Violation of Universality of Free Fall by Fast-moving Test Bodies

Hui Peng, Davidpeng949@hotmail.com

ORCID: 0000-0002-1844-31633

35964 Vivian PL, Fremont, CA 94536, USA

Abstract

The Universality of Free Fall (UFF) and the Weak Equivalent Principle (WEP) are crucial for Einstein's theory of gravity. In this article, we show that fast-moving test bodies violate UFF. A well-known problem of WEP is its lack of clarity about the inertial masses, e.g., the gravitational mass is equal to the rest mass for test bodies whereas it is equal to the relativistic mass, $T^{\mu\nu}$, for the source of gravitational fields (S-GF). To resolve this problem we expand WEP into three Sub-WEPs: WEP-I, WEP-II, and WEP-III. This allows us to define a new precise WEP where WEP-I is for slow-moving test bodies/S-GFs and WEP-II or WEP-III is for fast-moving test bodies/S-GFs. Not only is WEP-I valid and equivalent to UFF for slow-moving test bodies, it is also valid for slow-moving S-GFs. Either WEP-II or WEP-III is applicable to fast-moving test bodies/S-GFs. A synchrotron-type experiment is designed to directly verify both the predicted violation of UFF and the un-demonstrated WEP-II or WEP-III. Experimental confirmation of WEP-II would lead to an internal U (1) symmetry of gravity.

Key Words:

Universality of Free Fall, Weak Equivalence Principle (WEP), Gravitational Experiment, Synchrotron-type Experiment, Einstein's theory of gravity

1. Introduction

Galileo and Newton had postulated the Universality of Free Fall (UFF) based on experiments of their times. UFF states that test bodies fall in a gravitational field with the same acceleration, which is independent of the nature of test bodies. The basic concept of the Weak Equivalence Principle (WEP) is to relate the gravitational mass to the inertial mass. One of expressions of WEP states that the gravitational mass of a body is equal to its inertial mass [1]. We refer this expression as the original expression of WEP. For slow moving, non-relativistic, test bodies, UFF can be mathematically derived from the original expression of WEP. In this case, UFF and the original expression of WEP are equivalent. Experiments to demonstrate UFF and the original expression of WEP have been conducted or proposed to being carried out with progressively improving sensitivity ([2] for reviews). These Experiments include: torsion-balance-type test [3], free-fall-type tests of both matter and anti-matter [4]. Moreover, the original expression of WEP predicted the gravitational red shift that has been demonstrated [5].

Significantly, the original expression of WEP had historically played dual roles: it determined not only the left-hand side, i.e., the geometry part, but also the right-hand side, i.e., the source part, of Einstein equation of gravity:

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \frac{8\pi G}{C^4} T^{\mu\nu}.$$

Firstly, Einstein adopted the UFF and the original expression of WEP [1], which have played the most crucial role in the development of gravitation theories. Indeed, they are believed to be the evidence that gravity is determined by the geometry of space-time. This implies that the UFF and the original expression of the WEP have determined the structural feature of the left-hand side of Einstein equation of gravity.

Secondly, this expression of WEP had led Einstein to postulate that the energy-momentum tensor $T^{\mu\nu}$ of a body is the Source of Gravitational Fields (S-GFs). He argued that the special relativity had led to the conclusion that the inertial mass's complete mathematical expression is a symmetrical tensor, i.e., the energy-momentum tensor. He then postulated that, based on the basic concept of WEP, the corresponding energy-momentum tensor $T^{\mu\nu}$ of a body would be employed as the S-GF in his field equation of gravity [1]. This implies that the original expression of WEP, but not UFF, has determined the right-hand side of Einstein equation of gravity. Therefore, without the basic concept of the WEP, Einstein equation of gravity would not have the energy-momentum tensor $T^{\mu\nu}$ as the term for S-GF.

This original expression of WEP equates the inertial mass m_i with the gravitational mass m_g. The problem with this expression of WEP is that there are two different inertial masses, the inertial rest and relativistic masses, and this original expression of WEP does not define inertial masses clearly.

One way to resolve this lack of clarity issue of masses is to drop the basic concept of the WEP. This approach, in its modern form, states that free fall acceleration of test bodies in a gravitational field is independent of their internal structures and compositions. We will refer this expression as the modern expression of WEP. But this modern expression of WEP has three problems. Firstly, Einstein's postulation of the $T^{\mu\nu}$ of a body as the S-GF was based on the basic concept of WEP, without it this Einstein's postulation would lose its foundation. Secondly, this modern expression of WEP cannot predict the gravitational red shift [5] as the original expression of WEP did. Thirdly, UFF cannot be derived mathematically from this modern expression of WEP. Actually, the modern expression of WEP and UFF are the same. Hereafter, we denote both UFF and the modern expression of WEP as "UFF" and do not distinguish them.

In order to not only resolve the clarity issue of inertial masses of the original expression of WEP but also avoid the drawbacks of the modern expression of WEP, in this article, we will take a different approach, i.e., we keep the basic concept of the WEP but expand it to three different and precise Sub-WEPs each with its own clearly defined inertial mass.

To discuss the inertial mass, we consider situations with non-relativistic vs. relativistic test bodies and S-GFs. Because no relativistic test bodies were involved in any of above-mentioned testing experiments, three questions need to be answered. Firstly, is UFF still valid for relativistic test bodies? The answer to this question would relate to the structural feature of theories of gravity. Secondly, since the original expression of WEP has been applied only to test bodies, is the original expression of WEP valid for relativistic S-GFs? The answer to this question would relate to the nature of S-GFs. Thirdly, is the gravitational mass of a relativistic test body and a relativistic S-GF equal to either its inertial rest or to its relativistic mass? Note the latter two masses can only be distinguished for relativistic bodies. The answer to this question would disclose the natures of both S-GFs and gravity.

It was UFF that had led Einstein to propose the Einstein Equivalence Principle (EEP), which is the basis for his concept that gravity is a consequence of the geometry of space-time. Thus, any violation of UFF would have profound consequences for geometric theories of gravity. Possible violations of UFF and the original expression of WEP in quantum regime have been discussed and tested [6, 7]. We have discussed the possible violation of the EEP by quantum effects [8]. In this paper, we show a possible violation of UFF in non-quantum regime by relativistic test bodies and a synchrotron-type experiment is proposed to verify it. Therefore we will not use geometric concepts and terms to describe the behavior of relativistic test bodies and S-GFs in this article.

2. Precise Expressions of WEP

In order to answer the three questions about UFF and the original expression of WEP raised in Introduction, let's begin by reviewing the original expression of WEP [1], which states that the gravitational mass, m_g, of a body is equal to its inertial mass, m_i, i.e., $m_g = m_i$. The word "body" denotes either a test body or a body as a S-GF unless specified.

This original expression of WEP does not distinguish non-relativistic from relativistic bodies, i.e., it does not distinguish explicitly the inertial rest mass, m_{i0}, from the inertial relativistic mass, m_{ir}. Indeed, it has been pointed out that what happens to high-energy, i.e., relativistic, bodies might be different from the laws for low-energy, i.e., non-relativistic, bodies [9].

In order to differentiate situations where non-relativistic vs. relativistic bodies are involved we expand the original expression of WEP into three precise Sub-WEPs. Note all movements of bodies in different situations are relative to and measured by the observers.

For non-relativistic bodies we propose WEP-I as a Sub-WEP, where

$m_g = m_{i0}$.

For relativistic bodies, due to the current lack of experimental data, we propose WEP-II and WEP-III as two contrary and alternative Sub-WEPs, where WEP-II posits that

$m_g = m_{i0}$;

and WEP-III assumes that

$m_g = m_{ir} = \gamma m_{i0}$.

Note WEP-II implies that the gravitational mass is invariant like the electric charge. In contrast, WEP-III implies that the energy-momentum $T^{\alpha\beta}$ is the S-GF. Presently the $T^{\alpha\beta}$ of nucleons and electrons contains their mass-energy and the binding energies of their electromagnetic, weak and strong interactions, as well as their internal kinetic energy [7].

From now on we only employ these three precise Sub-WEPs, instead of the original expression of WEP, to describe the relations between the gravitational mass and the inertial rest and relativistic masses of test bodies and S-GFs. Moreover, in this article, we will only consider situations where the masses of the test bodies are negligible compared with that of the S-GFs and the observer moves at speeds negligible compared with that of light relative to either S-GF or test bodies. Note the former situations are the conditions under which all the reported past and current experiments have been and are being carried out.

Under these constrains, WEP-I and WEP-II/WEP-III generate two different cases.

Case (A) is a situation where Observer O_A is either at rest on or in non-relativistic motion relative to S-GF, e.g. Earth, or relative to experimental apparatuses, e.g. satellite, containing test bodies T_A. T_A is either at rest on or in non-relativistic motion relative to both S-GF and Observer O_A.

Here, WEP-I is applicable for both T_A and S-GF. To O_A, the gravitational mass m_g of T_A is equal to its inertial rest mass m_{i0}. For this situation, UFF and WEP-I are equivalent and have been experimentally demonstrated precisely [2-4]. UFF can be mathematically derived from the combination of WEP-I and

Newtonian mechanics.

Case (B) is a situation where Observer O_B is at rest on or in non-relativistic motion relative to S-GF. Relativistic test bodies T_B with rest masses are moving relative to both S-GF and Observer O_B.

To Observer O_B, WEP-I is applicable to S-GF. But, UFF and WEP-I are not applicable to T_B. Here, Observer O_B can apply one of two possible and contradicting Sub-WEPs. Applying WEP-II to T_B, Observer O_B has:

$$m_g = m_{i0}.$$

Applying WEP-III to T_B, Observer O_B has:

$$m_g = m_{ir}.$$

Clearly, either WEP-II or WEP-III, but not both, can be valid.

These three specified Sub-WEPs for non-relativistic and relativistic test bodies allowed us to narrow down future task of validating the different aspects of Sub-WEPs for situations that have not been explicitly explored, theoretically as well as empirically. In other words, for relativistic test bodies with rest mass, we need to validate two contradicting Sub-WEPs, i.e., WEP-II and WEP-III.

3. Violation of UFF by Relativistic Test Bodies

Before we can answer the question raised in Introduction whether UFF, WEP-II or WEP-III is valid for relativistic test bodies with rest mass, we need to show the differences such a relativistic test body would make from a non-relativistic test body. First, let's review the general situations of a test body with a force **F** acts on it.

Let's assume that the test body moves with the velocity **V** relative to a reference system K and an observer measures the mass and acceleration of the test body in the reference system K. For a non-relativistic test body Newton's second law is

$$m_{i0} \frac{d}{dt} V = \mathbf{F}, \tag{1}$$

whereas for a relativistic test body, it is

$$m_{i0} \frac{d}{dt} \frac{\mathbf{V}}{\sqrt{1 - \frac{V^2}{C^2}}} = \mathbf{F}. \tag{2}$$

For a situation where the acceleration and velocity of a test body are parallel to each other, Eq. (1) and Eq. (2) can be written, respectively, as the following

$$\mathbf{a}_N = \frac{d}{dt} V = \frac{\mathbf{F}}{m_{i0}}, \tag{3}$$

$$\mathbf{a}_R = \left(1 - \frac{V^2}{C^2}\right)^{\frac{3}{2}} \frac{\mathbf{F}}{m_{i0}}. \tag{4}$$

Moreover, Eq. (4) can also be written in term of "longitudinal mass" m_L as introduced by Einstein [1],

$$\mathbf{a}_R = \frac{\mathbf{F}}{m_L},$$

where

$$m_L \equiv \frac{m_{i0}}{\left(\sqrt{1-\frac{V^2}{C^2}}\right)^3}.$$

Obviously, the accelerations of non-relativistic and relativistic test bodies, $\mathbf{a}_N$ and $\mathbf{a}_R$, respectively, are different from each other regardless of the nature of the force **F**. Because Eq. (4) reduces to Eq. (3) when V/C approaches zero, and acceleration $\mathbf{a}_R$ approaches zero when V approaches C, therefore, Eq. (4) shows that the acceleration of any test body is velocity-dependent, regardless of the test body's speed of movement. This means that no force **F** can accelerate the speed of a test body with rest mass to that of light. This conclusion is consistent with Special Relativity.

In the situation where a non-relativistic test body free falls in a weak longitudinal gravitational field, let's follow the historical argument, i.e., the movement of the test body is determined by combining Newton's equation, $\mathbf{F} = m_g\,\mathbf{g}(r)$, with Eq. (3), then the acceleration $\mathbf{a}_N$ is,

$$\mathbf{a}_N = \frac{m_g}{m_{i0}}\,\mathbf{g}(r). \qquad (5)$$

This is the situation of Case (A). Applying WEP-I, the gravitational mass is equal to the inertial rest mass, $m_g = m_{i0}$, Eq. (5) reduces to

$$\mathbf{a}_N = \mathbf{g}(r),$$

which is UFF. Thus UFF can be derived from WEP-I and is valid for non-relativistic test bodies. Indeed, for non-relativistic test bodies, UFF and WEP-I are equivalent to each other.

The original expression of WEP was only applied to and tested for non-relativistic test bodies but not for relativistic test bodies. For examples, Eötvös-type and free-fall-type experiments did not demonstrate that the gravitational mass of a relativistic test body is equal to either its rest mass or to its relativistic mass for the simple reason that in these experiments both observers and the test bodies were either at rest on or in non-relativistic motion relative to the S-GF. We are mindful of the fact that the validity of Newton's law of gravity for relativistic test bodies has been demonstrated by the gravitational red shift experiments [5]. However, we want to stress the fact that in these experiments the test bodies are photons, which have zero rest mass. Therefore, the validity of WEP-II or WEP-III in describing the movement of relativistic test bodies with rest mass in gravitational fields remains an unanswered question.

To examine whether the UFF is valid for a relativistic test body with rest mass in a longitudinal gravitational field, we need to establish a relation between the acceleration and the gravitational force **F**. A simple way to do this is to determine whether Eq. (2) is applicable to describe such a test body's movement in the gravitational field. In this situation, the left hand side of Eq. (2) would remain the same, since it describes the movement of the relativistic test bodies regardless of the nature of the force **F.** For the right hand side of Eq. (2), the expression of the gravitational force **F** needs to be determined. Theoretically, when the weak field approximations were made, various theories of gravity reduce to the Newton's law, **F** = m_g **g**(r). Empirically, the experiments demonstrating the gravitational red shift have shown that the effect of a weak gravitational field on relativistic particles without rest mass can be described by Newton's law, **g**(r)L. Let's assume that for relativistic test bodies with rest mass in a weak longitudinal gravitational field **g**(r), the gravitational force **F** is also equal to m_g **g**(r). Now, Eq. (2) and Eq. (4) become, respectively,

$$m_{i0}\frac{d}{dt}\frac{\mathbf{V}}{\sqrt{1-\frac{V^2}{C^2}}} = m_g\mathbf{g}_0. \tag{6}$$

and

$$\mathbf{a}_R = \left(1 - \frac{V^2}{C^2}\right)^{\frac{3}{2}}\frac{m_g}{m_{i0}}\mathbf{g}_0. \tag{7}$$

When the ratio, V/C, approaches zero, WEP-I becomes valid and Eq. (7) gives UFF. Therefore, we show again, for more general situations, UFF is equivalent to WEP-I and is valid only for non-relativistic test bodies.

In sharp contrast, for relativistic test bodies with rest mass, Eq. (7) is valid but is inconsistent with both Eq. (5) and the UFF, regardless of the applicability of WEP-II or WEP-III. Thus, the UFF is not equivalent to either WEP-II or WEP-III and it thus no longer holds true for relativistic test bodies either with or without rest mass.

Now we can examine whether the gravitational mass of a relativistic test body is equal to its relativistic mass or to its rest mass. WEP-II and WEP-III gives different predictions. If WEP-II is valid, Eq. (7) becomes

$$\mathbf{a}_{R\text{-}II} = \left(1 - \frac{V^2}{C^2}\right)^{\frac{3}{2}}\mathbf{g}_0. \tag{8}$$

If WEP-III is valid, Eq. (7) becomes

$$\mathbf{a}_{R\text{-}III} = \left(1 - \frac{V^2}{C^2}\right)\mathbf{g}_0. \tag{9}$$

These different qualitative predictions of WEP II vs. WEP III indicate that it might be possible to experimentally determine which of them is applicable to relativistic test bodies. We will come back to this issue later.

4. Non-Relativistic and Relativistic Sources of Gravitational Fields

After analyzing the validity of different Sub-WEPs for non-relativistic and relativistic test bodies in gravitational fields, we now do a similar analysis for non-relativistic and relativistic S-GFs. Here we denote various masses of S-GFs in capital letters M's to distinguish from those masses in lower case letters m's used for test bodies.

In the case of a non-relativistic S-GF, WEP-I is applicable and it gives $M_g = M_{i0}$. This means that the rest mass M_{i0} of the S-GF generates a gravitational field **g**(r). Moreover, WEP-I is consistent with Newton's law of gravity.

In the case of a relativistic S-GF, WEP-II posits that the gravitational mass M_g of the S-GF is equal to its inertial rest mass M_{i0}, $M_g = M_{i0}$. Intriguingly, as the relation, $M_g = M_{i0}$, is the same in both WEP-I and WEP-II for non-relativistic and relativistic (if WEP-II is proved to be valid) S-GFs, it can be interpreted as an indication of a gravitational invariant. We would call this invariant the "gravitational charge", as an analog to the electrical charge.

In contrast, WEP-III posits that the gravitational mass, M_g, of a relativistic S-GF is equal to its inertial relativistic mass M_{ir},

$$M_g = M_{ir} = \gamma M_{i0}, \tag{10}$$

where γ is the Lorentz factor. Note it was the concept expressed in Eq. (10) that had led Einstein to suggest that the energy momentum $T^{\alpha\beta}$ is S-GF.

In summary, the applicability of UFF and these three Sub-WEPs to non-relativistic and relativistic test bodies as well as to non-relativistic and relativistic S-GFs are listed in the Table below.

		Test Body		**Source of Gravitational Field**	
		Non-Relativistic	**Relativistic**	**Non-Relativistic**	**Relativistic**
UFF/modern WEP		**Valid**	**Violated**	**Not Applicable**	**Not Applicable**
Sub - WEP	**WEP-I**	**Valid**		**Valid**	
	WEP-II/ WEP-III		**Valid**		**Valid**

This table shows that our approach of expanding the original expression of WEP into three Sub-WEPs, each with its own clearly defined masses, broadens the range of situations where they are applicable and therefore would generate new insights into the natures of S-GFs and gravity itself, insights that UFF and its equivalent, the modern expression of WEP, are not capable of generating. Specifically, WEP-I is not only valid for non-relativistic test bodies but is also valid for non-relativistic S-GFs. In contrast, UFF/modern expression of WEP is valid only for non-relativistic test bodies, and therefore has no prediction about any kinds of S-GFs. Moreover, experimental confirmation of WEP-II might lead to a novel internal symmetry of gravity, which UFF/modern expression of WEP does not. Finally, experimental confirmation of WEP-III might lead to Einstein's postulation that the $T^{\mu\nu}$ of a body is the S-GF, which UFF/modern expression of

WEP does not.

This expansion of the Sub-WEP allows us to define a new precise WEP (P-WEP) that contains two parts: WEP-I for non-relativistic test bodies and S-GFs, WEP-II or WEP-III for relativistic test bodies and S-GFs. In other words, the P-WEP is either a combination of WEP-I and WEP-II or a combination of WEP-I and WEP-III. Which of these two combinations is valid should be determined experimentally.

Although WEP-II and WEP-III are presently mathematically extrapolated theories, given the potential new insights that they might generate while finalizing the precise form of P-WEP, it is critical to directly test the validity of WEP-II or WEP-III for relativistic test bodies and S-GFs.

5. Re-interpretation of Existing Experiments utilizing WEP-I and WEP-III

The above detailed analysis of the validity of the Sub-WEPs for various gravitational situations provide re-interpretations of existing experimental data.

In Eötvös-type and free falling-type experiments, the test bodies, the S-GF, e.g., the Earth or the Sun, and Observers are either at rest or in non-relativistic motion relative to each other. For Eötvös-type experiments of two test bodies, the dimensionless Eötvös parameter is

$$\eta = \frac{2[(^{m_g}/_{m_{io}})_1 - (^{m_g}/_{m_{io}})_2]}{(^{m_g}/_{m_{io}})_1 + (^{m_g}/_{m_{io}})_2}.$$

Note in Eötvös-type experiments, the test bodies are at rest relative to Observers. This is the situation of Case (A). Thus we can use the inertial rest mass m_{i0} instead of the inertial mass m_i in the Eötvös parameter η, and these experiments examined the ratio of gravitational mass to its inertial rest mass, m_{i0}, for test bodies 1 and 2. WEP-I predicts the null result. Experiments demonstrate WEP-I with high sensitivities.

Applying WEP-I, i.e., $m_g = m_{i0}$, Eötvös parameter can then be written in terms of acceleration, a_1 and a_2, of test bodies 1 and 2, respectively,

$$\eta = \frac{2[a_1 - a_2]}{a_1 + a_2}.$$

The null results of experiments in terms of thus defined Eötvös parameter leads us to UFF, i.e., $\mathbf{a}_N = \mathbf{g}$. The results of these experiments have demonstrated the validity and equivalence of WEP-I and UFF. However, these experiments did not demonstrate the validity of either Eq. (6) for relativistic test bodies or Eq. (10) for relativistic S-GFs.

In the gravitational red shift-type experiments the observers were at rest on the S-GF, e.g., the Earth, photons were the relativistic test bodies without rest mass. Photons' energy of movement were reduced by the Newtonian gravitational field $\mathbf{g}$(r), i.e., a red shift $E_1 = E_2 + \frac{E_2}{C^2}$ gL. This is the situation of Case (B) where WEP-III is applicable. Although gravitational red shift experiments validated WEP-III for test bodies without rest mass, whether they can validate either WEP-II or WEP-III for relativistic test bodies with rest mass is unclear.

6. Synchrotron-type experiment Designed to Test Both Predicted Violation of UFF and Validate WEP-II or WEP-III by Relativistic Test Bodies with Rest Mass

It is critical to empirically and directly verify whether relativistic test particles violate UFF. Moreover, despite of the fact that validation of either WEP-II or WEP-III for relativistic bodies with rest mass is crucial to disclose the intrinsic nature of the S-GFs, WEP-II and WEP-III are only extrapolated hypothesis yet to be supported directly by experimental evidence. We notice that for experimental test of the applicability of the UFF to non-relativistic test bodies, it is feasible to put such test bodies in a longitudinal gravitational field. However, an experimental test for relativistic test bodies, it is feasible by utilizing a transverse gravitational field, as illustrated in Fig. 1.

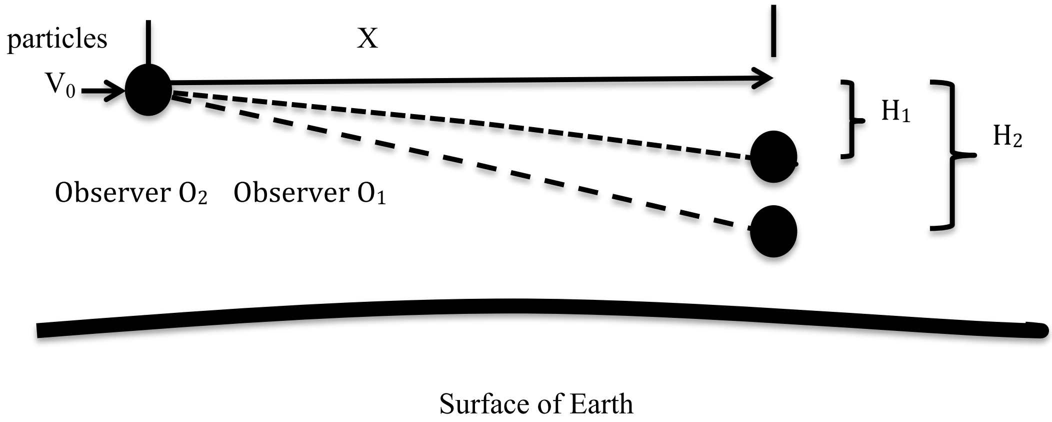

Fig.1 Observers O_1 and O_2 are at rest on Earth; test particles move a distance X in the x-axis, and free fall either a distance H_1 or a distance H_2 along the y-axis as predicted by Observer O_1 and O_2, respectively.

Thus, we propose a synchrotron-type experiment of measuring various parameters of the free fall of relativistic test bodies with rest mass in a weak constant transverse gravitational field, $\mathbf{g}_0$.

In this experiment, relativistic test particles with high initial velocity V_0, momentum P_0, and energy E_0 move along the x-axis. The x-axis is transverse to the Earth's gravitational field, which is along the y-axis. This situation on the surface of the Earth is analogous to that of a stellar light beam being deflected by the transverse gravitational force of the Sun. The test of the violation of UFF becomes a measurement of the test particles' free fall distances in the y-direction. Note all of measurements are performed on the Earth, i.e., the observer is at rest on the S-GF, thus this is the situation of Case (B).

In a manner entirely similar to the analysis of a relativistic charged particle in a constant transverse electrical field [10], we calculate the theoretical predictions of this proposed experiment by adopting the relations between the momentum **P** and the energy E of these relativistic test particles,

$$\mathbf{V} = \frac{\mathrm{P}\mathrm{C}^2}{\mathrm{E}}, \tag{11}$$

and

$$\mathrm{E} = \sqrt{\mathrm{m}_{\mathrm{i}0}^2\mathrm{C}^4 + \mathrm{P}^2\mathrm{C}^2}. \tag{12}$$

Note Eq. (11) and Eq. (12) are derived without any assumption about the nature of the external forces. In the presence of a gravitational field, the relativistic momentum **P** in both Eq. (11) and Eq. (12) is determined by Eq. (6), which can be expressed in term of the momentum **P** $\left(= \frac{\mathrm{m}_{\mathbf{i0}}\mathbf{V}}{\sqrt{1-\frac{\mathrm{V}^2}{\mathrm{C}^2}}}\right)$, i.e.,

$$\frac{\mathrm{d}\mathbf{P}}{\mathrm{dt}} = \mathrm{m}_{\mathrm{g}}\mathbf{g_0}.$$

Eq. (6) in this form describes the relation between the relativistic momentum **P** and the gravitational force, $\mathrm{m}_{\mathrm{g}}\mathbf{g_0}$.

This expression of relativistic momentum **P** together with Eq. (11) and Eq. (12) give us the travel distance X of the test particles along the x-axis, the time t_X required to travel the distance X, the angle $\theta\,(t_X)$ the trajectory makes with the x-axis, and the free fall distance H in the y-axis, which are correspondingly

$$\mathrm{X} = \frac{\mathrm{p_0\,C}}{\mathrm{m_g g_0}} \sinh^{-1} \frac{\mathrm{m_g g_0 t_X\,C}}{\mathrm{E_0}}, \tag{13}$$

$$\mathrm{t_X} = \frac{\mathrm{E_0}}{\mathrm{m_g g_0 C}} \sinh \frac{\mathrm{m_g g_0 X}}{\mathrm{P_0 C}}, \tag{14}$$

$$\tan\,\theta\,(\mathrm{t_X}) = \frac{\mathrm{m_g\,g_0\,t_X}}{\mathrm{P_0}}, \tag{15}$$

$$\mathrm{H} = \frac{\mathrm{E_0}}{\mathrm{m_g g_0}}\left[\sqrt{1 + \left(\frac{\mathrm{m_g g_0 t_X\,C}}{\mathrm{E_0}}\right)^2} - 1\right], \tag{16}$$

The theoretical predictions of the values of X, t_x, $\tan\,\theta\,(t_D)$, and H provide measurable variables in this experiment designed to validate UFF, WEP-II or WEP-III.

To determine directly whether the gravitational mass of a relativistic body equal to its inertial rest mass or to its relativistic mass, let's assume that Observer O_1 believes WEP-II, which states that the gravitational mass of the test particle is equal to its inertial rest mass, $\mathrm{m_g} = \mathrm{m_{i0}}$. Substituting m_{i0} for m_g in Eq. (16), Observer O_1 would predict its free fall

$$\mathrm{H_1} = \frac{\mathrm{E_0}}{\mathrm{m_{i0} g_0}}\left[\sqrt{1 + \left(\frac{\mathrm{m_{i0} g_0 t_X\,C}}{\mathrm{E_0}}\right)^2} - 1\right], \tag{17}$$

In contrast, let's assume Observer O_2 believes WEP-III, which states that the gravitational mass of the test particle is equal to its inertial relativistic mass, $\mathrm{m_g} = \gamma \mathrm{m_{i0}}$. Substituting $\gamma \mathrm{m_{i0}}$ for m_g in Eq. (16), Observer O_2 would predict its free fall,

$$\mathrm{H_2} = \frac{\mathrm{E_0}}{\gamma \mathrm{m_{i0} g_0}}\left[\sqrt{1 + \left(\frac{\gamma \mathrm{m_{i0} g_0 t_X\,C}}{\mathrm{E_0}}\right)^2} - 1\right], \tag{18}$$

Equations (17 and 18) show the value of H_2 as predicted by O_2 is different from that of H_1 as predicted by O_1. This difference is likely to be detectable in a real experiment.

Let's consider an electron accelerated in a synchrotron. The electron can travel at an initial horizontal speed V_0 = (C − 1m/s). This V_0 would give Lorentz factor $\gamma = 12{,}000$. This means that when the electron moves for 0.5 sec, the free fall distances predicted, respectively, by Observers O_1 and O_2 would be $H_1 \approx$ 0.1 mm, and $H_2 \approx$ 1 m. This discrepancy in the values of the vertical free fall, H_1 vs. H_2, should be detectable.

Note the calculation of this synchrotron-type experiment is made under the following assumed experimental conditions. Firstly, the effects of synchrotron radiation damping are ignored. Secondly, the electron is injected into the storage ring one at a time to avoid the electromagnetic interaction between them. Thirdly, a strong focusing mechanism is partially applied to tightly control the horizontal focusing in the storage ring so the horizontal component of the displacement of the electron from the equilibrium orbit is zero while the vertical focusing is turn off to allow the electron free fall in y-direction under the gravitational force of the Earth. Note H (Eq. 16) is the vertical component of the displacement of the electron from the equilibrium orbit. Lastly, additional magnetic/electric fields are employed to cancel all unwanted magnetic/electric fields, including the earth's magnetic field, along the path of the electron in the storage ring. Clearly, these are conditions in theory and particle physicists are better equipped to solve these and any other experimental complications.

Experimental confirmation of H_1 as given by Eq. (17) would suggest that WEP-II is valid, whereas experimental confirmation of a value of H_2 as given by Eq. (18) would suggest that WEP-III is valid. Furthermore, experimental confirmation of either Eq. (17) or Eq. (18) would indicate that Eq. (6) and Eq. (11 and 12) are applicable for describing the movement of relativistic test bodies with rest mass in a gravitational field. Clearly, this would simultaneously be an experimental demonstration of relativistic test bodies with rest mass violating UFF.

7. Conclusions and Discussions

In this paper, we have shown that when we expand the original expression of WEP into three Sub-WEPs, each with its own clearly defined masses, they can be applied to not only non-relativistic test bodies and S-GFs but also to relativistic test bodies and S-GFs (Table). Specifically, UFF is valid only for non-relativistic test bodies but not valid for either relativistic test bodies or any kind of S-GFs. In sharp contrast, WEP-I is applicable to both non-relativistic test bodies and non-relativistic S-GFs**.** Furthermore, either WEP-II or WEP-III is applicable to relativistic test bodies as well as relativistic S-GFs. Thus, compared to the narrow set of situations where UFF/modern WEP is valid, these three Sub-WEPs are potentially applicable to a wider range of situations including those hitherto unexamined**.** The broader scopes of these Sub-WEPs have the potential to generate new insights into the natures of the S-GFs and gravity.

This expansion of the original expression of WEP into three Sub-WEPs allows us to define a new precise WEP (P-WEP) that is either a combination of WEP-I and WEP-II or a combination of WEP-I and WEP-III.

Furthermore, we found that the acceleration of any test body is velocity-dependent and that weak gravitational fields cannot accelerate the speed of a test body to that of light. This is consistent with Special Relativity.

The results of our synchrotron-type experiment for relativistic test bodies with rest mass would potentially have any or a combination of the following implications.

(1) Because UFF hence the Einstein's EP (EEP) are the basis for the concept that gravity is a consequence of the geometry of space-time, confirmation of violation of UFF by relativistic test bodies would suggest a need for reconsideration of the structural feature of geometric theories of gravity.

(2) Confirmation of WEP-II, $m_g = m_{i0}$, would disclose a novel symmetry of gravity. In this case we will define a "gravitational charge" Q_g as $Q_g \equiv \sqrt{G}m_g$, where G is the Newtonian gravitational constant. Simultaneous validation of WEP-II and WEP-I would imply that Q_g is conserved. In analogy to QED, the conservation of Q_g would suggest that gravity might have an internal U (1) symmetry with the coupling constant Q_g. We have shown that Q_g is dimensionless when $\hbar = C = 1$ [11]. Experimental confirmation of WEP-II suggests that the P-WEP is the combination of WEP-I and WEP-II.

(3) Confirmation of WEP-III would confirm Einstein's postulation that the Energy-momentum $T^{\alpha\beta}$ of a body is the S-GF. Experimental confirmation of WEP-III suggests that the P-WEP is the combination of WEP-I and WEP-III.

(4) Nullification of all of Equations (6, 17, and 18) would indicate a need for a new physics beyond Einstein's theories.

We argue that the results of the designed experiment might suggest whether or not we need to reconsider both the nature of the S-GF and the nature of gravity. We hope that high-energy physicists would consider this experiment and find it worthy of being carried out.

Reference

[1] Einstein, A.: *The Principle of Relativity* (US, Dover Publications, Inc) (1920).

[2] Nobili, A. M., Lucchesi, D. M., Crosta, M. T., Shao, M., Turyshev, S. G., Peron, R., Catastini, G., Anselmi, A., Zavattini, G.: On

the universality of free fall, the equivalence principle, and the gravitational redshift. Am. J. Phys., 81, 527 (2013)

Cho, A.: DROP TEST: Physicists are challenging Einstein's theory in new versions of the most famous experiment that never happened. *Science* **347,** 1096-1097 (2015)

Thorne, K. S., and Blandford, R. D.: *Modern classical Physics* (Princeton: Princeton Univ. Press) (2015)

[3] Wagner, T. A., Schlamminger, S., Gundlach, J. H., Adelberger, E. G.: Torsion-balance tests of the weak equivalence principle. Class. Quantum Grav. **29,** 184002 (2012)

[4] Comandi, G. L., Nobili, A.M., Bramanti, D., Toncelli, R., Polacco, E., Chiofalo, M.L.: "Galileo Galilei (GG)" on the Ground-GGG: experimental results and perspecitves. Phys. Lett., **A318**, 213-222 (2003)

Kellerbauer, A., Amoretti, M., Belov, A.S., Bonomi, G., Boscolo, I., Brusa, R.S., Büchner, M., Byakov, V.M., Cabaret, L., Canali, C., Carraro, C., Castelli, F., Cialdi, S., de Combarieu, M.D., Comparat, D., Consolati, G., Djourelov, N., Doser, M., Drobychev, G., Dupasquier, A., Ferrari, G., et al.: Proposed antimatter gravity measurement with an antihydrogen beam. Nucl. Instrum. Methods, **B 266**, 351-356 (2008)

Amole, C., Ashkezari, M. D., […] Charman, A. E. (ALPHA Collaboration): An experiment limit on the charge of antihydrogen. Nature Communications, **5**, 3955 (2013)

[5] Pound, R. V., Rebka, G. A.:Apparent weight of photons. Phys. Rev. Let. **4**, 337 (1960)

Unnikrishnan, C. S., Gillies, G. T.: Reexamining the roles of gravitational and inertial masses in gravimetry with atom interferometers. Phys. Lett. **A377**, 60-63 (2012)

[6] Alvarez, C., Mann, R.: Testing the Equivalence Principle in the Quantum Regime Gen. Rel. Grav. **29**, 245-250 (1997)

Camacho, A., Camacho-Galvan, A.: Test of some fundamental principles in physics via quantum interference with neutrons and photons. Rep. Prog. Phys. **70**, 1937-1993 (2007)

Hernandez-Coronado, H., Okon, E.: Quantum equivalence principle without mass superselection. Phys. Lett. **A 377**, 2293-2297 (2013)

Aghion, S., Ahlén, O. […] Zmeskal, J.: A moiré deflectometer for antimatter. Nature Communications **5**, 4538 (2014)

[7] Hohensee, M. A., Muller, H.: Equivalence Principle and Bound Kinetic Energy Phys. Rev. Lett. **111**, 151102-151102-5 (2013)

[8] Peng, H.: A new statement of Einstein's equivalence principle and relevant experiments. Phys. Lett. **A 103**, 113-115 (1984)

[9] 't Hooft G：Local Conformal Symmetry: the Missing Symmetry Component for Space and Time, First Award from the *Gravity Research Foundation 2015 Awards for Essays on Gravitation* (2015).

[10] Landau, L. D., Lifshitz, E. M.: *The Classical Theory of Fields*. Chapter 20 (Elsevier, Singapore)(1975)

[11] Peng, H., Wang, K. S.: Dual Geometric-Gauge Field Aspects of Gravity. Intern. J. Theo. Phys. **31**, 1315-1324 (1992)