Electromagnetic wave function and equation in Rindler Space-time”

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ABSTRACT

In the general relativity theory, we find the electro-magnetic wave function and equation in Rindler space-time. Specially, this article is that electromagnetic wave equation is corrected by the gauge fixing equation in Rindler space-time.

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1. Introduction

In the general relativity theory, our article's aim is that we find the electro-magnetic wave equation and function in Rindler space-time. This article correct the article "Electromagnetic Field Equation and Lorentz Gauge in Rindler space-time" about the existence proof of electromagnetic wave function and equation.

The Rindler coordinate is

$$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c})$$

$$x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0} \quad , y = \xi^2, z = \xi^3 \qquad (1)$$

Therefore,

$$cdt = c\cosh(\frac{a_0\xi^0}{c})d\xi^0(1 + \frac{a_0}{c^2}\xi^1) + \sinh(\frac{a_0\xi^0}{c})d\xi^1$$

$$dx = c\sinh(\frac{a_0\xi^0}{c})d\xi^0(1 + \frac{a_0}{c^2}\xi^1) + \cosh(\frac{a_0\xi^0}{c})d\xi^1, dy = d\xi^2,\ dz = d\xi^3 \quad (2)$$

Hence,

$$\frac{1}{c}\frac{\partial}{\partial t} = \frac{c\partial\xi^0}{c\partial t}\frac{1}{c}\frac{\partial}{\partial\xi^0} + \frac{\partial\xi^1}{c\partial t}\frac{\partial}{\partial\xi^1}$$

$$= \frac{\cosh(\frac{a_0\xi^0}{c})}{(1 + \frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} - \sinh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}$$

$$\frac{\partial}{\partial x} = \frac{c\partial\xi^0}{\partial x}\frac{1}{c}\frac{\partial}{\partial\xi^0} + \frac{\partial\xi^1}{\partial x}\frac{\partial}{\partial\xi^1}$$

$$= -\frac{\sinh(\frac{a_0\xi^0}{c})}{(1 + \frac{a_0\xi^1}{c^2})}\frac{\partial}{c\partial\xi^0} + \cosh(\frac{a_0\xi^0}{c})\frac{\partial}{\partial\xi^1}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial z} = \frac{\partial}{\partial\xi^3} \qquad (3)$$

Hence,

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{1}{c^2(1 + \frac{a_0}{c^2}\xi^1)^2}(\frac{\partial}{\partial\xi^0})^2 - \nabla_\xi{}^2$$

$$\vec{\nabla} = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}), \quad \vec{\nabla}_{\xi} = (\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3})$$

(4)

2. Corrected electromagnetic wave equation in the Rindler space-time

The electro-magnetic field $(\vec{E}, \vec{B})$ is in the inertial frame,

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{c\partial t}, \vec{B} = \vec{\nabla} \times \vec{A} \quad (5)$$

Hence, we can define the electro-magnetic field $(\vec{E}_{\xi}, \vec{B}_{\xi})$ in Rindler space-time [1].

$$\vec{E}_{\xi} = -\frac{1}{(1+\frac{a_0\xi^1}{c^2})} \vec{\nabla}_{\xi} \{\phi_{\xi}(1+\frac{a_0\xi^1}{c^2})^2\} - \frac{1}{(1+\frac{a_0\xi^1}{c^2})} \frac{\partial \vec{A}_{\xi}}{c\partial \xi^0}$$

$$\vec{B}_{\xi} = \vec{\nabla}_{\xi} \times \vec{A}_{\xi}$$

In this time, $\vec{\nabla}_{\xi} = (\frac{\partial}{\partial \xi^1}, \frac{\partial}{\partial \xi^2}, \frac{\partial}{\partial \xi^3}), \vec{A}_{\xi} = (A_{\xi^1}, A_{\xi^2}, A_{\xi^3})$ (6)

Hence, Lorentz gauge condition is in Rindler space-time[1],

$$\phi_{\xi} \to \phi_{\xi} - \frac{1}{c}\frac{\partial \Lambda}{\partial \xi^0}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}, \quad \vec{A}_{\xi} \to \vec{A}_{\xi} + \vec{\nabla}_{\xi}\Lambda, \quad \Lambda \text{ is a scalar function.} \quad (7)$$

$$A^{\mu}{}_{;\mu} = \frac{\partial A^{\mu}}{\partial \xi^{\mu}} + \Gamma^{\mu}{}_{\mu\rho}A^{\rho} \to \partial_{\mu}(A^{\mu} + g^{\mu\upsilon}\partial_{\upsilon}\Lambda) + \Gamma^{0}{}_{01}(A^1 + \frac{\partial \Lambda}{\partial \xi^1}) \quad (8)$$

Lorentz gauge fix condition is in Rindler space-time[1],

$$0 = \frac{1}{c}\frac{\partial \phi_{\xi}}{\partial \xi^0} + \vec{\nabla}_{\xi} \cdot \vec{A}_{\xi} + \frac{A_{\xi^1}}{c^2}\frac{a_0}{(1+\frac{a_0\xi^1}{c^2})}$$

$$\to \frac{1}{c}\frac{\partial \phi_{\xi}}{\partial \xi^0} + \vec{\nabla}_{\xi} \cdot \vec{A}_{\xi} + \frac{A_{\xi^1}}{c^2}\frac{a_0}{(1+\frac{a_0\xi^1}{c^2})} - [\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial \xi^0})^2 - \nabla_{\xi}{}^2]\Lambda + \frac{\partial \Lambda}{\partial \xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}$$

$$= 0 \quad (9)$$

Hence, the gauge equation is

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial \xi^0})^2 - \nabla_{\xi}{}^2]\Lambda - \frac{\partial \Lambda}{\partial \xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})} = 0$$

(10)

We can use Eq(10) as an electromagnetic wave equation because we can apply electromagnetic wave function instead of the gauge function Λ to Eq(10) in Rindler space-time. Hence,

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_{\xi}{}^2]E_{\xi^1}-\frac{\partial E_{\xi^1}}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_{\xi}{}^2]B_{\xi^1}-\frac{\partial B_{\xi^1}}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0 \quad (11)$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_{\xi}{}^2]E_y-\frac{\partial E_y}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_{\xi}{}^2]B_y-\frac{\partial B_y}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0 \quad (12)$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_{\xi}{}^2]E_z-\frac{\partial E_z}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0$$

$$[\frac{1}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})^2}(\frac{\partial}{\partial\xi^0})^2-\nabla_{\xi}{}^2]B_z-\frac{\partial B_z}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0\xi^1}{c^2})}=0 \quad (13)$$

The electro-magnetic wave function is

$$E_x=E_{x0}\sin\Phi, E_y=E_{y0}\sin\Phi, E_z=E_{z0}\sin\Phi$$

$$B_x=B_{x0}\sin\Phi, B_y=B_{y0}\sin\Phi, B_z=B_{z0}\sin\Phi \quad (14)$$

$$E_{\xi^1}=E_X=E_{X0}\sin\Phi, B_{\xi^1}=B_X=B_{X0}\sin\Phi$$

$$E_{\xi^2}=E_y\cosh(\frac{a_0\xi^0}{c})-B_z\sinh(\frac{a_0\xi^0}{c}),$$

$$=(E_{y0}\sin\Phi)\cosh(\frac{a_0\xi^0}{c})-(B_{z0}\sin\Phi)\sinh(\frac{a_0\xi^0}{c})$$

$$B_{\xi^2}=B_y\cosh(\frac{a_0\xi^0}{c})+E_z\sinh(\frac{a_0\xi^0}{c})$$

$$=(B_{y0}\sin\Phi)\cosh(\frac{a_0\xi^0}{c})+(E_{z0}\sin\Phi)\sinh(\frac{a_0\xi^0}{c})$$

$$E_{\xi^3} = E_z \cosh(\frac{a_0\xi^0}{c}) + B_y \sinh(\frac{a_0\xi^0}{c})$$

$$= (E_{z0} \sin\Phi)\cosh(\frac{a_0\xi^0}{c}) + (B_{y0} \sin\Phi)\sinh(\frac{a_0\xi^0}{c})$$

$$B_{\xi^3} = B_z \cosh(\frac{a_0\xi^0}{c}) - E_y \sinh(\frac{a_0\xi^0}{c})$$

$$= (B_{z0} \sin\Phi)\cosh(\frac{a_0\xi^0}{c}) - (E_{y0} \sin\Phi)\sinh(\frac{a_0\xi^0}{c}) \qquad (15)$$

$$\Phi = \omega(t - l\frac{x}{c} - m\frac{y}{c} - n\frac{z}{c}) ,$$

$$= \omega\{(\frac{c}{a_0} + \frac{\xi^1}{c})(\sinh(\frac{a_0\xi^0}{c}) - l\cosh(\frac{a_0\xi^0}{c})) + \frac{c}{a_0} - m\frac{\xi^2}{c} - n\frac{\xi^3}{c}\},$$

$$l^2 + m^2 + n^2 = 1 \qquad (16)$$

3. Conclusion

We find the electro-magnetic wave equation and function in uniformly accelerated frame.

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