Electron carries "hidden" $31,6$ GW field energy vortex

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Abstract

An electron is enveloped by a "hidden" electromagnetic field-energy circulation vortex of $\approx 31,6$ GW passive power determined by the Poynting-vector field existing around an electron. The energy vortex is most intensive in the proximity of classical electron radius, with maximum in its equatorial plane.
A thoretical upper limit of such (non-usable) passive energy circulation is analytically determined by integration of the Poynting-vector field over a specific plane of reference. The result highlights a new singularity problem of classical electron theory.

1 Introduction

According to the Abraham-Lorentz conjecture [[1]] classical electron mass m_e is identical with its relativistic electromagnetic field-mass. One particular consequence is that its spin-angular momentum $\hbar/2$ is "hidden" electromagnetic field angular momentum. [2] [3] [4]
This is to focus on a similar implication resulting from the fundamental rule that superimposed electro- and magnetostatic fields $\vec{E}$ and $\vec{H}$ - like those surrounding an electron - generate electromagnetic energy-flux vortices of local intensity given by the Poynting-vector

$$\vec{S} = \vec{E} \times \vec{H} \quad [Wm^{-2}] \tag{1}$$

The task thus is to determine total electromagnetic field energy flux or passive power P [W] circulating around an electron as an energy-vortex, by integration of the Poynting-vector field (1) over a given reference plane p_e throughout the whole electromagnetic field surrounding the electron. The term "passive" power means idle power circulating loss-free in a closed (shortcut) loop through vacuum space. Passive power thus can neither be extracted nor technically used. Note that in the proximity of classical electron-radius r_e the electric and magnetic field strengths $\vec{E}$ and $\vec{H}$ are close to their theoretical upper limits (e.g. Schwinger-Limits) given by vacuum-polarization effects. [5]

2 Classical electromagnetic vacuum-field

The classical electron model which this article refers to is a point-like charge e of mass m_e and a magnetic dipole-moment μ_B (magneton), included in a sphere of a theoretically determined (classical) electron radius r_e. The electrons vacuum-field thus comprises an electrostatic Coulomb-field $\vec{E}$ superimposed to a magnetostatic dipole-field $\vec{H}$.

2.1 Frame of reference

For the analysis let a free electron be located at the origin $0,0,0$ of an orthonormal $x-y-z$ frame and its magnetic dipole-axis be aligned with the z-axis. The reference plane p_e for determination of total power-flux P is coplanar with the $x-z$ plane ($x>0$) with normal unit-vector of p_e $\vec{n} \parallel \vec{u}_y$. Further, the radius unit-vector $\vec{u}_r \perp \vec{n}$ and the angular unit-vector $\vec{u}_\Theta \perp \vec{u}_r$. The coordinates of a point in the $x-z$ plane will be given by their radial component $r=\sqrt{x^2+z^2}$ and their polar-angle $0 \leqq \Theta \leqq \pi$, where $x = r\sin\Theta$ and $y = r\cos\Theta$.

2.2 Electrostatic Coulomb-field

For a point-like object with charge e and radius r_e its electric vacuum-field can be approximated by the Coulomb-law

$$\vec{E} \approx \frac{e}{4\pi\epsilon_0 r_e^2}\vec{u_r} \tag{2}$$

2.3 Magnetostatic dipole field

The vacuum field $\vec{H}$ of a point-like dipole like the magneton μ_B is approximated by a classical dipole-field

$$\vec{H}_{(\vec{r},\vec{\Theta})} \approx \frac{\mu_B}{4\,\pi r^3}\Big(2\ cos\,\Theta\vec{u}_r\ +\ sin\,\Theta\vec{u}_\theta\Big) \tag{3}$$

μ_B is the electron-magneton.

2.4 Poynting-vector field

The total rate of electromagnetic energy flux or power P [W] around an electron results from its Poynting-vector field $\vec{S}_{(\vec{r},\vec{\Theta})} = \vec{E}_{(\vec{r},\vec{\Theta})} \times \vec{H}_{(\vec{r},\vec{\Theta})}$ (1) which has to be determined - at any point on the $x-z$ plane - as a function of r and Θ. Then, $\vec{S}$ has to be integrated over the whole $+x-z$ plane, with radial integration-limits $r_e \leq r < \infty$ and angular integration limits $0 \leq \Theta \leq \pi$.
Note that that $\vec{E} \perp \vec{n}$, $H_\Theta\,\vec{\Theta} \perp \vec{E}$ and $\vec{S} \parallel \vec{n}$ prevail in the $x-z$ plane.
It can be anticipated that the Poynting-vector field $\vec{S}$ resembles coaxial circles centered on the dipole-axis (z-axis) indicating circulating electromagnetic

power. These circles would pass $\perp$ through the $x-z$ plane

Let us now substitute with (2) and (3) in (1):

$$\vec{S}_{(r,\Theta)} = \frac{1}{16\pi^2}\Big(\frac{e}{\epsilon_0 r^2}\vec{u}_r \times \frac{\mu_B}{r^3}(2\cos\Theta\,\vec{u}_r + sin\,\Theta\,\vec{u}_\Theta)\Big) \tag{4}$$

3 Total electromagnetic passive power

To determine the total rate of electromagnetic energy-flux or passive power P [W] through the $x-z$ plane the energy-flux through each surface-element da has to be integrated over the whole $x-z$ plane, as a function of r and Θ.
Let

$$da = r\,d\Theta\,dr \tag{5}$$

Hence

$$dP = \vec{S}\,\vec{n}\,da = \vec{S}\,\vec{n}\,r\,dr\,d\Theta \tag{6}$$

Substitution with (4) in (6)

$$dP = \frac{1}{16\pi^2}\Big(\frac{e}{\epsilon_0 r^2}\vec{u}_r \times \frac{\mu_B}{r^3}(2\cos\Theta\,\vec{u}_r + sin\,\Theta\,\vec{u}_\Theta)\Big)\vec{n}\,r\,dr\,d\Theta \tag{7}$$

Substitution with

$$\mu_B = \frac{e\,\hbar}{2m_e} \tag{8}$$

in (7)

$$dP = \frac{e^2\hbar}{32\,\pi^2\,\epsilon_0\,r^4 m_e}\Big(\vec{u}_r \times \ (2\cos\Theta\,\vec{u}_r + sin\,\Theta\,\vec{u}_\Theta)\Big)\vec{n}\,dr\,d\Theta \tag{9}$$

Note that $(\vec{u_r} \times 2\cos\Theta\,\vec{u_r}) = 0$.
An inspection of (9) reveals that it includes factors common with the definition of classical electron radius $r_e = e^2/4\,\pi\,\epsilon_0\,m_e\,c^2$ which can be transformed into

$$\frac{e^2}{4\,\pi\,\epsilon_0\,m_e} = r_e c^2 \tag{10}$$

delivering a useful substitution in (9). Execution of the vector-product (9) yields

$$dP = \frac{r_e\,c^2\,\hbar}{8\pi\,r^4} sin\,\Theta\ \ dr\,d\Theta \tag{11}$$

Total electromagnetic power P thus is

$$P = \int_{r_e}^{\infty}\int_0^{\pi} dP = \frac{r_e\,c^2\,\hbar}{8\pi}\int_{r_e}^{\infty} r^{-4}\,dr \int_0^{\pi} \sin\Theta\,d\Theta \tag{12}$$

$$P = \frac{c^2\,\hbar}{12\,\pi\,r_e^2} \approx 31,6\,GW \tag{12a}$$

4 Conclusions and Comment

The unexpectedly large magnitude of a "hidden" electromagnetic field energy-circulation vortex power $P \approx 31,6\,GW$ is mainly attributable to the denominator in (7) (inverted square of classical electron radius): $(r_e^2)^{-1} = 1,26 \cdot 10^{31}$, as well as by the factor $c^2 \approx \cdot 10^{17}$ in the nominator of (11) and (12). Note that in (11) the denominator is $r_e{}^{-4} \approx 1,587 \cdot 10^{62}$. Three quarters of the energy vortex are located in a spherical shell between r_e and $2r_e$.
From a heuristic point of view it can be conjectured that electromagnetic field energy and its assignable photons propagate (rotate) with c, in accordance with the Poynting-vector. Hence vacuum field-energy and its relativistic (photon) mass would circulate around the electron with a maximum frequency f_e along a circle of radius r_e: $f_e \approx c/2\pi r_e \approx 3 \cdot 10^8 \cdot 0,57 \cdot 10^{15} \approx 1,7 \cdot 10^{22} s^{-1}$ This also means that roughly $1,7 \cdot 10^{22}$ electron-masses $m_e \approx 9,1 \cdot 10^{-31}$ kg would pass through the $x - z$- plane per second. The circulating power of $31,6 \cdot 10^9\,W$ would have a relativistic mass-flux equivalent of $\dot{m} \approx P/c^2 = 3,16 \cdot 10^{10}/10^{17} \approx 3,16 \cdot 10^{-7} kg\,s^{-1}$ or an equivalent number of electrons per second of $\dot{n} \approx 3 \cdot 10^{-7}/9,1 \cdot 10^{31} \approx 3,3 \cdot 10^{23} s^{-1}$ traversing the $x - z$ plane. Note that both of the above figures are of similar order of magnitude $\approx 10^{22}$ to $\approx 10^{23}$!
Obviously (6) defines a new singularity problem of classical electron-theory, which would lead to infinite amount of energy-circulation for $r_e \to 0$.
Consideration of vacuum-polarization might substantially reduce, but not eliminate above result.
The large amount of passive energy flux $\approx 31,6\,GW$ can be interpreted as a theoretical upper limit of electromagnetic energy flux.

References

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[5] [5] Schwinger-Limit;