Bell's inequality refuted on Bell's terms

Abstract Bell's famous inequality contradicts ours. But ours holds algebraically, experimentally, classically and quantum-mechanically: Bell's does not. So Bell's inequality is refuted on Bell's terms as we identify his naively-realistic error and correct it.

1. Introduction

1.1. We begin with some key facts: (i) Bell's famous 1964 inequality is violated by quantum theory (QT) and experiment. (ii) Bell's fame is partly due to those who share his dilemma about the physical significance of his work. Here's the essence of that dilemma, offset by Bell's related hope:

> (iii) 'I cannot say that action at a distance [AAD] is required in physics. I can say that you cannot get away with no AAD. The Einstein program fails; we might have to learn to accept not so much AAD, but [the] inadequacy of no AAD. That is the dilemma. I step back from asserting that there is AAD, I say only that you cannot get away with locality. You cannot explain things by events in their neighbourhood,' after Bell (1990:5-13).
>
> (iv) 'This AAD and no AAD business will pass. If we're lucky it will be to some big new development like the theory of relativity. Maybe someone will just point out that we were being rather silly. But I believe the questions will be resolved,' after Bell (1990:9).

1.2. We continue with some core claims that straddle Bell's 1990 dilemma: 'Bell's 1964 theorem is one of the most profound discoveries of science,' Stapp (1975:271). 'The Einstein-Podolsky-Rosen-Bohm experiment (EPRB)—in Bell's hands—demonstrates the impossibility of making QT the more complete representation of nature that Einstein wished it to be,' after Mermin (1988:14). 'Bell's 1964 theorem is false,' Watson (1989), replying to Mermin. 'Bell's theorem will be remembered in times to come as one of the few essential discoveries of twentieth century physics,' Van der Merwe *et al.* (1992:v). 'Ironically, Bell's theorem is "the most profound discovery of science" (after Stapp 1975) because it is *not* obeyed by the experimental facts,' Peres (1995:164): nor, as we show, by algebra.

1.3. Hence the primary focus of this essay: not only is Bell's inequality violated by QT and experiment, we show that it is false (and thus rather silly) under elementary algebra. This result leads to Watson 2018G (forthcoming). There—resolving Bell's contradictory dilemma ¶1.1(iii) via developments in line with Bell's hope ¶1.1(iv)—we find no need for AAD. For, consistent with Einstein's ideas, locality prevails: that is, we explain things by events in their neighbourhood; thereby confirming and correcting much Bellian confusion—and rejecting the naivety of his realism—with many questions resolved.

1.4. In short: via accepted facts and elementary algebra, we reinforce our 1989 claim—Bell's *theorem* is false—by now showing that Bell's *inequality* is algebraically false. As is Peres' related claim (1995:162) that 'Bell's theorem applies to any physical system with dichotomic variables'. See ¶5.Appendix.

[1] Corresponding author: eprb@me.com Subject line: 2018F-1Ax.

2. Analysis

2.1. Bell 1964 (freely available, see ¶6.References) defines the two key terms that we need. (i) In Bell 1964:(1), a result A is given by a function $A(\vec{a}, \boldsymbol{\lambda}) = \pm 1$, etc. (ii) In Bell 1964:(2), an expectation value $P(\vec{a}, \vec{b})$ is defined as the average over the product of paired-results $A(\vec{a}, \boldsymbol{\lambda})$ and $B(\vec{b}, \boldsymbol{\lambda})$. For future use and current clarity: let (14a)-(14c) identify the unlabelled relations between Bell's (14)-(15); the remainder being (15a), (21a)-(21e), (23).

2.2. So, from Bell 1964:(1) *and its preamble*, sharing Bell's indifference re $\boldsymbol{\lambda}$ for now (that is: until ¶2.16 when our algebraic studies are tested and confirmed via elementary classical engineering):

$$A(\vec{a}, \boldsymbol{\lambda}) = \pm 1, B(\vec{b}, \boldsymbol{\lambda}) = \pm 1; \text{ etc.} \tag{1}$$

2.3. And thus, in the closed interval $[-1, 1]$, the related expectation values that Bell uses are:

$$-1 \leq P(\vec{a}, \vec{b}) \leq 1, -1 \leq P(\vec{a}, \vec{c}) \leq 1, -1 \leq P(\vec{b}, \vec{c}) \leq 1. \tag{2}$$

2.4. So here's Bell's inequality, Bell 1964:(15), in two convenient forms for later comparisons—the leftmost designated (3)L , the rightmost (3)R—with immediate difficulties for Bell; ▲ denoting absurdity:

$$\text{(3)L: } \left|P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})\right| - P(\vec{b}, \vec{c}) \leq 1.\blacktriangle \quad \text{(3)R: } \left|P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})\right| - 1 \leq P(\vec{b}, \vec{c}).\blacktriangle \tag{3}$$

(i). (3) is absurd. For, from (2), given

$$P(\vec{a}, \vec{b}) = P(\vec{b}, \vec{c}) = -\tfrac{1}{2}, P(\vec{a}, \vec{c}) = \tfrac{1}{2}, \tag{4}$$

then Bell's inequality, Bell 1964:(15) as in (3), delivers these results:

$$\text{(3)L: } \tfrac{3}{2} \leq 1.\blacktriangle \qquad \text{(3)R: } 0 \leq -\tfrac{1}{2}.\ \blacktriangle \tag{5}$$

(ii). (5) is irrefutable, as is our critique of (3). For, with (1)-(2) being accepted facts, and with (3) and (4) delivering (5) via elementary algebra: our results are independent of any physical theory.

(iii). Moreover, (5) is supported experimentally. See Peres 1995:164 re the photonics in Aspect *et al.* (1982); a binary experiment against Bell that is also bound by, and thus consistent with our findings.

(iv). Further, $\vec{b}$ and $\vec{c}$ may be independently rotated in cones about the $\vec{a}$ direction, thereby holding $P(\vec{a}, \vec{b})$ and $P(\vec{a}, \vec{c})$ constant as $(\vec{b}, \vec{c})$ varies. Thus, in (3)R and via such analysis: given the *constants* on LHS (3)R, Bell's $P(\vec{b}, \vec{c})$ is a suspicious *variable* on RHS (3)R. Indeed, as we show, (5) is a forerunner of the fact that (3) is false almost everywhere; see the example in Fig.1 at ¶2.15.

2.5. Now (of course) our results would not hold if (4) were an impossible combination. But Bell is theorizing under the quantum-based EPRB experiment. So Bell's comparison-theory [QT; see Bell

(1964:195,199)] is equivalent to the QT-compatible proxies that we use in (14). Thus — ignoring the claimed *impossibility* below Bell 1964:(3); which we refute in 2018G, since Bell's claim is based on the inequality that we refute here — Bell in the line below Bell 1964:(2) allows via QT:

$$P(\vec{a},\vec{b}) = -\cos(\vec{a},\vec{b}), P(\vec{a},\vec{c}) = -\cos(\vec{a},\vec{c}), P(\vec{b},\vec{c}) = -\cos(\vec{b},\vec{c}); \tag{6}$$

so that, via our proxies (14) and as also independently via (6) from QT, (4) holds under

$$(\vec{a},\vec{b}) = 60^\circ, (\vec{a},\vec{c}) = 120^\circ, P(\vec{b},\vec{c}) = 60^\circ. \tag{7}$$

2.6. In short: via (6)-(7)'s validation of (4), (5) typifies the general failure of Bell's analysis in EPRB settings. For, with Bell (1964:195) citing EPR (1935)—whose 'elements of physical reality' we reject (Watson (1998)—and with Bell arguing from locality to a more complete specification of EPRB by means of parameters $\boldsymbol{\lambda}$ (which we support): Bell delivers (5)-style contradictions. And, as we show, it is in resolving such that we deliver the more complete specification.

2.7. So now, to find the source of Bell's problem, we use a relation between the numbers $p, q \in \mathbb{R}$:

$$\text{if } p \leq 1 \text{ and } 0 \leq q \text{ then } pq \leq q. \tag{8}$$

2.8. Thus, via (2) and (8): with $P(\vec{a},\vec{b})$ equivalent to p, and $1 + P(\vec{a},\vec{c})$ equivalent to q, we have

$$P(\vec{a},\vec{b})[1 + P(\vec{a},\vec{c})] \leq 1 + P(\vec{a},\vec{c}). \tag{9}$$

$$\therefore P(\vec{a},\vec{b}) - P(\vec{a},\vec{c}) + P(\vec{a},\vec{b})P(\vec{a},\vec{c}) \leq 1. \tag{10}$$

2.9. Then, bringing absolute brackets to (10), we have relations to test Bell 1964:(15) as in (3).

$$\text{(11)L: } 0 \leq \left|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})\right| + P(\vec{a},\vec{b})P(\vec{a},\vec{c}) \leq 1. \text{ (11)R: } \left|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})\right| - 1 \leq -P(\vec{a},\vec{b})P(\vec{a},\vec{c}). \tag{11}$$

where, using (2) and proof by exhaustion (and thus independent of any theory): (i) (11)L's lower bound is 0 when $P(\vec{a},\vec{b}) = P(\vec{a},\vec{c}) = 0$; (ii) (11)L's upper bound is 1 when $P(\vec{a},\vec{b}) = 0, P(\vec{a},\vec{c}) = 1$, etc; (iii) with variable limits within these static bounds, since $(\vec{a},\vec{b})$ and $(\vec{a},\vec{c})$ may be chosen freely.

2.10. So we now compare (3)R with (11)R via their common terms. Since $(\vec{a},\vec{b})$ and $(\vec{a},\vec{c})$ may be freely chosen, $P(\vec{b},\vec{c})$ in (3)R will breach that irrefutable number on RHS of (11)R and be false if

$$P(\vec{b},\vec{c}) \neq -P(\vec{a},\vec{b})P(\vec{a},\vec{c}). \tag{12}$$

2.11. Moreover, against Bell: our (11)L, never false by virtue of the exhaustive use of (2) — and clearly independent of $P(\vec{b},\vec{c})$ — nevertheless holds for all $(\vec{b},\vec{c})$. For, in exhausting $P(\vec{a},\vec{b})$ and $P(\vec{a},\vec{c})$, we exhaust all values of $(\vec{b},\vec{c})$. That is, via ¶2.4(iv) and (say) a geometry G (with examples),

$$(\vec{b},\vec{c}) = G\left[(\vec{a},\vec{b}),(\vec{a},\vec{c})\right]. \quad G_1 : (\vec{b},\vec{c}) = (\vec{a},\vec{b}) - (\vec{a},\vec{c}). \quad G_2 : (\vec{b},\vec{c}) = (\vec{a},\vec{b}) = (\vec{a},\vec{c}). \tag{13}$$

2.12. So we are now free to test many proxies for $P(\vec{a}, \vec{b})$ and $P(\vec{a}, \vec{c})$; linear, nonlinear, etc. Here, under EPRB but derived in 2018G:¶4 without QT, we use the negative-cosine function as the proxy. In this way our expectation values range from -1 to 1 over $[0°, 180°]$, etc:

$$-1 \leq P(\vec{a}, \vec{b}) = -\cos(\vec{a}, \vec{b}) \leq 1, -1 \leq P(\vec{a}, \vec{c}) = -\cos(\vec{a}, \vec{c}) \leq 1, -1 \leq P(\vec{b}, \vec{c}) = -\cos(\vec{b}, \vec{c}) \leq 1. \quad (14)$$

2.13. So our (11)L may be written

$$0 \leq \left|\cos(\vec{a}, \vec{c}) - \cos(\vec{a}, \vec{b})\right| + \cos(\vec{a}, \vec{b})\cos(\vec{a}, \vec{c}) \leq 1, \quad (15)$$

while, via our independent (14) or via (6) under QT from Bell: Bell's inequality as in (3)L becomes

$$\left|\cos(\vec{a}, \vec{c}) - \cos(\vec{a}, \vec{b})\right| + \cos(\vec{b}, \vec{c}) \leq 1. \quad (16)$$

2.14. Thus, via (12) and (14), (16) will be false when

$$\cos(\vec{b}, \vec{c}) \neq \cos(\vec{a}, \vec{b})\cos(\vec{a}, \vec{c}); \quad (17)$$

which, via (13) and its two geometries, reduces to Bell's inequality being false everywhere that:

$$\text{Under } G_1: \ \sin(\vec{a}, \vec{b})\sin(\vec{a}, \vec{c}) \neq 0. \ \text{ Under } G_2: \cos(\vec{b}, \vec{c}) \neq \cos^2(\vec{b}, \vec{c}). \ \text{ Etc.} \quad (18)$$

2.15. We conclude that Bell's inequality is false almost everywhere. A fact illustrated by another example: Fig.1 shows our (15) and Bell's inequality (16) under G_1 with the simplification $(\vec{a}, \vec{c}) = \frac{\pi}{2}$.

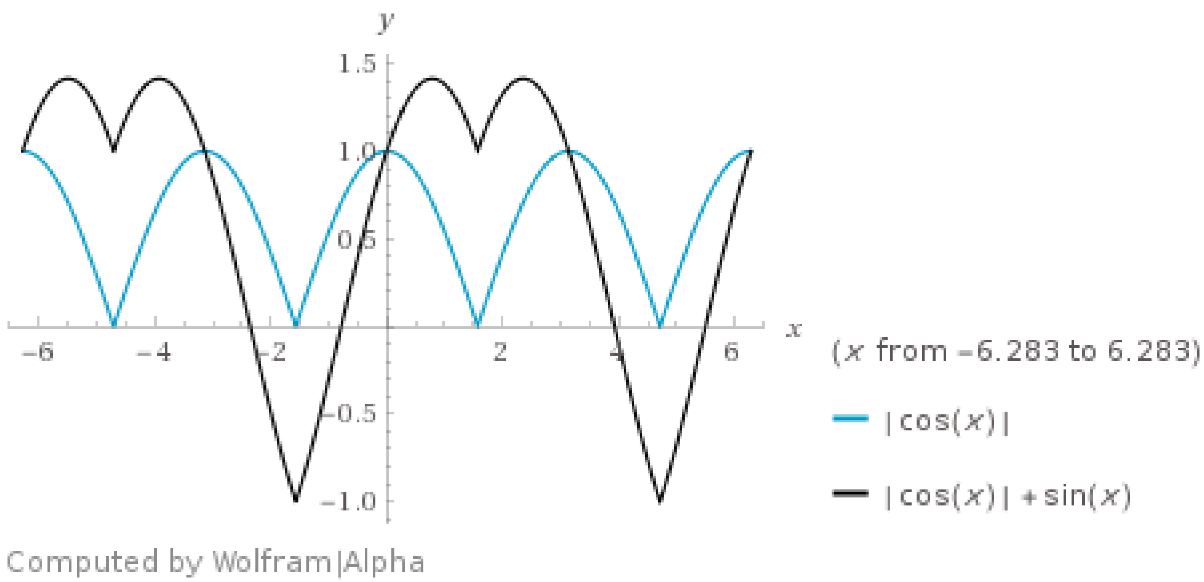

Fig.1. A comparison of Bell's inequality (16)-black and our (15)-blue: under G_1 from (13); with $x = (\vec{a}, \vec{b})$ and y = output when $(\vec{a}, \vec{c}) = \frac{\pi}{2}$ and (thus) $\cos(\vec{b}, \vec{c}) = \sin(x)$. Note: Bell's *own* upper bound—see (3)L—is 1; so on that bound alone (on Bell's terms alone) Bell's inequality is 50% false here. (In degrees, the x-axis is from $-360°$ to $360°$.)

2.16. So we now seek the source of $P(\vec{b},\vec{c})$ in (3). To this end (as in Watson 2017d) we allow each $\boldsymbol{\lambda}$ (a Bellian beable, a thing which exists) to be a discrete variable in 3-space: that is, the orientation of a particle's total angular momentum; pairwise correlated by the conservation of total angular momentum. Thus, indifferent as we have been to $\boldsymbol{\lambda}$'s nature thus far (see ¶2.2): now, from Bell 1964:(14) via discrete parameters $\boldsymbol{\lambda}_i$ and n sufficient to provide adequate accuracy, we can write:

$$P(\vec{a},\vec{b}) = -\int d\lambda\,\rho(\boldsymbol{\lambda})A(\vec{a},\boldsymbol{\lambda})A(\vec{b},\boldsymbol{\lambda}) = -\frac{1}{n}\sum_{i=1}^{n}[A(\vec{a},\boldsymbol{\lambda}_i)A(\vec{b},\boldsymbol{\lambda}_i)];\ \text{etc}, \tag{19}$$

as we move to analyze each outcome, particle-pair by particle-pair under strict pairwise-tracking.

2.17. Then, seeking to maximize generality via diverse sampling, let's randomly distribute $2n$ particle-pairs from $2n$ sources over $2n$ detector-pairs: with randomly paired detector-settings equating to $(\vec{a},\vec{b}) = \alpha\,$(say), $(\vec{a},\vec{c}) = \gamma$, as at ¶2.4(iv) above. And to match particle-pairs to their paired-outcomes, let's allow each particle-pair to be uniquely indexed — $[i = 1, 2, ..., n$ over the settings $(\vec{a},\vec{b}) = \alpha$; $j = n{+}i$ over the settings $(\vec{a},\vec{c}) = \gamma]$ — and reasonably unique: since the probability of $\boldsymbol{\lambda}_i$ being replicated by $\boldsymbol{\lambda}_j$ is negligible; each being a random orientation in 3-space (¶2.16). We then allow n to be such that, to an adequate accuracy (satisfying serious critics) and for convenience in presentation:

$$\text{LHS Bell 1964:(14a)} = P(\vec{a},\vec{b}) - P(\vec{a},\vec{c}) \tag{20}$$

$$= \frac{1}{n}\sum_{i=1}^{n}[A(\vec{a},\boldsymbol{\lambda}_{n+i})A(\vec{c},\boldsymbol{\lambda}_{n+i}) - A(\vec{a},\boldsymbol{\lambda}_i)A(\vec{b},\boldsymbol{\lambda}_i)],\ \text{using our (19).} \tag{21}$$

$$\therefore\ \left|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})\right| = \left|\frac{1}{n}\sum_{i=1}^{n} A(\vec{a},\boldsymbol{\lambda}_i)A(\vec{b},\boldsymbol{\lambda}_i)[A(\vec{a},\boldsymbol{\lambda}_i)A(\vec{b},\boldsymbol{\lambda}_i)A(\vec{a},\boldsymbol{\lambda}_{n+i})A(\vec{c},\boldsymbol{\lambda}_{n+i}) - 1]\right| \tag{22}$$

$$\leq \frac{1}{n}\sum_{i=1}^{n}[1 - A(\vec{a},\boldsymbol{\lambda}_i)A(\vec{b},\boldsymbol{\lambda}_i)A(\vec{a},\boldsymbol{\lambda}_{n+i})A(\vec{c},\boldsymbol{\lambda}_{n+i})], \tag{23}$$

$$\because\ \left|\frac{1}{n}\sum_{i=1}^{n} A(\vec{a},\boldsymbol{\lambda}_i)A(\vec{b},\boldsymbol{\lambda}_i)\right| \leq 1. \tag{24}$$

$$\therefore 0 \leq \left|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})\right| + P(\vec{a},\vec{b})P(\vec{a},\vec{c}) \leq 1, \tag{25}$$

reducing (23) via (19) to (25); which is (11)L, with the same bounds via the same exhaustion of (2). Elementary classical engineering thus delivers the algebraically irrefutable (11) independently.

2.18. And we now see the use that Bell makes of his cryptic remark "using (1)" below Bell 1964:(14b). For reverse-engineering requires—*and thus exposes*—that, in studying the quantum-based EPRB, Bell effectively reduces rigorous (23) to defective (3) by naively using

$$\boldsymbol{\lambda}_i = \boldsymbol{\lambda}_{n+i}\ \text{[sic]} : \tag{26}$$

that is, equating the $\boldsymbol{\lambda}_i$ used in the $P(\vec{a},\vec{b})$-run (of tests) with the $\boldsymbol{\lambda}_{n+i}$ used in the $P(\vec{a},\vec{c})$-run, and now having them available for the $P(\vec{b},\vec{c})$-run. Bell thereby concludes, using (26) in (23):

$$\therefore \left|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})\right| \le \frac{1}{n}\sum_{i=1}^{n}[1 - A(\vec{a},\boldsymbol{\lambda}_i)A(\vec{b},\boldsymbol{\lambda}_i)A(\vec{a},\boldsymbol{\lambda}_i)A(\vec{c},\boldsymbol{\lambda}_i)] \text{ [sic]}. \tag{27}$$

$$\therefore \left|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})\right| \le 1 + P(\vec{b},\vec{c}) \text{ [sic]}, \tag{28}$$

which is that algebraically false Bell-(15), our (3), again: Bell reducing false (27) to false (28) via

$$A(\vec{a},\boldsymbol{\lambda}_i)A(\vec{a},\boldsymbol{\lambda}_i) = 1 \text{ and } -\frac{1}{n}\sum_{i=1}^{n}[A(\vec{b},\boldsymbol{\lambda}_i)A(\vec{c},\boldsymbol{\lambda}_i)] = P(\vec{b},\vec{c}). \tag{29}$$

2.19. Note re $P(\vec{b},\vec{c})$ in (3) and (28): in our algebraic derivation of (11)—and in our independent classical-engineering derivation of the same, (25)—we find, instead of $P(\vec{b},\vec{c})$, the constant $P(\vec{a},\vec{b})P(\vec{a},\vec{c})$. Our results thus hold for all $(\vec{b},\vec{c})$ per ¶2.11. Further, we comply with an implicit requirement in (1): if $\boldsymbol{\lambda}$ is taken to be continuous [which is Bell's choice in (1)], then the implicit A_i and B_i in (1) — from the ith test — must be pairwise matched [as in 2018G:¶4] via delta-functions to $\boldsymbol{\lambda}_i$: the relevant element of the continuum. That is, for us and in agreement with Bell:

> "In a complete physical theory of the type envisaged by Einstein, the hidden variables would have dynamical significance and laws of motion; our $\boldsymbol{\lambda}$ can then be thought of as initial values of these variables at some suitable instant," Bell (1964:196).

2.20. Also: Bell's 1964:(15), in (3) or (28), is based on another unphysical assumption. That is: under EPRB, Bell's use of (1) via $\boldsymbol{\lambda}_i = \boldsymbol{\lambda}_{n+i}$ in (23) implies that our randomizing in ¶2.17 has somehow produced three identical cohorts of ordered particle-pairs. To the contrary, our fair randomizing produces representative cohorts of similar particle-pairs: which cohorts, if exchanged, would leave the results unchanged to an adequate accuracy; a true measure of fair sampling. The result is that Bell's (14b) does not equal Bell's (14a). See 2018G for further analysis and discussion.

2.21. Now some suggest privately that our use of (6) to move (3)L to (16) is invalid because Bell (against QT) wants to keep open the possibility that the *quantum correlations* like (6) are false. And we find some merit in this objection. To us it's a possible explanation for Bell's strange claim of "*not possible*" below his 1964:(3). That is—though it is shown to be false at Watson 2018G:¶4—Bell claims that a standard formula for an average in Bell 1964:(2) somehow doesn't work.

2.22. What's more: as shown via our new approach to EPRB's physics in Watson 2018G, we derive (6) without QT. So though Bell and private others may have reservations re (6), it is clearly available to us: with QT's endorsement; and via QT alone if necessary today.

2.23. So, with other Bellian claims refuted in Watson 2018G, we close our analysis here by making one final point: wanting to be clear and minimize a common misunderstanding. Though our theorizing can be independent of QT, QT is of course (given its remarkable history) an adequate theory in our terms. But in Watson 2018G we bring a new approach and additional understanding to physics in line with Einstein's ideas, Bell's expectation ¶1.1(iv), and Born's (1954:266) view of QT:

> "The lesson to be learned is that probable refinements of mathematical methods will not suffice to produce a satisfactory theory, but that somewhere in our doctrine is hidden a concept, unjustified by experience, which we must eliminate to open up the road."

2.24. Thus, combining the findings in ¶5 with the related findings in 2018G: the hidden-concept that Born seeks is a *naive understanding of true realism,* for it hides in plain sight here and in physics widely: via Bellian realism, Bell's inequality, Bell's theorem, Bell's dilemma, local realism, locally realistic, realism. Also via invalid counterfactual reasoning wherein—neglecting the fact that beables may change interactively, such changes experienced even classically—pre-measurement properties are taken to be the same as post-measurement properties, or vice-versa.

3. Conclusions

3.1. The basis for Bell's original inequality is widely discussed up to the present day. But now, whatever that basis: Bell 1964:(15) is algebraically false; refuted on Bell's own terms—in seeking a more complete specification of EPRB as at ¶2.6—and replaced by an inequality that holds for any EPRB-style experiment with binary outcomes (thus refuting Peres' claim at ¶1.4).

3.2. Moreover, Bell 1964:(15) is also unphysical—see Watson 2018G:¶5—akin to inferring that naive analysis might succeed in a highly-correlated quantum-setting (EPRB, the setting for Bell 1964 and thus for us here) and *not* then dropping such naivety when it fails so badly.

3.3. Further, as we show in ¶5, Bell's ideas also fail classically. So whatever triggered Bell's naive inferencing, Bell 1964:(15) is also unwarranted. Moreover, against Bell here, Watson 2018G (via true local realism, and without QT) delivers the same results as QT and observation. This shows independently that Bell 1964:(2) equals Bell 1964:(3): at the same time refuting Bell's theorem—that is, Bell's *"not possible"* below Bell 1964:(3)—and thus Bell's EPRB theorizing generally.

3.4. Thus, advancing our 1989 position: for us, true local realism (TLR) is the union of true locality and true realism. *True locality* insists that no influence propagates superluminally, after Einstein. *True realism* insists that some beables *may* change interactively, after Bohr. *Naive-realism* is then any brand of *realism* that negates or neglects that 'may' when relevant.

3.5. Then, re further consequences of this work: Einstein's program does not fail with us; for against Bell in ¶1.1(iii), Watson 2018G does get away with locality and local explanations. Thus, dismissing AAD, we refute claims like those in ¶1.2 (except our own) and these:

> 'Einstein maintained that quantum metaphysics entails spooky actions at a distance; experiments show that what bothered Einstein is not a debatable point but the observed behaviour of the real world,' after Mermin (1985:38). 'Our world is non-local,' after Davies (1984:48), Goldstein *et al.* (2011:1), Maudlin (2014:25), Bricmont (2016:112). '... the predictions of quantum theory cannot be accounted for by any local theory,' after Brunner *et al.* (2014:1), Norsen (2015:1); 'the search for such models is hopeless,' Aspect (2002: 9).

3.6. Finally: Bell's inequality, algebraically false independent of its failings under QT and repeated experiments, represents an interesting phase in the history of science. We suggest that this essay and 2018G resolve this phase and Bell's dilemma in a way that Einstein, Born and Bell might like:

> "... you make a very thorough analysis of EPR-Bell. As you still remain a 'realist' and refer to Bell's beables when you resolve Bell's dilemma, Bell might have liked your approach, who knows." (R. Bertlmann 2017, pers. comm. 26 June); see Watson (2017d).

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5. Appendix: Bell's inequality fails classically under experiments Ω and Ω'

> 5.0. 'The first moral of this story is just a practical one. Always test your general reasoning against simple models,' Bell (1982:12).

5.1. Against Bell generally, also Peres at ¶1.4 and Aspect *et al.* (1982: 1804)—'Bell's inequalities apply to any correlated measurement on two correlated systems'—we show that Bell's inequality fails under classical experiments Ω and Ω'. Modifying EPRB, Ω and Ω' are *classical* in that we can derive the correct expectation-values by validly extending Malus' classical law for linearly-polarized light to linearly-polarized spin-half particles (intrinsic-spin $s = \frac{1}{2}$) and photons ($s = 1$).

5.2. For experiment Ω, we modify EPRB in just two ways: (i) We sandwich the source between two aligned and coupled dichotomic linear-polarizers which step (rotate in unison) randomly between emissions about the line-of-flight axis. The separated particles are thus pairwise anti-correlated in a state of opposite polarization in direction $0 \leq \phi < 2\pi$ (orthogonal to the line-of-flight). (ii) We consequently constrain the principal axis of each EPRB detector to be orthogonal to the line-of-flight.

5.3. Thus, validly extending Malus' Law to such polarized particles, expectations P_Ω under Ω are:

$$P_\Omega(\vec{a},\vec{b}) = -\tfrac{1}{2\pi}\int_0^{2\pi} d\phi\,[\cos^2\tfrac{1}{2}(\vec{a},\phi) - \sin^2\tfrac{1}{2}(\vec{a},\phi)][\cos^2\tfrac{1}{2}(\vec{b},\phi) - \sin^2\tfrac{1}{2}(\vec{b},\phi)] = -\tfrac{1}{2}\cos(\vec{a},\vec{b});\text{ etc.} \tag{30}$$

5.4. So, using (30) with (11)R and (3)R respectively:

$$\left|P_\Omega(\vec{a},\vec{b}) - P_\Omega(\vec{a},\vec{c})\right| - 1 \leq -P_\Omega(\vec{a},\vec{b})P_\Omega(\vec{a},\vec{c}) = -\tfrac{1}{4}\cos(\vec{a},\vec{b})\cos(\vec{a},\vec{c}); \tag{31}$$

$$\left|P_\Omega(\vec{a},\vec{b}) - P_\Omega(\vec{a},\vec{c})\right| - 1 \leq P_\Omega(\vec{b},\vec{c}) = -\tfrac{1}{2}\cos(\vec{b},\vec{c}). \tag{32}$$

5.5. Then, as at ¶2.10) via their common terms [since $(\vec{a},\vec{b})$ and $(\vec{a},\vec{c})$ may be freely chosen]: $P_\Omega(\vec{b},\vec{c})$ in (32) will breach that irrefutable number on RHS (31) if

$$\cos(\vec{b},\vec{c}) \neq 2\cos(\vec{a},\vec{b})\cos(\vec{a},\vec{c}); \tag{33}$$

that is, under G_1 from (13), with $\vec{a},\vec{b},\vec{c}$ coplanar, orthogonal to the line-of-flight—and consistent (as expected) with our result at (17)—Bell's inferencing is classically false almost everywhere.

5.6. Moreover, the source in Aspect (2004) can be similarly modified to form experiment Ω'. Malus' (original classical) Law then delivers the correct expectations $P_{\Omega'}$ via

$$P_{\Omega'}(\vec{a}, \vec{b}) = \tfrac{1}{2\pi} \int_0^{2\pi} d\phi' \, [\cos^2(\vec{a}, \phi') - \sin^2(\vec{a}, \phi')][\cos^2(\vec{b}, \phi') - \sin^2(\vec{b}, \phi')] = \tfrac{1}{2} \cos 2(\vec{a}, \vec{b}); \text{ etc.} \tag{34}$$

5.7. Now there can be no doubting that the results under the above classicalities are consistent with TLR (¶3.4): with special emphasis on *locality*. For the paired outcomes arise *probabilistically* from local interactions between detectors and classically-correlated particle-pairs.

5.8. Moreover, still under TLR and with the same emphasis on *locality*: the expectations must change [*they double*] when the sandwiching-polarizers are removed. For the particle-detector interactions and outcomes then arise locally via the spherically-symmetric correlation of the pristine particles [pairwise correlated via the conservation of total angular momentum] and the related *deterministic dynamics.* Akin to the well-known classical determinism of spin, torque and precession, these dynamics then lead to correlated polarizations reported by the EPRB/Aspect detectors [polarizer-analyzers].

5.9. So, with our results again fully consistent with TLR, and again in line with Bell's hope ¶1.1(iv): *Bell's analysis again fails via the naivety of his realism*; and again, with our emphasis on *locality*, this failure has nothing to do with that false notion, AAD.

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