A Theory of Special Relativity Based on Four-Displacement of Particles Instead of Minkowski Four-Position

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ABSTRACT

All Lorentz four-vectors of Special Relativity (SR) are derived from a basic Lorentz four-position in a Minkowski space. This article explores use of a Lorentz four-displacement, describing translatory motion of particles in a 4-dimensional space irrespective of position, as a basic four-vector. Derivations based on this assumption have striking outcomes and impact: All four-vectors of SR associated with frame invariants are intact and valid. All postulates and predictions of SR are intact and valid, with two exceptions - Length contraction in the boost direction and relativity of simultaneity are both obviated. Notably, these two aspects of SR have never been definitively proven in experiments and can therefore be challenged. We propose an adapted SR theory, based on four-displacement of particles, predicting the following revolutionary and far-reaching consequences:

1. 4D-space is absolute and Euclidean relative to which all particles, mass and massless, are never at rest and perform an equal distance of translatory displacement at a universal displacement rate equivalent to the speed of light. Inertial frames of 3D-space+time remain relative.
2. What appear as Lorentz four-vectors in a Minkowski metric of a relative spacetime are actually Lorentz four-vectors in an absolute Euclidian 4D-space.
3. Special Relativity actually emerges from describing properties of particles in 4 momentum-space, irrespective of position and localization in space. In this form, SR achieves inherent compatibility with Quantum theories, reflecting the particle-wave duality of particle fields.
4. Time is a property of mass particles associated with their displacement in 4D-space, as opposed to a Minkowski-space property.
5. Frames of 3D-space+time represent mixed domain coordinates: 3 spatial coordinates of position-space and a temporal coordinate of momentum-space, unlike the position-space status of 4 spacetime coordinates in conventional SR.

I. INTRODUCTION

In Einstein's initial presentation of Special Relativity in 1905[1] he proposed two fundamental postulates of SR as follows:

1) The Principle of Relativity - The laws by which the states of physical systems undergo change are not affected, whether these changes of states be referred to the one or the other of two systems in uniform translatory motion relative to each other.

2) The Principle of Invariant Light Speed – "…light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body". That is, light in vacuum propagates with the speed c (a fixed constant independent of direction) in at least one system of inertial coordinates (the stationary system) regardless of the state of motion of light source.

The theory is "Special" because it applies to special cases where the curvature of spacetime due to gravity is negligible. The modern accepted version of Einstein's two postulates of SR can be summarized as follows:

1. The laws of physics are invariant in all inertial frames.

2. The speed of light in a vacuum is the same for all observers, regardless of the motion of the light source.

Mathematically, SR can be based on a single postulate of a flat 4-dimensional Minkowski four-position, in which the time coordinate has an opposite polarity signature to the 3 space coordinates.

SR implies a wide range of consequences, including lack of an absolute reference frame, time dilation, length contraction, mass-energy equivalence, a universal speed of light, and relativity of simultaneity. Most predictions of SR have been proven experimentally. However, despite various attempts, velocity dependent length contraction and relativity of simultaneity have never been experimentally proven.

Thus, it may be considered possible that these two predicted phenomena of SR do not reflect reality. This enables consideration of a different theory of SR that retains all proven predictions and obviates the unproven predictions. Since the unproven predictions are keystones of Minkowski spacetime, a paradigm shift in the basics of conventional SR is expected with any new theory. A proposed, new theory of SR is presented herein. We start with a short introduction to conventional SR and associated terminology, as a reference for the proposed SR theory.

II. THE LORENTZ TRANSFORMATION OF SPACETIME

Based on the two postulates described above, Einstein derived the Lorentz Frame Transformation of spacetime events from which the entire theory of Special Relativity emerged.

Assume an event with spacetime coordinates (ct,x,y,z) in frame F and (ct',x',y',z') in frame F' moving at velocity v with respect to frame F in the +x-direction. The Lorentz transformation specifies that these coordinates are related as follows:

(1.1)

$$\begin{aligned} ct' &= \gamma(ct - \beta x) & ct &= \gamma(ct' + \beta x') \\ x' &= \gamma(x - \beta ct) & x &= \gamma(x' + \beta ct') \\ y' &= y & y &= y' \\ z' &= z & z &= z' \end{aligned}$$

Where c is the speed of light and we define a normalized velocity $\beta \stackrel{\text{def}}{=} v/c$ and a related Lorentz factor $\gamma \stackrel{\text{def}}{=} 1/\sqrt{1-\beta^2}$. In this form the time coordinate (ct) has a status of position–space coordinate specified also in units of distance, and the transformation remains invariant under exchange of space (x) and time (ct).

Writing the Lorentz transformation and its inverse in terms of spacetime coordinate difference, where for instance one event has coordinates (ct_1, x_1, y_1, z_1) and $(ct'_1, x'_1, y'_1, z'_1)$, another event has coordinates (ct_2, x_2, y_2, z_2) and $(ct'_2, x'_2, y'_2, z'_2)$, and the differences are defined as

(1.2)

$$\begin{aligned} \Delta ct' &= ct'_2 - ct'_1 & \Delta ct &= ct_2 - ct_1 \\ \Delta x' &= x'_2 - x'_1 & \Delta x &= x_2 - x_1 \\ \Delta y' &= y'_2 - y'_1 & \Delta y &= y_2 - y_1 \\ \Delta z' &= z'_2 - z'_1 & \Delta z &= z_2 - z_1 \end{aligned}$$

We get the Lorentz transformation of spacetime interval

(1.3)

$$\begin{aligned} \Delta ct' &= \gamma(\Delta ct - \beta \Delta x) & \Delta ct &= \gamma(\Delta ct' + \beta \Delta x') \\ \Delta x' &= \gamma(\Delta x - \beta \Delta ct) & \Delta x &= \gamma(\Delta x' + \beta \Delta ct') \\ \Delta y' &= \Delta y & \Delta y &= \Delta y' \\ \Delta z' &= \Delta z & \Delta z &= \Delta z' \end{aligned}$$

An important conclusion is that the transformation of a spacetime interval is independent of the absolute spacetime values of the events composing the interval. It is only dependent on the spacetime distance between the events.

III. FOUR-VECTORS OF CONVENTIONAL SR

In the mathematical terminology of four-vectors[2] in Minkowski spacetime, which transform according to Lorentz transformation, the position four-vector is:

(1.4)

$$\mathbf{R} = (ct, \mathbf{r}) = (ct, x, y, z)$$

This represents four components of the position four-vector of a spacetime event. The displacement four-vector is defined to be an "arrow linking two spacetime events, i.e. a spacetime interval:

(1.5)

$$\Delta\mathbf{R} = (c\Delta t, \Delta\mathbf{r}) = (c\Delta t, \Delta x, \Delta y, \Delta z)$$

Or expressed in differential form, the displacement four-vector is:

(1.6)

$$d\mathbf{R} = (cdt, d\mathbf{r}) = (cdt, dx, dy, dz)$$

Its square norm is Lorentz invariant representing a proper time interval $d\tau$:

(1.7)

$$\|d\mathbf{R}\|^2 = (cdt)^2 - d\mathbf{r} \cdot d\mathbf{r} = (cd\tau)^2$$

The velocity four-vector is:

(1.8)

$$\mathbf{U} = \frac{d\mathbf{R}}{d\tau} = \frac{d\mathbf{R}}{dt}\frac{dt}{d\tau} = (c, \mathbf{u}) \cdot \gamma = \gamma(c, u_x, u_y, u_z)$$

Where $\beta = u/c$. Its square norm is Lorentz invariant representing the speed of light:

$$\|\mathbf{U}\|^2 = (\gamma \mathrm{c})^2 - (\gamma \mathrm{u})^2 = \mathrm{c}^2 \quad (1.9)$$

Therefore the magnitude of the four-velocity for any object is always a fixed constant.

The acceleration four-vector is:

$$\mathbf{A} = \frac{\mathrm{d}\mathbf{U}}{\mathrm{d}\tau} = \gamma\left(\frac{\mathrm{d}\gamma}{\mathrm{dt}}\mathrm{c}, \frac{\mathrm{d}\gamma}{\mathrm{dt}}\mathbf{u} + \gamma\frac{\mathrm{d}\mathbf{u}}{\mathrm{dt}}\right) \quad (1.10)$$

Its square norm is Lorentz invariant as follows:

$$\|\mathbf{A}\|^2 = \gamma^4 \mathbf{a}\cdot\mathbf{a} + \gamma^6(\boldsymbol{\beta}\cdot\mathbf{a})^2$$

Where $\mathbf{a} \equiv \mathrm{d}\mathbf{u}/\mathrm{dt}$ is the regular acceleration and $\boldsymbol{\beta} = \mathbf{u}/\mathrm{c}$ is the normalized velocity vector, both are spatial vectors.

The energy-momentum four-vector is:

$$\mathbf{P} = \mathrm{m}_0\mathbf{U} = \mathrm{m}_0\gamma(\mathrm{c}, \mathbf{u}) = (\mathrm{E}/\mathrm{c}, \mathbf{p}) = (\mathrm{E}/\mathrm{c}, \mathrm{p}_\mathrm{x}, \mathrm{p}_\mathrm{y}, \mathrm{p}_\mathrm{z}) \quad (1.11)$$

Where $\mathrm{E} = \gamma \mathrm{m}_0 \mathrm{c}^2$ and $\mathbf{p} = \gamma \mathrm{m}_0 \mathbf{u}$ are respectively the relativistic energy and momentum of an object with rest mass m_0. Its square norm is Lorentz invariant associated with the rest mass of the object:

$$\|\mathbf{P}\|^2 = (\mathrm{E}/\mathrm{c})^2 - \mathbf{p}\cdot\mathbf{p} = (\mathrm{m}_0\mathrm{c})^2 \quad (1.12)$$

Yielding the relativistic energy-momentum expression applied to particle physics as follows:

$$\mathrm{E}^2 = (\mathrm{pc})^2 + (\mathrm{m}_0\mathrm{c}^2)^2 \quad (1.13)$$

Making the link to Quantum theories, the energy of a particle is related to the angular frequency of an associated wave by $\mathrm{E} = \hbar\omega$, which is a reciprocal to time t. Similarly the momentum of a particle is related to the wavevector of an associated wave by $\mathbf{p} = \hbar\mathbf{k}$, which is reciprocal to spatial space $\mathbf{r}$. These conjugate variables are related by Fourier transform. The four-wavevector is:

$$\mathbf{K} = (\omega/\mathrm{c}, \mathbf{k}) = (\omega/\mathrm{c}, \mathrm{k}_\mathrm{x}, \mathrm{k}_\mathrm{y}, \mathrm{k}_\mathrm{z}) \quad (1.14)$$

Its square norm is Lorentz invariant associated with the rest mass of the particle:

$$\|\mathbf{K}\|^2 = (\omega/\mathrm{c})^2 - \mathbf{k}\cdot\mathbf{k} = (\mathrm{m}_0\mathrm{c}/\hbar)^2 \quad (1.15)$$

This represents the matter wave analog of the four-momentum. For massless photons $\mathrm{m}_0 = 0$, therefore Eq. (1.15) yields $\mathrm{k} = \omega/\mathrm{c}$ commonly associated with electromagnetic radiation.

IV. THE DISPLACEMENT AS A BASIC LORENTZ FOUR-VECTOR

One can see that the displacement four-vector $\Delta\mathbf{R}$ in Eq. (1.5) is a basic Lorentz four-vector from which all other following four-vectors and associated Lorentz invariant scalars can be derived. Thus, we have chosen to explore the outcome if a Lorentz four-displacement $\Delta\mathbf{R}$ was assumed as a basic 4-vector instead of a Lorentz four-position $\mathbf{R}$.

It is important to note that, with the obviation of the Lorentz four-position $\mathbf{R}$, the Lorentz four-displacement $\Delta\mathbf{R}$ cannot be derived from the geometry of spacetime. Instead, the four-displacement $\Delta\mathbf{R}$ describes the translatory motion of particles in 4D-space, irrespective of their position and localization in space, only the translatory distance of displacement matters. The four-displacement $\Delta\mathbf{R}$ of mass particles also describes mass objects, irrespective of their structure or spatial distribution in space. Assuming four-displacement of particles $\Delta\mathbf{R}$ as a basic Lorentz four-vector results in the following two consequences:

A) The two basic postulates of SR continue to be valid because they are both inherently related to particles in free or bound states, and can be expressed as follows:

   1) The law of physics [*i.e. of matter and radiation representing particles*], are invariant in all inertial frames.
   2) The speed of light in a vacuum [*i.e. of massless photon particles*], is the same for all observers, regardless of the motion of the light source.

B) Except for the Lorentz four-position $\mathbf{R}$ which is completely obviated, all other four-vectors and associated Lorentz invariant scalars of SR continue to be valid (this will be shown in subsequent sections of this article).

What can we learn from this assumption which leaves the postulates and major outcomes of SR intact and valid, but makes no conditions on the geometry of 4D-space? The remainder of this article is an analysis of this scenario, yielding a derivation of an adapted SR theory with revolutionary consequences.

V. FOUR-DISPLACEMENT OF PARTICLES

Hereinafter and in order to avoid ambiguity, we use a notation ($\mathbf{d}$) for four-displacement of particles. The purpose is to prevent confusion with four-displacement of spacetime intervals ($\Delta\mathbf{R}$) derived from four-position ($\mathbf{R}$). The new notation ($\mathbf{d}$) symbolizes that we solely refer to translatory distance of displacements, irrespective of position. The displacement $\Delta\mathbf{R}$ is equivalent to displacement $\mathbf{d}$ for time-like world-lines ($\|\Delta\mathbf{R}\|^2 \geq 0$) of point mass particles which are represented at the origin of their rest frames.

In the following derivations we refer to inertial frames. In the absence of gravity, 4D-space is flat, and the translatory motion of particles is uniform with straight line trajectories in 4D-space. When referring to mass particles, we also consider macroscopic mass objects of any structure. Four-displacement of a particle or an object follow time-like world-lines, expressed as follows:

$$\mathbf{d} = (d_t, d_x, d_y, d_z) \quad \text{In frame F}$$
$$\mathbf{d}' = (d'_t, d'_x, d'_y, d'_z) \quad \text{In frame F'} \tag{1.16}$$

Frame F' is assumed to have a Lorentz boost $\beta = v/c$ in x-direction in frame F. The Lorentz boost is associated with a Lorentz factor $\gamma = 1/\sqrt{1-\beta^2}$. The four-displacements $\mathbf{d}$ and $\mathbf{d}'$ are transformed between frames F and F' by the Lorentz transformation and the inverse Lorentz transformation, employing the Lorentz matrix $\boldsymbol{\mathcal{L}}$ and its inverse $\boldsymbol{\mathcal{L}}^{-1}$ as follows:

$$\mathbf{d} = \boldsymbol{\mathcal{L}}\,\mathbf{d}' \qquad \mathbf{d}' = \boldsymbol{\mathcal{L}}^{-1}\mathbf{d} \tag{1.17}$$

$$\boldsymbol{\mathcal{L}} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \boldsymbol{\mathcal{L}}^{-1} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The four-displacements $\mathbf{d}'$ and $\mathbf{d}$ contain information regarding the translatory distance of displacement in the temporal and the spatial coordinates of frame F' and frame F, respectively. These distances are specified irrespective of position or spatial distribution of the object in any of the frames. Since this Lorentz transformation ignores position, frame synchronization is accomplished by resetting all displacement components in the two frames. This is naturally ensured by resetting the temporal displacement in the two frames, i.e. $d_t = d'_t = 0$. During synchronization the two frames can arbitrarily be separated in position. The local time elapse in frame F ($d_t \equiv c\Delta t$) represents the translatory displacement in the temporal coordinate of mass particles which are at rest in the 3 spatial coordinates of reference frame F (i.e. $\Delta x \equiv d_x = 0,\ \Delta y \equiv d_y = 0,\ \Delta z \equiv d_z = 0$). Similarly, the local time elapse in frame F' ($d'_t \equiv c\Delta t'$) represents the translatory displacement in the temporal coordinate of mass particles which are at rest in the 3 spatial coordinates of reference frame F' (i.e. $\Delta x' \equiv d'_x = 0,\ \Delta y' \equiv d'_y = 0,\ \Delta z' \equiv d'_z = 0$).

We initially analyze the meaning of the Lorentz transformation of four-displacement of particles (Eq. (1.17)) in a special case where a mass particle or an object is at spatial rest in frame F'. This particle or object is displacing in frame F at normalized velocity $\beta = v/c = d_x/d_t$. For this case the four-displacement in frame F' is $\mathbf{d}' = (d'_t, 0, 0, 0)$ and in frame F is $\mathbf{d} = (d_t, d_x = \beta d_t, 0, 0)$. By the definition of the Lorentz transformation, the square norm of the four-displacement in the two frames is a Lorentz frame invariant scalar, as follows:

$$\|\mathbf{d}\|^2 = {d_t}^2 - {d_s}^2 = \|\mathbf{d}'\|^2 = {d'_t}^2$$
$${d_s}^2 \equiv {d_x}^2 + {d_y}^2 + {d_z}^2 \tag{1.18}$$

Where d_s represents the overall spatial displacement of the particle or the object in frame F, and d'_t represents the proper time of the particle or the object in frame F'. Eq. (1.18) can be written in a different form as follows:

$${d_t}^2 = {d'_t}^2 + {d_s}^2 \tag{1.19}$$

The temporal displacement d_t represents the total translatory displacement d_T made in frame F by mass particles that are at rest in the spatial coordinates of frame F. With no spatial displacement, the temporal displacement d_t is by definition the total displacement of these mass particles (i.e. $d_t = d_T$). Thus, Eqs. (1.18-1.19) get a new meaning:

$${d_T}^2 = {d'_t}^2 + {d_s}^2, \qquad {d_T}^2 - {d_s}^2 = {d'_t}^2 \tag{1.20}$$

The total displacement d_T of mass particles which are at spatial rest in frame F is equal to the total displacement (in frame F) of mass particles which are at spatial rest in frame F'. The last particles are observed in reference frame F as having two displacement components normal to each other and obey a <u>Pythagorean relation</u>. One displacement component d'_t is along the temporal coordinate of frame F and has the same value in frame F' and in frame F. The second displacement component (d_s) is orthogonal extending in the spatial coordinates of frame F (i.e. $d_s = d_x = \beta d_t = v\Delta t$). The left Pythagorean expression in Eq.(1.20) represents a <u>Euclidian metric of 4D-space</u>. Moreover, it states that the total distance of displacement of all mass particles (at spatial rest or not) is <u>identical</u> in any inertial frame of reference F. The right expression in Eq.(1.20) has the same meaning but expressed as a Lorentz frame invariant.

We now analyze the meaning of the Lorentz transformation of four-displacement of particles (Eq. (1.17)) in the general case whereby the spatial displacement of particles in frame F' is non-zero, i.e. ${d'_s}^2 \equiv {d'_x}^2 + {d'_y}^2 + {d'_z}^2 \neq 0$ irrespective of position in frame F'. A spatial displacement in frame F' can be the translatory motion of mass particles or alternatively of massless photon particles. Since light speed is postulated to be a universal constant (c) in any inertial frame, the spatial displacements of <u>massless</u> photon particles in frame F' obey:

$$d'_s/\Delta t' = c \quad \rightarrow \quad d'_s = d'_t \tag{1.21}$$

where $\Delta t'$ is the time elapse in frame F' during the spatial displacement (d'_s) of photon particles. This holds true irrespective of position, only the spatial distance of displacement matters.

For <u>mass</u> particles which undergo a spatial displacement d'_s in frame F', an analogy to Eqs. (1.18-1.20) in frame F' can be expressed as follows:

$$\|\mathbf{d}'\|^2 = {d'_t}^2 - {d'_s}^2 = \|\mathbf{d}''\|^2 = {d''_t}^2$$
$${d'_t}^2 = {d''_t}^2 + {d'_s}^2 = {d'_T}^2 \tag{1.22}$$

Where $d''_t \equiv c\Delta t''$ is a proper time representing the temporal displacement of mass particles at spatial rest in a different frame F'' (having a Lorentz boost in frame F'). These particles are observed in reference frame F' as having two displacement components normal to each other and obey a <u>Pythagorean relation</u>. One displacement component d''_t is along the temporal coordinate of frame F' and has the same value in frame F'' and in frame F'. The second displacement component (d'_s) is orthogonal extending in the spatial coordinates of frame F' (i.e. $d'_s = \beta' d'_t$). d'_t represents the temporal displacement in frame F' of mass particles which are at spatial rest in frame F', With no spatial displacement, the temporal displacement d'_t is by definition the total displacement of these mass particles ($d'_t = d'_T$). Therefore, according to Eq. (1.22) d'_T is also the total displacement (in frame F') of mass particles which are at spatial rest in frame F'' (but have in addition a spatial displacement d'_s in frame F').

Recall, frame F' has a Lorentz boost $\beta = v/c$ with respect to frame F in the +x-direction. Using the Lorentz transformation in Eq. (1.17) to transform the four-displacement **d'** in frame F' to a corresponding four-displacement **d** in reference frame F, we obtain:

$$d_t = \gamma d'_t + \gamma\beta d'_x$$
$${d_s}^2 = (\gamma d'_x + \gamma\beta d'_t)^2 + {d'_y}^2 + {d'_z}^2 \tag{1.23}$$

For <u>massless</u> photon particles, substituting ${d'_y}^2 + {d'_z}^2 = {d'_t}^2 - {d'_x}^2$ based on Eq. (1.21) into Eq. (1.23), we obtain:

$$\boxed{d_s = \gamma d'_t + \gamma\beta d'_x = d_t = d_T} \qquad \boxed{d'_s = d'_t = d'_T} \tag{1.24}$$

In frame F In frame F'

The right-hand expression was derived in Eq. (1.21) and confirms that the total displacement of mass particles at spatial rest in frame F' ($d'_t = d'_T$) is <u>equal</u> to the spatial displacement d'_s of photon particles in spatial dimensions of frame F'. The left-hand expression was derived by the Lorentz transformation of four-displacement **d'** to **d**, and confirms that although photon particles originated in frame F' (emitted from a stationary source in frame F'), their spatial displacement d_s in reference frame F is also <u>equal</u> to total displacement of mass particles at spatial rest in reference frame F ($d_t = d_T$). This confirms that photon particles move at the speed of light in any inertial frame of reference.

For <u>mass</u> particles substituting ${d'_y}^2 + {d'_z}^2 = {d'_t}^2 - {d''_t}^2 - {d'_x}^2$ from Eq. (1.22) into Eq. (1.23), we obtain:

$${d_s}^2 = (\gamma d'_t + \gamma\beta d'_x)^2 - {d''_t}^2 \tag{1.25}$$

$$\boxed{{d_t}^2 - {d_s}^2 = {d''_t}^2} \qquad \boxed{{d'_t}^2 - {d'_s}^2 = {d''_t}^2}$$

$${d_t}^2 = {d''_t}^2 + {d_s}^2 = {d_T}^2, \qquad {d'_t}^2 = {d''_t}^2 + {d'_s}^2 = {d'_T}^2$$

In frame F In frame F'

The right-hand top and bottom equivalent expressions are derived in Eq. (1.22) and confirm the <u>equality of total displacement</u> (d'_T) in frame F', between mass particles that are at spatial rest in frame F' ($d'_t = d'_T = c\Delta t'$) and mass particles that are at spatial rest in frame F'' (but have in addition a spatial displacement d'_s in frame F'). The velocity related to the total displacement (d'_T) of any particle in frame F' is equal to the speed of light ($d'_T/\Delta t' = c$), independent whether the particle is at spatial rest in frame F' or not.

The left top and bottom equivalent expressions are derived by the Lorentz transformation of four-displacement **d'** to **d**, and also confirm the <u>equality of total displacement</u> (d_T) in frame F, between mass particles that are at spatial rest in frame F ($d_t = d_T = c\Delta t$) and mass particles that are at spatial rest in frame F'' (but have in addition a spatial displacement d_s in frame F). The velocity related to the total displacement (d_T) of any particle in frame F is also equal to the speed of light ($d_T/\Delta t = c$), independent whether or not the particle is at spatial rest in frame F.

The two bottom equalities in Eq. (1.25) are Pythagorean expressions, representing a Euclidian 4D-space relative to which particle displacements are specified in frame F (left) and in frame F' (right). The two top expressions have the same meaning as the respective two bottom expressions, but are written in a Lorentz invariant form. The frame invariant (d''_t) is the proper time of mass particles at spatial rest in frame F". One can see that Eq. (1.25) reduces to Eq. (1.24) in the special case of zero temporal displacement $d''_t = 0$. Since Eq. (1.24) represents photon particles, we deduce that the temporal displacement of photon particles is zero in any frame of reference. This means no spatial rest frame exists for photon particles.

In conclusion:

a) The Lorentz transformation of four-displacement of particles unveils a fundamental universal principle: In any frame of reference all particles (mass and massless) perform an equal distance of total displacement, reflecting a universal translatory motion of particles in 4D-space. Thus, particles are never at rest in 4D-space, and universally move at the speed of light.
The Lorentz transformation of four-displacement and the equality of total displacement of particles in any frame are equivalent statements. From this fundamental principle, the constancy of the speed of light in any inertial frame emerges as a byproduct.

b) Although coordinate systems of inertial frames are relative, 4D-space is absolute and Euclidian, relative to which the translatory motion of particles is observable. Different inertial frames represent different Euclidian bases or perspectives of observing the universal displacement of particles in 4D-space.

c) In any inertial frame of reference F, the local time elapse d_t (i.e. the temporal displacement) represents the total displacement d_T of any mass particle which is at spatial rest in the reference frame F. Therefore, d_t becomes a measure of the total displacement d_T of every other particle, mass or massless, with spatial displacement in reference frame F. Substituting $d_t = d_T$ in the zero component of the four-displacement $\mathbf{d} = (d_T, d_x, d_y, d_z)$ yields that every particle or object has its own independent four-displacement representation in a frame of reference.

VI. FOUR-DISPLACEMENT OF PROPOSED SR THEORY VS CONVENTIONAL SR

When considering point mass particles at the origin of their spatial rest frames, the four-displacement $\mathbf{d}$ of proposed SR theory is equivalent to the four-displacement $\Delta\mathbf{R}$ of conventional SR, both are time-like world-lines. When particles are non-localized in space (quantum particles) and when considering mass objects that are spatially distributed, the four-displacement of the two SR theories reveal different properties related to relativity of simultaneity and length contraction in the direction of motion, as described in the following example:

Assume a rod of length L' is stationary in frame F'. The rod extends along the x' axis with one end of the rod at position x'_1 and the second end of the rod at position $x'_2 = x'_1 + L'$, both measured from the frame origin. Frame F' has a Lorentz boost β along the x axis in frame F. Frames F and F' are coordinate-aligned and synchronized, i.e. $x' = x = 0$ at $ct' = ct = 0$. This synchronization is relevant to conventional SR. The synchronization in proposed SR theory only requires $d'_t = d_t = d_T = 0$, i.e. synchronization is solely between temporal coordinates of the two frames, irrespective of position in order to reset the total translatory displacement of particles in both frames. The four-displacement of proposed SR theory does not contain any information about position which may be undefined (similar to Quantum free particles). During synchronization, the frames can arbitrarily be spatially separated in position in all spatial directions. The stationary rod in frame F' can be represented as composed of an array of infinitesimal segments. Every segment represents a set of localized bound mass particles with a defined position.

In proposed SR theory, the four-displacement of each segment of the rod is $\mathbf{d}' = (d'_t, 0,0,0)$. The four-displacement of all segments are identical, i.e. all segments are at spatial rest in frame F', simultaneously perform an identical temporal displacement, and Lorentz transform independent of their spatial position in frame F'. In frame F, the Lorentz transformation (Eq. (1.17)) of four-displacement of each rod segment yields $\mathbf{d} = (\gamma d'_t, \gamma\beta d'_t, 0,0)$. Thus, also in frame F, the four-displacement of all rod segments are identical. Therefore, all segments remain simultaneous in frame F, and simultaneously perform an identical temporal displacement $d_t = \gamma d'_t$ (reflecting a time dilation). Similarly, all segments simultaneously perform an identical spatial displacement $d_x = \gamma\beta d'_t = \beta d_t = v\Delta t$ (reflecting a spatial uniform motion of the entire rod). Moreover, since the position is unaffected by the Lorentz transformation, the rod length L' remain unaffected having the same absolute length in both frames ($L = L'$).

In conventional SR, the four-displacement of each segment of the rod depends on its position. To simplify the treatment, we refer to four-displacement of spacetime intervals tracing between the origin of frame F' and each of the two ends of the rod. Using the Lorentz transformation of spacetime we obtain: $x'_2 - x'_1 = \gamma(x_2 - \beta ct_2) - \gamma(x_1 - \beta ct_1)$. For a simultaneous ($ct_2 = ct_1$) measurement of the rod length (L) in frame F, we obtain: $x'_2 - x'_1 = L' = \gamma(x_2 - x_1) = \gamma L$ or $L = L'/\gamma$. This represents a velocity dependent length contraction of the rod in frame F. Similarly, using the Lorentz transformation of spacetime we obtain: $ct_2 - ct_1 = \gamma(ct'_2 + \beta x'_2) - \gamma(ct'_1 + \beta x'_1)$. Since the rod is simultaneous in frame F' ($ct'_2 = ct'_1$), therefore $ct_2 - ct_1 = \gamma\beta(x'_2 - x'_1) = \gamma\beta L'$. It is apparent that although the two ends of the rod are simultaneous in frame F' they are not simultaneous in Frame F. This demonstrates the relativity of simultaneity in conventional SR.

In conclusion:

d) In proposed SR theory, the Lorentz transformation of four-displacement of particles exhibits time dilation but does not reveal length contraction nor relativity of simultaneity, i.e. space dimensions and simultaneity are absolute in all inertial frames.

e) The Lorentz four-displacement is governed by motion properties of particles rather than geometric properties of spacetime. The four-displacement is affected by a Lorentz boost, but the Euclidian geometry of 4D-space is absolute and remains intact unaffected by a Lorentz boost. This is unlike the relative nature of spacetime in conventional SR, which is affected by a Lorentz boost.

VII. THE DISPLACEMENT-SPACE AND ASSOCIATED PROPERTIES

Lorentz four-displacement **d** is a property of particles, representing their motion in a Euclidian 4D-space. Thus, it is natural to define a 4D displacement-space whereby reference is made only to displacement distances of particles irrespective of position. The 4D displacement-space is associated with a 4D momentum-space, both of which are relative aspects of the same domain (explained in section-XI). Describing particles in displacement-space or in momentum-space irrespective of position is in line with Quantum theories which treat the position-space and the momentum-space as two Fourier reciprocal domains. This means that representation in momentum-space specifies the representation in position-space and vice-versa.

In the displacement space, any component of the four-displacement vector **d** in frame F can be expressed as a fraction D of the total displacement $d_T = \ell$ made by particles in frame F, as follows:

$$\mathbf{d} = (d_t, d_x, d_y, d_z) = (d_T, \mathbf{d}_s) = (\ell D_T, \ell \mathbf{D}_s)$$
$$d_t = d_T = \ell D_T = \ell, \quad \mathbf{d}_s = \ell \mathbf{D}_s, \quad d'_t = \ell D'_t \tag{1.26}$$

Where ℓ measures the accumulated total distance of translatory displacement made by any particle in frame F. ℓ is measured starting from frame synchronization, irrespective of position and localization in 4D-space. The expression of all displacement components of a particle as fractions of its total displacement ℓ gives to the fractional parameters D_T, D_s and D'_t a meaning of displacement rates or equivalently normalized velocities ($D = v/c$). Eq. (1.20) can be written using Eq. (1.26), as follows:

$$(\ell D_T)^2 = (\ell D'_t)^2 + (\ell D_s)^2$$
$$D_T^2 = D'^2_t + D_s^2 = 1 \tag{1.27}$$
$$0 \le D'_t \le 1, \quad 0 \le D_s \le 1, \quad D_T = 1$$

Where D_T, D_s and D'_t are dimensionless fractional displacement parameters identified as the displacement rates or equivalently the normalized velocities of a particle. They are associated with the various components of four-displacement in any frame of reference F, and respectively termed: the total, the spatial, and the temporal displacement rates or normalized velocities, as follows:

$$D_T = v_{\mathbf{T}}/c, \quad D'_t = v'_t/c, \quad D_s = v_{\mathbf{s}}/c = \beta \tag{1.28}$$

It is apparent from this description that a mass particle has an inherent temporal displacement rate or "velocity" ($D'_t = v'_t/c$) in the temporal coordinate, and may acquire a spatial displacement rate or "velocity" ($D_{\mathbf{s}} = \beta = v_{\mathbf{s}}/c$) due to a Lorentz boost. Eq. (1.27) represents a Pythagorean expression of "velocities" reflecting the Euclidian metric of 4D-space. Any increase in the spatial displacement rate $D_{\mathbf{s}}$ of a particle in reference frame F is at the expense of reducing its temporal displacement rate D'_t in the temporal coordinate of the reference frame F. In a specific inertial frame of reference, the displacement rates D_T, D_s and D'_t of a particle are constant. Since in synchronization we require $d'_t = d_t = d_T = 0$, the parameter which resets during frame synchronization is $\ell = 0$, thereby nullifying all components of four-displacement **d** irrespective of position and localization in 4D-space (see Eq. (1.26)). This represents frame synchronization in displacement-space, unlike frame synchronization in four-position of conventional SR.

The two "velocity" components of a particle (D'_t, D_s) in Eq. (1.27) modify in different frames, but the particle total "velocity" ($D_{\mathbf{T}} = v_{\mathbf{T}}/c = 1$) remains invariant in all inertial frames. This implies that all particles have a maximum total displacement rate equivalent to the speed of light ($v_{\mathbf{T}} = c$) relative to an absolute Euclidian 4D-space. In a frame of mass particles (representing a spatial rest frame with $D_s = 0 \rightarrow D'_t = D_{\mathbf{T}} = 1$), the total displacement (d_T) is experienced as space displacement over the mass particles at the speed of light ($v_{\mathbf{T}} = \mathrm{c}$). The total space displacement in this case is purely temporal (no spatial component), sensed and measurable as proper time elapse ($\mathrm{d_T} = \mathrm{d'_t} = \mathrm{c}\Delta\mathrm{t'}$) of the mass particles.

From Eq. (1.26) we deduce $\mathrm{d'_t} = \mathrm{d_t D'_t}$. From Eqs. (1.27-1.28) we deduce $\mathrm{D'_t} = \sqrt{1-\beta^2} = 1/\gamma$, i.e. the Lorentz boost reduces the temporal displacement rate of mass particles from a maximum value $\mathrm{D'_t} = 1$ in spatial rest frame F' (termed the proper displacement rate) to a lower value in a reference frame F. This mechanism of "velocity" reduction in the temporal coordinate due to a Lorentz boost yields the known relativity of time dilation between frames:

$$\boxed{\mathrm{d_t} = \gamma \mathrm{d'_t} \rightarrow \ \Delta\mathrm{t} = \gamma\Delta\mathrm{t'}} \tag{1.29}$$

VIII. THE DISPLACEMENT RATE FOUR-VECTOR

Based on Eq. (1.26-1.27) we can formulate the displacement rate or the "velocity" four-vector, which transforms between frames by the Lorentz transformation, as follows:

$$\mathbf{D} = \gamma(\mathrm{D_T}, \mathrm{D_x}, \mathrm{D_y}, \mathrm{D_z}) = \gamma(\mathrm{D_T}, \mathbf{D}_\mathrm{s}) \tag{1.30}$$

$$\mathrm{D_s}^2 \equiv \mathrm{D_x}^2 + \mathrm{D_y}^2 + \mathrm{D_z}^2$$

The square norm of **D** is Lorentz invariant representing a <u>constant</u> displacement rate, equivalent to the speed of light, by which all particles displace in 4D-space, as follows:

$$\|\mathbf{D}\|^2 = \gamma^2\left(\mathrm{D_T}^2 - \ \mathrm{D_s}^2\right) = \gamma^2 \mathrm{D'_t}^2 \tag{1.31}$$

For mass particles and mass objects the frame invariant in any frame of reference F is $\gamma^2 \mathrm{D'_t}^2 = \gamma^2/\gamma^2 = 1$. The frame invariant is equal to a maximal <u>temporal</u> displacement rate in a spatial rest frame F' ($\mathrm{D_s} = 0 \rightarrow \ \mathrm{D'_t} = \mathrm{D_T} = 1$), i.e. the proper displacement rate. For a photon particle, in any frame of reference F, the total displacement is purely spatial $\mathrm{d_s} = \mathrm{d_t} = \mathrm{d_T}$ (see Eq. (1.24)). This yields $\mathrm{D_s} = \mathrm{D_T} = 1$, representing a maximal <u>spatial</u> displacement rate in any inertial frame. The frame invariant is $\mathrm{D'_t} = 0$, meaning the temporal displacement rate of photons is zero, yielding zero time elapse ($\mathrm{D'_t} = 0 \ \rightarrow \mathrm{d'_t} = \ell \mathrm{D'_t} = 0$) in any inertial frame of reference F. Eqs. (1.30-1.31) are equivalent to the velocity four-vector **U** of conventional SR in Eqs. (1.8-1.9).

The four-displacement **d** can now be expressed in terms of the Displacement rate or the "velocity" four-vector **D**, as follows:

$$\mathbf{d} = (\mathrm{d_T}, \mathbf{d}_\mathrm{s}) = \ell(\mathrm{D_T}, \mathbf{D}_\mathrm{s}) = \gamma \mathrm{d'_t}(\mathrm{D_T}, \mathbf{D}_\mathrm{s}) = \mathrm{d'_t}\mathbf{D} \tag{1.32}$$

It differs from the "velocity" four-vector only by a scalar $\mathrm{d'_t}$, reconfirming the temporal displacement $\mathrm{d'_t}$ is Lorentz invariant. It represents the total displacement of a mass particle or an object in a spatial rest frame, termed the proper displacement (or proper time). Eq. (1.32) is equivalent to the displacement four-vector $\Delta\mathbf{R}$ of conventional SR (in Eqs. (1.5-1.7)) when restricted to time-like world-lines ($\|\Delta\mathbf{R}\|^2 \geq 0$) of point mass particles represented at the origin of their rest frames.

IX. NEW STRUCTURE OF LORENTZ FOUR-VECTORS

In Eqs. (1.30-1.32), it is apparent that the four-vectors **d** and **D** have new component structure which will be shown to be common to all Lorentz four-vectors of the proposed SR theory. The zero entry component in the new structure actually represents the total magnitude of the vector, rather than a temporal component as in conventional SR. The total magnitude is a scalar containing no information about direction.

The meaning of this new structure is analyzed hereinbelow. The square norm of a general four-vector of this type $\mathbf{V} = (\mathrm{V_T}, \mathbf{V}_\mathrm{s})$ is the inner product of the four-vector with itself. This is explicitly expressed using contravariant components (V^μ) and covariant components (V_μ) related by the metric ($\eta_{\mu\nu}$). With the new meaning of the component structure of four-vectors, the relevant metric for the inner product in Euclidian space is the Minkowski metric, as follows:

$$\|\mathbf{V}\|^2 = \mathbf{V}\cdot\mathbf{V} = \mathrm{V}^\mu \eta_{\mu\nu} \mathrm{V}^\nu = \mathrm{V}_\nu \mathrm{V}^\nu = \mathrm{V}^\mu \mathrm{V}_\mu = \tag{1.33}$$

$$(\mathrm{V_0})^2 - (\mathrm{V_1})^2 - (\mathrm{V_2})^2 - (\mathrm{V_3})^2 =$$

$$(\mathrm{V_T})^2 - (\mathrm{V_x})^2 - (\mathrm{V_y})^2 - (\mathrm{V_z})^2 = (\mathrm{V_T})^2 - (\mathrm{V_s})^2 = (\mathrm{V'_t})^2$$

where the Minkowski metric tensor is $\eta_{\mu\nu} = \mathrm{diag}[+1, -1, -1, -1]$. With the Minkowski metric, the inner product yields a Pythagorean relation between the vector components ($\mathrm{V_T}^2 - \mathrm{V_s}^2 = \mathrm{V'_t}^2$), in agreement with previously derived expressions in Eqs. (1.30-1.32). The actual temporal component ($\mathrm{V'_t}$) of the vector is implied and not an explicit component of the four-vector. The temporal component ($\mathrm{V'_t}$) is a Lorentz frame invariant. Only the spatial component ($\mathrm{V_s}$) varies with a Lorentz boost, correspondingly modifying the total magnitude ($\mathrm{V_T}$) of the vector. In a spatial rest frame, the temporal component is equal to the total magnitude of the vector ($\mathrm{V_s} = 0 \ \rightarrow \mathrm{V'_t} = \mathrm{V_T}$). It is thus termed the proper value of the vector. The proper value represents a fundamental frame invariant measure of the vector. Since the temporal component (V'_t) of the vector is frame invariant, representing the proper value, we maintain its primed notation (in a spatial rest frame F') also in any other unprimed reference frame F.

An alternative standard definition of a four component vector ($\mathbf{V} = (V'_t, \mathbf{V}_s)$) employing a Euclidian metric, would result in the same Pythagorean expression for the inner product ($V'^2_t + V_s^2 = V_T^2$), but in a form which is not frame invariant.

<u>In conclusion</u>:

f) With the zero component of any four-vector representing the total magnitude of the vector rather than a temporal component, the Minkowski metric of the inner product represents a Euclidian 4D-space. What appear in conventional SR as Lorentz four-vectors in a Minkowski metric of a relative spacetime are actually Lorentz four-vectors in an absolute Euclidian 4D-space.

g) The implied temporal component (V'_t) of any four-vector is frame invariant. Only in a spatial rest frame of a mass particle, the temporal component is equal to the total magnitude of the vector ($V_s = 0 \rightarrow V'_t = V_T$), therefore it represents a proper value of the vector. The proper value of any four-vector that represents a specific particle property is a fundamental frame-invariant measure of this particle property.

X. THE FOUR-MOMENTUM

In reference frame F, a particle has energy ε and four-displacement **d**. Each component of four-displacement is associated with a corresponding displacement rate or "velocity". Therefore, each displacement component is also associated with a corresponding momentum. The momentum components are newly defined in analogy to Eq. (1.26), as follows:

$$p_T = \varepsilon D_T, \qquad \mathbf{p}_s = \varepsilon \mathbf{D}_s, \qquad p'_t = \varepsilon D'_t \tag{1.34}$$

Where p_T, p_s and p'_t are respectively termed: the total, the spatial, and the temporal momentum. All components of four-momentum in Eq. (1.34) are expressed in instrumental units of energy. No mass is involved in this definition, only energy matters, therefore all types of particles receive equal treatment. Momentum is defined herein in energy units and is equal to conventional momentum normalized by the speed of light, i.e. Ƥ(conventional) = P/c. In Eq. (1.34) one can see that the total momentum of a particle (mass or massless) in a reference frame is equal to its energy ($D_T = 1$, $p_T = \varepsilon$). From this we can deduce that free-particles of non-zero energy always bear a momentum. This is well known for photons, and is now shown to be also valid for mass particles, reflecting universal motion in 4D-space of all particles at the speed of light. The total momentum of a particle, which is a vector in 4 momentum-space, is the fundamental property of the particle. The energy of the particle simply represents a scalar magnitude of the total momentum vector .

Based on a Euclidian 4D-space, the momentum components of a particle should follow:

$$p_T^2 = p'^2_t + p_s^2 \tag{1.35}$$

This Pythagorean expression naturally emerges also from combining Eq. (1.34) with Eq. (1.27). The momentum four-vector is formulated based on the new component structure, whereby the zero component entry stands for the total momentum of the particle, as follows:

$$\mathbf{P} = (p_T, p_x, p_y, p_z) = (p_T, \mathbf{p}_s) = \varepsilon(D_T, \mathbf{D}_s) = \varepsilon'\mathbf{D} \tag{1.36}$$

$$P_s^2 \equiv P_x^2 + P_y^2 + P_z^2 = \varepsilon^2(D_x^2 + D_y^2 + D_z^2)$$

If P is a true Lorentz four-vector, it should differ from the "velocity" four-vector D only by a scalar ε'. Based on Eq. (1.30), in order to fulfil the equality in Eq. (1.36) we require $\varepsilon = \gamma\varepsilon'$. The square norm of P is a Pythagorean expression of a Lorentz invariant, as follows:

$$\|\mathbf{P}\|^2 = p_T^2 - p_s^2 = p'^2_t = \varepsilon'^2 \tag{1.37}$$

The temporal momentum (p'_t) of a particle in any reference frame is frame invariant. For mass particles it is equal to total momentum in a spatial rest frame ($p_s = 0 \rightarrow p_T = p'_t$), termed the proper momentum. This reconfirms that in a spatial rest frame ($\gamma = 1$), the rest energy of a mass particle ($\varepsilon = \gamma\varepsilon' = \varepsilon'$) is equal to the magnitude of its total momentum in the spatial rest frame, i.e. $p_T = p'_t = \varepsilon'$. This can be deduced also from Eq. (1.34), yielding $p'_t = \varepsilon D'_t = \gamma\varepsilon'/\gamma = \varepsilon'$. For a photon particle, since $D_s = D_T = 1$ in any inertial frame, the total momentum of the photon is always purely spatial $p_T = p_s$, yielding a frame invariant of zero temporal momentum ($p'_t = 0$).

An alternative approach to derive the expression $\varepsilon = \gamma\varepsilon'$, with new insight, is as follows: the temporal momentum ($p'_t = \varepsilon'$) is the proper momentum of mass particles in a spatial rest frame. When introducing a Lorentz boost to a particle, it acquires spatial momentum p_s in a spatial direction normal to the temporal coordinate. Therefore, the temporal momentum should not be affected by the Lorentz boost. However, the temporal "velocity" in the reference frame F is reduced by the boost $D'_t = 1/\gamma$. In order to maintain the temporal momentum unaffected in the reference frame, we require $p'_t = \varepsilon' = \varepsilon D'_t = \varepsilon/\gamma$, i.e. the particle energy in the reference frame ($\varepsilon = \gamma\varepsilon'$) must be increased by a factor γ.

According to proposed SR theory, the temporal momentum of a particle is what defines the mass of the particle. The temporal momentum is frame invariant, indicating the particle has a spatial rest frame where the temporal momentum is equal to the proper momentum of the particle. The energy of the particle (ε') in the spatial rest frame defines the mass of the particle, as follows: the temporal momentum $\mathcal{P}'_t$ (in units of momentum) is conventionally expressed by the displacement velocity in the temporal coordinate of the spatial rest frame ($D'_t = D_{\mathbf{T}} = 1 \rightarrow \; \mathcal{v}'_t = c$) times the particle's rest mass (m_0), i.e. $p'_t = \mathcal{P}'_t c = (m_0 \mathcal{v}'_t)c = m_0 c^2 = \varepsilon'$. Using Eq. (1.34), Eq. (1.35) can be written for mass particles as follows:

$$\varepsilon^2 = \varepsilon'^2 + p_s{}^2 \;\; \rightarrow \qquad \varepsilon^2 = (m_0 c^2)^2 + p_s{}^2 \tag{1.38}$$

$$p_s = \gamma\varepsilon' D_s = (\gamma m_0 \mathcal{v}_s)c, \qquad \varepsilon = \gamma m_0 c^2$$

For photon particles, the temporal momentum in any inertial frame is zero ($p'_t = \varepsilon' = 0$), indicating no spatial rest frames exist and defining zero rest energy and mass ($m = 0$) for photon particles. Eq. (1.38) is equivalent to the relativistic energy-momentum expression of conventional SR in Eq. (1.13). They only differ by the units of the spatial momentum term ($p_s = \mathcal{P}c$), expressed herein in energy units. With proper conversion of units, the four-momentum **P** in Eqs. (1.36-1.37) is equivalent to the energy-momentum four-vector of conventional SR in Eqs. (1.11-1.12).

<u>In conclusion</u>:

h) In any frame of reference, the energy of a particle is equal to its total momentum ($p_T = \varepsilon$). This is well known for photons in conventional SR ($\varepsilon = p_T = \mathcal{P}_T c$). It is shown in proposed SR theory to be also valid for all particles mass and massless.

i) Mass particles have an inherent temporal momentum. The translatory displacement in the temporal coordinate generated by the temporal momentum is experience as proper time elapse. Therefore, time is a displacement property of mass particles in 4D-space. This differs from conventional SR, whereby the temporal coordinate has a status of a position coordinate, and time elapse is a Minkowski-space property.

XI. MOMENTUM-SPACE AND POSITION-SPACE

The four-displacement **d** and the four-momentum **P** can be regarded as describing particles in the same domain. Both **P** and **d** are specified in terms of the displacement rate four-vector **D**, and differ only by scalars (see Eqs. (1.32), and Eq. (1.36)). The four-vectors **P** and **d** are related properties in the same domain. Actually, when normalized by the total value of the vector they become identical, i.e. $\mathbf{P}/p_T = \mathbf{d}/d_T$. In the proposed SR theory, all properties of particles are represented in displacement-space or equivalently in the momentum-space, irrespective of position and localization in 4D-space. In order to represent particles in position-space, we have to follow procedures of Quantum theories. In Quantum, the momentum-space (or k-space) is reciprocal to the position-space. The momentum-space and the position-space are dual reciprocal domains, related by Fourier transform of conjugate variables. This dictates that momentum and conjugate position are non-commuting, yielding the Heisenberg uncertainty principle which sets a limit on simultaneous definition of position and momentum.

For mass particles, the <u>spatial</u> momentum ($p_s = \beta\gamma\varepsilon'$) in any reference frame, can have a wide spectrum of momentum eigenvalues (different values of β yield different values of p_s). Each eigenvalue (p_s) corresponds to an associated spatial momentum eigenstate $[p_s\rangle$. According to Quantum theories, superposition of spatial momentum eigenstates is a valid state. The Fourier transform of the superposition yields a probability wave-function in position-space, describing the localization of the mass particle in spatial coordinates. The expectation value of superposition $\langle p_s \rangle$ corresponds to an effective Lorentz boost β_g, reflecting the group velocity of the particle. A superposition of spatial momenta with zero expectation value $\langle p_s \rangle = 0$ describes the spatial localization of the mass particle in a spatial rest frame. In the spatial coordinates localization can be developed such as in bound states of particles. Therefore, in the 3 spatial coordinates of any reference frame, localizations and bound states of mass particles are possible, allowing position accessibility in spatial coordinates. Position accessibility allows observance of particles in both domains; the spatial position-space or the spatial displacement space.

By contrast, the <u>temporal</u> momentum of mass particles is a single eigenvalue ($p'_t = \varepsilon'$), irrespective of its spatial momentum. Therefore, a mass particle is always in a eigenstate of temporal momentum. The Fourier transform from momentum-space to position-space thus yields a plane wave in the position-space of the temporal coordinate, with no possibility of localization. Therefore, with no localization whatsoever, the position-space along the temporal coordinate is not accessible.

This lack of accessibility in the temporal coordinate leads to the conclusion that in a spatial rest frame of mass particles, the temporal coordinate can only be represented by a displacement-space or a momentum-space coordinate, rather than a position-space coordinate as implied in the Minkowski four-position of conventional SR. This is believed to be the reason the temporal coordinate is experienced so differently from other spatial coordinates.

The spatial velocity of photon particles, in any inertial frame of reference, is a single constant value ($D_s = 1$, $v_s = c$). Correspondingly, the spectrum of spatial momentum of a photon ($p_s = p_T = \varepsilon$) is a single eigenvalue, related to the energy of the photon in the frame of reference. Therefore, for photon particles no superposition of spatial momenta can be developed, yielding a plane EM wave in the reciprocal spatial position dimension of frames.

In the proposed SR theory, frames have 3 spatial coordinates (x,y,z) of position-space and a single temporal coordinate (d_t) of displacement-space, representing frames of 3D-space+time. Although the combined dimensions of the temporal position (x_t) with the other 3 spatial positions (x_t/x,y,z) are accessible in 4D-space, they are not accessible in frames of 3D-space+time. This means that the total displacement vector ($\mathbf{d}_T$) of particles in 4D-space, cannot be directly observed in frames of 3D-space+time. The only observables are a scalar magnitude of the vector and its projection components into the spatial and the temporal coordinates of the reference frame ($d_T, \mathbf{d}_s, d'_t$). This explains why four-vectors in frames of 3D-space+time don't contain explicit information regarding the direction of the vector in 4D-space (see section IX). For photon particles the total displacement vector is purely spatial ($\mathbf{d}_T = \mathbf{d}_s$), i.e. directed in accessible spatial position dimensions. Therefore the total displacement of photons can be observed as a spatial vector and not by projection components, as for mass particles.

In conclusion:

j) Coordinates of frames are mixed domains: 3 spatial coordinates of position-space and a single temporal coordinate of displacement-space. Therefore, in spatial coordinates of any frame of reference, particles can be observed either as waves in position-space or as momentum particles in displacement-space. In the temporal coordinate they can be observed solely as momentum particles in displacement-space with no accessibility to position.

XII. THE RELATIVISTIC DOPPLER SHIFT

In the following we derive a general frame transformation of energy and momentum in displacement-space relevant to all particles. This is particularly instrumental for massless photons which are typically treated in conventional SR as radiation waves in position-space. A position-space derivation based on a Lorentz transformation of four-position is obviated in the proposed SR theory. The following ratio is universal and valid for any particle between any two inertial frames F and F':

$$\boxed{\varepsilon/\varepsilon' = P_T/P'_T = d_T/d'_T} \tag{1.39}$$

Where the unprimed energy ε, total momentum p_T, and total displacement d_T of a particle are observed in a reference frame F. The primed energy ε', total momentum p'_T, and total displacement d'_T of a particle are observed in frame F', which has a Lorentz boost relative to reference frame F in the +x or in the opposite –x direction. The ratio in Eq. (1.39) is a direct outcome of the Lorentz transformation of four-displacement and four-momentum between frames and using Eq. (1.34). d_T is deduced by the Lorentz transformation of four-displacement in Eq. (1.17), as follows: For mass particles at spatial rest in frame F', we deduce $d'_T = d'_t$ and $d_T = d_t = \gamma d'_t$ independent of the frame boost direction +β or –β. For a photon displacing in frame F' in +x' direction ($Ph_{x'}$), we deduce $d'_T = d'_t = d'_x$ and $d_T = d_t = d_x = \gamma d'_x \pm \gamma\beta d'_t$ which is dependent on the frame boost direction +β or –β. For a photon displacing in frame F' in +y' direction ($Ph_{y'}$), we deduce $d'_T = d'_t = d'_y$ and $d_T = ({d_y}^2 + {d_x}^2)^{\frac{1}{2}} = d_t$ independent of the frame boost direction. For a photon displacing in frame F' in +z' direction ($Ph_{z'}$), we deduce $d'_T = d'_t = d'_z$ and $d_T = ({d_z}^2 + {d_x}^2)^{\frac{1}{2}} = d_t$ independent of the frame boost direction. Employing these considerations to Eq. (1.39), we obtain the observed energy in reference frame F of each particle class, as follows:

$$\text{Mass particles} \rightarrow \quad \varepsilon = \varepsilon'(\gamma d'_t)/d'_t = \boxed{\gamma\varepsilon'} \tag{1.40}$$

$$Ph_{x'} \rightarrow \quad \varepsilon = \varepsilon'(\gamma d'_x \pm \gamma\beta d'_t)/d'_x = \boxed{\gamma(1 \pm \beta)\varepsilon'} \tag{1.41}$$

$$Ph_{y'} \rightarrow \quad \varepsilon = \varepsilon'({d'_y}^2 + \gamma^2\beta^2{d'_t}^2)^{\frac{1}{2}}/d'_y = \boxed{\gamma\varepsilon'} \tag{1.42}$$

$$Ph_{z'} \rightarrow \quad \varepsilon = \varepsilon'({d'_z}^2 + \gamma^2\beta^2{d'_t}^2)^{\frac{1}{2}}/d'_z = \boxed{\gamma\varepsilon'} \tag{1.43}$$

Eq. (1.40) represents an energy increase over the rest energy or rest mass ($\varepsilon' = m_0 c^2$) of a mass particle due to a Lorentz boost. Eq. (1.41) represents a photon energy increase or decrease related to longitudinal Doppler shift.

The plus or minus sign is respectively related to frame F' (where the photon emitting source is stationary) approaching or receding a stationary observer in reference frame F. The longitudinal Doppler shift is generated due to a displacement component in the direction of the frame boost ($d'_x \neq 0$), which effectively induces a displacement term ($\pm\gamma\beta d'_x$) in the temporal coordinate. For $P_{x'}$ photons this component is maximal $d'_x = d'_t$, therefore the longitudinal Doppler shift is maximal ($\pm\gamma\beta$). For $P_{y'}$ or $P_{z'}$ photons this component is zero yielding zero longitudinal Doppler shift. Eqs. (1.42-1.43) represent the energy increase of $P_{y'}$ or $P_{z'}$ photons related to transverse Doppler shift which is identical for opposing frame boost directions $\pm\beta$. These results are equivalent to those derived in position-space by conventional SR, treating a vast plurality of photons as waves propagating radially in all space directions. Here the derivation is entirely in displacement-space where photons with specific momentum directions are treated as momentum particles. Using the linear dependence of photon energy on associated wave frequency ($\varepsilon \propto f$), Eqs. (1.41-1.43) can be expressed by frequencies, replacing everywhere ε' by f ' and ε by f.

For a general displacement direction of photons in frame F' the same derivation principles are used, yielding:

$$\boxed{\varepsilon/\varepsilon' = P_T/P'_T = d_T/d'_T = \gamma(1 \pm \beta\cos\alpha)} \tag{1.44}$$

Where $\cos\alpha = d'_x/d'_T$ accounts for the projection component of the total displacement d'_T into the x' coordinate of frame F'. This component is in the direction of the Lorentz boost, therefore responsible for a longitudinal Doppler shift. For mass particles (at spatial rest in frame F') and for $P_{y'}$ or $P_{z'}$ photons this displacement component is zero, yielding $\varepsilon/\varepsilon' = \gamma$ as appears in Eqs.(1.40, 1.42, 1.43).

XIII. THE FOUR-CURRENT

In any inertial frame of reference, the electric four-current **J** form a Lorentz four-vector with the new structure of Eq. (1.33). The four-current is expressed in terms of the electric charge density ρ_0 in a spatial rest frame and the "velocity" four-vector **D**, as follows:

$$\mathbf{J} = (j_T, j_x, j_y, j_z) = (J_T, \mathbf{J}_s) = c\rho_0\gamma(D_T, \mathbf{D}_s) = c\rho_0\mathbf{D} \tag{1.45}$$

$$J_T = (c\gamma\rho_0)D_T, \quad \mathbf{J}_s = (c\gamma\rho_0)\mathbf{D}_s, \quad j'_t = (c\gamma\rho_0)D'_t = c\rho_0$$

Where $(c\gamma\rho_0)$ is the total magnitude of the four-current. As a position-space property, the charge density ρ_0 remains constant in all frames (in conventional SR, the charge density is modified due to length contraction). The square norm of **J** is a Pythagorean expression of a Lorentz invariant, as follows:

$$\|\mathbf{J}\|^2 = {J_T}^2 - {J_s}^2 = {j'_t}^2 = (c\rho_0)^2 \tag{1.46}$$

The temporal current j'_t is frame invariant, equal to the total current of an object in a spatial rest frame ($J_s = 0 \rightarrow J_T = j'_t$), termed the proper current. Since particles are never at rest in 4D-space, no charge can exists without generating a current. We could define the current components without the extra factor γ, as their natural meaning, i.e. $J_T = c\rho_0 D_T$, $\mathbf{J}_s = c\rho_0\mathbf{D}_s$, $j'_t = c\rho_0 D'_t$. The identity ${J_T}^2 - {J_s}^2 = {j'_t}^2$ would then be still valid but not Lorentz invariant. In this case, the frame invariant would be the total current ($J_T = c\rho_0$) rather than the temporal current (j'_t). By multiplying both sides of the identity by the factor γ^2, the current components compose a Lorentz four-current with the temporal component being Lorentz invariant of the same value $j'_t = c\rho_0$. The same approach was taken for the "velocity" four-vector in Eq. (1.30).

XIV. THE FOUR-WAVEVECTOR

Making a link to Quantum theories, the total energy E of a particle is related to a wave angular frequency of the particle field by $E = \hbar\omega$. Since for free-particles interaction energy is zero, therefore $E = \varepsilon = p_T = c\hbar k_T$ yielding $k_T = \omega/c$. Similarly the spatial momentum of a particle (in energy units) is related to a spatial wavevector of the particle field by $\mathbf{p}_s = c\hbar\mathbf{k}_s$. They are all components of four-wavevector **K**, linking between particle properties in momentum-space and wave properties of particle field in position-space, as follows:

$$\mathbf{K} = (k_T, k_x, k_y, k_z) = (k_T, \mathbf{k}_s) = \frac{1}{c\hbar}\mathbf{P} = \frac{\varepsilon'}{c\hbar}\mathbf{D} = k'_t\mathbf{D} \tag{1.47}$$

$$k_T = (\varepsilon/c\hbar)D_T, \quad \mathbf{k}_s = (\varepsilon/c\hbar)\mathbf{D}_s, \quad k'_t = (\varepsilon/c\hbar)D'_t$$

The four-wavevector **K** differs from the "velocity" or the displacement rate four-vector **D** only by a scalar. Therefore, its square norm is a Pythagorean expression of a Lorentz invariant as follows:

$$\|\mathbf{K}\|^2 = (k_T)^2 - (k_s)^2 = {k'_t}^2 = (\varepsilon'/c\hbar)^2 \tag{1.48}$$

The temporal wavevector k'_t is frame invariant, associated with the proper momentum of the particle in a spatial rest frame ($k'_t = \varepsilon'/c\hbar = p'_t/c\hbar$), For mass particles the frame invariant is $\varepsilon'/c\hbar = m_0c/\hbar$. For massless photons the frame invariant is $\varepsilon' = 0$, yielding $k_T = \omega/c = k_s$ associated with Electromagnetic field of photons. We conclude: the total wavevector of any particle, mass or massless, is non-dispersive ($k_T = \omega/c$). Only the spatial wavevector of mass particles is dispersive, i.e. ${k_s}^2 = (\omega/c)^2 + (m_0 c/\hbar)^2$. Eqs. (1.47-1.48) are equivalent to the four-wavevector $\mathbf{K}$ of conventional SR in Eqs. (1.14-1.15).

XV. THE FOUR-MOMENTUM OPERATOR

In the proposed SR theory, particles are described in the domain of displacement-space associated with the momentum-space, irrespective of position and localization in position-space. This exhibits inherent compatibility with Quantum theories, which provide procedures to transform representations in momentum-space to equivalent representations in position-space, and vice-versa. These two domains are 4-dimensional reciprocal Fourier domains, yielding a Heisenberg uncertainty of conjugate position and momentum extended to 4D-space. Therefore, any representation in position-space bears properties of momentum-space. The four-momentum operator is represented in the momentum-space as a multiplicative operator:

$$\widehat{\mathbf{P}} = (\hat{p}_T, \hat{p}_x, \hat{p}_y, \hat{p}_z) = (\hat{p}_T, \widehat{\mathbf{p}}_s) = (\hat{E}, \widehat{\mathbf{p}}_s) \tag{1.49}$$

Since $\varepsilon = p_T$, and for free particles $\varepsilon = E$, the total momentum operator $\hat{p}_T$ (observed in herein units of energy) is equivalent to the total energy operator ($\hat{p}_T = \hat{E}$). According to Quantum theories, the four-momentum operator in Eq. (1.49) has an equivalent representation in position-space as a four-gradient $\boldsymbol{\partial}$ (expressed by contravariant components), as follows:

$$\widehat{\mathbf{P}} = ic\hbar\left(\frac{\partial}{\partial d_T}, -\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}, -\frac{\partial}{\partial z}\right) = ic\hbar\left(\frac{\partial}{\partial d_T}, -\boldsymbol{\nabla}\right) \tag{1.50}$$

$$\hat{p}_T = ic\hbar\frac{\partial}{\partial d_T} = i\hbar\frac{\partial}{\partial t}\ , \quad \widehat{\mathbf{p}}_s = -ic\hbar\boldsymbol{\nabla}, \qquad \widehat{\mathbf{P}} \equiv ic\hbar\boldsymbol{\partial}$$

Where $\boldsymbol{\nabla}$ is the spatial gradient operator, $\hbar$ is the reduced Planck constant, and i is the imaginary unit. In a covariant representation all components of the four-gradient have a positive signature. The four-momentum operator should be expressed in position-space as a partial derivative with respect to all relevant positions of 4D-space. However, the partial derivative of the zero component (representing the total momentum operator $\hat{p}_T$) is based on displacement (∂d_T) rather than on position (∂x_T) as for other spatial components ($\partial x, \partial y, \partial z$). As previously explained, the total displacement d_T is equal to the temporal displacement d_t in any frame of reference ($d_T = d_t = ct$). Since in frames of 3D-space+time there is no position accessibility in the temporal coordinate, the relevant physical observable in the temporal coordinate is the temporal displacement $d_t = ct$ rather than the temporal position x_t. As intervals they are equivalent to each other ($\Delta d_t = \Delta x_t = \Delta d_T = \Delta x_T$). This explains the definition of the total momentum operator $\hat{p}_T$ in Eq. (1.50).

This explains why there is no time operator $\hat{t}$ in quantum theories. As opposed to position operators in spatial coordinates, which have observables that can take on a multitude of statistical values in an experiment, time is an independent displacement variable that increases with absolute certainty. Operators of Fourier conjugate observables don't commute. A spatial position operator doesn't commute with a conjugate spatial momentum operator, i.e. $[\hat{x}, \hat{p}_x] \neq 0,\ [\hat{y}, \hat{p}_y] \neq 0,\ [\hat{z}, \hat{p}_z] \neq 0$. No such relation exists between the total momentum operator $\hat{p}_T$ and time (or equivalently, between the total energy $\hat{E}$ or the Hamiltonian $\hat{H}$ operator and time). This is simply because time (understood as temporal displacement $d_t = ct$) is in the same domain as the momentum and not in a reciprocal domain of position. By contrast, in conventional SR time has a status of a position coordinate. So there is no convincing explanation in conventional SR why time cannot be a Quantum operator.

XVI. POSITION-SPACE WAVE EQUATIONS

The four-gradient $\boldsymbol{\partial}$, as a representation of the Lorentz four-momentum, is a basic component in various Quantum wave equations in position-space. The four-gradient borrows the Minkowski metric of the four-momentum as defined in Eq. (1.33). Recall, when a zero component of the four-vector represents the total magnitude of the vector, the Minkowski metric $\eta_{\mu\nu}$ yields a Pythagorean expression for the inner product, representing a Lorentz invariant and indicating that the 4D-space is Euclidian and absolute. This understanding is also applicable to four-operators, i.e. the inner product of the four-gradient with itself is the four-Laplacian operator, yielding a Lorentz invariant as follows:

$$\widehat{\mathbf{P}} \cdot \widehat{\mathbf{P}} = -(c\hbar)^2 \boldsymbol{\partial} \cdot \boldsymbol{\partial} = \varepsilon'^2 \tag{1.51}$$

$$\boldsymbol{\partial} \cdot \boldsymbol{\partial} = \partial^\mu \eta_{\mu\nu}\, \partial^\nu = \partial_\nu\, \partial^\nu = \partial^\mu\, \partial_\mu =$$

$$\frac{\partial^2}{\partial {d_t}^2} - \boldsymbol{\nabla}^2 = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla}^2 = -\frac{\varepsilon'^2}{(c\hbar)^2}$$

Two examples of wave equations in position-space, based on the inner product of the four-momentum, are brought in the following. For a scalar field (Ψ), Eq. (1.51) yields the most basic relativistic Quantum wave equation known as the Klein-Gordon wave equation[3] for spin-zero mass bearing free-particles ($\varepsilon' = m_0 c^2$), as follows:

$$\left[(\boldsymbol{\partial}\cdot\boldsymbol{\partial})+\left(\frac{\mathrm{m_0 c}}{\hbar}\right)^2\right]\Psi=0 \tag{1.52}$$

For a four-vector field ($\mathbf{A}$) in a Lorentz-gauge ($\boldsymbol{\partial}\cdot\mathbf{A}=\partial_\mu \mathrm{A}^\mu=0$), Eq. (1.51) yields the free Maxwell wave equation in vacuum for massless ($\varepsilon'=0$) photon field, as follows:

$$[\boldsymbol{\partial}\cdot\boldsymbol{\partial}]\mathbf{A}=0 \tag{1.53}$$

Based on an electromagnetic four-potential $\mathbf{A}$ of the same four-vector new structure in Eq. (1.33), the full Maxwell Equations[4] comprising sources are expressed (in SI units), as follows:

$$[\boldsymbol{\partial}\cdot\boldsymbol{\partial}]\mathbf{A}=\mu_0\mathbf{J} \tag{1.54}$$

$$\frac{1}{\mathrm{c}^2}\frac{\partial^2\mathrm{A}^\mu}{\partial \mathrm{t}^2}-\boldsymbol{\nabla}^2\mathrm{A}^\mu=\mu_0\mathrm{J}^\mu$$

$$\mathbf{A}=\left(\mathrm{A_T},\mathrm{A_x},\mathrm{A_y},\mathrm{A_z}\right)=(\mathrm{A_T},\mathbf{A}_\mathrm{s})=(\Phi/\mathrm{c},\mathbf{A}_\mathrm{s})$$

Where μ_0 is the permittivity of the vacuum and $\mathbf{J}$ is the four-current derived in Eq. (1.45). The electromagnetic four-potential $\mathbf{A}$ has the new component structure, composed of a total magnitude component $\mathrm{A_T}$ and 3 spatial components $\mathbf{A}_\mathrm{s}$. The total component is identified as the electric scalar potential $\mathrm{A_T}=\Phi/\mathrm{c}$ of conventional SR. The spatial components $\mathbf{A}_\mathrm{s}$ are the conventional magnetic vector potential. Since $\boldsymbol{\partial}\cdot\boldsymbol{\partial}$ is a Lorentz invariant scalar operator, and $\mathbf{J}$ is a four-vector, $\mathbf{A}$ is also a four-vector with the same new component structure as the four-current $\mathbf{J}$. An explicit expression of the four-potential, providing insight to its new component structure, is derived in the following.

In the Lorentz-gauge of Eq. (1.54), the four-potential $\mathbf{A}$ can be calculated as four-retarded potential[5] from four-retarded current distribution, as follows:

$$\mathbf{A}_\mathrm{s}(\mathbf{r},\mathrm{t})=\frac{\mu_0}{4\pi}\int_V\frac{\mathbf{J}_\mathrm{s}(\mathbf{r}_r,\mathrm{t}_r)}{|\mathbf{r}-\mathbf{r}_r|}\mathrm{d}^3\mathbf{r}_r \tag{1.55}$$

$$\mathrm{A_T}(\mathbf{r},\mathrm{t})=\frac{\mu_0}{4\pi}\int_V\frac{\mathrm{J_T}(\mathbf{r}_r,\mathrm{t}_r)}{|\mathbf{r}-\mathbf{r}_r|}\mathrm{d}^3\mathbf{r}_r$$

Where the four-potential components at spatial position $\mathbf{r}$ and time t, in a reference frame, are calculated from a relevant current source at distant spatial position $\mathbf{r}_r$ at an earlier (retarded) time $\mathrm{t}_r=\mathrm{t}-|\mathbf{r}-\mathbf{r}_r|/\mathrm{c}$. Integration is taken over all current sources in a relevant volume V. This simply reflects that changes in current sources propagate at the speed of light, and what matters is the relative distance from the source ($|\mathbf{r}-\mathbf{r}_r|$) and the relevant component of the four- current. It is explicitly apparent in Eq. (1.55), that the four-potential $\mathbf{A}$ is a position-space field that has the same component structure as the four-current $\mathbf{J}=\mathrm{c}\rho_0\mathbf{D}$, i.e. it borrows the Lorentz four-vector structure and the metric as all four-vectors in proposed SR theory.

In conclusion:

k) In proposed SR theory, Maxwell Equations are Lorentz covariant, i.e. retain the same form in all inertial frames. This Lorentz covariance is not due to the Lorentz transformation of spacetime, but due to Lorentz transformation of four-vectors representing the electromagnetic sources and fields in the momentum-space.

XVII. TRANSFORMATION OF SPATIAL VELOCITIES

For the completeness of the proposed SR theory, we now derive the frame transformation of displacement rates (D) or normalized velocities ($v/\mathrm{c}=\mathrm{u}=\mathrm{D}$) of mass particles based on Lorentz four-displacement of particles. Consider that some mass particles have a uniform normalized velocity u' in frame F', composed of the velocity components:

$$\mathrm{u'_x}=\mathrm{d'_x}/\mathrm{d'_t},\quad \mathrm{u'_y}=\mathrm{d'_y}/\mathrm{d'_t},\quad \mathrm{u'_z}=\mathrm{d'_z}/\mathrm{d'_t} \tag{1.56}$$

Frame-F' has a Lorentz boost ($v/\mathrm{c}=\beta$) in x-direction in reference frame F. Using the four-displacement transformation in Eq. (1.17), we derive the transformation of velocity components in Eq. (1.56) to the corresponding velocity components observed in reference-frame F, as follows:

$$\mathrm{u_x}=\frac{\mathrm{d_x}}{\mathrm{d_t}}=\frac{\gamma\mathrm{d'_t}(\mathrm{u'_x}+\beta)}{\gamma\mathrm{d'_t}(1+\beta\mathrm{u'_x})}=\boxed{\frac{\mathrm{u'_x}+\beta}{1+\beta\mathrm{u'_x}}} \tag{1.57}$$

$$\mathrm{u_y}=\frac{\mathrm{d_y}}{\mathrm{d_t}}=\frac{\mathrm{d'_y}}{\gamma\mathrm{d'_t}(1+\beta\mathrm{u'_x})}=\boxed{\frac{\mathrm{u'_y}}{\gamma(1+\beta\mathrm{u'_x})}}$$

$$\mathrm{u_z}=\frac{\mathrm{d_z}}{\mathrm{d_t}}=\frac{\mathrm{d'_z}}{\gamma\mathrm{d'_t}(1+\beta\mathrm{u'_x})}=\boxed{\frac{\mathrm{u'_z}}{\gamma(1+\beta\mathrm{u'_x})}}$$

We now pursue the inverse transformation. Consider that some mass particles have a uniform normalized velocity u in frame F, composed of the velocity components:

$$\mathrm{u_x}=\mathrm{d_x}/\mathrm{d_t},\quad \mathrm{u_y}=\mathrm{d_y}/\mathrm{d_t},\quad \mathrm{u_z}=\mathrm{d_z}/\mathrm{d_t} \tag{1.58}$$

Frame F has now a Lorentz boost $(-\nu/c = -\beta)$ in –x direction in reference frame F'. Using Eq. (1.17), we derive the transformation of velocity components in Eq. (1.58) to the corresponding velocity components observed in reference frame F', as follows:

$$u'_x = \frac{d'_x}{d'_t} = \frac{\gamma d_t(u_x - \beta)}{\gamma d_t(1 - \beta u_x)} = \boxed{\frac{u_x - \beta}{1 - \beta u_x}}$$
$$u'_y = \frac{d'_y}{d'_t} = \frac{d_y}{\gamma d_t(1 - \beta u_x)} = \boxed{\frac{u_y}{\gamma(1 - \beta u_x)}}$$
$$u'_z = \frac{d'_z}{d'_t} = \frac{d_z}{\gamma d_t(1 - \beta u_x)} = \boxed{\frac{u_z}{\gamma(1 - \beta u_x)}} \quad (1.59)$$

The frame transformation of displacement rates or normalized velocities are pure expressions of displacement-space irrespective of position and localization in 4D-space. These expressions are equivalent to those of conventional SR. They indicates that at no frame can the transformed normalized velocity exceed a numerical value of one $(u \leq 1,\ u' \leq 1)$, i.e. the speed of light is a maximum velocity limit which cannot be exeeded.

XVIII. SUMMARY

A proposed theory of Special Relativity (SR) is derived in momentum-space and described above. The proposed SR theory is based on Lorentz four-displacement of particles instead of a Lorentz four-position in Minkowski-space. One result is that the two known postulates of conventional SR are replaced by a single, predicted universal principle: in any frame of reference, the total distance of displacement of all particles in the universe is identical. This principle unveils the source of the constancy of speed of light in inertial frames. It predicts that although inertial frames of 3D-space+time are relative, 4D-space is absolute and Euclidian. Furthermore, what appear in conventional SR as Lorentz four-vectors in a Minkowski metric of a relative spacetime are actually Lorentz four-vectors in an absolute Euclidian 4D-space. Moreover, the theory provides a fundamental new understanding of time, and retains most predictions of Special Relativity with inherent compatibility and integration to Quantum theories. Length-contraction and relativity of simultaneity are proposed to be obviated. Moreover, they are both inconsistent with quantum theories of bound-states, and have never been proven experimentally.

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Table-1 summarizes the equivalence and the differences between conventional SR based on Minkowski four-position and proposed SR derived herein based on four-displacement of particles. The main observable disagreement between theories is the prediction that length contraction in the direction of motion and relativity of simultaneity does not occur. Absence or existence of these two phenomena is strong evidence by which the proposed SR theory can be experimentally verified. Currently there are no published experiments which definitively demonstrate any of these two phenomena[6]. Considering the complexity of such experiments, and the fact that many other aspects of SR have been experimentally proven, no compelling motivation has existed to further perform relevant experiments. With the benefit of achieving simplified SR theory inherently compatible with Quantum theories, motivation will be naturally raised to carry out experiments for proving or disproving the existence of length contraction and relativity of simultaneity. Such experiments will provide an evidentiary basis whether Special Relativity is the outcome of Lorentz four-position in a Minkowski-space, or an outcome of Lorentz four-displacement of particles in an absolute Euclidian space, as proposed herein.

Table 1: Equivalence and difference between proposed SR theory versus conventional SR theory

Topic	**SR based on Four-displacement of particles**	**SR based on Minkowski four-position**
Domain of SR Theory	Displacement/momentum-space A Fourier reciprocal to 4D position-space of Euclidian metric: $d'_t = ct \rightarrow p'_t(\mathcal{F})\, x_0 \quad (+)$ $d_x \rightarrow p_x\,(\mathcal{F})\, x \quad (+)$ $d_y \rightarrow p_y\,(\mathcal{F})\, y \quad (+)$ $d_z \rightarrow p_z\,(\mathcal{F})\, z \quad (+)$	Position-space Four-position of a Minkowskian metric: $x_0 = ct \quad (+)$ $x_1 = x \quad (-)$ $x_2 = y \quad (-)$ $x_3 = z \quad (-)$
Time & spatial coordinates	Time ($d_t = ct$) is a property of mass particles in temporal coordinate, generated by the inherent proper momentum of mass particles. Frame coordinates are mixed domains: 3 spatial coordinates of accessible position-space and a temporal coordinate of displacement-space.	Time ($x_0 = ct$) is a position-space like coordinate with polarity signature opposite to the other spatial coordinates of Minkowski four-position. Time and space are intertwined, placed on equal footing, all Frame coordinates are of position-space.
Universal governing law	Predicted In any frame, all particles perform an equal distance of total displacement irrespective of position and localization in position-space. Constancy of speed of light is a byproduct.	Postulated Speed of light in vacuum is a universal constant in any inertial frame of reference and in any direction.
4D-space	Absolute and Euclidian The 4D Euclidian space is absolute for all inertial frames, unaffected by a Lorentz boost.	Relative Minkowskian The 4D-Minkowskian space is relative. Spacetime of frames is affected by a Lorentz boost.
Fundamental frame	Spatial rest frame of mass particles $P_s = 0, \quad P'_t \neq 0$	Rest-frame of mass particles $P = 0$
Frame synchronization	By four-displacement Irrespective of position of frames and particles in 4D position-space $\ell = 0 \rightarrow \quad ct = ct' = 0$	By four-position Dependent on relative position and velocity of frames and particles in spacetime $x = x' = 0, y = y' = 0, z = z' = 0, t = t' = 0$
4-vector structure	$\mathbf{V} = (V_0, V_1, V_2, V_3) = (V_T, V_x, V_y, V_z)$ $V_0 = V_T$ - the total magnitude of the vector	$\mathbf{V} = (V_0, V_1, V_2, V_3) = (V_t, V_x, V_y, V_z)$ $V_0 = V_t$ - the time-like component of the vector
4-vector Inner product	$\mathbf{V}\cdot\mathbf{V} = (V_T)^2 - (V_x)^2 - (V_y)^2 - (V_z)^2 = (V'_t)^2$ Pythagorean expression of a Lorentz invariant, representing the temporal component of the vector. The invariant is equal to the total magnitude of the vector in a spatial rest frame, termed the proper value	$\mathbf{V}\cdot\mathbf{V} = (V_t)^2 - (V_x)^2 - (V_y)^2 - (V_z)^2 = LI$ Minkowskian expression of a Lorentz invariant (LI)
Velocity 4-Vector	$\mathbf{D} = \gamma(D_T,\ \mathbf{D}_s), \quad (D_T = v_T/c = 1,\ \mathbf{D}_s = \boldsymbol{v}_s/c)$ $\mathbf{D}\cdot\mathbf{D} = (\gamma D'_t)^2 = 1$	$\mathbf{U} = \Delta\mathbf{R}/\Delta\boldsymbol{\tau} = \gamma(c,\ \boldsymbol{v})$ $\mathbf{U}\cdot\mathbf{U} = c^2$
Displacement 4-Vector	$\mathbf{d} = (d_T,\ \mathbf{d}_s) = d'_t\mathbf{D}$ $\mathbf{d}\cdot\mathbf{d} = {d'_t}^2 \geq 0$ Displacement of particles	$\Delta\mathbf{R} = (c\Delta t,\ \Delta\mathbf{r})$ $\Delta\mathbf{R}\cdot\Delta\mathbf{R} \geq 0$ Timelike, $\Delta\mathbf{R}\cdot\Delta\mathbf{R} \leq 0$ Spacelike
Momentum 4-Vector	$\mathbf{P} = (P_T,\ \mathbf{P}_s) = \varepsilon'\mathbf{D}$ Energy units $\mathbf{P}\cdot\mathbf{P} = {P'_t}^2 = \varepsilon'^2$	$\mathbf{P} = m_0\gamma(c,\ \boldsymbol{v}) = m_0\mathbf{U}$ $\mathbf{P}\cdot\mathbf{P} = (m_0 c)^2$
Position 4-Vector	Obviated	$\mathbf{R} = (ct,\ \mathbf{r})$
Quantum compatibility	Inherent compatibility Non deterministic theory compatible with the formalism of Quantum theories.	Limited compatibility Deterministic theory with areas of inconsistency with the formalism of Quantum theories.
Measurable disagreements	Absolute simultaneity Absolute length	Relative simultaneity Length contraction

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