



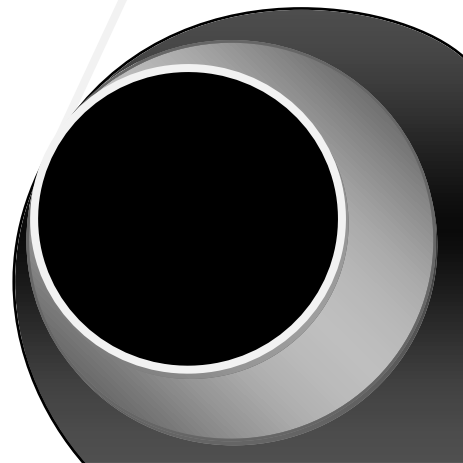
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MALE-FEMALE WAGE DIFFERENTIALS: DISCRIMINATION OR QUALITY DIFFERENTIALS?

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Abstract

This paper presents a quality theory for labor that is offered for obtaining estimates of the demand for labor quality and testing whether there is feminist wage discrimination and whether labor quality depends on sex. The approach is instructive for understanding the workings of a hedonic labor market framework, offers closed form solutions, and allows the introduction of nonlinearities in the analysis.

Keywords: hedonic pricing, quality differentials, product quality, labor quality, equilibrium model

JEL Classification: J2, J3, D11

Introduction

Tinbergen (1959) and Epple (1984) introduced a general strategy for obtaining closed form solutions to a class of hedonic equilibrium models that had several restrictive features. This methodology was extended by Giannias (1989) to obtain analytical solutions for a hedonic housing model that was used to estimate the consumer benefit from marginal and non-marginal improvements in the air quality

of Houston, Texas. Giannias (1991) presents a 4-step estimation procedure that is suitable for the analytical approach introduced in Giannias (1989). The Giannias's (1989) approach is employed in this paper to present a labor quality theory that can explain wage differentials. This theory is offered for testing whether there is feminist wage discrimination and whether labor quality depends on sex. The theory assumes that each labor unit can be fully described by a vector of objectively measured characteristics (e.g., education, experience, ...) and its sex. The vector of objectively measured characteristics determines the quality of labor which is assumed to be a scalar.

Firms are assumed to pay the same wage for labor units of equal quality. If the labor quality does not depend on sex, a man and a woman with an identical vector of objectively measured characteristics should receive the same wage. If they do not receive the same wage (given that the labor quality does not depend on sex), there is feminist wage discrimination. The theory presents a model that can be estimated and address the latter issue.

The paper is organized as follows. Section II introduces the economic model. Section III discusses the labor quality equation. Section IV specifies the econometric model. Concluding remarks are presented in Section V.

The economic model

I consider a competitive economy in which labor is heterogeneous. Firms produce the good x . A labor unit can be accurately described by a $(1 \times m)$ vector, a , of objectively measured characteristics and by its sex. So, a labor unit can be described by (a, d) , where $d = 1$ if male and $d = 0$ if female. It is assumed that firms care only about the quality, s , of the labor that they hire. The labor quality s is a

scalar and a function of (a,d). The labor quality index is discussed in the next section in more detail.

Firms are assumed to use constant returns to scale technologies and have different unit costs. These technologies can be described by a vector of technological parameters. b is the $(1 \times k)$ vector of technological parameters that describes the technology of a firm. It is assumed that b follows an exogenous normal distribution, that is,

$$b \sim N(m(b), v(b))$$

where, $m(b)$ is the mean of b , and $v(b)$ is the variance-covariance matrix of b .

$C(s,b)$ is the unit cost given b and s -type (quality) of labor.

$$C(s,b) = \text{ADMC}(s,b) + L(s,b) w(s) \quad (1)$$

where, $w(s)$ is the equilibrium wage equation (a function of labor quality), $L(s,b)$ is the number of s -type labor units needed to produce 1 unit of the product given b , and $\text{ADMC}(s,b)$ includes administrative and capital costs for producing 1 unit of the product given b and s -type of labor.

The $\text{ADMC}(s,b)$ and $L(s,b)$ functional forms are given next.

$$L(s,b) = N_0 + N_1 s \quad (2)$$

$$\text{ADMC}(s,b) = c_1 b' + c_2 b' s + 0.5 c_3 s^2 \quad (3)$$

where, N_0, N_1, c_3 are parameters, and c_1 , and c_2 are $(1 \times k)$ vectors of parameters.

A cost minimizing firm solves the following optimization problem.

$$\begin{aligned} \min \text{ADMC}(s,b) + L(s,b) w(s) & \quad (4) \\ \text{with respect to } s & \end{aligned}$$

The optimum decisions of producers depend on the equilibrium wage equation. The equilibrium wage equation is determined so that the producers and the suppliers of labor are perfectly matched. In equilibrium, none of the firms can improve its position, all of their optimum decisions are feasible, and the wage equation $w(s)$ is determined by the distribution of the supply for labor quality, the distribution of the vector of technological parameters b , and the exogenous parameters of the model.

The supply for labor quality is assumed to follow a normal distribution¹. Let it be:

$$s \sim N(M(s), V(s))$$

where, $M(s)$ is the mean, and $V(s)$ is the variance.

The condition for an equilibrium in the market described above is: aggregate demand for labor quality = aggregate supply for labor quality. Solving the profit maximisation problem (1) it is obtained that the demand for labor quality follows a normal distribution, since it is linear in the vector of technological parameters b . The following equation, equation (2), is the equilibrium wage equation because it sets the mean and the variance of the aggregate demand for s equal to the mean and variance of the aggregate supply for s .

$$w(s) = n_0 + n_1 s \quad (5)$$

where²,

$$n_0 = -0.5 [(R - c_3) N_0 + 2 N_1 c_2 m(b)] + 2 N_1 R M(s) / N_1^2 \quad (6)$$

$$n_1 = 0.5 (R - c_3) / N_1, \text{ and} \quad (7)$$

$$R = (c_2 v(b) c_2' / V(s))^{0.5}$$

The labor quality index equation

The labor quality index is assumed to be a scalar and a function of (a,d). Let this function be: $s = s(a,d)$ where, $d = 1$ if male and $d = 0$ if female, and a is the (1xm) vector of objectively measured characteristics that can accurately describe a labor unit.

The Taylor's theorem³ implies that $s(a,d)$ can be written as an nth-degree polynomial around the mean of a , $m(a)$, in the following way:

$$s(a,d) = k_0(d) + \sum_i k_i(d) [(a - m(a))_i]^i \quad (8)$$

where, $k_0(d) = s(m(a),d)$, $k_i(d)$ is the appropriately specified vector of the i-th order partial derivatives of $s(a,d)$ with respect to the elements of the (1xm) vector of labor characteristics a , evaluated at $(m(a),d)$, and $(a - m(a))_i$ is an appropriately specified vector whose elements are the i-th order products of the vector $(a - m(a))$.

For example,

$$k_1(d) (a - m(a))'_1 = s_1 (a - m(a))'$$

where, s_1 is the $(1 \times m)$ vector of the first partial derivatives of $s(a,d)$ with respect to the elements of the $(1 \times m)$ vector of labor characteristics, a , evaluated at $(m(a),d)$, and

$$k_2(d) (a - m(a))'_2 = 0.5 (a - m(a))' s_2 (a - m(a))'$$

where, s_2 is the $(m \times m)$ matrix of the second order partial derivatives of $s(a,d)$ with respect to a , evaluated at $(m(a),d)$. The latter implies that for the case that a is a (1×2) vector:

$$k_2(d) = (s_{11}(y) \ s_{22}(y) \ s_{12}(y)), \text{ and}$$

$$(a - m(a))'_2 = (0.5 (A_1)^2 \ 0.5 (A_2)^2 \ (A_1) (A_2)),$$

where, $s_{ij}(y)$ is the second partial derivative of $s(a,d)$ with respect to a_i and a_j , evaluated at $y = (m(a),d)$, and A_i is the deviation of the i -th element of a from its mean, for all i and j , $i = 1,2$ and $j = 1,2$.

We can now see that the complete model consists of equations (5), (8), and the equilibrium demand for labor quality:

$$s = g_0 + g_1 b' \tag{9}$$

where,

$$g_0 = M(s) + c_2 m(b)' / R, \text{ and}$$

$$g_1 = - c_2 / R$$

The demand for labor quality is the solution to the optimization problem (4). The equilibrium demand for labor quality (9) is obtained after substituting the equilibrium wage equation $w(s)$ (given in the Proposition above) into the demand for labor quality.

The econometric model

Assuming an additive error term on equations (5) and (9) and substituting equation (8) into (5), I obtain:

$$w = n_0 + \acute{O}_i D_i [(a - m(a))_i]' + u_1 \quad (10)$$

$$s = g_0 + g_1 b' + u_2 \quad (11)$$

where, $i= 1,2, \dots, m$, u_1 and u_2 are the econometric errors of equations (10) and (11) respectively,

$$D_i = N_1 k_i$$

for $i = 1, 2, \dots, m$.

It is assumed that $k_i(d)$ satisfies:

$$k_i(d) = k_{0i} + k_{1i} d \quad \text{for all } i = 0,1,2,\dots,m \quad (12)$$

where, k_{0i} and k_{1i} are exogenous vectors of parameters (they are scalars only for $i = 0$).

The quality of labor is a latent variable. Without loss of generality, the labor quality can be normalized by setting $k_{10} = 1$, and $k_{00} = 0$.

The wage equation can now be written in the following way:

$$w = n_0 + \acute{O}_i (D_{0i} + D_{1i} d) [(a - m(a))_i]' + u_1 \quad (13)$$

where, $D_{0i} = n_1 k_{0i}$, $D_{1i} = n_1 k_{1i}$.

The complete model consists of equations (13), (11), and (8). To estimate the model, I can apply the 4-step estimation procedure that is introduced in Giannias (1991). The following restrictions among the parameters will have to be used to obtain estimates for the parameters of the labor quality index equation (8):

$$k_{0i} = D_{0i}/D_{10} \text{ and} \quad (14)$$

$$k_{1i} = D_{1i}/D_{10} \quad (15)$$

To be more specific, first OLS is applied to estimate (13). The parameter estimates of (13) together with (14), (15), and (12) yield the estimates for the parameters of equation (8). The latter is used to obtain an estimated series for labor quality which is subsequently used to estimate (14).

Conclusions

We can estimate the model and test whether the parameters d_{1i} (for all i) equal zero, that is, we can test whether wages do not depend on sex. If this hypothesis is accepted, the model suggests that labor quality does not depend on sex. Consequently, a man and a woman with an identical vector of objectively measured characteristics should receive equal wages because they are of equal labor quality. If they do not there is feminist wage discrimination. Finally, the model is instructive for understanding the workings of the hedonic labor framework and is capable of introducing non-linearities in the analysis.

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ENDNOTES

1. The normality of the supply is justified using a central limit theorem argument which can be justified by sufficient regularity conditions, see Greenberg and Webster (1983).
2. There are two solutions that satisfy the equilibrium condition. One of them is rejected because it does not satisfy the second-order conditions for cost minimization.
3. See Chiang (1984).

