Looking to the Bohr's Atomic Model

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Abstract:

Bohr in 1913 introduced new atomic model based on quantum theory that explained the limitations of the previously existing Rutherford's atomic model. Through this article we have tried to provide an outlook to the Bohr's model and studies how it helped to describe the atomic structure of an atom.

Introduction:

Rutherford's atomic model was one of the earliest atomic model that described the atomic structure. It was commonly known as planetary model, as it assumed that the electrons revolve around the nucleus continuously. However, it failed to reconcile itself with the Maxwell's laws of electrodynamics as well as failed to explain the spectra of hydrogen atom. These were overcome by Neil Bohr, who applied quantum theory to improve the failures on Rutherford's model. Hence, Bohr's model was the first atomic model to explain the atomic structure on the basis of quantum theory.

Bohr's Atomic Model:

Neil Bohr in 1913 proposed an atomic model by applying quantum theory and gave a new atomic model. The major postulates of Bohr's Theory are

1. The electrostatic force of attraction between nucleus and electron must be balanced by the centrifugal force due to rotation of electron.

i.e. $$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}=\frac{mv^2}{r} \quad \text{(for H-atom)} \qquad \text{(Eqn 1)}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}=\frac{mv^2}{r}$$ (for other atoms) (Eqn 2)

2. Electrons revolve around the nucleus only in those fixed orbits, where the momentum of electrons is integral multiple of $\frac{h}{2\pi}$. It is known as Bohr's quantum condition and is given by

$$mvr=\frac{h}{2\pi} \qquad \text{(Eqn 3)}$$

3. When electron gets transition from one energy level to another energy level, then the absorption or emission of energy takes place in the form of radiation. i.e. If E_1 and E_2 are two energy levels then,
$E_1 - E_2 = h\nu$, where ν is the frequency of radiation.

From Bohr's first postulates, since the centrifugal force is balanced by the electrostatic attraction,

$$\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n^2}=\frac{mv_n^2}{r_n}$$ where, r_n is the radius of the nth orbit and v_n is the velocity at r_n

Or, $$mv_n^2=\frac{Ze^2}{4\pi\varepsilon_0 r_n} \qquad \text{(Eqn 4)}$$

From Bohr's second postulate we have,

$$v_n=\frac{nh}{2\pi m r_n}$$

Then (eqn 4) becomes,

$$\frac{mn^2h^2}{4\pi^2m^2r_n^2}=\frac{Ze^2}{4\pi\varepsilon_0 r_n}$$

Which gives, $r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m Z e^2}$

For Hydrogen atom (Z = 1), hence

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2} \qquad \text{(Eqn 5)}$$

(Eqn 5) yields that $v_n \propto n^2$, and the radii of the orbits are in the ratio 1:4:9:16:25 etc.

Thus, the radius of the first orbit for Hydrogen atom is calculated to be
$r_1 = 0.53$ Å.

This is called Bohr's radius of H-atom.

For an atom having atomic number Z, it is given by $r_n = 0.53 \frac{n^2}{Z}$ Å ,

And the velocity of the electron is obtained as,

$$v_n = \frac{e^2}{2 \varepsilon_0 n h} \qquad \text{(Eqn 6)}$$

Here, (Eqn 6) gives the velocity of the electron in nth orbit and clearly shows that $v_n \propto \frac{1}{n}$, which means that the electron closer to nucleus move faster than those lying further away.

For the total energy of the electron, we obtain it by the sum of kinetic energy and potential energy.

K.E. of an electron is given by $E_k = \frac{1}{2} m v^2$, with the help of (Eqn 6), it becomes

$E_k = \frac{m e^4}{8 \varepsilon_0^2 n^2 h^2}$. The potential energy of the electron in an orbit is obtained by calculating the amount of work done in it by bringing it from infinity to that orbit.

i.e. Potential Energy $E_p = \int_{\infty}^{r_n} \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_n^2}$, which gives

$$E_p = \frac{-me^4}{4\varepsilon_0^2 n^2 h^2}$$

Thus the total energy is given by,

$E = E_k + E_p$

$$= \frac{m e^4}{8\varepsilon_0^2 n^2 h^2} - \frac{me^4}{4\varepsilon_0^2 n^2 h^2}$$

$$E = \frac{-m e^4}{8\varepsilon_0^2 n^2 h^2} \qquad \text{(Eqn 7)}$$

Eqn 7 gives the energy of the electron revolving in the nth orbit of Hydrogen atom. As the value of n increases the value of energy, too increases which shows that, the outermost orbits have greater energy than the inner orbits. Here '-ve' sign, indicates that the force is attractive.

Upon calculating the energy for energy levels 1, 2, 3, 4, …….. and so on, we get

$E_1 = -13.6$ eV

$E_2 = -3.40$ eV

$E_3 = -1.51$ eV

$E_4 = -0.85$ eV

And so on, at $E_\infty = 0$ eV.

Bohr's Correspondence Principle

According to the classical physics an accelerated charged particle emits electromagnetic radiation, but in the case of Bohr's atomic model electron, on revolving around the circular orbit, doesnot radiate any radiation. Thus, to solve this problem Bohr's proposed an correspondence principle, stating that quantum theory must agree with classical theory in the limit of large quantum numbers.

For an atom in classical condition an electron moving in a circle radiates energy at a frequency equal to the frequency of it's rotation, and the time period of revolution is given by,

$$T = \frac{Distance\,travelled \in one\,revolution}{velocity}$$

i.e. $T = \frac{2\pi r}{v}$ (Eqn 8)

From (Eqn 1), v can be calculated as

$$v^2 = \frac{e^2}{4\pi\varepsilon_0 mr} \quad \text{(Eqn 9)}$$

From (Eqn 2), we have,

$$v = \frac{nh}{2\pi mr} \quad \text{(Eqn 10)}$$

Putting the value of r from (Eqn 5) into (Eqn 10), we get,

$$v = \frac{e^2}{2\varepsilon_0 nh} \quad \text{(Eqn 11)}$$

Thus, by using (Eqn 5) and (Eqn 11), (Eqn 8) reduces to,

$$T = \frac{4\varepsilon_0^2 n^3 h^3}{me^4}$$ (Eqn

12)

Also, the frequency of radiation is given by,

$$\nu = \frac{1}{T} = \frac{me^4}{4\varepsilon_0^2 n^3 h^3}$$

Hence, a classical electron at radius r_n would radiate at its frequency at its

frequency $\nu_n = \frac{me^4}{4\varepsilon_0^2 h^3}\frac{1}{n^3}$ (Eqn 13)

Then, the frequency of the emitted radiation when the electron jumps from state n to state (n-1) is given by,

$$\nu = \frac{me^4}{4\varepsilon_0^2 h^3}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= \frac{me^4}{4\varepsilon_0^2 h^3}\left(\frac{2n-1}{n^2(n-1)^2}\right)$$

For large n, we can assume (n-1) by n, and (2n-1) by 2n, which yields,

$$\nu \approx \frac{me^4}{8\varepsilon_0^2 h^3}\frac{2n}{n^4}$$

Thus, $\nu = \frac{me^4}{4\varepsilon_0^2 n^3 h^3}$ which is identical with (Eqn 13)

Thus, in the case of classical condition, electron spirals slowly towards the nuclear whereas in the quantum condition, the electron jumps from the orbit n to (n – 1), then, (n-2) and so on, concluding that for large quantum numbers, quantum theory must agree with classical theory.

Effect of Nuclear Motion on Atomic Spectra

Bohr's model had assumed the nucleus to be fixed at the centre of it's circular orbit. If the nucleus have finite mass M, then both the nucleus, as well as the electron revolves around their common centre of mass acquiring same angular velocity ω .

Then the total angular momentum of the atom about the common centre of mass is given by,

$h = mr^2\omega$, where $\mu = \frac{m}{1+\frac{m}{M}}$ is the reduced mass of electron, and M is the mass of nucleus.

Then from Bohr's second postulates,

$$Mr^2\omega = \frac{nh}{2\pi} \quad \text{(Eqn 14)}$$

In the absence of nuclear motion, the equation becomes,

$mr^2\omega = \frac{nh}{2\pi}$, which shows that it is identical with (Eqn 14)

Then, now the energy levels of hydrogen atoms are given by,

$$E_n = \frac{\mu Z^2 e^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2}$$

Upon the transition of element from Bohr's orbit n_2 to n_1 , the frequency of emitted radiation is given by,

$$v = \frac{\mu Z^2 e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

And it's corresponding wavenumber is,

$$\bar{v} = \frac{\mu Z^2 e^4}{8\varepsilon_0^2 h^2 c}\left(\frac{1}{n_1^2} - \frac{1}{n_1^2}\right)$$

Then the Rydberg's constant for any element Z is given by,

$$R=\frac{\mu e^4}{8\varepsilon_0^2 h^3 c}=\frac{e^4}{8\varepsilon_0^2 h^3 c}\times\frac{M_z m}{(M_z+m)}$$

$$=\frac{M e^4}{8\varepsilon_0^2 h^3 c}\times\frac{M_z}{(M_z+m)}$$

$\therefore R=R_z\left(\frac{1}{1+\frac{m}{M_z}}\right)$ where, $R_z=\frac{M e^4}{8\varepsilon_0^2 h^3 c}$ is the value of Rydberg's constant when the mass of the nucleus is infinite.

Now, $\bar{v}=Z^2 R_z\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$ (Eqn 15)

It is the expansion for the corresponding wavenumber for the nucleus with infinite mass.

Failures and Conclusion:

Though Bohr's Model did explain the hydrogen spectra, it failed to explain the fine and hyperfine structure of spectral lines. It assumed that electrons revolved around the nucleus in circular orbits, however later on Sommerfeld found that the circular orbits were only the special cases. Similarly it also failed to explain the Zeeman effect and Stark effect. It also doesn't consider the spin and relativistic motion of the electron and is unable to describe the many atom systems.

Though Bohr's model had certain failure, it paved the foundations for the study of atomic structure on the basis of quantum theory. It revolutionized our understanding of energy packets commonly known as photons, and their way of interfering with atomic nucleus, by either being absorbed or released, opening the door to many new phenomenon in physics.

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