Proof of the Second Landau's Problem

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***Abstract*—This article describes how, using a formula found as a result of a careful review of the initial conditions, this problem was solved.**

***Index Terms*—algorithm**

I. Definition of the Second Landau's Problem

Definition: Is there infinitely sequence of "simple twins" - primes (y_o), the difference between them being 2?

II. Algorithm for Proof of the Second Landau's Problem

A. Types of Pairs of Odd Numbers

The sequence of odd numbers $\{y\}$ consists of pairs of odd numbers of the following sequence after the separation of the sequence $\{3y\}$:

$$\left\{y_n, (y_n+2) \mid \frac{y_n}{3} \notin \mathbb{N}^*,\ \frac{y_n+2}{3} \notin \mathbb{N}^*\right\}, \tag{1}$$

where $\mathbb{N}^*$ be natural numbers without zero.

The sequence of pairs (1) consists only of "simple twins" after the separation $\{3y\}$.
Then "simple twins" are beat out by intersecting sequences of composite odd numbers (y_{comp}) of the following form:

$$\left\{y_{on}y \mid y \geq y_{on}, \frac{y}{3} \notin \mathbb{N}^*\right\}. \tag{2}$$

After the successive interaction of (2), (1) will have the form shown in Tables I-IV. Thus, after the selection of the sequence $\{3y\}$ in (1) $\Sigma_A = 100\%, \Sigma_{BCD} = 0\%$; where Σ_A is appearance of the the set A in (1), Σ_{BCD} is appearance of the the set $\{B;C;D\}$ in (1) (see Tables I-IV).

TABLE I
PRIME PAIRS

A	
first prime	second prime

TABLE II
PAIRS WITH THE FIRST PRIME

B	
prime	composite number

TABLE III
PAIRS WITH THE SECOND PRIME

C	
composite number	prime

TABLE IV
PAIRS WITH COMPOUND NUMBERS

D	
first composite number	second composite number

B. Pairs of the Sequence of Odd Numbers of the form 5y

Let's start in succession from the sequence of composite numbers $\{5y \mid y \geq 5,\ y/3 \notin \mathbb{N}^*\}$. The distribution of this sequence in (1) is repeated in rows of five pairs according to the scheme as shown in Table V. The frequency of the appearance of the the set $\{B;C;D\}$ (see Tables II-IV) in (1) will now be determined according to Table V:

$$\Sigma_{BCD} = 100\% \cdot \frac{2}{5} = 40\%. \tag{3}$$

TABLE V
THE DISTRIBUTION OF THE SET $5y$ IN THE ROW OF THE SET (1)

1 pair	**2 pair**	**3 pair**	**4 pair**	**5 pair**
B	A	C	A	A

C. Pairs of the Sequence of Odd Numbers of the form 7y

The distribution of the next sequence of composite numbers $\{7y \mid y \geq 7,\ y/3 \notin \mathbb{N}^*\}$ in pairs (1) is repeated in rows of seven pairs according to the scheme, as shown in Table VI:

TABLE VI
THE DISTRIBUTION OF THE SET $7y$ IN THE ROW OF THE SET (1)

1 pair	**from 2 pair to 5 pair**	**6 pair**	**7 pair**
B	A	C	A

D. Pairs of the Sequence of Odd Numbers of the form 11y

The distribution of the next sequence of composite numbers $\{11y \mid y \geq 11,\ y/3 \notin \mathbb{N}^*\}$ in (1) is repeated in rows of eleven pairs according to the scheme, as shown in Table VII:

TABLE VII
THE DISTRIBUTION OF THE SET $11y$ IN THE ROW OF THE SET (1)

1 pair	from 2 pair to 4 pair	5 pair	from 6 to 11 pairs
B	A	C	A

E. Pairs of the Sequence of Odd Numbers of the form 13y

The distribution of the next sequence of composite numbers $\{13y \mid y \geq 13,\ y/3 \notin \mathbb{N}^*\}$ in (1) is repeated in rows of thirteen pairs according to the scheme, as shown in Table VIII:

TABLE VIII
THE DISTRIBUTION OF THE SET $13y$ IN THE ROW OF THE SET (1)

1 pair	from 2 pair to 9 pair	10 pair	from 11 to 13 pairs
B	A	C	A

F. Regularities of Knocking out Simple Twins from Pairs

According to subsections *B-E*, a separate knocking out "of simple twins " from (1) by (2) has the following regularities:

1. The repetition of the appearance of y_{comp} from (2) in (1) by rows with the number of pairs equal to y_{on} ;
2. Filling in a repeating rows with only one pair B (see Table II) and one pair C (see Table III);
3. B (see Table II) and C (see Table III) are not located in adjacent pairs;
4. B (see Table II) is always located in the first pair.

(4)

Starting with subsection *B* the knocking out of "simple twins" from (1) by the sequences (2) is superimposed on the pairs already filled with the previous sequence (2). The regularity changes in this case. Repetition can now be represented by areas where the number of consecutively filled columns from right to left is y_{on}, and the number of rows filled from top to bottom is $y_{o(n-1)}$, where $y_{o(n-1)}$ is a prime number in the sequence of primes just before y_{on}. Now there is a knocking out of "simple twins" from (1) according to the type D (see Table IV), which means intersecting sequences.
Let's calculate the frequency of the appearance of the set $\{B; C; D\}$ (see Tables II-IV) in subsequent pairs (1) for (3) and the new sequence $\{7y \mid y \geq 7,\ y/3 \notin \mathbb{N}^*\}$ with allowance for intersecting sequences:

$$\begin{aligned}\Sigma_{BCD} &= 100\% \cdot \left(\frac{2}{5} + \frac{2}{7} - \frac{4}{35}\right) = \\ &= 100\% \cdot \frac{4}{7} \approx 57{,}1429\%.\end{aligned} \tag{5}$$

Let's calculate the frequency of the appearance of the set $\{B; C; D\}$ (see Tables II-IV) in subsequent pairs (1) for (5) and the new sequence $\{11y \mid y \geq 11,\ y/3 \notin \mathbb{N}^*\}$ with allowance for intersecting sequences:

$$\begin{aligned}\Sigma_{BCD} &= 100\% \cdot \left(\frac{4}{7} + \frac{2}{11} - \frac{6}{77}\right) = \\ &= 100\% \cdot \frac{52}{77} \approx 67{,}5325\%.\end{aligned} \tag{6}$$

Let's calculate the frequency of the appearance of the set $\{B; C; D\}$ (see Tables II-IV) in subsequent pairs (1) for (6) and the new sequence $\{13y \mid y \geq 13,\ y/3 \notin \mathbb{N}^*\}$ with allowance for intersecting sequences:

$$\begin{aligned}\Sigma_{BCD} &= 100\% \cdot \left(\frac{52}{77} + \frac{2}{13} - \frac{102}{1001}\right) = \\ &= 100\% \cdot \frac{104}{143} \approx 72,7272\%.\end{aligned} \tag{7}$$

G. Expression for the Set $\{B; C; D\}$

Let's represent (5), (6) and (7) in a different form.
Let's (5) as follows:

$$\Sigma_{BCD} = 100\% \cdot \left(\frac{2}{5} + \frac{2 - \frac{4}{5}}{7}\right). \tag{8}$$

Let's (6) as follows:

$$\Sigma_{BCD} = 100\% \cdot \left(\frac{4}{7} + \frac{2 - \frac{6}{7}}{11}\right). \tag{9}$$

Let's (7) as follows:

$$\Sigma_{BCD} = 100\% \cdot \left(\frac{\frac{52}{7}}{11} + \frac{2 - \frac{102}{77}}{13}\right). \tag{10}$$

According (8), (9) and (10), the expression for the frequency of the appearance of the set $\{B; C; D\}$ in the sequence of subsequent pairs (1) under the action of (2) can be represented as follows:

$$\Sigma_{BCD} = 100\% \cdot \left(\frac{K_{y_{on}}}{y_{o(n-1)}} + \frac{2 - R_{y_{on}}}{y_{on}}\right), \tag{11}$$

where, without delving into the formula, we can distinguish the following conditions:

$$\begin{aligned}
&1.\quad K_{y_{on}} \notin \mathbb{N}^*;\\
&2.\quad y_{o(n-1)} - K_{y_{on}} > 2,\\
&\qquad (y_{o(n-2)} - K_{y_{o(n-1)}}) \leq (y_{o(n-1)} - K_{y_{on}});\\
&3.\quad \frac{K_{y_{o(n-1)}}}{y_{o(n-2)}} < \frac{K_{y_{on}}}{y_{o(n-1)}};\\
&4.\quad R_{y_{on}} \notin \mathbb{N}^*;\\
&5.\quad 0 < R_{y_{on}} < 2, \quad \frac{2 - R_{y_{o(n-1)}}}{y_{o(n-1)}} < \frac{2 - R_{y_{on}}}{y_{on}};\\
&6.\quad \frac{K_{y_{o(n-1)}}}{y_{o(n-2)}} + \frac{2 - R_{y_{o(n-1)}}}{y_{o(n-1)}} < \frac{K_{y_{on}}}{y_{o(n-1)}} + \frac{2 - R_{y_{on}}}{y_{on}}.
\end{aligned} \tag{12}$$

H. New Definition of the Second Landau's Problem

From (11) follows a new definition of the Second Landau's Problem:
Is it possible that with the successive filling of pairs (1) with the next sequence (2), the frequency of the appearance of the set $\{B; C; D\}$ (see Tables II-IV) in (11) reaches 100%?

I. Proof of the Second Landau's Problem by contradiction

Let's assume that when the pairs (1) are successively filled with some sequence (2) in (11), $\Sigma_{BCD} = 100\%$. Then:

$$1 - \frac{K_{y_{on}}}{y_{o(n-1)}} = \frac{2 - R_{y_{on}}}{y_{on}}. \tag{13}$$

But in the sequence of primes:

$$y_{o(n-1)} < y_{on}. \tag{14}$$

According to condition 5 in (12):

$$\frac{2 - R_{y_{on}}}{y_{on}} < \frac{2}{y_{on}}. \tag{15}$$

According to condition 2 in (12):

$$1 - \frac{K_{y_{on}}}{y_{o(n-1)}} > \frac{2}{y_{o(n-1)}}. \tag{16}$$

Because of (14), (15) and (16), expression (13) becomes invalid and takes the following form:

$$1 - \frac{K_{y_{on}}}{y_{o(n-1)}} > \frac{2 - R_{y_{on}}}{y_{on}}. \tag{17}$$

According to (17), when pairs (1) are filled with the next sequence (2), always:

$$\Sigma_{BCD} < 100\%. \tag{18}$$

Consequently, the sequence of "simple twins" is infinite.

(2015 year)

ACKNOWLEDGMENT

REFERENCES