The Chronology Protection Conjecture and the Formation of the Kerr Black Hole through Gravitational Collapse

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Abstract

Exterior of the Kerr black hole is considered as the likely spacetime geometry around a black hole formed by gravitational collapse. However the Kerr black hole has chronology violating Closed Time-like Curves (CTCs), near the ring singularity in interior. Hawking has considered formation of CTCs using a scalar field and shown that back reaction of nature - divergence of energy-momentum tensor - would prevent formation of CTCs. The question being raised here is whether the physics behind Hawking's concept of "chronology protection", will actually prevent formation of the Kerr black hole - through gravitational collapse. On the other hand, if the Kerr black hole indeed gets formed by gravitational collapse, then one has a counter example to Chronology Protection Conjecture. Chronology protection thus imposes the constraint that formation of a rotating black hole through gravitational collapse should be such that the interior metric of the black hole does not match that of the Kerr black hole, or any other black hole with CTCs. It is possible that the quantum gravity effects rule out formation of CTCs during gravitational collapse - as they are expected to remove singularities. There may however be classical solutions to the problem. Angular momentum orginating from the rotation and the formation of CTCs are intimately related - through the off diagonal term $g_{t\phi}$ i.e., the coefficient of the $dtd\phi$ term (corresponding to the temporal coordinate t and the longitudinal angular coordinate ϕ) - of the metric tensor. A possible solution is that during the gravitational collapse, all the angular momentum of the star, is radiated away by gravitational waves - leaving behind as residual, the Schwarzschild black hole.

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Schwarzschild [1] worked out an exact solution of the Einstein's Field Equations which described space time geometry of a point mass. A spherically symmetric gravitational collapse is generally considered to lead to the formation of a Schwarzschild black hole. However, a more realistic scenario would be when the collapsing matter has non-zero angular momentum. The end point of such a collapse would be a rotating black hole - notably, the one worked out by Kerr [2]. Carter [3] showed that the Kerr black hole had causality violating Closed Timelike Curves (CTCs). Uniqueness of a black hole is provided by the no hair theorem [4, 5, 6] whereas latter investigations provided counter examples [7, 8, 9].

In a landmark paper, Hawking put forth the "Chronology Protection Conjecture" [10]. This conjecture states that -

"The laws of physics do not allow the appearance of closed timelike curves".

Using the evolution of a scalar field, Hawking showed, that if through some mechanism, a CTC is about to form, the energy-momentum tensor of the field would diverge - which in turn would prevent the formation of the CTCs.

On the other hand, if the Kerr black hole does indeed get formed by gravitational collapse, then it is a counter example to the Hawking's chronology protection conjecture.

Nakamura [15] initiated numerical general relativistic studies of stellar collapse and black hole formation. Biotti et. al. [16] have done three dimensional relativistic simulations of rotating neutron star collapse to a Kerr black hole. It is expected that after a black hole has formed and settled down, its multipole moments should be identical to the source multipoles of a Kerr black hole. Schnetter, Krishnan and Beyer [17] using numerical general relativity, show that, it is in principle, possible to verify this conjecture and to calculate the rate at which a black hole approaches equilibrium. Given these numerical studies, one is lead to expect that from a classical view point, stellar collapse would indeed lead to formation of the Kerr black hole.

Aside from numerical studies, there are analytical studies of gravitational collapse. Most studies of gravitational collapse start with a spherically symmetric metric. There also exist various perturbations of the Kerr solution, and without the CTCs would be the likely candidates for end point of gravitational collapse. A number of authors have considered evolution of a scalar field in gravitational collapse and formation of black hole.

For various rotating solutions of Einstein's Field Equations, - the off diagonal term $g_{t\phi}$ i.e., the coefficient of $dtd\phi$ term (corresponding to the temporal coordinate t and the angular coordinate ϕ) of the metric tensor, is related to the spatial rotation as well as the CTCs. For instance, this term is given as follows -

1. for the Kerr Black hole -

$$g_{t\phi} = 2aMr\sin^2\theta/\rho^2 \tag{1}$$

2. for the Gödel universe [11] -

$$g_{t\phi} = 2\sqrt{2}\sinh^2 r \tag{2}$$

3. for the van Stockum's cylinder [12, 13] -

$$g_{t\phi} = M(r) \tag{3}$$

4. and for the Tomimatsu-Sato solution [14] -

$$g_{t\phi} = 2f\omega \tag{4}$$

An interesting possibility, to avoid the CTC generating ring singularity of the Kerr black hole - is to impose the condition that, the gravitational waves carry away the complete angular momentum of the collapsing star, leaving behind a residual Schwarszchild black hole. Garecki [18] shows that gravitational waves indeed carry away angular momentum. Owen et. al. [19] have shown that angular momentum J of a young hot rapidly rotating star is carried away by gravitational waves, as follows -

$$\frac{dJ}{dt} = 4\pi D^2 \frac{m}{\omega}\frac{1}{16\pi}\omega^2\left\langle h_+^2 + h_\times^2\right\rangle \tag{5}$$

where, D is the distance to source, h_+^2 and $h_\times^2$ are amplitudes of the two polarizations of gravitational waves and $\langle ... \rangle$ denotes an average over the orientation of the source and its location on the observers sky. The loss of angular momentum in turn accelerates the gravitational collapse.

Quantum gravity effects can also be expected to rule out singularities. Campiglia et. al. [20] show that singularity at origin is avoided in gravitating quantum matter. This result however, is for the Schwarzschild black hole. The result is due to discretization of the spacetime. It is not immediately clear, if some form of ring singularity of the Kerr black hole would also get removed. Despite the discretization, some form of CTCs can be still present. An example is the 2-D Ising model of Politzer [21] which models a time machine - and is based upon discrete spacetime.

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