Linear & Fifths Based Tuning Systems

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Abstract: I would like to share a few alternatives for twelve tone tuning, which may be used to tune any instrument with a fixed value for each note, such as a keyed or fretted instrument. I do not claim these methods to be superior in any way to other existing tuning methods.

Throughout this document, I will use sharps (A,A#,B,C,C#,D,D#,E,F,F#,G,G#), regardless of the origin or order of notes in any sequence. None of these patterns utilize the standard concert A=440 Hz. I don't have anything against 440 Hz, it just happens that these particular patterns are formed by other numbers. The pitch A=432 Hz is consistent in each of these systems. For comparison, the values below represent one full octave in Hz (A=432 Hz), using the standard 12-tone equal temperament ratio.

```
432.00 Hz A
457.69 Hz A#
484.90 Hz B
513.74 Hz C
544.29 Hz C#
576.65 Hz D
610.94 Hz D#
647.27 Hz E
685.76 Hz F
726.53 Hz F#
769.74 Hz G
815.51 Hz G#
864.00 Hz A
```

System I (linear)

The first begins with A=432 Hz, and ends with A=864 Hz. The difference between tones 432 Hz & 457 Hz is 25 Hz; the difference between subsequent tones increases by 2 Hz every increment.

```
432 Hz A
          (+ 25 Hz)
457 Hz A#
          (+ 27 Hz)
484 Hz B
          (+ 29 Hz)
513 Hz C
          (+ 31 Hz)
544 Hz C#
          (+ 33 Hz)
577 Hz D
          (+ 35 Hz)
612 Hz D#
          (+ 37 Hz)
649 Hz E
          (+ 39 Hz)
688 Hz F
          (+ 41 Hz)
729 Hz F#
          (+ 43 Hz)
772 Hz G
          (+ 45 Hz)
817 Hz G#
          (+ 47 Hz)
864 Hz A
```

Below, is a second linear sequence of tones. There are fourteen tones in the following pattern (octaves 210 Hz to 420 Hz & 222 Hz to 444 Hz). The difference between tones 210 Hz & 222 Hz is 12 Hz; the difference between subsequent tones increases by 1 Hz every increment.

```
210 Hz
       (+ 12 Hz)
222 Hz
       (+ 13 Hz)
235 Hz
       (+ 14 Hz)
249 Hz
       (+ 15 Hz)
264 Hz
       (+ 16 Hz)
280 Hz
       (+ 17 Hz)
297 Hz
       (+ 18 Hz)
315 Hz
       (+ 19 Hz)
334 Hz
       (+ 20 Hz)
354 Hz
       (+ 21 Hz)
375 Hz
       (+ 22 Hz)
397 Hz
       (+ 23 Hz)
420 Hz
       (+ 24 Hz)
444 Hz
```

The tones above are quarter tones between each half tone in the first linear sequence. When each pair of quarter tones are summed in numeric order, the values from the first linear sequence are shown.

```
210 Hz + 222 Hz = 432 Hz A
222 Hz + 235 Hz = 457 Hz A#
235 Hz + 249 Hz = 484 Hz B
249 Hz + 264 Hz = 513 Hz C
264 Hz + 280 Hz = 544 Hz C#
280 Hz + 297 Hz = 577 Hz D
297 Hz + 315 Hz = 612 Hz D#
315 Hz + 334 Hz = 649 Hz E
334 Hz + 354 Hz = 688 Hz F
354 Hz + 375 Hz = 729 Hz F#
375 Hz + 397 Hz = 772 Hz G
397 Hz + 420 Hz = 817 Hz G#
420 Hz + 444 Hz = 864 Hz A
```

System II (fifths based)

The second system consists of two scales with notes forming a circle of fifths. The first scale begins with C=1 Hz and contains seven notes (C,G,D,A,E,B,F#). The scale is formed by multiplying the pitch 1 Hz by 3, 6 times.

```
    1 Hz = C
  1 Hz X 3 = 3 Hz G
  3 Hz X 3 = 9 Hz D
  9 Hz X 3 = 27 Hz A
 27 Hz X 3 = 81 Hz E
 81 Hz X 3 = 243 Hz B
243 Hz X 3 = 729 Hz F#
```

These seven notes form the G major diatonic scale.

```
384 Hz G
432 Hz A
486 Hz B
512 Hz C
576 Hz D
648 Hz E
729 Hz F#
```

The five remaining notes may be found by summing the notes surrounding each pitch that is absent from the diatonic scale above.

```
384 Hz + 432 Hz = 816 Hz G#
432 Hz + 486 Hz = 918 Hz A#
512 Hz + 576 Hz = 1088 Hz C#
576 Hz + 648 Hz = 1224 Hz D#
648 Hz + 729 Hz = 1377 Hz F
```

When each note is reduced to its lowest whole octave and placed in numeric order, we can see that the second scale follows the same pattern as the first scale. The second scale begins with C#=17 Hz and contains five notes (C#,G#,D#,A#,F). The scale is formed by multiplying the pitch 17 Hz by 3, 4 times.

```
   17 Hz = C#
 17 Hz X 3 = 51 Hz G#
 51 Hz X 3 = 153 Hz D#
153 Hz X 3 = 459 Hz A#
459 Hz X 3 = 1377 Hz F
```

System III (hybrid)

This system consists of two scales, each containing six whole tones, forming one twelve tone scale. Some of these numbers differ from the numbers in the first two systems, but many of the numbers are the same. The scale sounds "in tune" to my ears, so I'll share it along with the others. It begins with a linear sequence of whole tones. The difference between tones 51 Hz & 57 Hz is 6 Hz; the difference between subsequent tones increases by 1 Hz every increment.

```
51 Hz G#
          (+ 6 Hz)
57 Hz A#
          (+ 7 Hz)
64 Hz C
          (+ 8 Hz)
72 Hz D
          (+ 9 Hz)
81 Hz E
         (+ 10 Hz)
91 Hz F#
```

When each of these notes is multiplied by 3, the six remaining notes are shown.

```
51 Hz X 3 = 153 Hz D#
57 Hz X 3 = 171 Hz F
64 Hz X 3 = 192 Hz G
72 Hz X 3 = 216 Hz A
81 Hz X 3 = 243 Hz B
91 Hz X 3 = 273 Hz C#
```

The difference between tones 153 Hz & 171 Hz is 18 Hz; the difference between subsequent tones increases by 3 Hz every increment.

```
153 Hz D#
          (+ 18 Hz)
171 Hz F
          (+ 21 Hz)
192 Hz G
          (+ 24 Hz)
216 Hz A
          (+ 27 Hz)
243 Hz B
          (+ 30 Hz)
273 Hz C#
```

Comparison

System I (linear)

```
 432 Hz A
 457 Hz A#
 484 Hz B
 513 Hz C
 544 Hz C#
 577 Hz D
 612 Hz D#
 649 Hz E
 688 Hz F
 729 Hz F#
 772 Hz G
 817 Hz G#
 864 Hz A
```

System II (fifths based)

```
 432 Hz A
 459 Hz A#
 486 Hz B
 512 Hz C
 544 Hz C#
 576 Hz D
 612 Hz D#
 648 Hz E
688.5 Hz F
 729 Hz F#
 768 Hz G
 816 Hz G#
 864 Hz A
```

System III (hybrid)

```
 432 Hz A
 456 Hz A#
 486 Hz B
 512 Hz C
 546 Hz C#
 576 Hz D
 612 Hz D#
 648 Hz E
 684 Hz F
 729 Hz F#
 768 Hz G
 816 Hz G#
 864 Hz A
```