GAME OF INTEGRALS

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July 10 , 2018

Abstract
This note presents some trigonometric integrals

Key words and phrases
Trigonometric integrals, number pi, radicals

SOME TRIGONOMETRIC INTEGRALS

$$\int_0^{\pi} x\sqrt{1+\sqrt{1+\cos^4 x}}\,\sin x\,dx =$$

$$= \frac{\pi}{4}\left\{2\sqrt{1+\sqrt{2}}+2\tan^{-1}\left(\sqrt{-1+\sqrt{2}}\right)+\ln\left(1+\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}\right)\right\} \tag{1}$$

$$\int_0^{\pi} x\sqrt{2+\cos^2 x+2\sqrt{1+\cos^2 x+\cos^4 x}}\,\sin x\,dx =$$

$$= \frac{\pi}{4}\left\{2\sqrt{3+2\sqrt{3}}+2\tan^{-1}\left(\sqrt{-1+\frac{2}{\sqrt{3}}}\right)+\sqrt{3}\ln\left(1+\sqrt{3}+\sqrt{3+2\sqrt{3}}\right)\right\} \tag{2}$$

$$\int_0^{\pi} x\sqrt{2-\cos^2 x+2\sqrt{1-\cos^2 x+\cos^4 x}}\,\sin x\,dx = \frac{\pi}{8}\left\{4\sqrt{3}+\pi\sqrt{3}+2\ln\left(2+\sqrt{3}\right)\right\} \tag{3}$$

$$\int_0^{\pi} x\sqrt{25+7\cos^2 x+5\sqrt{25+14\cos^2 x+25\cos^4 x}}\,\sin x\,dx =$$

$$= \frac{\pi}{\sqrt{2}}\left\{6+3\tan^{-1}\frac{1}{2}+2\ln 5\right\} \tag{4}$$

$$\int_0^{\pi} x\sqrt{2+\sqrt{2}\cos^2 x+2\sqrt{1+\sqrt{2}\cos^2 x+\cos^4 x}}\,\sin x\,dx =$$

$$= \frac{\pi}{4}\left\{2\sqrt{2+\sqrt{2}+2\sqrt{2+\sqrt{2}}}+2\sqrt{2-\sqrt{2}}\,\tan^{-1}\left(\sqrt{-1+\sqrt{4-2\sqrt{2}}}\right)+\right. \tag{5}$$

$$\left.+\sqrt{2+\sqrt{2}}\ln\left(1+\sqrt{2+\sqrt{2}}+\sqrt{2+\sqrt{2}+2\sqrt{2+\sqrt{2}}}\right)\right\}$$

$$\int_0^{\pi} x\sqrt{-1+\sqrt{1+\cos^4 x}}\,\sin x\,dx = \\ = \frac{\pi}{4}\left\{2\sqrt{-1+\sqrt{2}}+2\tan^{-1}\left(\sqrt{-1+\sqrt{2}}\right)-\ln\left(1+\sqrt{2}+\sqrt{2+2\sqrt{2}}\right)\right\} \tag{6}$$

$$\int_0^{\pi} x\sqrt{-2-\cos^2 x+2\sqrt{1+\cos^2 x+\cos^4 x}}\,\sin x\,dx = \\ = \frac{\pi}{4}\left\{2\sqrt{-3+2\sqrt{3}}+2\sqrt{3}\tan^{-1}\left(\sqrt{-1+\frac{2}{\sqrt{3}}}\right)+\ln\left(1+\sqrt{3}-\sqrt{3+2\sqrt{3}}\right)\right\} \tag{7}$$

$$\int_0^{\pi} x\sqrt{-2+\cos^2 x+2\sqrt{1-\cos^2 x+\cos^4 x}}\,\sin x\,dx = \frac{\pi}{8}\left\{4+\pi-2\sqrt{3}\ln\left(2+\sqrt{3}\right)\right\} \tag{8}$$

$$\int_0^{\pi} x\sqrt{-25-7\cos^2 x+5\sqrt{25+14\cos^2 x+25\cos^4 x}}\,\sin x\,dx = \\ = \frac{\pi}{2\sqrt{2}}\left\{4+8\tan^{-1}\frac{1}{2}-3\ln 5\right\} \tag{9}$$

$$\int_0^{\pi} x\sqrt{-2-\sqrt{2}\cos^2 x+2\sqrt{1+\sqrt{2}\cos^2 x+\cos^4 x}}\,\sin x\,dx = \\ = \frac{\pi}{4}\left\{2\sqrt{-2-\sqrt{2}+2\sqrt{2+\sqrt{2}}}+2\sqrt{2+\sqrt{2}}\tan^{-1}\left(\sqrt{-1+\sqrt{4-2\sqrt{2}}}\right)\right. \\ \left.-\sqrt{2-\sqrt{2}}\ln\left(1+\sqrt{2+\sqrt{2}}+\sqrt{2+\sqrt{2}+2\sqrt{2+\sqrt{2}}}\right)\right\} \tag{10}$$

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