | (0.6, 0.7, 0) |
| x_{10} | (0.5, 0.7, 0.4) | (0.9, 0, 0.3) | (1, 0, 0) |

Table 2. Cont.

| U | e ₄ | e ₅ | e ₆ |
|-----------------|-----------------|-----------------|-----------------|
| x ₁ | (0.9, 0.4, 0.2) | (0.6, 0.4, 0.7) | (0.6, 0.5, 0.4) |
| x ₂ | (0.6, 0.4, 0.3) | (0.7, 0.5, 0.4) | (0.7, 0.8, 0.9) |
| x ₃ | (0.5, 0.5, 0.3) | (0.6, 0.8, 0.6) | (0.7, 0.2, 0.5) |
| x ₄ | (0.9, 0.4, 0.2) | (0.7, 0.3, 0.5) | (0.6, 0.4, 0.4) |
| x ₅ | (0.7, 0.5, 0.2) | (0.7, 0.5, 0.6) | (0.6, 0.7, 0.8) |
| U | e ₄ | e ₅ | e ₆ |
| x ₆ | (0.4, 0.8, 0) | (0.7, 0.4, 0.2) | (0.5, 0.6, 0.3) |
| x ₇ | (0.3, 0.5, 0.1) | (0.6, 0.3, 0.6) | (0.5, 0.2, 0.6) |
| x ₈ | (0.7, 0.3, 0.6) | (0.6, 0.8, 0.5) | (0.6, 0.2, 0.4) |
| x ₉ | (0.7, 0.4, 0.3) | (0.6, 0.6, 0.7) | (0.7, 0.3, 0.2) |
| x ₁₀ | (0.5, 0.6, 0.7) | (0.5, 0.2, 0.7) | (0.8, 0.4, 0.1) |

4.2. Adaptation of the Algorithm to Non-Integrated Weight Measure

In this section, we present an adaptation of our algorithm introduced in Section 4.1 to cases where only the objective weights or subjective weights of the attributes are taken into consideration. The results obtained via these two new variants are then compared to the results obtained via the original algorithm in Section 4.1. Further, we also compare the results obtained via these two new variants of the algorithm to the results obtained via the other methods in literature that are compared in Section 5.

To adapt our proposed algorithm in Section 3 for these special cases, we hereby represent the objective-only and subjective-only adaptations of the algorithm. This is done by taking only the objective (subjective) weight is to be used, then simply take $w_j = \theta_j$ ($w_j = h_j$). The two adaptations of the algorithm are once again applied to the dataset for SVNS A given in Table 2.

4.2.1. Objective-Only Adaptation of Our Algorithm

All the steps remain the same as the original algorithm; however, only the objective weights of the attributes are used, i.e., we take $w_j = \theta_j$.

The results of applying this variant of the algorithm produces the ranking given below:

$$x_9 > x_1 > x_4 > x_{10} > x_7 > x_6 > x_5 > x_8 > x_3 > x_2.$$

Therefore, if only the objective weight is to be considered, then the optimal decision is to select supplier x_9 .

4.2.2. Subjective-Only Adaptation of Our Algorithm

All the steps remain the same as the original algorithm; however, only the subjective weights of the attributes are used, i.e., we take $w_j = h_j$.

The results of applying this variant of the algorithm produces the ranking given below:

$$x_1 > x_9 > x_4 > x_7 > x_5 > x_2 > x_6 > x_{10} > x_8 > x_3$$

Therefore, if only the objective weight is to be considered, then the optimal decision is to select supplier x_1 .

From the results obtained above, it can be observed that the ranking of the alternatives are clearly affected by the decision of the decision maker to use only the objective weights, only the subjective weights of the attributes, or an integrated weight measure that takes into consideration both the objective and subjective weights of the attributes.

5. Comparatives Studies

In this section, we present a brief comparative analysis of some of the recent works in this area and our proposed method. These recent approaches are applied to our Example 1, and the limitations that exist in these methods are elaborated, and the advantages of our proposed method are discussed and analyzed. The results obtained are summarized in Table 3.

5.1. Comparison of Results Obtained Through Different Methods

Table 3. The results obtained using different methods for Example 1.

| Method | The Final Ranking | The Best Alternative |
|---|--|----------------------|
| Ye [39] | | |
| (i) WAAO * | $x_1 > x_4 > x_9 > x_5 > x_7 > x_2 > x_{10} > x_8 > x_3 > x_6$ | x_1 |
| (ii) WGAO ** | $x_{10} > x_9 > x_8 > x_1 > x_5 > x_7 > x_4 > x_2 > x_6 > x_3$ | x_{10} |
| Ye [10] | | |
| (i) Weighted correlation coefficient | $x_1 > x_4 > x_5 > x_9 > x_2 > x_8 > x_7 > x_3 > x_6 > x_{10}$ | x_1 |
| (ii) Weighted cosine similarity measure | $x_1 > x_9 > x_4 > x_5 > x_2 > x_{10} > x_8 > x_3 > x_7 > x_6$ | x_1 |
| Ye [11] | $x_1 > x_9 > x_4 > x_7 > x_5 > x_2 > x_8 > x_6 > x_3 > x_{10}$ | x_1 |
| Huang [14] | $x_1 > x_9 > x_4 > x_5 > x_2 > x_7 > x_8 > x_6 > x_3 > x_{10}$ | x_1 |
| Peng et al. [40] | | |
| (i) GSNNWA *** | $x_9 > x_{10} > x_8 > x_6 > x_1 > x_7 > x_4 > x_5 > x_2 > x_3$ | x_9 |
| (ii) GSNNWG **** | $x_1 > x_9 > x_4 > x_5 > x_7 > x_2 > x_8 > x_3 > x_6 > x_{10}$ | x_1 |
| Peng & Liu [15] | | |
| (i) EDAS | $x_1 > x_4 > x_6 > x_9 > x_{10} > x_3 > x_2 > x_7 > x_5 > x_8$ | x_1 |
| (ii) Similarity measure | $x_{10} > x_8 > x_7 > x_4 > x_1 > x_2 > x_5 > x_9 > x_3 > x_6$ | x_{10} |
| Maji [41] | $x_5 > x_1 > x_9 > x_6 > x_2 > x_4 > x_3 > x_8 > x_7 > x_{10}$ | x_5 |
| Karaaslan [42] | $x_1 > x_9 > x_4 > x_5 > x_7 > x_2 > x_8 > x_3 > x_6 > x_{10}$ | x_1 |
| Ye [43] | $x_1 > x_9 > x_4 > x_5 > x_7 > x_2 > x_8 > x_3 > x_6 > x_{10}$ | x_1 |
| Biswas et al. [44] | $x_{10} > x_9 > x_7 > x_1 > x_4 > x_6 > x_5 > x_8 > x_2 > x_3$ | x_{10} |
| Ye [45] | $x_9 > x_7 > x_1 > x_4 > x_2 > x_{10} > x_5 > x_8 > x_3 > x_6$ | x_9 |
| Adaptation of our algorithm (objective weights only) | $x_9 > x_1 > x_4 > x_{10} > x_7 > x_6 > x_5 > x_8 > x_3 > x_2$ | x_9 |
| Adaptation of our algorithm (subjective weights only) | $x_1 > x_9 > x_4 > x_7 > x_5 > x_2 > x_6 > x_{10} > x_8 > x_3$ | x_1 |
| Our proposed method (using integrated weight measure) | $x_1 > x_9 > x_4 > x_7 > x_5 > x_2 > x_{10} > x_6 > x_8 > x_3$ | x_1 |

* WAAO = weighted arithmetic average operator; ** WGAO = weighted geometric average operator; *** GSNNWA = generalized simplified neutrosophic number weighted averaging operator; **** GSNNWG = generalized simplified neutrosophic number weighted geometric operator.

5.2. Discussion of Results

From the results obtained in Table 3, it can be observed that different rankings and optimal alternatives were obtained from the different methods that were compared. This difference is due to a number of reasons. These are summarized briefly below:

- (i) The method proposed in this paper uses an integrated weight measure which considers both the subjective and objective weights of the attributes, as opposed to some of the methods that only consider the subjective weights or objective weights.
- (ii) Different operators emphasizes different aspects of the information which ultimately leads to different rankings. For example, in [40], the GSNNWA operator used is based on an arithmetic average which emphasizes the characteristics of the group (i.e., the whole information), whereas the GSNNWG operator is based on a geometric operator which emphasizes the characteristics of each individual alternative and attribute. As our method places more importance on the characteristics of the individual alternatives and attributes, instead of the entire information

as a whole, our method produces the same ranking as the GSNNWG operator but different results from the GSNNWA operator.

5.3. Analysis of the Performance and Reliability of Different Methods

The performance of these methods and the reliability of the results obtained via these methods are further investigated in this section.

Analysis

In all of the 11 papers that were compared in this section, the different authors used different types of measurements and parameters to determine the performance of their respective algorithms. However, all of these inputs *always* contain a tensor with at least three degrees. This tensor can refer to different types of neutrosophic sets depending on the context discussed in the respective papers, e.g., simplified neutrosophic sets, single-valued neutrosophic sets, neutrosophic sets, or INSs. For the sake of simplicity, we shall denote them simply as S .

Furthermore, all of these methods consider a weighted approach i.e., the weight of each attribute is taken into account in the decision-making process. The decision-making algorithms proposed in [10,11,14,39,40,43,45] use the subjective weighting method, the algorithms proposed in [42,44] use the objective weighting method, whereas only the decision-making methods proposed in [15] use an integrated weighting method which considers both the subjective and objective weights of the attributes. The method proposed by Maji [41] did not take the attribute weights into consideration in the decision-making process.

In this section, we first apply the inputs of those papers into our own algorithm. We then compare the results obtained via our proposed algorithm with their results, with the aim of justifying the effectiveness of our algorithm. The different methods and their algorithms are analyzed below:

- (i) The algorithms in [10,11,39] all use the data given below as inputs

$$S = \left\{ \begin{array}{l} [0.4, 0.2, 0.3], [0.4, 0.2, 0.3], [0.2, 0.2, 0.5] \\ [0.6, 0.1, 0.2], [0.6, 0.1, 0.2], [0.5, 0.2, 0.2] \\ [0.3, 0.2, 0.3], [0.5, 0.2, 0.3], [0.5, 0.3, 0.2] \\ [0.7, 0.0, 0.1], [0.6, 0.1, 0.2], [0.4, 0.3, 0.2] \end{array} \right\}$$

The subjective weights w_j of the attributes are given by $w_1 = 0.35$, $w_2 = 0.25$, $w_3 = 0.40$. All the five algorithms from papers [10,11,39] yields either one of the following rankings:

$$A_4 > A_2 > A_3 > A_1 \quad \text{or} \quad A_2 > A_4 > A_3 > A_1$$

Our algorithm yields the ranking $A_4 > A_2 > A_3 > A_1$ which is consistent with the results obtained through the methods given above.

- (ii) The method proposed in [44] also uses the data given in S above as inputs but ignores the opinions of the decision makers as it does not take into account the subjective weights of the attributes. The algorithm from this paper yields the ranking of $A_4 > A_2 > A_3 > A_1$. To fit this data into our algorithm, we randomly assigned the subjective weights of the attributes as $w_j = \frac{1}{3}$ for $j = 1, 2, 3$. A ranking of $A_4 > A_2 > A_3 > A_1$ was nonetheless obtained from our algorithm.
- (iii) The methods introduced in [14,43,45] all use the data given below as input values:

$$S = \left\{ \begin{array}{l} [0.5, 0.1, 0.3], [0.5, 0.1, 0.4], [0.7, 0.1, 0.2], [0.3, 0.2, 0.1] \\ [0.4, 0.2, 0.3], [0.3, 0.2, 0.4], [0.9, 0.0, 0.1], [0.5, 0.3, 0.2] \\ [0.4, 0.3, 0.1], [0.5, 0.1, 0.3], [0.5, 0.0, 0.4], [0.6, 0.2, 0.2] \\ [0.6, 0.1, 0.2], [0.2, 0.2, 0.5], [0.4, 0.3, 0.2], [0.7, 0.2, 0.1] \end{array} \right\}$$

The subjective weights w_j of the attributes are given by $w_1 = 0.30$, $w_2 = 0.25$, $w_3 = 0.25$ and $w_4 = 0.20$.

In this case, all of the three algorithms produces a ranking of $A_1 > A_3 > A_2 > A_4$.

This result is however not very reliable as all of these methods only considered the subjective weights of the attributes and ignored the objective weight which is a vital measurement of the relative importance of an attribute e_j relative to the other attributes in an objective manner i.e., without “prejudice”.

When we calculated the objective weights using our own algorithm we have the following objective weights:

$$a_j = [0.203909, 0.213627, 0.357796, 0.224667]$$

In fact, it is indeed $\langle 0.9, 0.0, 0.1 \rangle$ that mainly contributes to the largeness of the objective weight of attribute e_3 compared to the other values of e_j . Hence, when we calculate the integrated weight, the weight of attribute e_3 is still the largest.

Since $[0.9, 0.0, 0.1]$ is in the second row, our algorithm yields a ranking of $A_2 > A_1 > A_3 > A_4$ as a result.

We therefore conclude that our algorithm is more effective and the results obtained via our algorithm is more reliable than the ones obtained in [14,43,45], as we consider both the objective and subjective weights.

- (iv) It can be observed that for the methods introduced in [10,11,39,44], we have $0.8 \leq T_{ij} + I_{ij} + F_{ij} \leq 1$ for all the entries. A similar trend can be observed in [14,43,45], where $0.6 \leq T_{ij} + I_{ij} + F_{ij} \leq 1$ for all the entries. Therefore, we are not certain about the results obtained through the decision making algorithms in these papers when the value of $T_{ij} + I_{ij} + F_{ij}$ deviates very far from 1.

Another aspect to be considered is the weighting method that is used in the decision making process. As mentioned above, most of the current decision making methods involving SVNSe use subjective weighting, a few use objective weighting and only two methods introduced in [15] uses an integrated weighting method to arrive at the final decision. In view of this, we proceeded to investigate if all of the algorithms that were compared in this section are able to produce reliable results when both the subjective and objective weights are taken into consideration. Specifically, we investigate if these algorithms are able to perform effectively in situations where the subjective weights clearly prioritize over the objective weights, and vice-versa. To achieve this, we tested all of the algorithms with three sets of inputs as given below:

Test 1: A scenario containing a very small value of $T_{ij} + I_{ij} + F_{ij}$.

$$S_1 = \left\{ \begin{array}{l} A_1 = ([0.5, 0.5, 0.5], [0.9999, 0.0001, 0.000]) \\ A_2 = ([0.5, 0.5, 0.5], [0.9999, 0.0001, 0.0001]) \\ A_3 = ([0.5, 0.5, 0.5], [0.9999, 0.0000, 0.0001]) \\ A_4 = ([0.5, 0.5, 0.5], [0.0001, 0.0000, 0.000]) \end{array} \right\}$$

The subjective weight in this case is assigned as: $a_j = [0.5, 0.5]$.

By observation alone, it is possible to tell that an effective algorithm should produce A_4 as the least favoured alternative, and A_2 should be second least-favoured alternative.

Test 2: A scenario where subjective weights *prioritize* over objective weight.

$$S_2 = \left\{ \begin{array}{l} A_1 = ([0.80, 0.10, 0.10], [0.19, 0.50, 0.50]) \\ A_2 = ([0.20, 0.50, 0.50], [0.81, 0.10, 0.10]) \end{array} \right\}$$

The subjective weight in this case is assigned as: $a_j = [0.99, 0.01]$.

By observation alone, we can tell that an effective algorithm should produce a ranking of $A_1 > A_2$.

Test 3: This test is based on a real-life situation.

Suppose a procurement committee is looking to select the best supplier to supply two raw materials e_1 and e_2 . In this context, the triplet $[T, I, F]$ represents the following:

T : the track record of the suppliers that is approved by the committee

I : the track record of the suppliers that the committee feels is questionable

F : the track record of the suppliers that is rejected by the committee

Based on their experience, the committee is of the opinion that raw material e_1 is slightly more important than raw material e_2 , and assigned subjective weights of $w_1^{sub} = 0.5001$ and $w_2^{sub} = 0.4999$.

After an intensive search around the country, the committee shortlisted 20 candidates (A_1 to A_{20}). After checking all of the candidates' track records and analyzing their past performances, the committee assigned the following values for each of the suppliers.

$$S_3 = \left\{ \begin{array}{l} A_1 = ([0.90, 0.00, 0.10], [0.80, 0.00, 0.10]), A_2 = ([0.80, 0.00, 0.10], [0.90, 0.00, 0.10]) \\ A_3 = ([0.50, 0.50, 0.50], [0.00, 0.90, 0.90]), A_4 = ([0.50, 0.50, 0.50], [0.10, 0.90, 0.80]) \\ A_5 = ([0.50, 0.50, 0.50], [0.20, 0.90, 0.70]), A_6 = ([0.50, 0.50, 0.50], [0.30, 0.90, 0.60]) \\ A_7 = ([0.50, 0.50, 0.50], [0.40, 0.90, 0.50]), A_8 = ([0.50, 0.50, 0.50], [0.50, 0.90, 0.40]) \\ A_9 = ([0.50, 0.50, 0.50], [0.60, 0.90, 0.30]), A_{10} = ([0.50, 0.50, 0.50], [0.70, 0.30, 0.90]) \\ A_{11} = ([0.50, 0.50, 0.50], [0.70, 0.90, 0.30]), A_{12} = ([0.50, 0.50, 0.50], [0.00, 0.30, 0.30]) \\ A_{13} = ([0.50, 0.50, 0.50], [0.70, 0.90, 0.90]), A_{14} = ([0.50, 0.50, 0.50], [0.70, 0.30, 0.30]) \\ A_{15} = ([0.50, 0.50, 0.50], [0.60, 0.40, 0.30]), A_{16} = ([0.50, 0.50, 0.50], [0.50, 0.50, 0.30]) \\ A_{17} = ([0.50, 0.50, 0.50], [0.40, 0.60, 0.30]), A_{18} = ([0.50, 0.50, 0.50], [0.30, 0.70, 0.30]) \\ A_{19} = ([0.50, 0.50, 0.50], [0.20, 0.80, 0.30]), A_{20} = ([0.50, 0.50, 0.50], [0.10, 0.90, 0.30]) \end{array} \right\}$$

The objective weights for this scenario was calculated based on our algorithm and the values are $w_1^{obj} = 0.1793$ and $w_2^{obj} = 0.8207$.

Now it can be observed that suppliers A_1 and A_2 are the ones that received the best evaluation scores from the committee. Supplier A_1 received better evaluation scores from the committee compared to supplier A_2 for attribute e_1 . Attribute e_1 was deemed to be more important than attribute e_2 by the committee, and hence had a higher subjective weight. However, the objective weight of attribute e_2 is much higher than e_1 . This resulted in supplier A_2 ultimately being chosen as the best supplier. This is an example of a scenario where the objective weights are prioritized over the subjective weights, and has a greater influence on the decision-making process.

Therefore, in the scenario described above, an effective algorithm should select A_2 as the optimal supplier, followed by A_1 . All of the remaining choices have values of $T < 0.8$, $I > 0.0$ and $F > 0.1$. As such, an effective algorithm should rank all of these remaining 18 choices behind A_1 .

We applied the three tests mentioned above and the data set for S_3 given above to the decision-making methods introduced in the 11 papers that were compared in the previous section. The results obtained are given in Table 4.

Thus it can be concluded that our proposed algorithm is the most effective algorithm and the one that yields the most reliable results in all the different types of scenario. Hence, our proposed algorithm provides a robust framework that can be used to handle any type of situation and data, and produce accurate and reliable results for any type of situation and data.

Finally, we look at the context of the scenario described in Example 1. The structure of our data (given in Table 2) is more generalized, by theory, having $0 \leq T_{ij} + I_{ij} + F_{ij} \leq 1$ and $0 \leq T_{ij} + I_{ij} + F_{ij} \leq 3$, and is similar to the structure of the data used in [15,40–42]. Hence, our choice of input data serves as a more faithful indicator of how each algorithm works under all sorts of possible conditions.

Table 4. Compliance to Tests 1, 2, and 3.

| Paper | | Test 1 Compliance | Test 2 Compliance | Test 3 Compliance |
|--|------------------------------------|----------------------|----------------------|----------------------|
| Ye [39] | WAAO * | Y | Y | N |
| | WGAO * | N | Y | N |
| Ye [10] | Weighted correlation coefficient | Y | Y | N |
| | Weighted cosine similarity measure | N | Y | N |
| Ye [11] | | Y | Y | N |
| Huang [14] | | Y | Y | N |
| Peng et al. [40] | GSNNWA ** | Y | Y | N |
| | GSNNWG ** | Y | Y | N |
| Peng & Liu [15] | EDAS | Y | Y | N |
| | Similarity measure | N | Y | Y |
| Maji [41] | | N | N | N |
| Karaaslan [42] | | Y | Y | N |
| Ye [43] | | Y | Y | N |
| Biswas et al. [44] | | Y | N | Y |
| Ye [45] | | Y | Y | N |
| Adaptation of our proposed algorithm (objective weights only) | | Y | N | Y |
| Adaptation of our proposed algorithm (subjective weights only) | | Y | Y | N |
| Our proposed algorithm | | Y | Y | Y |

Remarks: Y = Yes (which indicates compliance to Test); N = No (which indicates non-compliance to Test); * WAAO = weighted arithmetic average operator; * WGAO = weighted geometric average operator; ** GSNNWA = generalized simplified neutrosophic number weighted averaging operator; ** GSNNWG = generalized simplified neutrosophic number weighted geometric operator.

6. Conclusions

The concluding remarks and the significant contributions that were made in this paper are expounded below.

- (i) A novel TOPSIS method for the SVNS model is introduced, with the maximizing deviation method used to determine the objective weight of the attributes. Through thorough analysis, we have proven that our algorithm is compliant with all of the three tests that were discussed in Section 5.3. This clearly indicates that our proposed decision-making algorithm is not only an effective algorithm but one that produces the most reliable and accurate results in all the different types of situation and data inputs.
- (ii) Unlike other methods in the existing literature which reduces the elements from single-valued neutrosophic numbers (SVNNs) to fuzzy numbers, or interval neutrosophic numbers (INNs) to neutrosophic numbers or fuzzy numbers, in our version of the TOPSIS method the input data is in the form of SVNNs and this form is maintained throughout the decision-making process. This prevents information loss and enables the original information to be retained, thereby ensuring a higher level of accuracy for the results that are obtained.
- (iii) The objective weighting method (e.g., the ones used in [10,11,14,39,40,43,45]) only takes into consideration the values of the membership functions while ignoring the preferences of the decision makers. Through the subjective weighting method (e.g., the ones used in [42,44]), the attribute weights are given by the decision makers based on their individual preferences and experiences. Very few approaches in the existing literature (e.g., [15]) consider both the objective and subjective weighting methods. Our proposed method uses an integrated weighting model that considers both the objective and subjective weights of the attributes, and this accurately reflects the input values of the alternatives as well as the preferences and risk attitude of the decision makers.

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