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Structural Damage Information Decision Based on Z-numbers

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Abstract

Structural health monitoring (SHM) has grate economic value and research value because of the application of finite element model technology, structural damage identification theory, intelligent sensing system, signal processing technology and as so on. A typical SHM system involved three major subsystems: a sensor subsystem, a data processing subsystem and a health evaluation subsystem. It is significance of sensor data fusion for the data processing subsystem. In this paper, considering the fuzziness and reliability of the data, the method based on Z-numbers is proposed in the damage information fusion for decision level, which is a softer method and avoids the severe effect of a small data on the fusion result. The result given by the simulation example of space structure shows the effectiveness of this method.

Keywords

Z-number, Structural health monitoring, Information fusion, Decision making, Ordered weighted average

Introduction

Civil engineering structures will inevitably be damaged by various natural and human factors during long-term use [1–4]. Structural damage is the changes in the material and geometric properties of the whole or parts of the structure, these changes may affect the future use of the structural system [5–7]. For example, the degradation or decline of a structure in terms of stiffness, strength, boundaries connection. Therefor, in order to avoid heavy casualties and property losses, it is necessary to detect and predict the structural damage in time.

The structural health monitoring (SHM) is to monitor the physical mechanical properties of the structure, monitor the whole behavior of the structure in real time, diagnose the damage position and damage level of structure, make the intelligent evaluations of service condition, reliability, durability and carrying capacity. A typical SHM system involves three major subsystems: a sensor subsystem, a data processing subsystem and a health evaluation subsystem [8]. A typical SHM processing includes four main steps: damage detection, damage location, damage classification, and damage severity evaluation [8–11]. In the SHM system of large and complex engineering structure, the type and number of sensors are quite large, include temperature sensors, force sensors, strain sensors, displacement sensors, vibrating sensors and as so on. How to model and fuse the data from a large number of different types of sensors are critical to structural damage decision making [12–22].

In many practical cases, structural damage is a fuzzy state, because there is no obvious and clear grading mark in the actual health monitoring [23]. There are many methods to model uncertain information, such as probability distribution [24], Dempster-Shafer evidence theory [25–28], fuzzy set [29–31], Z-numbers [32], rough sets [33] etc. Fuzzy set theory is widely used in SHM [34–36]. The fuzzy decision fusion method is based on the principle of fuzzy transformation and the principle of maximum membership degree, and synthetically considers all factors related to the evaluation object to realize the fusion [37, 38]. However, the information collected by sensors is not just fuzzy. The sensor may not accurately detect the actual data when affected by natural or human factors. We must consider the partially reliability and fuzziness of the information synchronously [39–44]. Thus, it is more objective and scientific for modeling the structural damage information by introducing the concept of Z-number. A Z-number is an ordered pair $Z = (A, B)$, where A is an fuzzy number playing a role of a fuzzy constraint on values that a random variable X, and B is a fuzzy number with a membership function $\mu_B : [0, 1] \to [0, 1]$, expresses the reliability of A [32]. The concept of Z-number is a step that formalizing the extraordinary ability of human beings to make rational decisions in imprecise and uncertain environment [45–47]. In this paper, a structural damage information decision model based on the Z-number is proposed. In this model, the data collected by sensors is represented by the Z-number, and refer to the evidence combination used soft likelihood, which avoids just one low value for any evidence will greatly reduce the final fusion result [48]. The ordered weighted averaging (OWA) aggregation operator plays an important role in this method. The OWA aggregation operator is a n dimensional vector W, which elements w_i are referred to as the OWA weights, such that: $w_i \in [0, 1]; \sum_i w_i = 1$ and

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where $OWA(a_1, \cdots, a_n) = \sum_{i=1}^{n} w_i a_{\lambda(i)}$ with $a_{\lambda(i)}$ is the ith largest value of a_j [49]. Other weighting operators can also be used in different environments [50, 51].

The main critical problems that naturally arises in the proposed method are computation and reasoning with Z-numbers. Rafi k A Aliev *et al.* developed a universal approach to compute with Z-number in [45, 52]. It is characterized by propagation restrictions. The Z-number algorithm is not only the "mechanical sum" algorithm of probability algorithm and fuzzy algorithm, but also the cooperation of these algorithms. In probability algorithm, the well-known probabilistic arithmetic is used. Fuzzy arithmetic deals with possibility constraints, which describe the object as a class with "unsharpened" boundary [45]. In this paper, we mainly use the addition, multiplication and ranking of Z numbers.

The rest part of this paper is organized as follows. The definition and arithmetic of probability distribution, fuzzy number and Z-number are briefly introduce in Section 2. The OWA operator is also introduced in this section. Section 3 introduced the structural damage information fusion based on Z-number. Section 4 presents a simulation example of SHM. Finally, this paper is concluded in Section 5.

Preliminaries

The arithmetic of probability distribution

The problem of arithmetic of probability distribution is widely appeared in engineering problems. When random variable is numeral, numerical methods are used and analytical solutions are used when random variable is not numeral. In this subsection, we only consider the numerical methods for arithmetic of probability distribution. For two probability distributions p_1, p_2, on random variables x_1 and x_2 respectively, for several typical binary operations [24, 53]:

Addition $p_{12} = p_1 + p_2$:

$$p_{12}(x) = \sum_{x=x_{1i}+x_{2j}} p_1(x_{1i}) p_2(x_{2j}). \quad (1)$$

Subtraction $p_{12} = p_1 - p_2$:

$$p_{12}(x) = \sum_{x=x_{1i}-x_{2j}} p_1(x_{1i}) p_2(x_{2j}). \quad (2)$$

Multiplication $p_{12} = p_1 - p_2$:

$$p_{12}(x) = \sum_{x=x_{1i}\cdot x_{2j}} p_1(x_{1i}) p_2(x_{2j}). \quad (3)$$

Division $p_{12} = p_1 - p_2$:

$$p_{12}(x) = \sum_{x=x_{1i}/x_{2j}} p_1(x_{1i}) p_2(x_{2j}). \quad (4)$$

The arithmetic of fuzzy number

Fuzzy set is new mathematical tool for dealing with uncertain information introduced by Zadeh [29]. Before introducing the arithmetic of fuzzy number, we review the definition of fuzzy number.

Definition 1. *Let universe, $X \in \mathcal{R}$, whose elements are denoted x. A fuzzy subset A on X with membership function $\mu_A : X \to [0, 1]$ is a fuzzy number if [29]*

a) A is a convex fuzzy set;

b) α-cut of A, A^α is a closed interval for every $\alpha \in (0,]$;

c) the support of A is bounded,

where α-cut of A, $A^\alpha = \{x \in X, \mu_A(x) \geq \alpha\}$.

Let $A, B \subset \mathcal{R}$ be two fuzzy numbers and $*$ denote any of $+, -, \cdot, /$, for each $\alpha \in (0, 1]$, the α-cut of $A * B$ is [29, 45, 54]:

$$(A * B)^\alpha = A^\alpha * B^\alpha, \quad (5)$$

when $* = /$, $0 \notin supp(B)$. The fuzzy number $A * B$ is defined as

$$A * B = \bigcup_{\alpha \in [0,1]} \alpha (A * B)^\alpha. \quad (6)$$

Let A and B are two fuzzy number and their α-cut, A^α, B^α are [45]:

$$A^\alpha = [a_1, a_2]; \quad B^\alpha = [b_1, b_2].$$

Then addition:

$$(A + B)^\alpha = [a_1, a_2] + [b_1, b_2] = [a_1 + b_1, a_2 + b_2]. \quad (7)$$

Subtraction:

$$(A - B)^\alpha = [a_1, a_2] - [b_1, b_2] = [a_1 - b_2, a_2 - b_1]. \quad (8)$$

Multiplication:

$$(A \cdot B)^\alpha = [a_1, a_2] \cdot [b_1, b_2] = [a_1 \cdot b_1, a_2 \cdot b_2]. \quad (9)$$

Division:

$$(A \cdot B)^\alpha = [a_1, a_2] \cdot [b_1, b_2] = [a_1 / b_2, a_2 / b_1]. \quad (10)$$

The arithmetic of Z-number

Definition 2. *A Z-number is an ordered pair of fuzzy numbers $Z = (A, B)$, where A is a fuzzy number playing a role of a fuzzy constraint on values that a random variable X may take:*

$$X is A$$

and B is a fuzzy number with the membership function $\mu_B : [0, 1] \to [0, 1]$, playing a role of a fuzzy constraint on the probability measure of A:

$$P(A) is B,$$

where $P(A) = \int_{\mathcal{R}} \mu_A(x) p(x), P(A) \in supp(B)$ with $p(x)$ is the probability distribution of X.

Let $Z_1 = (A_1, B_1)$ and $Z_2 = (A_2, B_2)$ be two Z-numbers and $Z_{12} = (A_{12}, B_{12}) = Z_1 + Z_2 = (A_1, B_1) + (A_2, B_2)$. Their Z^+-numbers are $Z_1^+ = (A_1, R_1)$, $Z_2^+ = (A_2, R_2)$ and $Z_{12}^+ = (A_{12}, R_{12})$, R plays the role of the probability distribution p. The sum operation over Z-numbers is defined as following [45]:

$$A_{12} = A_1 + A_2, \quad (11)$$

$$p_{R_1+R_2}(v) = p_{R_1} + p_{R_2}, \quad (12)$$

$$\mu_{p_{12}}(p_{12}) = sup_{p_1,p_2}(\mu_{B_1}(\int_{\mathcal{R}} \mu_{A_x}(u)p_1(u)du) \wedge \mu_{B_2}(\int_{\mathcal{R}} \mu_{A_2}(u)p_2(u)du)) \tag{13}$$

$$\mu_{B_{12}}(w) = sup_{p_{12}}(\mu_{p_{12}}(p_{12})), \tag{14}$$

$$s.t.w = \int_{\mathcal{R}} \mu_{A_{12}} p_{12}(u)du. \tag{15}$$

The multiplication operation over Z-numbers is defined as following [45]:

$$A_{12} = A_1 \cdot A_2, \tag{16}$$

$$p_{R_1 \cdot R_2}(v) = p_{R_1} \cdot p_{R_2}, \tag{17}$$

$$\mu_{p_{12}}(p_{12}) = sup_{p_1,p_2}(\mu_{B_1}(\int_{\mathcal{R}} \mu_{A_x}(u)p_1(u)du) \wedge \mu_{B_2}(\int_{\mathcal{R}} \mu_{A_2}(u)p_2(u)du)) \tag{18}$$

$$\mu_{B_{12}}(w) = sup_{p_{12}}(\mu_{p_3 12}(p_{12})), \tag{19}$$

$$s.t.w = \int_{\mathcal{R}} \mu_{A_{12}} p_{12}(u)du. \tag{20}$$

For any Z-numbers, Z_1, Z_2, and Z_3, have [45]

(1) (addition associative law)
$((A_1, B_1) + (A_2, B_2)) + (A_3, B_3) = (A_1, B_1) + ((A_2, B_2) + (A_3, B_3))$;
(2) (multiplication association law)
$((A_1, B_1) \cdot (A_2, B_2)) \cdot (A_3, B_3) = (A_1, B_1) \cdot ((A_2, B_2) \cdot (A_3, B_3))$;
(3) (addition commutative law)
$(A_1, B_1) + (A_2, B_2) = (A_2, B_2) + (A_1, B_1)$;
(4) (multiplication commutative law)
$(A_1, B_1) \cdot (A_2, B_2) = (A_2, B_2) \cdot (A_1, B_1)$;
(5) (addition identity law)
$(A_1, B_1) + 0 = (A_1, B_1)$;
(6) (multiplication identity law)
$(A_1, B_1) \cdot 1 = (A_1, B_1)$.

There are many methods to rank Z-numbers, such as comparison of Z-numbers on the base of FPO principle proposed by [55] and the multi-layer methodology for ranking Znumbers from [56]. The first method suggest to consider a Z-number as a pair of values of two attributes: one attribute measures value of variable, the other one measures the associated reliability. Then it will be adequate to compare Z-numbers as multi-attribute alternatives [55]. The second method is to transform the Z-numbers into fuzzy numbers, then rank the transformed fuzzy numbers. The second method is simpler, but equally effective. This paper mainly used the second method to rank Z-numbers. This method consists two main steps: converting Znumbers into fuzzy numbers and ranking the fuzzy numbers after converting [56].

For a Z-number $Z = (A, B)$, converting Z into fuzzy numbers [57].

$$\alpha = \frac{\int_{\mathcal{R}} x\mu_B(x)dx}{\int_{\mathcal{R}} \mu_B(x)dx}, \tag{21}$$

α represents the weight of the reliability.

$$Z^{\alpha} = \{< x, \mu_{A^{\alpha}}(x) > | \mu_{A^{\alpha}}(x) = \alpha\mu_A(x), x \in [0,1]\}. \tag{22}$$

$$Z' = \{< x, \mu_{Z'}(x) > | \mu_{Z'}(x) = \mu_A(\frac{x}{\sqrt{\alpha}}), x \in [0,1]\}. \tag{23}$$

Ranking the fuzzy numbers after converting, $Z' = [a, b, c, d; w]$ [58].

$$x^*_{Z'} = \frac{\int_{\mathcal{R}} x\mu(x)dx}{\int_{\mathcal{R}} \mu(x)dx}, \tag{24}$$

$x^*_{z'}$ represents the horizontal centroid Z' and $x^*_{Z'} \in [0, 1]$.

$$y^*_{Z'} = \frac{\int_0^w \alpha|Z'_{\alpha}|d\alpha}{\int_0^w |Z'_{\alpha}|d\alpha}, \tag{25}$$

where $|Z'_{\alpha}|$ is the length of α-cuts of Z', $y^*_{Z'}$ is the vertical centroid of Z' and $y^*_{Z'} \in [0, w]$.

$$i_{Z'} = dist(d - a) = |d - a|, \tag{26}$$

$i_{Z'}$ is the distance along x-axis from $x^*_{Z'}$ and $i_{Z'} \in [0, 1]$.

$$ii_{Z'} = y^*_{Z'}, \tag{27}$$

$ii_{Z'}$ is the distance along the vertical y-axis from $y^*_{Z'}$ and $ii_{Z'} \in [0, 1]$.

$$s_{Z'} = i_{Z'} \times ii_{Z'} \tag{28}$$

$s_{Z'}$ is spread of Z' and $s_{Z'} \in [0, 1]$.

$$CPS_Z(Z) = x^*_{Z'} \times y^*_{Z'} \times (1 - s_{Z'}). \tag{29}$$

$CPS_Z(Z') \in [0, 1]$.

If $CPS_Z(Z_A) > CPS_Z(Z_B)$, then Z_A is greater than Z_B.

If $CPS_Z(Z_A) < CPS_Z(Z_B)$, then Z_A is lesser than Z_B.

If $CPS_Z(Z_A) = CPS_Z(Z_B)$, then Z_A is equal to Z_B.

Ordered weight averaging operator

Ordered weight averaging (OWA) operator is widely used in data fusion due to it allows assign weights according to the quantifiers.

Definition 3. *An ordered weight averaging operator of n dimension is a mapping $OWA : [0, 1]^n \to [0, 1]$, which weighting vector W is [49]*

$$W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

such that

$$w_i \in [0,1], \quad \sum_{i=1}^{n} w_i = 1,$$

and where $OWA(a_1, \cdots, a_n) = \sum_{i=1}^{n} w_i a_{\lambda(i)}$ with $a_{\lambda(i)}$ is the ith largest value of $a_1, \cdots, a_n$.

Structural damage information decision model based on the Z-number

In the structural damage information decision model, there are q influencing factors of the structure construct the factor set $U = \{U_1, U_2, \cdots, U_q\}$. The decision set is denoted as $V = \{V_1, V_2, \cdots, V_n\}$. The data fusion process of the model is shown in Figure 1.

Step 1: The data collected by sensors construct the Z-numbers decision matrix $\mathcal{Z}$:

$$\mathcal{Z} = \begin{bmatrix} Z_{11} & Z_{12} & \ddots & Z_{1n} \\ Z_{21} & Z_{22} & \ddots & Z_{2n} \\ \vdots & \vdots & Z_{ij} & \vdots \\ Z_{q1} & Z_{q2} & \ddots & Z_{qn} \end{bmatrix}$$

$Z_{ij} = (A_{ij}, B_{ij})$, the fuzzy number A_{ij} expresses the fuzzy number of the V_j is determined according to U_i, its membership function, $\mu_{A_{ij}} : [0,1] \to [0,1]$, the fuzzy number B_{ij} expresses the reliability of A_{ij}, its membership function, $\mu_{B_{ij}} : [0,1] \to [0,1]$.

Step 2: Rank $Z_{ij}, (i = 1, \cdots, 4)$ for all $j = i = 1, \cdots, 4$ to obtain index function λ_j.

λ_j is an index function so that $\lambda_j(k)$ is the index of kth largest Z-number associated with decision alternatives V_j. Thus the Z-number $Z\lambda_{j(k)i}$ is the kth largest Z-number of $Z_{j1}, Z_(j2), \cdots, Z_{jq}$. Here we used the method in Section to determine the order of Z-numbers.

Step 3: Calculate the product of the i largest Z-numbers.

Using the index function λ_j we let

$$Prod_j(i) = \prod_{k=1}^{i} Z_{j\lambda_j(k)} \tag{30}$$

$Prod_j(i)$ is the product of the i largest Z-numbers. It is obvious that $Prod_j(i+1) = Prod_j(i) Z_{(i+1)j}$.

Step 4: Calculate the fusion results Z-numbers of each decision alternative V_j with OWA operator.

The fusion result Z-number of V_j based on an OWA aggregation with weighting vector W and $Prod_j(i)$ denoted as $Z_{j,W}$. W is the q dimensional OWA weighting vector, consists of the weight w_i for $j = 1, \cdots, q$ such that $w_i \in [0,1]$ and $\sum_{i=1}^{q} w_i = 1$ [49].

Using OWA weighting vector, the fusion result Z-number of decision alternative V_j is defined as

$$Z_{j,W} = OWA_W(Prod_j(1), \cdots, Prod_j(q)). \tag{31}$$

It is means that we need to rank $Prod_j(1), \cdots, Prod_j(q)$.

Step 5: Rank the fusion result Z-numbers of all decision alternatives V_j to find the final decision.

Simulation Example

Taking a space structure as an example, the change of deflection (U_1) and the change of strain (U_2), predicts damage values by SVM (U_3) and modal curvature rate (U_4) are the main parameters accurately reflect the structural damage information and they are easy to collect. The factor set is $U = \{U_1, U_2, U_3, U_4\}$. The structural health status is divided into 4 grades: intact (V_1), minor damage (V_2), moderate damage (V_3), and severe damage (V_4). The decision set is $V = \{V_1, V_2, V_3, V_4\}$. Assume the OWA vector is $W = [0.2, 0.25, 0.4, 0.15]$. The Z-numbers relation matrix is constructed as

$$R = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} =$$

$$\begin{bmatrix} ([0, 0.002, 0.005], sure) & ([0.1, 0.225, 0.3], very\ sure) \\ ([0, 0.110, 0.155], sure) & ([0.886, 0.967, 1], sure) \\ ([0, 0, 0], very\ sure) & ([0, 0, 0], very\ sure) \\ ([0, 0, 0], sure) & ([0, 0, 0], sure) \end{bmatrix.$$

$$\begin{matrix} ([0.5, 0.735, 0.9], very\ sure) & ([0, 0.112, 0.22], sure) \\ ([0.110, 0.209, 0.298], very\ sure) & ([0, 0.057, 0.1], sure) \\ ([0.35, 0.5, 0.55], sure) & ([0.4, 0.5, 0.6], sure) \\ ([0.8, 0.955, 1], very\ sure) & ([0, 0.191, 0.223], very\ sure) \end{matrix} \right]$$

where $sure$ is the fuzzy number $[0.6, 0.7, 0.8]$, $very\ sure$ is the fuzzy number $[0.8, 0.9, 1]$.

Calculate $CPS_Z(Z_{ij})$ for $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$ by using Eq. (21)-(29). The result is shown in Table 1.

Table 1. $CPS_Z(Z_{ij})$ for $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$.

$CPS_Z(Z_{ij})$	j=1	j=2	j=3	j=4
i=1	0.00065	0.06171	0.19658	0.02897
i=2	0.02357	0.25678	0.06117	0.01419
i=3	0	0	0.12288	0.13167
i=4	0	0	0.27204	0.04056

Thus, we can obtain the index function λ_j:
$\lambda_1(1) = 2, \lambda_1(2) = 1, \lambda_1(3) = 3, \lambda_1(4) = 4.$
$\lambda_2(1) = 2, \lambda_2(2) = 1, \lambda_2(3) = 3, \lambda_2(4) = 4.$
$\lambda_3(1) = 4, \lambda_3(2) = 1, \lambda_3(3) = 3, \lambda_3(4) = 2.$
$\lambda_4(1) = 3, \lambda_4(2) = 4, \lambda_4(3) = 1, \lambda_4(4) = 2.$

Then, Calculate $Prod_j(g)$ for $g = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$ by using Eq. (30). Taking an example with $Prod_3$:

$$Prod_3(1) = Z_{34},$$

$$Prod_3(2) = (A_{p32}, B_{p32}) = Z_{34} \cdot Z_{31}.$$

$$Prod_3(3) = (A_{p33}, B_{p33}) = Z_{34} \cdot Z_{31} \cdot Z_{33}.$$

$$Prod_3(4) = (A_{p34}, B_{p34}) = Z_{34} \cdot Z_{31} \cdot Z_{33} \cdot Z_{34}.$$

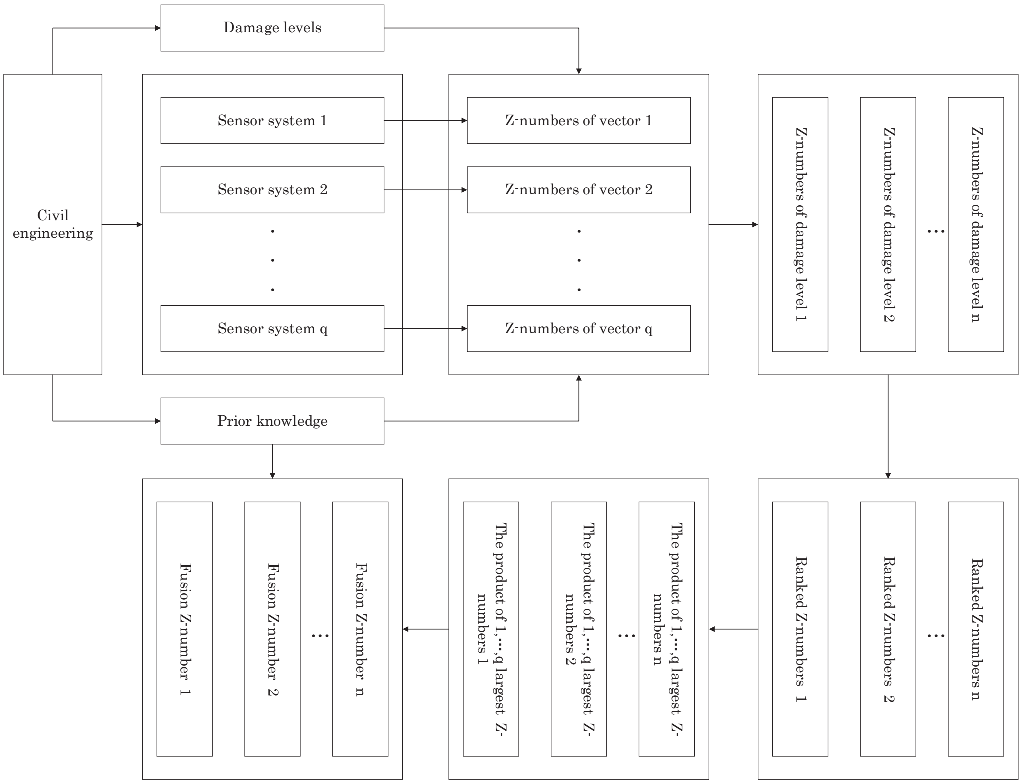


Figure 1. The model of structural damage information decision based on the Z-number.

The membership functions of A_{34}, B_{34} and A_{31}, B_{31} can express as:

$$\mu_{A_{34}} = \begin{cases} \dfrac{x-0.8}{0.955-0.8}, & 0.8 \le x \le 0.955 \\ \dfrac{1-x}{1-0.955}, & 0.955 \le x \le 1. \end{cases}$$

$$\mu_{A_{31}} = \begin{cases} \dfrac{x-0.5}{0.735-0.5}, & 0.5 \le x \le 0.735 \\ \dfrac{0.9-x}{0.9-0.735}, & 0.735 \le x \le 0.9. \end{cases}$$

$$\mu_{B_{34}} = \mu_{B_{34}} = \begin{cases} \dfrac{x-0.8}{0.9-0.8}, & 0.8 \le x \le 0.9 \\ \dfrac{1-x}{1-0.9}, & 0.9 \le x \le 1. \end{cases}$$

So,
$A^{\alpha}_{34} = [\alpha(0.955-0.8)+0.8, 1-\alpha(1-0.955)]$,
$A^{\alpha}_{31} = [\alpha(0.735-0.5)+0.5, 0.9-\alpha(0.9-0.735)]$.
Using Eq. (9) we can obtain:
$A^{\alpha}_{p32} = [(\alpha(0.955-0.8)+0.8)(\alpha(0.735-0.5)+0.5), (1-\alpha(1-0.955))(0.9-\alpha(0.9-0.735))]$.
A_{p32} is shown in Figure. 3(a).

For simplicity, we consider normal distributions, taking into account that μ_{ij} are fixed due to compatibility conditions.

$$p_{34} = \frac{1}{\sqrt{2\Pi}\sigma_{34}} e^{-\frac{x-0.955^2}{2\sigma_{34}^2}},$$

$$p_{31} = \frac{1}{\sqrt{2\Pi}\sigma_{31}} e^{-\frac{x-0.735^2}{2\sigma_{31}^2}},$$

From the definition of z number, It can be known

$$\int_{supp(A_{34})} \mu_{A_{34}}(x) \frac{1}{\sqrt{2\Pi}\sigma_{34}} e^{-\frac{x-0.955^2}{2\sigma_{34}^2}} dx\ is B_{34}.$$

$$\int_{supp(A_{31})} \mu_{A_{31}}(x) \frac{1}{\sqrt{2\Pi}\sigma_{31}} e^{-\frac{x-0.735^2}{2\sigma_{31}^2}} dx\ is B_{34}.$$

The relationship between $supp(B_{34})$ and σ_{34} is shown in Figure. 2(a), the relationship between $supp(B_{31})$ and σ_{31} is shown in Figure. 2(b).

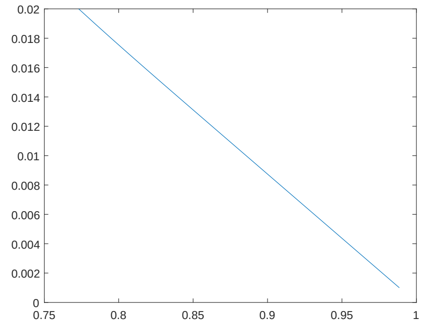
$$p_{p32} = N(\mu_{34}\mu_{31}, \sqrt{\mu_{34}^2\sigma_{31}^2 + \mu_{31}^2\sigma_{34}^2 + \sigma_{34}^2\sigma_{31}^2}).$$

Thus we can get B_{p32} by using Eq. (18)-(20), it is shown in Figure. 3(b).

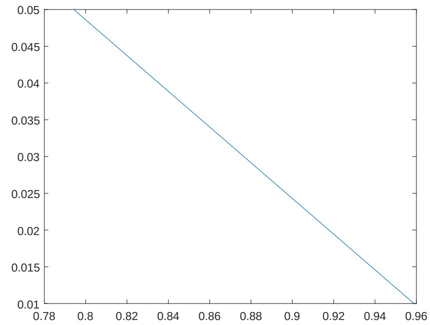
In the same way, we can get $Prod_3(3)$ shown in Figure. 4 and $Prod_3(4)$ shown in Figure. 5.

Finally, Using the OWA vector W to calculate the fusion result Z-number, $Z_{3,W}$ of V_3. The result is shown in Figure. 6.

Similarly, $Z_{1,W}, Z_{2,W}, Z_{4,W}$ are shown in Figure. 7-9.

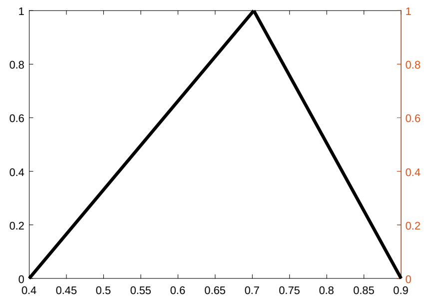


(a) The relationship between $supp(B_{34})$ and σ_{34}.

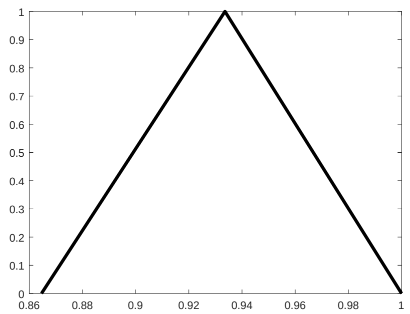


(b) The relationship between $supp(B_{31})$ and σ_{31}

Figure 2



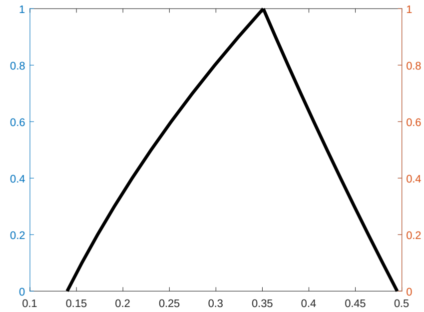
(a) A_{p32}



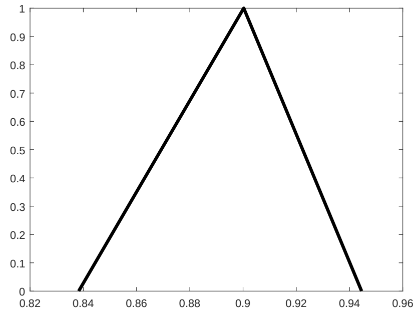
(b) B_{p32}

Figure 3. $Prod_3(2)$

Then, $CPS_Z(Z_{1,W}) \approx 0.0030$, $CPS_Z(Z_{2,W}) \approx 0.0587$, $CPS_Z(Z_{3,W}) \approx 0.1340$, $CPS_Z(Z_{4,W}) \approx 0.0263$. It can be seen Z_3 is maximum, so the moderate damage is judged to occur, which is consistent with the actual situation.

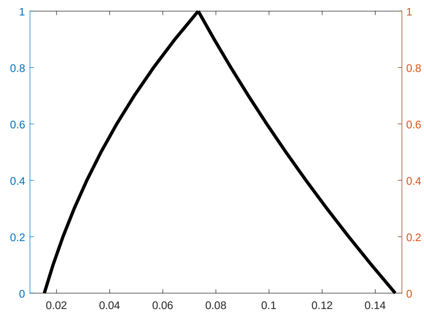


(a) A_{p33}

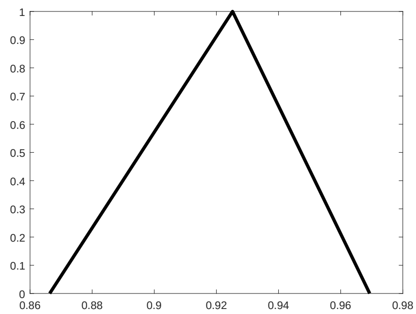


(b) B_{p33}

Figure 4. $Prod_3(3)$



(a) A_{p34}

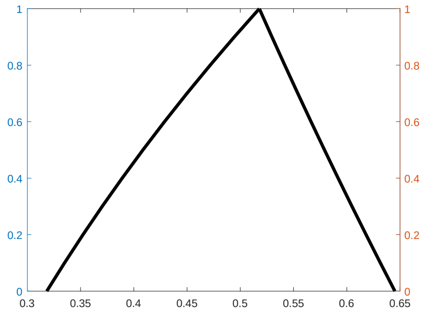


(b) B_{p34}

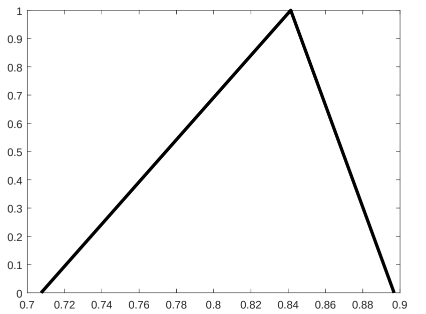
Figure 5. $Prod_3(4)$

Conclusions

In SHM system, the numbers and kinds of sensors are very large. This has led to a lot of data that engineers need to deal with. The real-world information is characterized by

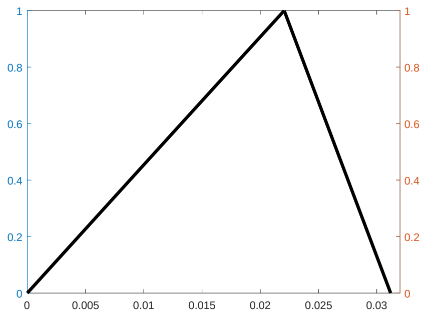


(a) $A_{3,W}$

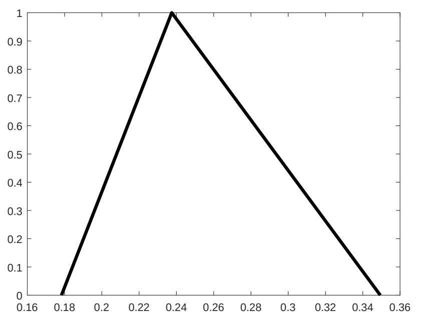


(b) $B_{3,W}$

Figure 6. $Prod_{3,W}$

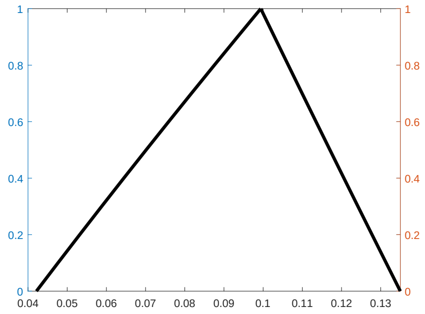


(a) $A_{1,W}$

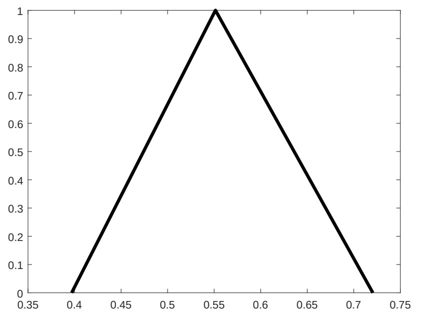


(b) $B_{1,W}$

Figure 7. $Z_{1,W}$

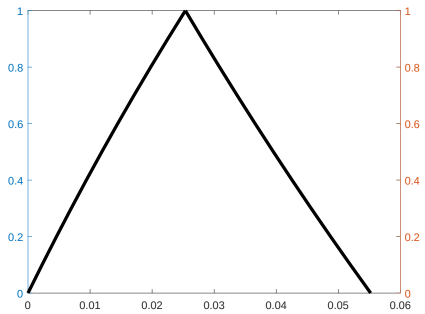


(a) $A_{2,W}$

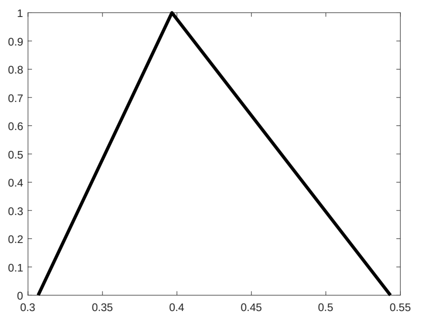


(b) $B_{2,W}$

Figure 8. $Z_{2,W}$



(a) $A_{4,W}$



(b) $B_{4,W}$

Figure 9. $Z_{4,W}$

fuzziness and partial reliability. In order to better model and fuse the data collected by a large number of sensors, we proposed the structural damage information fusion model based on Z-numbers. Z-numbers have a great advantage in presenting the real-world information. Z-numbers contain more information than other methods of expressing the uncertain information. We also used a softer method to fuse the sensor data represented by Z-numbers, which avoids the severe effect of a small data on the fusion result. Additionally, the method is more flexible by instructing the

OWA operator. The method also involves a lot of arithmetics of Z-numbers. Fortunately, the theory of Z-arithmetic is comprehensive, self-contained and has been studied in many existing literatures. We must point that many uncertain information models, such as probability distribution, fuzzy number and D-S evidence theory, their arithmetics are much simpler than computing with Z-numbers.

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