An image for $f(z)=\ln(2z+\tan z)$

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Abstract. This note presents an image related with the function $f(z)=\ln(2z+\tan z)$.

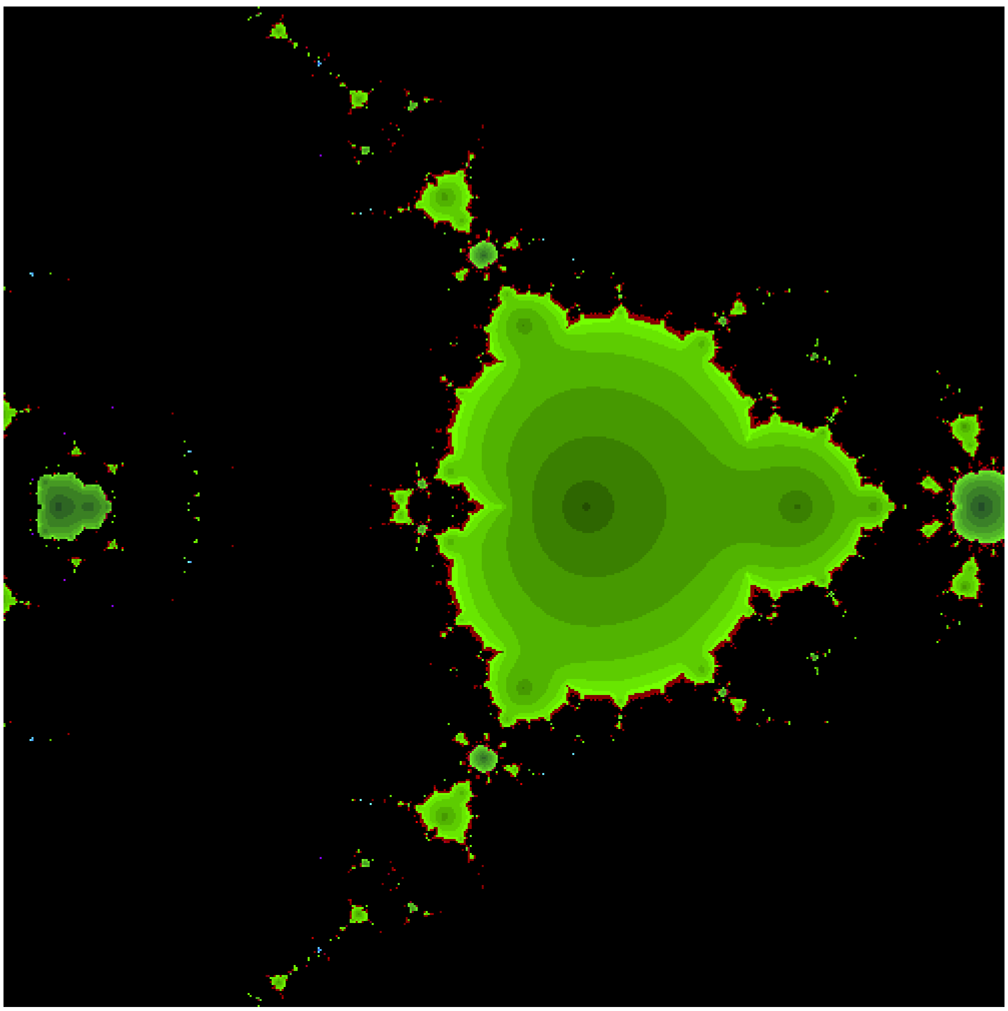

Figure 1. $f(z)=\ln(2z+\tan z)$.

Brief Note

Entry 1:

$$f(z)=\ln(2z+\tan z)=0\Rightarrow 2z+\tan z=1 \tag{1}$$

$$2z+\tan z=1\Rightarrow z=\frac{1}{2}-\frac{1}{2}\tan\left(\frac{1}{2}-\frac{1}{2}\tan\left(\frac{1}{2}-\frac{1}{2}\tan\left(\frac{1}{2}-...\right)\right)\right) \tag{2}$$

$$z=0.3291899... \tag{3}$$

$$x_{n+1}=\frac{1}{2}-\frac{1}{2}\tan x_n \quad ,x_0=0\Rightarrow x_n\to z=0.329189... \tag{4}$$

$$x_{n+1}=\frac{1+x_n-\tan x_n}{3} \quad ,x_0=0\Rightarrow x_n\to z=0.329189... \tag{5}$$

Entry 2: If $z=\frac{1}{2}-\frac{1}{2}\tan\left(\frac{1}{2}-\frac{1}{2}\tan\left(\frac{1}{2}-\frac{1}{2}\tan\left(\frac{1}{2}-...\right)\right)\right)$, then

$$\pi=8z+4\sum_{n=1}^{\infty}z^{n+1}\sum_{k=0}^{[n/2]}\frac{(-1)^k}{2k+1}\binom{n}{n-2k} \tag{6}$$

where π is the pi constant:

$$\pi=4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}=3.1415926535... \tag{7}$$

Entry 3: If $f_n=\sum_{k=0}^{[n/2]}\frac{(-1)^k}{2k+1}\binom{n}{n-2k}$ $,n=1,2,3,...$, then

$$f_n=\left\{1,\frac{2}{3},0,-\frac{4}{5},-\frac{4}{3},-\frac{8}{7},0,\frac{16}{9},\frac{16}{5},\frac{32}{11},0,...\right\} \tag{8}$$

$$f_{4n+3}=0 \quad ,n=0,1,2,3,... \tag{9}$$

References

1. Barnsley, M.F. and Rising, H.: Fractals Everywhere, 2nd ed. Boston, MA: Academic Press, 1993.
2. Beckmann, P.: A History of Pi, 3rd ed. New York: Dorset Press, 1989.
3. Blatner, D.: The Joy of Pi. New York: Walker , 1997.
4. Castellanos, D.: The Ubiquitous Pi. Part I. Math. Mag. 61, 67-98,1988a.
5. Castellanos, D.: The Ubiquitous Pi. Part II. Math. Mag. 61, 148-163, 1988b.
6. Dalzell, D.P.: On 22/7. J. London Math. Soc. 19, 133-134, 1944.
7. Dalzell, D.P.: On 22/7 and 355/113. Eureka 34, 10-13,1971.
8. Devaney, R.L.: Complex Dynamical Systems: The Mathematics Behind the Mandelbrot and Julia Sets. Providence, RI: Amer. Math. Soc., 1994.

9. Finch, S.R.: Mathematical Constants. Cambridge, England: Cambridge University Press,2003.
10. Gourdon, X. and Sebah, P.: Collection of Series for π . http://numbers.computation.free.fr/Constants/Pi/piSeries.html.
11. Hardy, G.H.: Ramanujan: Twelve Lectures on Subjects Suggested by His Life and Work, 3rd ed. New York: Chelsea, 1999.
12. Smith, D.: History of Mathematics, Vol. 2. New York: Dover, 1953.
13. Valdebenito, E.: The Cubic: $x^3 + x^2 + 1 = 0$. http://vixra.org/pdf/1705.0461v1pdf .
14. Valdebenito, E.: The quintic: $z^5 + z^4 - 1 = 0$. http://vixra.org/pdf/1706.0205v1pdf .
15. Valdebenito, E.: Complex Continued Fractions , Numbers , Sequences , Number Pi . http://vixra.org/pdf/1706.0206v1pdf .
16. Valdebenito, E.: Ramanujan Trigonometric Formula. http://vixra.org/pdf/1707.0241v1pdf .
17. Valdebenito, E.: Question 416: Pi , Integral Representations. http://vixra.org/pdf/1711.0303v1pdf .
18. Valdebenito, E.: Dalzell Integral . http://vixra.org/pdf/1711.0310v1pdf .