SPACETIME STRUCTURES OF ELECTROMAGNETIC AND MATTER WAVES

Vu B Ho

Advanced Study, 9 Adela Court, Mulgrave, Victoria 3170, Australia

Email: vuhbo@bigpond.net.au

Abstract: In this work we discuss the possibility to construct spacetime structures for electromagnetic and matter waves in which the universal speed in relativity plays a decisive role. Based on our recent works on physical interactions that are associated with the geometric and topological structures of the spatiotemporal manifold, we suggest that when the spatiotemporal manifold decomposes *n*-cells at each point on the manifold then it can be regarded as a fiber bundle where the base space is the spatiotemporal manifold and the fiber is the *n*-cells. We will discuss the case when the *n*-cells are *n*-spheres therefore the total spatiotemporal continuum will be regarded as an *n*-sphere bundle, in particular when *n* equals 6. We will show that the universal speed is the speed at which there is a conversion between the temporal and spatial submanifolds of the spatiotemporal manifold. Since in both Maxwell and Dirac fields there is a conversion between the temporal and spatial forms of matter and since temporal matter is assumed to be associated with the temporal submanifold of the spatiotemporal manifold and spatial matter with the spatial submanifold, therefore in order to describe the conversion of matter we need to formulate the corresponding conversion between the spatial and temporal submanifolds. As a result, the 6-sphere fibers form the required medium for the electromagnetic and matter waves. From the description of waves in terms of spacetime structures it is reasonable to state that both matter and electromagnetic waves are oscillations of the intrinsic geometric structures of the total spatiotemporal manifold.

Despite the electromagnetic field is well-known, thoroughly studied and widely applied in both forms of quantum and classical, the nature of its physical formation still remains a mystery. Besides that, we are even more puzzled with the fact that the electromagnetic field seemingly propagates with a universal speed in all inertial frames, and even though being classically formulated as a wave it does not require a medium to transport energy. Furthermore, perhaps, the most perplexed feature that relates to the electromagnetic field is its exhibition of wave-particle duality. On the other hand, in quantum physics, matter wave is less well-known as a physical wave and it has an unknown physical formulation in terms of the speed of transmission and its wave components, it also shows the property of wave-particle duality, which in fact can partially be shown, in terms of geometric structures, to be due to the quantum manifestation of an extended rather than a localised physical object [1]. Are all of these seemingly unexplainable properties of electromagnetic and matter waves related and determined by the same intrinsic geometric structure of the spatiotemporal manifold? In this work we will address this question by showing that the electromagnetic and matter waves can be described classically as disturbances moving through a medium which is composed of decomposed cells of the spatiotemporal manifold which possesses the geometric and topological structure of an *n*-sphere bundle [2,3], and in which the universal speed in relativity will play a decisive role. It is worth mentioning here that the postulate of the universal speed of light in vacuum is one of the two foundational pillars in Einstein theory of special relativity and then also in general relativity [4]. However, the question of whether the speed of light should be universal with a global character has still been raised and discussed [5,6]. Because the question about the unaccountable properties of electromagnetic and matter waves is more concerned with the global formulation of general relativity, in order to answer this question we would need to know about the geometric character and possible relationships between the electromagnetic and matter fields and the gravitational field, all of which are assumed to be related to the geometric and topological structures of the spatiotemporal manifold. In fact, this question has been investigated since the publication of Einstein general relativity but only with attempts to unify physical fields in different unified mathematical formulations which specify physical fields as different mathematical objects possessed by the spatiotemporal manifold rather than different physical formations of the same mathematical structure. For example, it is quite reasonable to suggest that the electromagnetic field is related to the quantum structure of the spatiotemporal manifold and the gravitational field is related to its global structure which exhibits physical laws on macroscopic scale. In the following we will discuss the possibility to construct spacetime structures for electromagnetic and matter waves based on our recent works on the geometric and topological structures of the spatiotemporal manifold by suggesting that when the manifold decomposes *n*-cells at each point then it can be regarded as a fiber bundle where the base space is the spatiotemporal manifold and the fibers are the *n*-cells. However, we will only discuss the case when the *n*-cells are *n*-spheres therefore the total spatiotemporal continuum M will be regarded as an *n*-sphere bundle, in particular when n equals 6. We will show that the universal speed is the speed at which there is a conversion between the temporal and spatial submanifolds of the spatiotemporal manifold. Since in both Maxwell and Dirac fields there is a conversion between temporal and spatial forms of matter and since temporal matter is assumed to be associated with the temporal submanifold of the spatiotemporal manifold and spatial matter with the spatial submanifold, therefore in order to describe the conversion of matter we need to formulate the corresponding conversion between the spatial and temporal submanifolds. With the view that the electromagnetic and matter waves are disturbances that convert the spatiotemporal geometric structures between temporal and spatial manifolds, as a result, we assume that the 6-sphere fibers form the required medium for the electromagnetic and matter waves. From the description of waves in terms of spacetime structures it is reasonable to state that both matter and electromagnetic waves are oscillations of the intrinsic geometric structures of the total spatiotemporal manifold. In brief, we showed in our previous works on Dirac and Maxwell fields that temporal matter and spatial matter are converted from one form to another and the conversions manifest as physical fields [7,8]. It was shown that both Dirac and Maxwell field equations can be formulated from a general system of linear first order partial differential equations

$$\left(\sum_{i=1}^{n} A_i \frac{\partial}{\partial x_i}\right)\psi = k_1\sigma\psi + k_2 J \tag{1}$$

where $\psi = (\psi_1, \psi_2, \dots, \psi_n)^T$, $\partial\psi/\partial x_i = (\partial\psi_1/\partial x_i, \partial\psi_2/\partial x_i, \dots, \partial\psi_n/\partial x_i)^T$, A_i, σ and J are matrices representing the undetermined physical quantities, and k_1 and k_2 are undetermined constants. In order for the above systems of partial differential equations to be applied to physical phenomena, the matrices A_i must be determined. For the case of Dirac and Maxwell field equations, the matrices A_i must take a form so that Equation (1) reduces to a wave equation

$$\left(\sum_{i=1}^{n} {A_i}^2 \frac{\partial^2}{\partial x_i^2}\right)\psi = k_1^2\sigma^2\psi + k_1 k_2 \sigma J + k_2 \sum_{i=1}^{n} A_i \frac{\partial J}{\partial x_i} \tag{2}$$

With $\psi = \left(E_x, E_y, E_z, B_x, B_y, B_z\right)^T = (\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6)^T$ and $J = (j_1, j_2, j_3, j_4, j_5, j_6)^T$ is the electromagnetic current in which the electric current is $j_e = (j_1, j_2, j_3)$ and the magnetic current is $j_m = (j_4, j_5, j_6)$, Maxwell field equations that are derived in classical physics from Faraday and Ampere laws can be written the following matrix form

$$\left(A_1 \frac{\partial}{\partial t} + A_2 \frac{\partial}{\partial x} + A_3 \frac{\partial}{\partial y} + A_4 \frac{\partial}{\partial z}\right)\psi = A_5 J \tag{3}$$

where the matrices A_i are given as

$$A_1 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad A_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad A_5 = \begin{pmatrix} \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{4}$$

Furthermore, if an additional condition that imposes on the function ψ that requires that it also satisfies the wave equation given by Equation (2) then Gauss's laws will be recovered. We also showed that the electromagnetic field can be viewed as a conversion between the density of the temporal matter defined as magnetic charge q_m, and the density of the spatial matter defined as electric charge q_e. In particular, we showed that the electric charge and the magnetic charge can be viewed as topological structures of the spatiotemporal manifold and they relate to each other via Dirac relation $\hbar c/q_e q_m = 2$. On the other hand, Dirac equation can be derived from Equation (1) by simply imposing the following conditions on the matrices A_i

$$A_i^2 = \pm 1 \tag{5}$$

$$A_iA_j + A_jA_i = 0 \quad \text{for} \quad i \neq j \tag{6}$$

For the case of $n = 4$, the matrices A_i can be shown to take the form

$$A_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad A_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \qquad A_4 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{7}$$

Dirac field of massive particles may also be viewed as being composed of two different physical fields, namely the field (ψ_1, ψ_2) which is associated with the spatial mass m and the field (ψ_1, ψ_2) which is associated with the temporal mass $D = -m$.

Now, there are two possibilities that we may consider for the formulation of electromagnetic and matter waves as spacetime structures, one with a global decomposition of *n*-cells which may be used to describe gravity and one with a local decomposition which may be used to describe quantum physical fields. If we assume that the spatiotemporal manifold is described by a six-dimensional differentiable manifold M which is composed of a three-dimensional spatial manifold and a three-dimensional temporal manifold, in which all physical objects are embedded, then the manifold M can be decomposed in the form $M = M \# S_S^3 \# S_T^3$, where S_S^3 and S_T^3 are spatial and temporal spheres, respectively. Even though this form of decomposition can be used to describe gravity as a global structure it cannot be used as a medium for any other physical fields which possess a wave character. Therefore we would need to devise different types of decomposition to account for these physical fields. In classical physics, the formation of a wave requires a medium which is a collection of physical objects therefore with this classical picture in mind we may assume that the medium for the electromagnetic and matter waves is composed of quantum particles which have the geometric and topological structures of spatiotemporal *n*-cells that are decomposed from the spatiotemporal manifold at each point of the spatiotemporal continuum. This is equivalent to considering the spatiotemporal manifold as a fiber bundle $E = B \times F$, where B is the base space, which is the spatiotemporal continuum, and the fiber F, which is the *n*-cells. In the following we will only consider an *n*-cell as an *n*-sphere S^n and the total spatiotemporal manifold M will be regarded as an *n*-sphere bundle. It is reasonable to suggest that there may exist physical fields that are associated with different dimensions of the *n*-spheres, however, as an illustration, we will consider only the case with $n = 6$ so that S^6 is homeomorphic to $S_S^3 \times S_T^3$, hence the medium of the electromagnetic and matter waves will be assumed to be composed of $S_S^3 \times S_T^3$ cells at each point of the spatiotemporal manifold. In other words, the 6-sphere fibers form the required medium for the electromagnetic and matter waves. Consequently, the problem that we want to address reduces to the problem of the conversion between the spatial and temporal manifolds S_S^3 and S_T^3. It is expected that the formulation of such conversion should be derived from a general line element $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$. As examples, we will show in the following that the conversion of between the spatial and temporal manifolds S_S^3 and S_T^3 can be described by assuming the general line element to take the form of either a centrally symmetric metric or the Robertson-Walker metric [9]. A general six-dimensional centrally symmetric metric can be written as

$$ds^2 = e^{\psi}c^2dt^2 + c^2t^2\left(d\theta_T{}^2 + \sin^2\theta_T d\phi_T{}^2\right) - e^{\chi}dr^2 - r^2\left(d\theta_S{}^2 + \sin^2\theta_S d\phi_S{}^2\right) \quad (8)$$

If we rearrange the (θ, ϕ) directions of both the spatial and the temporal cells so that they coincide, $\theta_S = \theta_T = \theta$ and $\phi_S = \phi_T = \phi$, then we have

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 - (r^2 - c^2t^2)(d\theta^2 + \sin^2\theta d\phi^2) \quad (9)$$

There are profound differences in the structure of the spatiotemporal manifold that arise from the line element given in Equation (9). The line element in Equation (9) can be re-written in the form

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 - r^2\left(1 - \frac{c^2}{v^2}\right)(d\theta^2 + \sin^2\theta d\phi^2) \quad (10)$$

where we have defined the new quantity that has the dimension of speed as $v = r/t$. It is seen from Equation (10) that if $v > c$ then the line element given in Equation (10) can lead to the conventional structure of spacetime in which, effectively, space has three dimensions and time has one dimension, and that if $v < c$ then the line element given in Equation (10) can lead to the conventional structure of spacetime in which time has three dimensions and space has one. However, for the purpose of discussing a conversion between the temporal manifold and the spatial manifold of spacetime we would need to consider possible relationship between space and time and how they change with respect to each other continuously. In order to fulfil this task we need to utilise the results obtained in our previous work on temporal geometric interactions that show that there are various temporal forces associated with the decomposed *n*-cells from which, by applying Newton's law of dynamics, different possible relationships between space and time could be derived [10]. For example, by applying the temporal Newton's second law for radial motion to the force that is associated with decomposed 1-cells we obtain

$$D\frac{d^2t}{dr^2} = h_1 t \quad (11)$$

General solutions to the equation given in Equation (11) are

$$t = c_1 e^{\sqrt{h_1/D}r} + c_2 e^{-\sqrt{h_1/D}r} \quad (12)$$

If $D = -m$ and $h_1 > 0$ then the following solution can be obtained

$$t = A\sin(\omega r) \quad (13)$$

where $\omega = \sqrt{h_1/D}$. By differentiation we have

$$\frac{dt}{dr} = A\omega\cos(\omega r) \tag{14}$$

If we assume a linear approximation between space and time for the values of $v \sim c$, i.e., $dr/dt \sim r/t = v$, then Equation (10) becomes

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 - r^2\big(1 - c^2A^2\omega^2\cos^2(\omega r)\big)(d\theta^2 + \sin^2\theta d\phi^2) \tag{15}$$

It is seen from Equation (15) that if $1 - c^2A^2\omega^2\cos^2(\omega r) > 0$ then effectively spacetime appears as a spatial manifold in which there are three spatial dimensions and one temporal dimension. Therefore it is expected that for $1 - c^2A^2\omega^2\cos^2(\omega r) < 0$ spacetime would appear as a temporal manifold. This is in fact the case as can be shown as follows. Instead of the metric form given in Equation (10), the line element given in Equation (9) can also be re-written in a different form as follows

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 + c^2t^2\left(1 - \frac{v^2}{c^2}\right)(d\theta^2 + \sin^2\theta d\phi^2) \tag{16}$$

Using Equation (14) we obtain

$$ds^2 = e^{\psi}c^2dt^2 + c^2t^2\left(1 - \frac{1}{c^2A^2\omega^2\cos^2(\omega r)}\right)(d\theta^2 + \sin^2\theta d\phi^2) - e^{\chi}dr^2 \tag{17}$$

Therefore, if the condition $1 - c^2A^2\omega^2\cos^2(\omega r) < 0$ is satisfied then Equation (17) is reduced to a line element for the spatiotemporal manifold which effectively has three temporal dimensions and one spatial dimension. For the case $r^2 - c^2t^2 \neq 0$ the line element given in Equation (9) can be determined by applying Einstein field equations of general relativity [11]

$$ds^2 = \left(1 - \frac{2GM}{c^2r}\right)c^2dt^2 - \left(1 - \frac{2GM}{c^2r}\right)^{-1}dr^2 - (r^2 - c^2t^2)(d\theta^2 + \sin^2\theta d\phi^2) \tag{18}$$

It should also be mentioned here that for the case $r^2 - c^2t^2 = 0$, the line element given in Equation (9) reduces to the simple form

$$ds^2 = e^{\psi}c^2dt^2 - e^{\chi}dr^2 \tag{19}$$

and as discussed in our previous works that spacetime that is endowed with this particular metric appears to behave as a wave where the functions ψ and χ satisfy the wave equation

$$\frac{\partial^2\psi}{c^2\partial t^2} - \frac{\partial^2\psi}{\partial r^2} = 0 \tag{20}$$

Now, we consider the case when the decomposed $S_S^3 \times S_T^3$ cells from the spatiotemporal manifold are furnished with the Robertson-Walker metric. In the spatiotemporal manifold which has three spatial dimensions and one temporal dimension, the Robertson-Walker metric is given as

$$ds^2 = c^2dt^2 - S^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) \tag{21}$$

With the decomposition of $S_S^3 \times S_T^3$ cells from the spatiotemporal manifold which has the mathematical structure of an *n*-sphere bundle, the Robertson-Walker metric is assumed to be extended to a six-dimensional line element of the form

$$ds^2 = S^2(r)\left(\frac{dt^2}{1-k_Tt^2} + t^2\left(d\theta_T{}^2 + \sin^2\theta_T d\phi_T{}^2\right)\right) - S^2(t)\left(\frac{dr^2}{1-k_Sr^2} + r^2\left(d\theta_S{}^2 + \sin^2\theta_S d\phi_S{}^2\right)\right) \tag{22}$$

If we also arrange the (θ, ϕ) directions of both spatial and the temporal manifolds so that $\theta_S = \theta_T = \theta$ and $\phi_S = \phi_T = \phi$ then the general space-time metric given in Equation (22) becomes

$$ds^2 = \frac{S^2(r)dt^2}{1-k_Tt^2} - \frac{S^2(t)dr^2}{1-k_Sr^2} - \left(r^2S^2(t) - t^2S^2(r)\right)(d\theta^2 + \sin^2\theta d\phi^2) \tag{23}$$

Equation (23) can be rewritten in the following form

$$ds^2 = \frac{S^2(r)dt^2}{1-k_Tt^2} - \frac{S^2(t)dr^2}{1-k_Sr^2} - \left(S^2(t) - \frac{1}{v^2}S^2(r)\right)r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{24}$$

where we have also defined $v = r/t$. Now, we need to look for possible relationships between space and time so that they can show a conversion between the temporal component S_T^3 and the spatial component S_S^3 of the decomposed spatiotemporal cells $S_S^3 \times S_T^3$. Even though the conditions that will be imposed are rather arbitrarily they do show that the temporal manifold S_T^3 and the spatial manifold S_S^3 can actually be converted into one another. It should also be mentioned that these are not the only conditions that can give rise to a conversion between space and time and, as shown in our works on Euclidean relativity, Euclidean special relativity also produces such conversion [11]. Now, if we impose the following condition

$$\frac{S^2(r)}{1-k_Tt^2} = c^2 \tag{25}$$

then the line element given in Equation (24) reduces to

$$ds^2 = c^2dt^2 - \frac{S^2(t)dr^2}{1-k_Sr^2} - \left(S^2(t) - \frac{c^2}{v^2}(1-k_Tt^2)\right)r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{26}$$

Equation (25) describes particular structures of the temporal manifold with respect to the change of the spatial manifold. Using a linear approximation between space and time for the values of $v \sim c$, then from the relation $1/v^2 = A^2\omega^2\cos^2(\omega r)$, Equation (26) becomes

$$ds^2 = c^2dt^2 - \frac{S^2(t)dr^2}{1-k_Sr^2} - S^2(t)\left(1 - \frac{c^2A^2\omega^2\cos^2(\omega r)(1-k_Tt^2)}{S^2(t)}\right)r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (27)$$

If we further impose the condition

$$\frac{S^2(t)}{1-k_Tt^2} = 1 \quad (28)$$

then we obtain

$$ds^2 = c^2dt^2 - \frac{S^2(t)dr^2}{1-k_Sr^2} - S^2(t)\left(1 - c^2A^2\omega^2\cos^2(\omega r)\right)r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (29)$$

It is seen from the line element given in Equation (29) that if $\left(1 - c^2A^2\omega^2\cos^2(\omega r)\right) > 0$ then effectively the spatiotemporal manifold behaves as a spatial manifold endowed with the Robertson-Walker metric given in Equation (21). On the other hand, the six-dimensional Robertson-Walker metric given in Equation (23) can also be written as

$$ds^2 = \frac{S^2(r)dt^2}{1-k_Tt^2} - \frac{S^2(t)dr^2}{1-k_Sr^2} - \left(v^2S^2(t) - S^2(r)\right)t^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (30)$$

If we impose the following condition

$$\frac{S^2(t)}{1-k_Sr^2} = {c_T}^2 \quad (31)$$

then we obtain

$$ds^2 = \frac{S^2(r)dt^2}{1-k_Tt^2} - {c_T}^2dr^2 + S^2(r)\left(1 - \frac{v^2{c_T}^2(1-k_Sr^2)}{S^2(r)}\right)t^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (32)$$

From the linear approximation $1/v^2 \sim A^2\omega^2\cos^2(\omega r)$, Equation (32) becomes

$$ds^2 = \frac{S^2(r)dt^2}{1-k_Tt^2} - {c_T}^2dr^2 + S^2(r)\left(1 - \frac{{c_T}^2(1-k_Sr^2)}{A^2\omega^2\cos^2(\omega r)S^2(r)}\right)t^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (33)$$

If we further impose the condition

$$\frac{S^2(r)}{1-k_Sr^2} = {c_T}^2c^2 \quad (34)$$

then we obtain

$$ds^2 = \frac{S^2(r)dt^2}{1 - k_T t^2} - c_T{}^2 dr^2 + S^2(r)\left(1 - \frac{1}{c^2 A^2 \omega^2 \cos^2(\omega r)}\right) t^2(d\theta^2 + \sin^2\theta d\phi^2) \qquad (35)$$

Therefore if $\left(1 - c^2 A^2 \omega^2 \cos^2(\omega r)\right) < 0$ then effectively the spatiotemporal manifold behaves as a temporal manifold endowed with the temporal Robertson-Walker metric [12]

$$ds^2 = S^2(r)\left(\frac{dt^2}{1 - k_T t^2} + t^2(d\theta^2 + \sin^2\theta d\phi^2)\right) - c_T{}^2 dr^2 \qquad (36)$$

It is also noted from the line element given in Equation (23) that when space and time satisfy the condition $r^2 S^2(t) - t^2 S^2(r) = 0$ then we have

$$ds^2 = \frac{S^2(r)dt^2}{1 - k_T t^2} - \frac{S^2(t)dr^2}{1 - k_S r^2} \qquad (37)$$

The metric given in Equation (37) is a particular form of the general line element given in Equation (19) with $S^2(r)/(1 - k_T t^2) = e^{\psi} c^2$ and $S^2(t)/(1 - k_S r^2) = e^{\chi}$, therefore the wave motion of spacetime which is endowed with the Roberson-Walker metric also occurs at the position of conversion between the temporal and spatial manifolds.

References

[1] Vu B Ho, *Spacetime Structures of Quantum Particles* (Preprint, ResearchGate, 2017), viXra 1708.0192v2, Int. J. Phys. vol 6, no 4 (2018): 105-115. doi: 10.12691/ijp-6-4-2.

[2] Allen Hatcher, *Algebraic Topology*, 2001.

[3] S. S. Chern and E. Spanier, *The Homology Structure of Sphere Bundles*, Proc. NAS, vol. 36 (1950).

[4] A. Einstein, *The Principle of Relativity* (Dover Publications, New York, 1952).

[5] Rossler O E, 2015 *Progress in Physics* **11**, 340-343.

[6] Rossler O E, 2009 *Chaos, Solitons and Fractals*.

[7] Vu B Ho, *On Dirac Negative Mass and Magnetic Monopole* (Preprint, ResearchGate, 2018), viXra 1806.0319v1.

[8] Vu B Ho, *On the Principle of Least Action*, ResearchGate (2016), viXra 1708.0226v2, Int. J. Phys, vol. 6, no. 2 (2018): 47-52. doi: 10.12691/ijp-6-4-2.

[9] Ray D'Inverno, *Introducing Einstein's Relativity* (Clarendon Press, Oxford, 1992).

[10] Vu B Ho, *Temporal Geometric Interactions* (Preprint, ResearchGate, 2018), viXra 1807.0134v1.

[10] Vu B Ho, *A Quantum Structure of Spacetime* (Preprint, ResearchGate, 2017).

[11] Vu B Ho, *Euclidean Relativity* (Preprint, ResearchGate, 2017), viXra 1710.0302v1.

[12] Vu B Ho, *A Theory of Temporal Relativity* (Preprint, ResearchGate, 2016), viXra 1708.0196v1.