Intuitionistic evidence sets

Yangxue Li[a], Yong Deng[a,*]

[a] *Institute of Fundamental and Frontier Science, University of Electronic Science and Technology of China, Chengdu, 610054, China*

Abstract

Dempster-Shafer evidence theory can express and deal with uncertain and imprecise information well, which satisfies the weaker condition than the Bayes probability theory. The traditional single basic probability assignment only considers the degree of the evidence support the subsets of the frame of discernment. In order to simulate human decision-making processes and any activities requiring human expertise and knowledge, intuitionstic evidence sets (IES) is proposed in this paper. It takes into account not only the degree of the support, but also the degree of non-support. The combination rule of intuitionstic basic probability assignments (IBPAs) also be investigated. Feasibility and effectiveness of the proposed method are illustrated using an application of multi-criteria group decision making.

Keywords: Dempster-Shafer evidence theory, Intuitionstic evidence sets, Combination rule, Group decision making, Intuitionstic fuzzy sets.

1. Introduction

Dempster-Shafer evidence theory is the generalization of Bayes probability theory, which is mainly carried out by using Bayes conditional probability in probability theory [1–3]. The D-S evidence theory does not need to know the priori probability and can

*Corresponding author, Yong Deng, Institute of Fundamental and Frontier Science, University of Electronic Science and Technology of China, Chengdu, 610054, China

Email address: dengentropy@uestc.edu.cn (Yong Deng)

express ”uncertainty” well. It is widely used to deal with uncertain information. Such as information fusion [4–9], decision making [10–12], risk assignment [13, 14], performance assessment [15–20], target recognition and tracking [21], fault diagnosis [22–24] and pattern classification [25–27]. Nevertheless, the initial D-S evidence theory is not perfect enough. For example, the counterintuitive results may be obtained when the fused evidence are highly conflict each other [28].

There are many methods are proposed to improve the D-S evidence theory. Some methods improve the Dempster combination rule, such as the new combination rule proposed by Yager [29], the rule of combination proposed by Smets [30], the combination operator proposed by Dubois and Prade [31] and as so on [32–35]. Some methods to correct the source of evidence, such as the discounting coefficients method [36, 37], Murphy's average approach [38], Deng *et al.*'s modified average approach [39] etc. [40–43]. Other methods is to improve the way of the D-S evidence theory modeling uncertain information, such as the generalized evidence theory [44, 45], interval-valued evidence theory [46–51] and D numbers [52–59].

The traditional single basic probability assignment only considers the degree of the evidence support the subsets of the frame of discernment. But in practice, the information of non-support is equally important. For example, in the problem of sensor information fusion, some the collected data show the degree of support for the target, other data may show the degree of non-support. In this case, the single basic probability assignment is not sufficient to handle this information. In fuzzy logic, intuitionstic fuzzy sets (IFS) is the extension of traditional fuzzy sets [60], which considers three aspects of the degree of membership, the degree of non-membership and hesitancy [61]. Therefore, it is more flexible and practical than the traditional fuzzy set in dealing with uncertain information. Based on the concept of IFS, the intuitionstic evidence sets (IES) is presented in this paper, which considers the degree of the evidence support the subset of the frame of discernment and the degree of the evidence non-support the subset. The sums of all the degrees of support is one and the sum of all the degrees of non-support is one. Except the frame of discernment, for each subset, the support degree and the non-support degree can not be one at the same time. Because it's impossible to be completely positive and completely negative simultaneously. When the support degree and non-support degree of the frame of discernment is one at the same time, this case is completely uncertain.

The Dempster combination rule plays an important role in D-S evidence theory [1, 62–64]. The combination rule of IES also be discussed in this paper. A intuitionstic basic probability assignment (IBPA) can be regarded as two traditional BPAs. One of them indicates the BPA of support and another indicates the BPA of non-support. Then combining two classes BPAs of all IBPA, respectively, using Dempster combination rule.

This paper is organized as follows. The definitions and properties of Dempster-Shafer evidence theory and intuitionstic fuzzy sets are briefly introduced in Section 2. The definition and properties of IES and its combination rule are proposed in Section 3. In Section 4, the method of multi-criteria group decision making is discussed. A numerical example is given in Section 5. Finally, this paper is concluded in Section 6.

2. Background

In this section, the background material of Dempster-Shafer evidence theory [1, 2] and intuitionstic fuzzy set [61] will be briefly introduced.

2.1. Dempster-Shafer evidence theory

Dempster-Shafer theory was defined on a finite set of mutually exclusive elements. This finite set is called the frame of discernment denotes as Θ, it's power set denotes as 2^{Θ}. Evidence theory allows belief to be assigned to not only the single subsets of the frame of discernment, but also the multiple subsets. The evidence theory will be degenerated as the probability theory when the belief only to be assigned to the single subsets [1, 2].

Definition 1. Let the frame of discernment is $\Theta = \{h_1, h_2, \cdots, h_n\}$. The power set of Θ is 2^Θ, $2^\Theta = \{\emptyset, \{h_1\}, \cdots, \{h_n\}, \{h_1, h_2\}, \cdots, \{h_1, h_2, \cdots, h_i\}, \cdots, \Theta\}$. A basic probability assignment (BPA) function m is a mapping of 2^Θ to a interval $[0, 1]$, defined as [1, 2]:

$$m: \quad 2^\Theta \to [0, 1] \tag{1}$$

which satisfies the two following conditions:

$$m(\emptyset) = 0 \quad \sum_{A \in 2^\Theta} m(A) = 1 \tag{2}$$

where $\emptyset$ is an empty set, and A is any element of 2^Θ. The mass $m(A)$ shows the degree of the evidence support A.

Definition 2. For a BPA m on Θ, each element of 2^Θ such as $m(A) > 0$ is called a focal element of m [1, 2].

Definition 3. For a BPA m on Θ, the belief function Bel and the plausibility function pl are defined, respectively, as [1, 2]

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{3}$$

$$Pl(A) = \sum_{A \cap B \neq \emptyset} m(B) = 1 - \sum_{A \cap B = \emptyset} m(B) \tag{4}$$

The belief function be also called lower function, which can be interpreted as the belief of "A is true". The plausibility function be also called upper function, which can be interpreted as the belief of "A is not false". So the possibility of a subset A lies the interval, $Poss(A) \in [Bel(A), Pl(A)]$. It is obviously that $Bel(\emptyset) = Pl(\emptyset) = 0$, $Pl(\Theta) = Pl(\Theta) = 1$ and If $A \subseteq B \subseteq \Theta$ then $Pl(A) \leq Pl(B)$ and $Bel(A) \leq Bel(B)$.

Definition 4. The pignistic measure of a BPA m on Θ is defined as [30, 65]

$$BetP(A) = \sum_{B \in 2^\Theta} \frac{|A \cap B|}{|B|} \frac{m(B)}{1 - m(\emptyset)}, \forall A \in 2^\Theta \tag{5}$$

where $|A|$ is the cardinality of A. The pignistic measure is a probability distribution on Θ with

$$p_k = BelP(h_k) = \sum_{h_k \in B} \frac{m(B)}{|B|} \tag{6}$$

Definition 5. Assume m is a BPA on Θ, the cost of a focal element A, $A \subseteq \Theta$, is defined as

$$Cost(A) = \frac{n - |A|}{n - 1} \tag{7}$$

where $|A|$ is the cardinality of A.

The cost is a decreasing function of the cardinality, the smaller the cardinality the most the cost. We can know $Cost(\Theta) = \frac{n-n}{n-1} = 0$, it is the least costly. $Cost(h_k) = \frac{n-1}{n-1} = 1$, it is the most costly.

Definition 6. Assume m is a BPA on Θ, the cost of m is defined as

$$Cost(m) = \sum_{A \subseteq \Theta} Cost(A)m(A) \tag{8}$$

When m is a pure probability distribution, m is the most costly to use with $Cost(m) = 1$. When m with one focal element $m(\Theta) = 1$, m is the least costly with $Cost(m) = 0$. It can be seen that more imprecise the BPA the less costly, the more precise the focal elements the more costly.

Definition 7. Assume m is a BPA on Θ, the specificity of m is defined as

$$Sp(m) = \sum_{A \subseteq \Theta, A \neq \emptyset} \frac{m(A)}{|A|} \tag{9}$$

where $|A|$ is the cardinality of A.

When m with one focal element $m(\Theta) = 1$, the specificity of m has minimal value $Sp(m) = \frac{m(\Theta)}{|\Theta|} = \frac{1}{n}$. When m is a pure probability distribution, the specificity of m has maximal value $Sp(m) = \sum_{i=1}^{n} m(\{h_i\}) = 1$.

Definition 8. Given two BPA m_1 and m_2 on Θ, the Dempster combination rule is used to fuse BPAs. The result BPA is denoted as $m_1 \otimes m_2$, is given by [1]:

$$\begin{cases} m_1 \otimes m_2(\emptyset) = 0 \\ m_1 \otimes m_2(A) = \frac{\sum\limits_{B \bigcap C = A} m_1(B)m_2(C)}{1-K} \end{cases} \tag{10}$$

where $K = \sum\limits_{B \bigcap C = \emptyset} m_1(B)m_2(C)$.

The counterintuitive results may be obtained by Dempster combination rule when the fused evidence are highly conflict each other. In order to address this problem, an average method is proposed by Murphy [38]. This method average the masses and then calculate the combined masses by combining the average masses multiple times using classical Dempster combination rule. However, the importance of evidence may not be equal. The weighted average method is proposed by Deng *et al.* based on Murphy's average method [39].

Definition 9. Given two BPA m_1 and m_2 on Θ, the importance weights of two BPAs is w_1, w_2. The weighted average combination method is defined as

$$m_1 \otimes m_2(A) = AVE(m_1 \otimes m_2) \otimes AVE(m_1 \otimes m_2), \quad \forall A \in 2^{\Theta} \tag{11}$$

where

$$AVE(m_1 \otimes m_2)(A) = m_1(A) \times w_1 + m_2(A) \times w_2, \quad \forall A \in 2^{\Theta} \tag{12}$$

2.2. *Intuitionstic fuzzy sets*

Intuitionistic fuzzy sets (IFS) introduced by Atanassov [61] is the extension of traditional fuzzy set, which considers three aspects of the degree of membership, the degree of non-membership and hesitancy. Therefore, it is more flexible and practical than the traditional fuzzy sets in dealing with uncertain information. IFS have been applied in many fields, such as decision making [66, 67], pattern recognition [67, 68], medical diagnosis [69] and as so on [70].

Definition 10. Let $X = \{x_1, x_2, \cdots, x_n\}$ be a universe set, then an intuitionstic fuzzy set (IFS) in X is defined as [61]:

$$A = \{\langle x, \mu_A(x), v_A(x)\rangle | x \in X\} \tag{13}$$

where $\mu_A(x) : X \to [0,1]$ and $v_A(x) : X \to [0,1]$ are the degree of membership and the degree of non-membership, respectively, such that

$$0 \leq \mu_A(x) + v_A(x) \leq 1 \tag{14}$$

The third parameter of IFS is the degree of hesitancy, $\pi_A(x)$:

$$\pi_A(x) = 1 - (\mu_A(x) + v_A(x)) \tag{15}$$

It is obviously that $0 \leq \pi_A(x) \leq 1, \forall x \in X$. When $\pi_A(x) = 0$, the IFS degenerates into the classical fuzzy set. The classical fuzzy set has the form $\{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in X\}$.

3. Intuitionstic evidence sets

The intuitionstic basic probability assignment (IBPA), like BPA, is defined on a finite set of mutually exclusion elements, known as the frame of discernment.

Definition 11. Assume a frame of discernment $\Theta = \{h_1, h_2, \cdots, h_3\}$, its power set is $2^\Theta = \{\emptyset, \{h_1\}, \cdots, \{h_n\}, \{h_1, h_2\}, \cdots, \{h_1, h_2, \cdots, h_i\}, \cdots, \Theta\}$. An intuitionstic basic probability assignment on 2^Θ is defined as

$$m(A) = \langle m^+(A), m^-(A)\rangle \tag{16}$$

where $m^+(A) : 2^\Theta \to [0,1]$ and $m^- : 2^\Theta \to [0,1]$ are support degree and non-support degree, respectively. A is any element of 2^Θ. They must satisfy the following conditions:

(1) $m^+(\emptyset) = 0$ and $m^-(\emptyset) = 0$;

(2) $\sum_{A \in 2^\Theta} m^+(A) = 1$ and $\sum_{A \in 2^\Theta} m^-(A) = 1$;

(3) If $m^+(A) = 1$ then $m^-(A) \neq 1$, if $m^-(A) = 1$, then $m^+(A) \neq 1, \forall A \subset \Theta$.

Definition 12. For an IBPA m on Θ, each subset A of θ such as $m^+(A) > 0$ or $m^-(A) > 0$ is called a focal element of m.

Definition 13. Let m be an IBPA on Θ with intuitionstic probability masses $m(A) = \langle m^+(A), m^-(A) \rangle$, and $A \in 2^\Theta$. The belief function and plausibility function of A also have two components, defined respectively as:

$$Bel(A) = \langle Bel^+(A), Bel^-(A) \rangle \tag{17}$$

$$Pl(A) = \langle Pl^+(A), Pl^-(A) \rangle \tag{18}$$

where

$$Bel^+(A) = \sum_{B \subseteq A} m^+(B) \tag{19}$$

$$Bel^-(A) = \sum_{B \subseteq A} m^-(B) \tag{20}$$

$$Pl^+(A) = \sum_{B \cap A \neq \emptyset} m^+(B) \tag{21}$$

$$Pl^-(A) = \sum_{B \cap A \neq \emptyset} m^-(B) \tag{22}$$

The possibility of a subset A also has two components $Poss(A) = \langle Poss^+(A), Poss^-(A) \rangle$, $Poss^+(A) \in [Bel^+(A), Pl^+(A)]$ and $Poss^-(A) \in [Bel^-(A), Pl^-(A)]$.

Definition 14. Given an IBPA m on Θ with intuitionstic probability masses $m(A) = \langle m^+(A), m^-(A) \rangle$, and $A \in 2^\Theta$. The pignistic probability function of m is defined by:

$$BetP(A) = \langle BelP^+(A), BelP^-(A) \rangle, \forall A \in 2^\Theta \tag{23}$$

where

$$BetP^+(A) = \sum_{B \in 2^\Theta} \frac{|A \cap B|}{|B|} \frac{m^+(B)}{1 - m^+(\emptyset)} \tag{24}$$

$$BetP^-(A) = \sum_{B \in 2^\Theta} \frac{|A \cap B|}{|B|} \frac{m^-(B)}{1 - m^-(\emptyset)} \tag{25}$$

Definition 15. Assume m_1 and m_2 are two IBPAs on Θ such that for each subset A they have $Bel_2^+(A) \leq Bel_1^+(A)$, $Bel_2^-(A) \leq Bel_1^-(A)$, $Pl_2^+(A) \geq Pl_1^+(A)$ and $Pl_2^-(A) \geq Pl_1^-(A)$, $[Bel_1^+(A), Pl_1^+(A)] \subseteq [Bel_2^+(A), Pl_2^+(A)]$ and $[Bel_1^-(A), Pl_1^-(A)] \subseteq [Bel_2^-(A), Pl_2^-(A)]$, we say that m_1 entail m_2, denoted $m_1 \subset m_2$.

If m is an IBPA on Θ, so for each subset A, $Poss^+(A) \in [Bel^+(A), Pl^+(A)]$ and $Poss^-(A) \in [Bel^-(A), Pl^-(A)]$. Assume $Bel^+(A) = a^+$, $Bel^-(A) = a^-$, $Pl^+(A) = b^+$ and $Pl^-(A) = b^-$, $Poss^+(A) \in [a^+, b^+]$ and $Poss^-(A) \in [a^-, b^-]$. If $c^+ \leq a^+$, $c^- \leq a^-$, $d^+ \geq b^+$ and $d^- \geq b^-$, then $Poss^+(A) \in [c^+, d^+]$ and $Poss^-(A) \in [c^-, d^-]$.

Theotem 1. Let m be an IBPA on Θ with intuitionstic probability masses $m(A) = \langle m^+(A), m^-(A) \rangle$, and $A \in 2^\Theta$. The pignistic probability function of singleton of m is an IBPA on Θ.

Proof. m' denotes the pignistic probability transform of singleton of m, so $m'(A) = BetP(A) = \langle BelP^+(A),$
$BelP^-(A) \rangle, \forall A \in \Theta$.

From the definition of IBPA, we can see

$$\sum_{A \in 2^\Theta} m^+(A) = 1, \quad \sum_{A \in 2^\Theta} m^-(A) = 1$$

so,

$$0 \leq BetP^+(A) = \sum_{B \in 2^\Theta} \frac{|A \cap B|}{|B|} m^+(B) \leq 1, \quad 0 \leq BetP^-(A) = \sum_{B \in 2^\Theta} \frac{|A \cap B|}{|B|} m^-(B) \leq 1$$

$$\sum_{A \in \Theta} BetP^+(A) = \sum_{A \in \Theta} \sum_{B \in 2^\Theta} \frac{|A \cap B|}{|B|} m^+(B) = \sum_{B \in 2^\Theta} m^+(B) = 1$$

Similarly,

$$\sum_{A \in \Theta} BetP^-(A) = 1$$

When $BetP^+(A) = 1$,

$$BetP^+(A) = \sum_{B \in 2^\Theta} \frac{|A \cap B|}{|B|} m^+(B) = 1$$

so the BPA m must satisfy $|A \cap B| = |B|, \forall B \in 2^{\Theta}$ and $m(B) \neq 0$, so $m^{+}(A) = 1$. Similarly, when $BetP^{-}(A) = 1$, $m^{-}(A) = 1$. Because $m^{+}(A)$ and $m^{-}(A)$ can not be one at the same time, $BetP^{+}(A)$ and $BetP^{-}(A)$ will not be one at the same time.

Therefore, m' satisfies the three conditions in Definition 11, m' is an IBPA. $\square$

Definition 16. let an intuitionstic probability mass of A, $\langle m^{+}(A), m^{-}(A)\rangle$, in an IBPA m on Θ. The pure support degree is defined as

$$PS(A) = BetP^{+}(A) - BetP^{-}(A) \tag{26}$$

For the problem of target recognition, the bigger the pure support degree, the more likely the target is A. For the problem of decision making, the higher the pure support degree, the alternative A is more in line with requirements.

Definition 17. Assume m is an IBPA on Θ, the cost of a focal element A, $A \subseteq \Theta$, is defined as

$$Cost(A) = \frac{n - |A|}{n - 1} \tag{27}$$

where $|A|$ is the cardinality of A.

It can be seen that $Cost(A) \in [0, 1]$.

Definition 18. Assume m is a IBPA on Θ, the cost of m is defined as

$$Cost(m) = \sum_{A \subseteq \Theta} Cost(A) m^{+}(A) + \sum_{A \subseteq \Theta} Cost(A) m^{-}(A) \tag{28}$$

When all focal elements of m are singleton, the cost of m is the most cost, $Cost(m) = 2$. When m with one focal element $m(\Theta) = \langle 1, 1\rangle$, the cost of m is the least cost, $Cost(m) = 0$.

Definition 19. Assume m is an IBPA on Θ, the specificity of m is defined as

$$Sp(m) = \sum_{A \subseteq \Theta, A \neq \emptyset} \frac{m^{+}(A) + m^{-}(A)}{|A|} \tag{29}$$

where $|A|$ is the cardinality of A.

If all focal elements of m is singleton, the specificity of m is maximal $Sp(m) = 2$. If m only has one element $m(\Theta) = \langle 1, 1 \rangle$, the specificity of m is minimal $Sp(m) = \frac{2}{n}$.

An IBPA can transform into a traditional BPA, but a traditional BPA can not transform into an IBPA, because IBPA contains more information than BPA.

Definition 20. Assume an IBPA m on Θ with intuitionstic probability masses $m(A) = \langle m^+(A), m^-(A) \rangle$, and $A \in 2^\Theta$. m' is a BPA transformed by m, m can be calculate by

$$m(A) = \frac{m^+(A) - m^-(A)}{\sum_{B \subseteq \Theta}(m^+(B) - m^-(B))} \tag{30}$$

It is obviously that $\sum_{A \subseteq \Theta} m(A) = 1$.

An IBPA can be understood as consisting of two BPAs, one representing the support of the evidence and the other expressing the non-support of the evidence. This thinking be adopted in the combination of IBPA. The two BPAs of IBPAs are combined by the classical Dempster-Shafer combination rule.

Definition 21. Given two IBPA m_1 and m_2 on Θ, the combination result IBPA is denoted as $m_1 \otimes m_2$, is given by:

$$m_1 \otimes m_2(A) = \langle m^+(A), m^-(A) \rangle \tag{31}$$

where

$$\begin{cases} m^+(\emptyset) = 0 \\ m^+(A) = \frac{\sum\limits_{B \cap C = A} m_1^+(B) m_2^+(C)}{1 - \sum\limits_{B \cap C = \emptyset} m_1^+(B) m_2^+(C)} \end{cases} \tag{32}$$

$$\begin{cases} m^-(\emptyset) = 0 \\ m^-(A) = \frac{\sum\limits_{B \cap C = A} m_1^-(B) m_2^-(C)}{1 - \sum\limits_{B \cap C = \emptyset} m_1^-(B) m_2^-(C)} \end{cases} \tag{33}$$

Obviously, the problem of evidence conflict also exists in combination of IBPA, the weighted average combination method also can be used in IBPA.

Definition 22. Given two IBPA m_1 and m_2 on Θ, the weighted average combination result IBPA, $m_1 \otimes m_2(A) = \langle m^+(A), m^-(A) \rangle$, is given by:

$$
\begin{aligned}
m_1 \otimes m_2(A) &= AVE(m_1 \otimes m_2(A)) \otimes AVE(m_1 \otimes m_2(A)) \\
&= \langle AVE(m_1^+ \otimes m_2^+) \otimes AVE(m_1^+ \otimes m_2^+), AVE(m_1^- \otimes m_2^-) \otimes AVE(m_1^- \otimes m_2^-) \rangle \quad \forall A \in 2^{\Theta}
\end{aligned} \tag{34}
$$

4. Application in multi-criteria group decision making

Consider the problem of air-condition brands selection. Suppose there are five air-condition brands A_1, A_2, A_3, A_4 and A_5, the alternative set is $A = \{A_1, A_2, A_3, A_4\}$. In order to evaluate alternative air-condition brands, a decision group consists of three decision makers has been formed. The set of decision maker is $D = \{D_1, D_2, D_3\}$. Suppose three criteria C_1(quality), C_2(price), C_3(degree of satisfaction), C_4(function) are considered in the selection problem. The criteria set is $C = \{C_1, C_2, C_3, C_4\}$. Procedure for the selection problem is shown in Fig. 1 and contains the following steps:

Step 1. Determine the weights of criteria.

Evaluations of each criteria by decision makers are shown in Table 1.

The fused result m_c of decision makers' opinions can be calculate by Eq. (31-33)

$$m_c(\{C_1\}) = \langle 0.3810, 0.1250 \rangle,$$

$$m_c(\{C_2\}) = \langle 0.3175, 0.1250 \rangle,$$

$$m_c(\{C_3\}) = \langle 0.1586, 0.2500 \rangle,$$

$$m_c(\{C_4\}) = \langle 0.1429, 0.5000 \rangle$$

so, the weights of criteria $W_c = [w_{c,1}, w_{c,2}, w_{c,3}, w_{c,4}]$ can be calculated by:

$$w_{c,j} = \langle w_{c,j}^+, w_{c,j}^- \rangle = \langle BetP^+(C_j), BetP^-(C_j) \rangle, j = 1, \cdots, 4$$

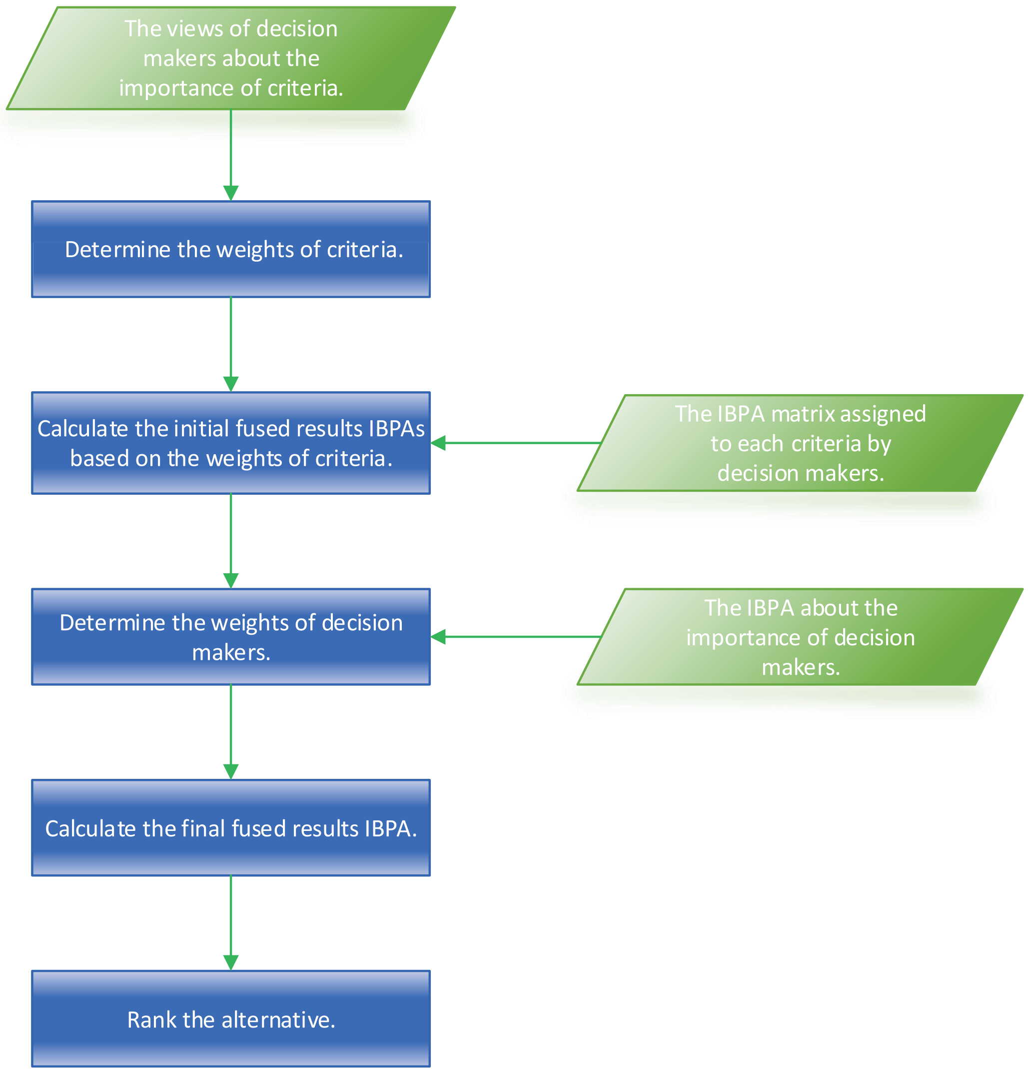

Figure 1: The procedure of proposed method.

D_1		D_2		D_3	
$\{C_1\}$	$\langle 0.3, 0.1\rangle$	$\{C_1\}$	$\langle 0.2, 0.2\rangle$	$\{C_1\}$	$\langle 0.3, 0.1\rangle$
$\{C_2\}$	$\langle 0.2, 0.2\rangle$	$\{C_2\}$	$\langle 0.2, 0.2\rangle$	$\{C_2\}$	$\langle 0.3, 0.1\rangle$
$\{C_3\}$	$\langle 0.2, 0.3\rangle$	$\{C_3\}$	$\langle 0.2, 0.2\rangle$	$\{C_3\}$	$\langle 0.1, 0.2\rangle$
$\{C_4\}$	$\langle 0.2, 0.3\rangle$	$\{C_4\}$	$\langle 0.1, 0.3\rangle$	$\{C_4\}$	$\langle 0.1, 0.2\rangle$
$\{C_1, C_4\}$	$\langle 0.1, 0.1\rangle$	$\{C_1, C_2, C_3\}$	$\langle 0.1, 0.1\rangle$	$\{C_1, C_4\}$	$\langle 0.1, 0.2\rangle$
		$\{C_2, C_3, C_4\}$	$\langle 0.2, 0\rangle$	$\{C_2, C_3\}$	$\langle 0.1, 0\rangle$
				$\{C_2, C_3, C_4\}$	$\langle 0, 0.2\rangle$

Table 1: Opinions of each criteria by decision makers.

So,

$$W_c = [\langle 0.3810, 0.1250\rangle, \langle 0.3175, 0.1250\rangle, \langle 0.1586, 0.2500\rangle, \langle 0.1429, 0.5000\rangle]$$

Step 2. Calculate the initial fused results IBPAs based on the weights of criteria.

The IBPA decision matrix M is

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \\ m_{41} & m_{42} & m_{43} \end{bmatrix}$$

where m_{jk} is an IBPA assigned to criteria C_j by decision maker D_k.

The IBPAs of decision matrix are shown in Tab. 2.

According Eq. (34) the initial fused results $M^{in} = [m_1^{in}, m_2^{in}, m_3^{in}]$ can be calculated by

$$AVE(m_k^{in}) = \sum_{j=1}^{4} m_{j,k} \times w_{cj}, \quad k = 1, 2, 3$$

$$m_k^{in} = AVE(m_k^{in}) \otimes AVE(m_k^{in}) \otimes AVE(m_k^{in}) \otimes AVE(m_k^{in}), \quad k = 1, 2, 3$$

Criteria	Subset of brands	Decision maker		
		D_1	D_2	D_3
	$\{A_1\}$	$\langle 0.22, 0.15\rangle$	$\langle 0.22, 0.12\rangle$	$\langle 0.21, 0.17\rangle$
	$\{A_2\}$	$\langle 0.18, 0.23\rangle$	$\langle 0.19, 0.22\rangle$	$\langle 0.15, 0.33\rangle$
C_1	$\{A_3\}$	$\langle 0.27, 0.08\rangle$	$\langle 0.23, 0.11\rangle$	$\langle 0.25, 0.08\rangle$
	$\{A_4\}$	$\langle 0.18, 0.23\rangle$	$\langle 0.19, 0.22\rangle$	$\langle 0.21, 0.17\rangle$
	$\{A_1, A_2\}$	$\langle 0, 0.31\rangle$	$\langle 0, 0.33\rangle$	$\langle 0, 0.25\rangle$
	$\{A_1, A_3\}$	$\langle 0.15, 0\rangle$	$\langle 0.17, 0\rangle$	$\langle 0.18, 0\rangle$
	$\{A_1\}$	$\langle 0.21, 0.18\rangle$	$\langle 0.23, 0.13\rangle$	$\langle 0.19, 0.23\rangle$
	$\{A_2\}$	$\langle 0.18, 0.23\rangle$	$\langle 0.20, 0.20\rangle$	$\langle 0.22, 0.15\rangle$
C_2	$\{A_3\}$	$\langle 0.29, 0.06\rangle$	$\langle 0.23, 0.13\rangle$	$\langle 0.25, 0.08\rangle$
	$\{A_4\}$	$\langle 0.18, 0.24\rangle$	$\langle 0.17, 0.27\rangle$	$\langle 0.19, 0.23\rangle$
	$\{A_1, A_2\}$	$\langle 0, 0.29\rangle$	$\langle 0, 0.27\rangle$	$\langle 0, 0.31\rangle$
	$\{A_1, A_3\}$	$\langle 0.14, 0\rangle$	$\langle 0.17, 0\rangle$	$\langle 0.15, 0\rangle$
	$\{A_1\}$	$\langle 0.21, 0.14\rangle$	$\langle 0.20, 0.20\rangle$	$\langle 0.24, 0.09\rangle$
	$\{A_2\}$	$\langle 0.18, 0.19\rangle$	$\langle 0.17, 0.30\rangle$	$\langle 0.18, 0.27\rangle$
C_3	$\{A_3\}$	$\langle 0.22, 0.14\rangle$	$\langle 0.23, 0.10\rangle$	$\langle 0.20, 0.18\rangle$
	$\{A_4\}$	$\langle 0.21, 0.14\rangle$	$\langle 0.20, 0.20\rangle$	$\langle 0.20, 0.18\rangle$
	$\{A_1, A_2\}$	$\langle 0, 0.29\rangle$	$\langle 0, 0.20\rangle$	$\langle 0, 0.28\rangle$
	$\{A_1, A_3\}$	$\langle 0.18, 0\rangle$	$\langle 0.20, 0\rangle$	$\langle 0.18, 0\rangle$
	$\{A_1\}$	$\langle 0.22, 0.17\rangle$	$\langle 0.22, 0.15\rangle$	$\langle 0.23, 0.13\rangle$
	$\{A_2\}$	$\langle 0.18, 0.25\rangle$	$\langle 0.15, 0.31\rangle$	$\langle 0.19, 0.20\rangle$
C_1	$\{A_3\}$	$\langle 0.24, 0.08\rangle$	$\langle 0.25, 0.08\rangle$	$\langle 0.23, 0.20\rangle$
	$\{A_4\}$	$\langle 0.21, 0.17\rangle$	$\langle 0.19, 0.23\rangle$	$\langle 0.19, 0.20\rangle$
	$\{A_1, A_2\}$	$\langle 0, 0.33\rangle$	$\langle 0, 0.23\rangle$	$\langle 0, 0.27\rangle$
	$\{A_1, A_3\}$	$\langle 0.15, 0\rangle$	$\langle 0.19, 0\rangle$	$\langle 0.16, 0\rangle$

Table 2: Opinions of each criteria by decision makers.

So,

$$m_1^{in}: m_1^{in}(\{A_1\}) = \langle 0.3534, 0.2805\rangle,$$
$$m_1^{in}(\{A_2\}) = \langle 0.0211, 0.6466\rangle,$$
$$m_1^{in}(\{A_3\}) = \langle 0.5893, 0.0003\rangle,$$
$$m_1^{in}(\{A_4\}) = \langle 0.0256, 0.0070\rangle,$$
$$m_1^{in}(\{A_1, A_2\}) = \langle 0, 0.0655\rangle,$$
$$m_1^{in}(\{A_1, A_3\}) = \langle 0.0100, 0\rangle,$$

$$m_2^{in}: m_2^{in}(\{A_1\}) = \langle 0.4385, 0.2167\rangle,$$
$$m_2^{in}(\{A_2\}) = \langle 0.0211, 0.7222\rangle,$$
$$m_2^{in}(\{A_3\}) = \langle 0.5007, 0.0008\rangle,$$
$$m_2^{in}(\{A_4\}) = \langle 0.0215, 0.0266\rangle,$$
$$m_2^{in}(\{A_1, A_2\}) = \langle 0, 0.0337\rangle,$$
$$m_2^{in}(\{A_1, A_3\}) = \langle 0.0182, 0\rangle,$$

$$m_3^{in}: m_4^{in}(\{A_1\}) = \langle 0.3973, 0.2605\rangle,$$
$$m_3^{in}(\{A_2\}) = \langle 0.0223, 0.6508\rangle,$$
$$m_3^{in}(\{A_3\}) = \langle 0.5330, 0.0084\rangle,$$
$$m_3^{in}(\{A_4\}) = \langle 0.0316, 0.0162\rangle,$$
$$m_3^{in}(\{A_1, A_2\}) = \langle 0, 0.0641\rangle,$$
$$m_3^{in}(\{A_1, A_3\}) = \langle 0.0158, 0\rangle,$$

Step 3. Determine the weights of decision makers.

The IBPA of importance of decision makers m_d is

$$m_d : m_d(\{D_1\}) = \langle 0.4, 0.2 \rangle$$
$$m_d(\{D_2\}) = \langle 0.2, 0.35 \rangle$$
$$m_d(\{D_3\}) = \langle 0.3, 0.25 \rangle$$
$$m_d(\{D_1, D_3\}) = \langle 0.1, 0.1 \rangle$$
$$m_d(\{D_2, D_3\}) = \langle 0, 0.1 \rangle$$

So the set of weights of decision makers, $W_d = [w_{d1}, w_{d2}, w_{d3}]$ can be calculated by

$$w_{dk} = \langle w_{dk}^{+}, w_{dk}^{-} \rangle = \langle BetP^{+}(D_k), BetP^{-}(D_k) \rangle, k = 1, 2, 3$$

So,

$$W_d = [\langle 0.45, 0.25 \rangle, \langle 0.2, 0.4 \rangle, \langle 0.35, 0.35 \rangle].$$

Step 4. Calculate the final fused results IBPA.

The final fused results IBPA m^{fi} is calculated by

$$AVE(m^{fi}) = \sum_{k=1}^{3} m_{k^{in}} \times w_{dk}$$
$$m^{fi} = AVE(m^{fi}) \otimes AVE(m^{fi}) \otimes AVE(m^{fi})$$

So,

$$m^{fi} : m^{fi}(\{A_1\}) = \langle 0.2606, 0.0646 \rangle,$$
$$m^{fi}(\{A_2\}) = \langle 0, 0.9350 \rangle,$$
$$m^{fi}(\{A_3\}) = \langle 0.7393, 0 \rangle,$$
$$m^{fi}(\{A_4\}) = \langle 0.0001, 0 \rangle,$$
$$m^{fi}(\{A_1, A_2\}) = \langle 0, 0.0003 \rangle,$$

Step 5. Rank the alternative.

The pure support degree of each alternative in m^{fi} can be calculate by Eq. (23)-(26). Then rank the alternative according to the pure support degree, and the larger the pure support degree, the more forward the alternative is.

$$PS(A_1) = BetP^+(A_1) - BetP^-(A_1) = 0.2606 - 0.06475 = 0.19585,$$

$$PS(A_2) = BetP^+(A_2) - BetP^-(A_2) = 0 - 0.93515 = -0.93515,$$

$$PS(A_3) = BetP^+(A_3) - BetP^-(A_3) = 0.7393 - 0 = 0.7393,$$

$$PS(A_3) = BetP^+(A_3) - BetP^-(A_3) = 0.0001 - 0 = 0.0001.$$

The pure support degrees of alternatives were determined, and then four alternatives were ranked according to descending order of PS. The alternative were ranked as $A_3 > A_1 > A_4 > A_2$. So A_3 should be selected among the four air-condition brands.

5. Conclusion

Dempster-Shafer evidence can deal with uncertain information well, but it only considers the degree of the evidence support the subsets of the frame of discernment. According the concept of IFS, intuitionstic evidence sets (IES) is presented, it takes into account not only the degree of support, but also the degree of non-support. It can handel more information compared with Dempster-Shfer evidence theory. Therefore, in future, intuitionstic evidence theory can be used in other areas such as target recognition, pattern classification and fault diagnosis.

Acknowledgment

The work is partially supported by National Natural Science Foundation of China (Grant Nos. 61573290, 61503237).

Reference

[1] A. P. Dempster, Upper and lower probabilities induced by a multivalued mapping, The annals of mathematical statistics (1967) 325–339.

[2] G. Shafer, et al., A mathematical theory of evidence, Vol. 1, Princeton university press Princeton, 1976.

[3] W. Feller, An introduction to probability theory and its applications, Vol. 2, John Wiley & Sons, 2008.

[4] S. Le Hegarat-Mascle, I. Bloch, D. Vidal-Madjar, Application of dempster-shafer evidence theory to unsupervised classification in multisource remote sensing, IEEE transactions on geoscience and remote sensing 35 (4) (1997) 1018–1031.

[5] R. Sun, H.-Z. Huang, Q. Miao, Improved information fusion approach based on ds evidence theory, Journal of mechanical science and technology 22 (12) (2008) 2417–2425.

[6] J. Zhang, Y. Deng, A method to determine basic probability assignment in the open world and its application in data fusion and classification, Applied Intelligence 46 (4) (2017) 934–951.

[7] Y. Li, Y. Deng, Generalized ordered propositions fusion based on belief entropy, International Journal of Computers Communications & Control 13 (5) (2018) 771–786.

[8] F. Ye, J. Chen, Y. Li, J. Kang, Decision-Making Algorithm for Multisensor Fusion Based on Grey Relation and DS Evidence Theory, Journal of Sensorsdoi:10.1155/2016/3954573.

[9] P. Dutta, An uncertainty measure and fusion rule for conflict evidences of big data via dempstercshafer theory, International Journal of Image and Data Fusion 9 (2) (2018) 152–169. doi:10.1080/19479832.2017.1391336.

[10] J. Wang, Y. Hu, F. Xiao, X. Deng, Y. Deng, A novel method to use fuzzy soft sets in decision making based on ambiguity measure and Dempster-Shafer theory of evidence: An application in medical diagnosis, Artificial intelligence in medicine 69 (2016) 1–11.

[11] B. Kang, Y. Hu, Y. Deng, D. Zhou, A New Methodology of Multicriteria Decision-Making in Supplier Selection Based on Z-Numbers, Mathematical Problems in Engineeringdoi:10.1155/2016/8475987.

[12] V. Huynh, Y. Nakamori, T. Ho, T. Murai, Multiple-attribute decision making under uncertainty: The evidential reasoning approach revisited, IEEE Transaction on Systems Man and Cybernetics Part A-Systems and Humans 36 (4) (2006) 804–822.

[13] Y. Duan, Y. Cai, Z. Wang, X. Deng, A novel network security risk assessment approach by combining subjective and objective weights under uncertainty, Applied Sciences 8 (3) (2018) Article ID 428. doi:10.3390/app8030428.

[14] K. Chatterjee, E. K. Zavadskas, J. Tamo?aitien?, K. Adhikary, S. Kar, A hybrid mcdm technique for risk management in construction projects, Symmetry 10 (2) (2018) 46. doi:10.3390/sym10020046.
URL `http://www.mdpi.com/2073-8994/10/2/46`

[15] X. Zheng, Y. Deng, Dependence assessment in human reliability analysis based on evidence credibility decay model and iowa operator, Annals of Nuclear Energy 112 (2018) 673–684.

[16] H. Zheng, Y. Deng, Evaluation method based on fuzzy relations between dempster–shafer belief struc-

ture, International Journal of Intelligent Systems 33 (7) (2018) 1343–1363.

[17] X. Guo, Y. Zhou, J. Qian, Y. Deng, Using evidence credibility decay model for dependence assessment in human reliability analysis, Annals of Nuclear Energy 100 (2017) 107–118.

[18] L. Chen, X. Deng, A modified method for evaluating sustainable transport solutions based on ahp and dempstercshafer evidence theory, Applied Sciences 8 (4) (2018) Article ID 563. doi:10.3390/app8040563.

[19] X. Zhang, S. Mahadevan, Aircraft re-routing optimization and performance assessment under uncertainty, Decision Support Systems 96 (2017) 67–82.

[20] X. Xu, P. Liu, Y. Sun, C. Wen, Fault Diagnosis Based on the Updating Strategy of Interval-Valued Belief Structures, Chinese Journal of Electronics 23 (4) (2014) 753–760.

[21] Y. Li, J. Chen, F. Ye, D. Liu, The Improvement of DS Evidence Theory and Its Application in IR/MMW Target Recognition, Journal of Sensors 2016 (1903792).

[22] O. Basir, X. Yuan, Engine fault diagnosis based on multi-sensor information fusion using dempster–shafer evidence theory, Information Fusion 8 (4) (2007) 379–386.

[23] F. Xiao, A novel evidence theory and fuzzy preference approach-based multi-sensor data fusion technique for fault diagnosis, Sensors 17 (11) (2017) 2504.

[24] Y. Gong, X. Su, H. Qian, N. Yang, Research on fault diagnosis methods for the reactor coolant system of nuclear power plant based on D-S evidence theory, Annals of Nuclear Energy (2017) DOI: 10.1016/j.anucene.2017.10.026.

[25] X. Deng, Q. Liu, Y. Deng, S. Mahadevan, An improved method to construct basic probability assignment based on the confusion matrix for classification problem, Information Sciences 340 (2016) 250–261. doi:10.1016/j.ins.2016.01.033.

[26] Z. Liu, Q. Pan, J. Dezert, A. Martin, Combination of classifiers with optimal weight based on evidential reasoning, IEEE Transactions on Fuzzy Systems PP (99) (2017) 1–15. doi:10.1109/TFUZZ.2017.2718483.

[27] Z. Liu, Q. Pan, J. Dezert, J.-W. Han, Y. He, Classifier fusion with contextual reliability evaluation, IEEE Transactions on Cybernetics PP (99) (2017) 1–14. doi:10.1109/TCYB.2017.2710205.

[28] L. A. Zadeh, A simple view of the dempster-shafer theory of evidence and its implication for the rule of combination, Ai Magazine 7 (2) (1986) 85–90.

[29] R. R. Yager, On the dempster-shafer framework and new combination rules, Information sciences 41 (2) (1987) 93–137.

[30] P. Smets, The combination of evidence in the transferable belief model, IEEE Transactions on pattern

analysis and machine intelligence 12 (5) (1990) 447–458.

[31] D. Dubois, H. Prade, Representation and combination of uncertainty with belief functions and possibility measures, Computational intelligence 4 (3) (1988) 244–264.

[32] J. Qian, X. Guo, Y. Deng, A novel method for combining conflicting evidences based on information entropy, Applied Intelligence 46 (4) (2017) 876–888.

[33] J. Wang, F. Xiao, X. Deng, L. Fei, Y. Deng, Weighted Evidence Combination Based on Distance of Evidence and Entropy Function, International Journal of Distributed Sensor Networks 12 (7). doi:10.1177/155014773218784.

[34] F. Xiao, An improved method for combining conflicting evidences based on the similarity measure and belief function entropy, International Journal of Fuzzy Systems (2017) DOI: 10.1007/s40815–017–0436–5.

[35] K.-S. Chin, C. Fu, Weighted cautious conjunctive rule for belief functions combination, Information Sciences 325 (2015) 70–86.

[36] Z.-g. Liu, J. Dezert, Q. Pan, G. Mercier, Combination of sources of evidence with different discounting factors based on a new dissimilarity measure, Decision Support Systems 52 (1) (2011) 133–141.

[37] M. C. Florea, A.-L. Jousselme, É. Bossé, Dynamic estimation of evidence discounting rates based on information credibility, RAIRO-Operations Research 44 (4) (2010) 285–306.

[38] C. K. Murphy, Combining belief functions when evidence conflicts, Decision support systems 29 (1) (2000) 1–9.

[39] D. Yong, S. WenKang, Z. ZhenFu, L. Qi, Combining belief functions based on distance of evidence, Decision support systems 38 (3) (2004) 489–493.

[40] W. Jiang, S. Wang, X. Liu, H. Zheng, B. Wei, Evidence conflict measure based on OWA operator in open world, PloS one 12 (5) (2017) e0177828.

[41] W. Bi, A. Zhang, Y. Yuan, Combination method of conflict evidences based on evidence similarity, Journal of Systems Engineering and Electronics 28 (3) (2017) 503–513.

[42] Y. Zhao, R. Jia, P. Shi, A novel combination method for conflicting evidence based on inconsistent measurements, Information Sciences 367-368 (2016) 125–142.

[43] C. Fu, S. Yang, Conjunctive combination of belief functions from dependent sources using positive and negative weight functions, Expert Systems with Applications 41 (4) (2014) 1964–1972.

[44] Y. Deng, Generalized evidence theory, Applied Intelligence 43 (3) (2015) 530–543.

[45] F. Sabahi, A novel generalized belief structure comprising unprecisiated uncertainty applied to aphasia diagnosis, Journal of Biomedical Informatics 62 (2016) 66–77.

[46] Y. Song, X. Wang, L. Lei, A. Xue, Combination of interval-valued belief structures based on intuitionistic fuzzy set, Knowledge-Based Systems 67 (2014) 61–70.

[47] X. Deng, Analyzing the monotonicity of belief interval based uncertainty measures in belief function theory, International Journal of Intelligent Systems (2018) Published online, doi: 10.1002/int.21999.

[48] C. Fu, S. Yang, Group consensus based on evidential reasoning approach using interval-valued belief structures, Knowledge-Based Systems 35 (2012) 201–210.

[49] C. Fu, S. Yang, The conjunctive combination of interval-valued belief structures from dependent sources, International Journal of Approximate Reasoning 53 (5) (2012) 769–785.

[50] C. Fu, S. Yang, Analyzing the applicability of dempster's rule to the combination of interval-valued belief structures, Expert Systems with Applications 38 (4) (2011) 4291–4301.

[51] Y. Song, X. Wang, L. Lei, S. Yue, Uncertainty measure for interval-valued belief structures, Measurement 80 (2016) 241–250.

[52] Y. Deng, D numbers: Theory and applications, JOURNAL OF INFORMATION &COMPUTATIONAL SCIENCE 9 (9) (2012) 2421–2428.

[53] X. Zhou, Y. Hu, Y. Deng, F. T. Chan, A. Ishizaka, A dematel-based completion method for incomplete pairwise comparison matrix in ahp, Annals of Operations Research (2018) 1–22doi:10.1007/s10479-018-2769-3.

[54] X. Deng, Y. Deng, D-ahp method with different credibility of information, Soft Computing (2018) 1–9.

[55] T. Bian, H. Zheng, L. Yin, Y. Deng, Failure mode and effects analysis based on d numbers and topsis, Quality and Reliability Engineering International 34 (4) (2018) 501–515. doi:10.1002/qre.2268.

[56] T. Liu, Y. Deng, F. Chan, Evidential supplier selection based on dematel and game theory, International Journal of Fuzzy Systems 20 (4) (2018) 1321–1333.

[57] H. Mo, Y. Deng, A new aggregating operator in linguistic decision making based on d numbers, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 24 (6) (2016) 831–846.

[58] X. Zhou, Y. Shi, X. Deng, Y. Deng, D-DEMATEL: A new method to identify critical success factors in emergency management, Safety Science 91 (2017) 93–104.

[59] F. Xiao, An intelligent complex event processing with D numbers under fuzzy environment, Mathematical Problems in Engineering 2016 (1) (2016) 1–10.

[60] L. A. Zadeh, Fuzzy sets, in: Fuzzy Sets, Fuzzy Logic, And Fuzzy Systems: Selected Papers by Lotfi A Zadeh, World Scientific, 1996, pp. 394–432.

[61] K. T. Atanassov, Intuitionistic fuzzy sets., Fuzzy sets and Systems 20 (1) (1986) 87–96.

[62] H. Xu, Y. Deng, Dependent evidence combination based on shearman coefficient and pearson coeffi-

cient, IEEE Access 6 (2018) 11634–11640.

[63] X. Deng, D. Han, J. Dezert, Y. Deng, Y. Shyr, Evidence Combination From an Evolutionary Game Theory Perspective, IEEE TRANSACTIONS ON CYBERNETICS 46 (9) (2016) 2070–2082. doi:10.1109/TCYB.2015.2462352.

[64] R. R. Yager, On viewing fuzzy measures as fuzzy subsets, IEEE Transactions on Fuzzy Systems 24 (4) (2016) 811–818.

[65] P. Smets, R. Kennes, The transferable belief model, Artificial intelligence 66 (2) (1994) 191–234.

[66] H.-W. Liu, G.-J. Wang, Multi-criteria decision-making methods based on intuitionistic fuzzy sets, European Journal of Operational Research 179 (1) (2007) 220–233.

[67] T.-Y. Chen, The inclusion-based topsis method with interval-valued intuitionistic fuzzy sets for multiple criteria group decision making, Applied Soft Computing 26 (2015) 57–73.

[68] F. E. Boran, D. Akay, A biparametric similarity measure on intuitionistic fuzzy sets with applications to pattern recognition, Information Sciences 255 (2014) 45–57.

[69] N. T. Thong, et al., Intuitionistic fuzzy recommender systems: an effective tool for medical diagnosis, Knowledge-Based Systems 74 (2015) 133–150.

[70] X. Zhang, Y. Deng, F. T. Chan, P. Xu, S. Mahadevan, Y. Hu, IFSJSP: A novel methodology for the job-shop scheduling problem based on intuitionistic fuzzy sets, International Journal of Production Research 51 (17) (2013) 5100–5119.