Special relativity in complex space-time. Part 3.
Description of complex space-time phenomena in the real coordinate system of the observer.

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Abstract

The current article is a next stage of construction of the alternative Special Relativity. The problems of Special Theory of Relativity (STR) arise from their colliding assumptions. The correct postulates of STR do not match the seemingly obvious assumption that space-time is real. In our articles, we try to prove the hypothesis that space-time has a complex structure, and the real one is only locally in a system related to an observer. The misleading impression that space-time is real results from the fact that the information carrier is energy that is always real. This article shows how the complex space-time phenomena can be seen by the observer in his real coordinate system. The concept of realisation of the complex orthogonal paravector, which represents a compound boost, to the form of the real velocity paravector has been introduced on the basis of energy equivalence . The realisation of coordinates of the state paravectors of any object is interpreted as a choice of real coordinates in the observer's frame, which in classical mechanics corresponds to the choice of the object's axis of motion with which the coordinates of the motion of another object are distributed. Mathematical properties of realisation are studied and an attempt to apply it to describe the physical object states.

Keywords: *Complex space-time, alternative special relativity, paravectors, realisation*

Introduction

Until now, all the researchers have assumed that space-time is a real or quasi-real structure that is, the character of imaginary numbers have one kind of coordinates (spatial or time). In the presented model both time and space are complex. It turns out that expanding of domain creates an opportunity to build the theory alternative for the Special

Theory of Relativity (STR) that has been in force for 100 years. Using the paravector calculus for the STR studies it is clear that the ineligibility of the topical theory comes from its assumptions. The right postulate of limited and constant speed of light is at odds with the assumption that space-time has a real structure. Unfortunately, in order to see this, it is necessary to master the mathematical tool which is the paravector calculus that is why in all publications we refer to the article [3] where the paravectors are shown as complex 4x4 matrices reduced to two-element column matrices with special properties.

During the formation of STR, the tensor formalism developed in parallel, and it became an obligatory language of scientific publications describing both theories of relativity. It is a very universal tool so, as it usually does, it is not ergonomic. The main disadvantage of the tensor notation is its unintuitive nature, which is very important in physics. The tensor calculus is a purely calculative tool in which one uses indexes without seeing what lies under them. It does not matter when the concepts under the indexes have the same properties, but it is not so in space-time. In space one can stand still, move at different speeds and return to the starting point, and the time does not stop and does not go back but it passes continuously. Therefore, the components of the 4-vectors need to be distinguished and separated. On the other hand, it has long been known that all physical quantities have two fundamentally different features and therefore they are divided into scalars and vectors. Attempts to find the most pictorial record of four-dimensional quantities were made from the very beginning of the formulation of the concept of space-time (H. Minkowski 1909). Apart from the tensors, matrices, quaternions, biquaternions and multi-vectors were used for this purpose. At the same time, a whole large branch of mathematics developed - Geometric Algebra (GA). David Hestenes achieved very successful results of using GA in physics. Many others followed, including William Baylis to whose articles we refer many times. Both of them used the algebra of paravectors, the first of which did not call it paravectors and used the multi-vector formalism known from Grassmann's algebra [2]. William Baylis (Algebra of Physical Space) was already working on the concept of paravectors, who noticed that his students were much more easily absorbed the material written in this formalism than in the tensor formalism. We went even further and by modifying paravectors we brought them closer to vectors and matrices.

The current article is the sixth in the series, and the third directly dealing with the problems of STR in complex space-time. They all are written using paravector formalism. The first two articles constitute the mathematical basis necessary to describe physical issues. The article [3] discusses the algebra of paravectors. In the next one [4] the four transformation identities containing two operators of space-time differentiation (4-gradient and 4-divergence) were proved. Thanks to them, it is easy to show that the wave equation is invariant under the orthogonal paravector transformation. Since this transformation does not belong to the Lorentz group (due to the domain), then it was interesting to use it in physics [5] [6]. It turned out that it is possible to find physical interpretations of complex time-space, and thus that it is possible to construct an alternative STR. The article [7] shows that Lorentz transformation in a form proposed by W. Baylis [1] most likely has an error because it has been shown that Euclidean turn is a particular form of a compound boost. The articles [5] [6] show the complex space-time on specific examples (coordinates and transformation parameters are real) so that the obtained results have their interpretations in classical physics. However, it should be remembered that some sizes may be complex, and then they come to be not intuitive. In the current article, we will deal with a mathematical solution to the problem of the transition from the complex space-time to the local real one of the observer. It has been hypothesized that real local space-time of the observer is to cognition, because the only information carrier is energy, and this is always real. The observer can see only the projection of complex space-time phenomena onto his space-time.

Below there are the assumptions that are applied at this and all previous works:

1. Physical formulas are written in a natural system of units, that is:
   - the speed of light is equal to 1,
   - linear speed is a dimensionless quantity and its value is a fraction of the speed of light,
   - The electric permittivity and magnetic permeability of a vacuum are equal to 1.

2. the real quantities are described with Roman letters and the complex with Greek ones.

1 Equivalence classes of orthogonal transformations.

The electric field of the stationary charge in the paravector notation is described by the function bellow

$$\mathbb{F} = \begin{pmatrix} 0 \\ \mathbf{E}(\mathbb{X}) \end{pmatrix} \tag{1}$$

In the article [6] it was shown that this field seen from the frame moving at the speed $-\mathbf{v}$ changes according to the following formula:

$$\mathbb{F}' = V \begin{pmatrix} 0 \\ \mathbf{E}(V^{-}\mathbb{X}') \end{pmatrix} \qquad \text{where} \qquad V = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix} \tag{2}$$

We called V a velocity paravector [5]. The energy density of this field [6] is equal to $W' = \mathbb{F}'\mathbb{F}'^{*}/2$, that is

$$W' = \frac{1}{2} V \begin{pmatrix} 0 \\ \mathbf{E}(V^{-}\mathbb{X}') \end{pmatrix} \begin{pmatrix} 0 \\ \mathbf{E}(V^{-}\mathbb{X}') \end{pmatrix} V = \frac{\mathbf{E}^2}{2} V V \tag{3}$$

Since the energy density of stationary electric field is a scalar function that it can be moved outside the velocity paravectors product.

In the article [6] the transformations described by real orthogonal paravectors V were considered, which where interpreted as velocity. But the result of multiplication of such paravectors is the Λ one which has complex components, which is easy to verify. For the complex velocity the energy density of the field (1) is:

$$W = \frac{1}{2} \Lambda \begin{pmatrix} 0 \\ \mathbf{E}(\Lambda^{-}\mathbb{X}') \end{pmatrix} \begin{pmatrix} 0 \\ \mathbf{E}(\Lambda^{-}\mathbb{X}') \end{pmatrix} \Lambda^{*} = \frac{\mathbf{E}^2}{2} \Lambda \Lambda^{*} \tag{4}$$

The energy density of electric field is a real paravector because it is a result of the product of 4-vectors mutually conjugated. Looking at the equations of (3) and (4) it can be seen that the paravector W is always real, whether it concerns a complex or real field. This property of energy has suggested the following hypothesis:

The man experiences the world around him through received energy stimuli. What he hears is the energy of the sound waves received by his ears. He sees that his eye receptors receive the energy of electromagnetic waves. It is similarly with heat or touch. The same applies to laboratory tests. Measuring instruments record a piece of energy proportional to the tested quantity, strengthen it and transform it into information. The only information carrier is energy, and this is real always(!). From it comes the impression that the surrounding world is real (in the mathematical sense). As a result of the above considerations a doubt arises: Isn't the World perhaps more complex than we think?

We hypothesize that even if the relativistic phenomena take place in a complex space-time, there should be a way to project them onto the real space-time available to our cognition, so that energy is preserved. Having the above in mind, we will try to find a way to bring any orthogonal paravector Λ representing the complex speed to the real velocity paravector $V = \frac{1}{\sqrt{1-v^2}}\begin{bmatrix}1\\ \mathbf{v}\end{bmatrix}$.

As it appears from the formulas (3) and (4) the energy density 4-vector of the field coming from the charge in-motion is the product of scalar energy density of a stationary field and the VV paravector associated with the motion. If we inserted any other Λ orthogonal paravector in place of V, the value of energy density would not change if

$$VV = \Lambda\Lambda^* = \frac{\Gamma\Gamma^*}{\det\Gamma} \qquad \text{or} \qquad VV = \Lambda^*\Lambda \tag{5}$$

Definition 1.1. We call **the right realisation of any orthogonal paravector** Λ the transformation that assigns to it the velocity paravector $\mathrm{V} = \frac{1}{\sqrt{1-v^2}}\begin{bmatrix}1\\ \mathbf{v}\end{bmatrix}$ according to the relation

$$\underline{|\Lambda} := \mathrm{V} \qquad \Longleftrightarrow \qquad \mathrm{VV} = \Lambda\Lambda^*$$

Definition 1.2. We call **the left realisation of any orthogonal paravector** Λ the transformation that assigns to it the velocity paravector $\mathrm{V} = \frac{1}{\sqrt{1-v^2}}\begin{bmatrix}1\\ \mathbf{v}\end{bmatrix}$ according to the relation

$$\underline{\Lambda|} := \mathrm{V} \qquad \Longleftrightarrow \qquad \mathrm{VV} = \Lambda^*\Lambda$$

As it is not difficult to show for the paravector $\Lambda = \frac{\Gamma}{|\Gamma|} = \frac{1}{\sqrt{a^2-b^2+c^2-d^2}}\begin{bmatrix}a+id\\ \mathbf{b}+i\mathbf{c}\end{bmatrix}$ the left realisation is the transformation:

$$\underline{\frac{\Gamma}{|\Gamma|}\Bigg|} = \frac{1}{\sqrt{1-v^2}}\begin{bmatrix}1\\ \mathbf{v}\end{bmatrix}, \qquad \text{where} \qquad \mathbf{v} = \frac{a\mathbf{b}+d\mathbf{c}-\mathbf{b}\times\mathbf{c}}{a^2+c^2} \quad , \tag{6}$$

and in accordance with the right realisation it is

$$\mathbf{v} = \frac{a\mathbf{b} + d\mathbf{c} + \mathbf{b}\times\mathbf{c}}{a^2+c^2}. \tag{7}$$

Then, in both cases

$$v^2 = \frac{b^2+d^2}{a^2+c^2}. \tag{8}$$

Proof. of the formula (6)

We start from the definition 1.2

$$\frac{1}{1-v^2}\begin{bmatrix}1\\ \mathbf{v}\end{bmatrix}\begin{bmatrix}1\\ \mathbf{v}\end{bmatrix} = \frac{1}{a^2-b^2+c^2-d^2}\begin{bmatrix}a-id\\ \mathbf{b}-i\mathbf{c}\end{bmatrix}\begin{bmatrix}a+id\\ \mathbf{b}+i\mathbf{c}\end{bmatrix} \tag{9}$$

The above equality is transformed into the form:

$$\frac{1+v^2}{1-v^2}\begin{bmatrix}1\\ \frac{2\mathbf{v}}{1+v^2}\end{bmatrix} = \frac{1+\frac{b^2+d^2}{a^2+c^2}}{1-\frac{b^2+d^2}{a^2+c^2}}\begin{bmatrix}1\\ \frac{2\frac{a\mathbf{b}+d\mathbf{c}-\mathbf{b}\times\mathbf{c}}{a^2+c^2}}{1+\frac{b^2+d^2}{a^2+c^2}}\end{bmatrix} \tag{10}$$

So, if we assume that $\mathbf{v} = \frac{a\mathbf{b}+d\mathbf{c}-\mathbf{b}\times\mathbf{c}}{a^2+c^2}$ then we can check that $v^2 = \frac{b^2+d^2}{a^2+c^2}$. □

For formalities, we must check that $v^2 < 1$.

Proof.

From the equation (8) it follows that $v^2\left(a^2+c^2\right) = b^2+d^2$. Subtracting a^2+c^2 from both sides of the above equation and dividing the result by a^2+c^2 we get

$$v^2-1 = \frac{-a^2+b^2-c^2+d^2}{a^2+c^2},$$

hence

$$v^2 = 1-\frac{a^2-b^2+c^2-d^2}{a^2+c^2}$$

Since the paravector Γ is proper, i.e. $a^2-b^2+c^2-d^2 > 0$, and $a^2-b^2+c^2-d^2 < a^2+c^2$, so on the right of the last equality we have a real non-negative number which is less then one.

□

The proof of formula (7) we leave to the reader.

To remember in which case we write the „-" sign before the vector product, and in which case „+", we give the following rule:

- underscore is directed towards negative numbers (left) for the left realisation and before the vector product we write the sign „-”,
- the underlining is directed towards the right (positive numbers) for the right realisation and before the vector product we write the sign „+”.

Let us get back to the definition of the right realisation

$$\frac{1}{1-v^2}\begin{bmatrix}1\\ \mathbf{v}\end{bmatrix}\begin{bmatrix}1\\ \mathbf{v}\end{bmatrix}=\frac{1}{a^2-b^2+c^2-d^2}\begin{bmatrix}a+id\\ \mathbf{b}+i\mathbf{c}\end{bmatrix}\begin{bmatrix}a-id\\ \mathbf{b}-i\mathbf{c}\end{bmatrix} \tag{11}$$

and substitute $v^2=\frac{b^2+d^2}{a^2+c^2}$, then we obtain

$$\begin{bmatrix}1\\ \mathbf{v}\end{bmatrix}\begin{bmatrix}1\\ \mathbf{v}\end{bmatrix}=\frac{1}{a^2+c^2}\begin{bmatrix}a^2+b^2+c^2+d^2\\ 2(a\mathbf{b}+d\mathbf{c}+\mathbf{b}\times\mathbf{c})\end{bmatrix} \tag{12}$$

$$\begin{bmatrix}1+v^2\\ 2\mathbf{v}\end{bmatrix}=\begin{bmatrix}1+\frac{b^2+d^2}{a^2+c^2}\\ 2\frac{a\mathbf{b}+d\mathbf{c}+\mathbf{b}\times\mathbf{c}}{a^2+c^2}\end{bmatrix} \tag{13}$$

In view of the above, we can extend the definition of the realisation on singular paravectors and we can define its right realisation

$$\underline{|\Omega}=\begin{bmatrix}1\\ \frac{r\mathbf{u}+s\mathbf{w}+\mathbf{u}\times\mathbf{w}}{r^2+w^2}\end{bmatrix} \tag{14}$$

As a result of the realisation of the complex paravector representing the compound speed of light, we obtain the real vector of the speed of light

$$\mathbf{c}=\frac{r\mathbf{u}+s\mathbf{w}+\mathbf{u}\times\mathbf{w}}{r^2+w^2}=\frac{r\mathbf{u}+s\mathbf{w}+\mathbf{u}\times\mathbf{w}}{s^2+u^2} \tag{15}$$

Attention should be paid to consistency with the formulas obtained in a completely different way in the article [5].

The left realisation of the paravector Ω is

$$\underline{\Omega|}=\begin{bmatrix}1\\ \frac{r\mathbf{u}+s\mathbf{w}-\mathbf{u}\times\mathbf{w}}{r^2+w^2}\end{bmatrix} \tag{16}$$

Realisation is the monadic operation. Orthogonal paravectors are equivalent if they realise to the same velocity paravector. So we can create an equivalence class in the set of orthogonal paravectors because of realisation. As a physically interpretable representative of each class, we choose the velocity paravector (V).

2 Properties of realisation

Below we will provide some basic properties of the conjugation which we will not be able to prove because we know from the article [3] that paravectors have their matrix representation.

Theorem 2.1. The following statements are true:

1. $(\Gamma_1\Gamma_2)^* = \Gamma_2^*\Gamma_1^*$
2. $\Gamma\Gamma^* \in R_+ \times R^3$ or equivalently $\Gamma_1^*\Gamma_2 + \Gamma_2^*\Gamma_1 \in R_+ \times R^3$
3. There are paravectors for which it is true that $\Gamma^*\Gamma \neq \Gamma\Gamma^*$

Theorem 2.2. For any two paravectors is true that $(\Gamma_1,\Gamma_2\rangle^* = \left\langle\Gamma_1^*,\Gamma_2^*\right)^-$

Proof.

$$(\Gamma_1,\Gamma_2\rangle^* = (\Gamma_1,\Gamma_2^-)^* = \Gamma_2^{-*}\Gamma_1^* = \left(\Gamma_1^{-*},\Gamma_2^*\right)^- = \left\langle\Gamma_1^*,\Gamma_2^*\right)^-$$

□

Theorem 2.3. Let Γ_1 and Γ_2 are proper paravectors and $\Gamma_1 \parallel \Gamma_2$ then $\frac{\Gamma_1}{|\Gamma_1|} = \frac{\Gamma_2}{|\Gamma_2|}$.

Proof.

From the definition of parallelism of proper paravectors it follows that their integrated product is a number $\lambda \neq 0$.

$$\frac{\Gamma_1}{|\Gamma_1|} = \frac{\Gamma_1\Gamma_2^-\Gamma_2}{\sqrt{\Gamma_1\Gamma_1^-}\det\Gamma_2} = \frac{\lambda}{\sqrt{\Gamma_1\Gamma_1^-\Gamma_2\Gamma_2^-}}\frac{\Gamma_2}{|\Gamma_2|} = \frac{\lambda}{\sqrt{\Gamma_1\lambda\Gamma_2^-}}\frac{\Gamma_2}{|\Gamma_2|} = \frac{\lambda}{\sqrt{\lambda^2}}\frac{\Gamma_2}{|\Gamma_2|} = \frac{\Gamma_2}{|\Gamma_2|}$$

□

Conclusion 2.1. Any orthogonal paravectors are parallel if and only if they are equal.

Theorem 2.4. Let Λ be an orthogonal paravector

$$\Lambda = \frac{1}{\sqrt{a^2 - b^2 + c^2 - d^2}}\begin{bmatrix} a + id \\ \mathbf{b} + i\mathbf{c} \end{bmatrix},$$

then the realisation of the paravector Λ has the following properties:

1. $\underline{|\Lambda^-} = \underline{\Lambda|}^-$ and $(\underline{|\Lambda})^- = \underline{\Lambda^-|}$
2. $\underline{|\Lambda^*} = \underline{\Lambda|}$ and $\underline{\Lambda^*|} = \underline{|\Lambda}$

3. For each orthogonal paravector $\underline{\Lambda_1\Lambda_2|} = \underline{\underline{\Lambda_1|}\Lambda_2\Big|}$

   Warning! The order of realisation is important. First we realise the "earlier" paravectors (with a lower index) and then the next ones towards the "last" one. If the indexes grow from left to right, then we use the left realisation; if they grow from right to left, then we use the right realisation [1].

   Respectively for the right realisation we have $\underline{|\Lambda_2\Lambda_1} = \underline{\Big|\Lambda_2\underline{|\Lambda_1}}$

4. For any orthogonal paravector
   $\underline{\Lambda_1|} = \underline{\Lambda_2|}$ if and only if $(\Lambda_1, \Lambda_2\rangle^* = (\Lambda_1, \Lambda_2\rangle^-$
   $\underline{|\Lambda_1} = \underline{|\Lambda_2}$ if and only if $\langle\Lambda_1, \Lambda_2)^* = \langle\Lambda_1, \Lambda_2)^-$
   It follows that if $\underline{\Lambda_1|} = \underline{\Lambda_2|}$, then the integrated product $(\Lambda_1, \Lambda_2\rangle$ is the unitary paravector, and in case of the right realization, the left integrated product is an unitary paravector.

5. $\left(\Lambda, \underline{\Lambda|}\right\rangle = \frac{1}{\sqrt{a^2+c^2}}\begin{bmatrix} a \\ i\mathbf{c} \end{bmatrix}$ $\qquad$ $\left(\underline{\Lambda|}, \Lambda\right\rangle = \frac{1}{\sqrt{a^2+c^2}}\begin{bmatrix} a \\ -i\mathbf{c} \end{bmatrix}$

   So, the integrated product of any orthogonal paravector and its left realisation give the unitary paravector.

6. In the general case, the realisation of the orthogonal paravector does not preserve a scalar product which means that realisation is not orthogonal transformation, but it preserve the parallelism.

7. The realisation preserves the scalar product of the vigors of orthogonal paravectors, i.e.

$$\langle \mathrm{vig}\Lambda_1, \mathrm{vig}\Lambda_2\rangle = \langle \mathrm{vig}\underline{|\Lambda_1}, \mathrm{vig}\underline{|\Lambda_2}\rangle \tag{17}$$

8. $\frac{1}{a^2+c^2}\begin{bmatrix} a \\ -i\mathbf{c} \end{bmatrix}\underline{\Lambda|}\begin{bmatrix} a \\ i\mathbf{c} \end{bmatrix} = \frac{1}{a^2+c^2}\begin{bmatrix} a \\ i\mathbf{c} \end{bmatrix}\underline{\Lambda|}\begin{bmatrix} a \\ -i\mathbf{c} \end{bmatrix} = \underline{|\Lambda}$

9. For any rotation is true that : $\quad \underline{R^-\Lambda R|} = R^-\underline{\Lambda|}R, \qquad$ where $R = \frac{1}{\sqrt{r^2+s^2}}\begin{bmatrix} r \\ i\mathbf{s} \end{bmatrix}$.

Proof.

1. $\left(\underline{|\Lambda^-}\right)_V = \frac{a(-\mathbf{b})+d(-\mathbf{c})+(-\mathbf{b})\times(-\mathbf{c})}{a^2+c^2} = -\frac{a\mathbf{b}+d\mathbf{c}-\mathbf{b}\times\mathbf{c}}{a^2+c^2} = \left(\underline{\Lambda|}^-\right)_V$

2. $\left(\underline{|\Lambda^*}\right)_V = \frac{a\mathbf{b}+d\mathbf{c}-\mathbf{b}\times\mathbf{c}}{a^2+c^2} = \left(\underline{\Lambda|}\right)_V$

[1] Such and not the other order of increasing the value of the index results from the following reasoning: The phase difference $V^-\mathbb{X}$ after passing to the frame moving at the speed $-\mathbf{v}_1$ has the form $V^-V_1^-\mathbb{X}'$, where $X' = V_1\mathbb{X}$. After the realisation of compound boost, the last phase will have the form $V^-_{real}\mathbb{X}_{real} = \underline{V^-\Lambda_1^-\Lambda_2^-\Big|}\mathbb{X}_{real}$, or $V_{real} = \underline{|\Lambda_2\Lambda_1 V}$

3. $\underline{\Lambda_1\Lambda_2}| = \mathrm{V} \quad \Longleftrightarrow \quad (\Lambda_1\Lambda_2)^*(\Lambda_1\Lambda_2) = \mathrm{VV} \quad \Longleftrightarrow \quad \Lambda_2^*\Lambda_1^*\Lambda_1\Lambda_2 = \mathrm{VV}$
   and since $\Lambda_1^*\Lambda_1 = \mathrm{V}_1\mathrm{V}_1$, then $\Lambda_2^*\mathrm{V}_1\mathrm{V}_1\Lambda_2 = \mathrm{VV}$,
   or $(\mathrm{V}_1\Lambda_2)^*(\mathrm{V}_1\Lambda_2) = \mathrm{VV} \quad \Longleftrightarrow \quad \underline{\mathrm{V}_1\Lambda_2}| = \mathrm{V} \quad \Longleftrightarrow \quad \underline{\underline{\Lambda_1}|\Lambda_2}\Big| = \mathrm{V}$

4. $\langle\Lambda_1,\Lambda_2\rangle^* = \langle\Lambda_1,\Lambda_2\rangle^- \quad \Longleftrightarrow \quad \left(\Lambda_1\Lambda_2^-\right)^* = \left(\Lambda_1\Lambda_2^-\right)^- \quad \Longleftrightarrow$
   $\Lambda_2^{-*}\Lambda_1^* = \Lambda_2\Lambda_1^- \quad \Longleftrightarrow \quad \Lambda_1^*\Lambda_1 = \Lambda_2^*\Lambda_2$ or $\underline{\Lambda_1}| = \underline{\Lambda_2}|$

5. The appropriate quantities should be inserted into the definition of the integrated product and calculations should be made.

6. We check how the integrated product $\langle\Lambda_1,\Lambda_2\rangle$ changes.
   $\langle\Lambda_1,\Lambda_2\rangle = \Lambda_1\Lambda_2^- = \Lambda_1\underline{\Lambda_1}|^-\underline{\Lambda_1}|\underline{\Lambda_2}|^-\underline{\Lambda_2}|\Lambda_2^- = \Big\langle\Lambda_1,\underline{\Lambda_1}|\Big\rangle\Big\langle\underline{\Lambda_1}|,\underline{\Lambda_2}|\Big\rangle\Big\langle\Lambda_2,\underline{\Lambda_2}|\Big\rangle^-$
   from which it can be seen that realization does not keep the scalar product.

7. The definitions of vigor, of the integrated product and of the realization should be used.
   $\langle \mathrm{vig}\Lambda_1, \mathrm{vig}\Lambda_2\rangle = \langle \mathrm{vig}\Lambda_1, \mathrm{vig}\Lambda_2\rangle_s = [(\Lambda_1\Lambda_1^*)(\Lambda_2\Lambda_2^*)^-]_s = [(V_1V_1)(V_2V_2)^-]_s = \langle \mathrm{vig}\underline{\Lambda_1}, \mathrm{vig}\underline{\Lambda_2}\rangle$

8. Use the property 7.

9. If $R = \frac{1}{\sqrt{r^2+s^2}}\begin{bmatrix} r \\ i\mathbf{s}\end{bmatrix}$, then $R^* = R^-$
   $\underline{R^-\Lambda R}\Big| = V \Leftrightarrow (R^-\Lambda R)^* R^-\Lambda R = VV \Leftrightarrow R^-\Lambda^* R R^-\Lambda R = R^-\Lambda^*\Lambda R$
   or
   $\underline{R^-\Lambda R}\Big|\,\underline{R^-\Lambda R}\Big| = R^-\Lambda^*\Lambda R$
   On the other hand
   $\underline{\Lambda}\Big| = V' \Leftrightarrow \Lambda^*\Lambda = V'V'$
   so
   $\underline{R^-\Lambda R}\Big|\,\underline{R^-\Lambda R}\Big| = R^-\,\underline{\Lambda}\Big|\,RR^-\,\underline{\Lambda}\Big|\,R$
   from where it is already seen that $\underline{R^-\Lambda R}| = R^-\underline{\Lambda}|R$.

□

Theorem 2.5. Each orthogonal paravector

$$\Lambda = \frac{1}{\sqrt{a^2-b^2+c^2-d^2}}\begin{bmatrix} a+id \\ \mathbf{b}+i\mathbf{c}\end{bmatrix}$$

can be unambiguously presented as a product

$$\frac{1}{\sqrt{1^2-v^2}}\frac{1}{\sqrt{a^2+c^2}}\begin{bmatrix} a \\ i\mathbf{c}\end{bmatrix}\begin{bmatrix} 1 \\ \mathbf{v}\end{bmatrix} = RV \qquad \text{or} \qquad \frac{1}{\sqrt{1^2-w^2}}\frac{1}{\sqrt{a^2+c^2}}\begin{bmatrix} 1 \\ \mathbf{w}\end{bmatrix}\begin{bmatrix} a \\ i\mathbf{c}\end{bmatrix} = WR,$$

where the paravector V is the right realisation, and W is the left realisation of Λ.

Proof.

From Theorem 2.4.5 it follows that the orthogonal paravector $\Lambda = RV$, where

$$\Lambda = \frac{1}{\sqrt{a^2-b^2+c^2-d^2}}\begin{bmatrix} a+id \\ \mathbf{b}+i\mathbf{c}\end{bmatrix}, \qquad R = \frac{1}{\sqrt{a^2+c^2}}\begin{bmatrix} a \\ i\mathbf{c}\end{bmatrix} \qquad \text{i} \qquad V = \underline{|\Lambda} = \frac{1}{\sqrt{1-v^2}}\begin{bmatrix} 1 \\ \mathbf{v}\end{bmatrix}$$

Suppose there are two different vectors $\mathbf{v}$ that satisfy the condition

$$\frac{1}{\sqrt{1-v^2}}\frac{1}{\sqrt{a^2+c^2}}\begin{bmatrix} a \\ i\mathbf{c}\end{bmatrix}\begin{bmatrix} 1 \\ \mathbf{v}\end{bmatrix} = \Lambda,$$

then it would have to be $\frac{1}{\sqrt{1-v_1^2}}\frac{1}{\sqrt{a_1^2+c_1^2}}\begin{bmatrix} a_1 \\ i\mathbf{c}_1\end{bmatrix}\begin{bmatrix} 1 \\ \mathbf{v}_1\end{bmatrix} = \frac{1}{\sqrt{1-v_2^2}}\frac{1}{\sqrt{a_2^2+c_2^2}}\begin{bmatrix} a_2 \\ i\mathbf{c}_2\end{bmatrix}\begin{bmatrix} 1 \\ \mathbf{v}_2\end{bmatrix}$

Hence $\frac{1}{\sqrt{1-v_2^2}}\frac{1}{\sqrt{1-v_1^2}}\begin{bmatrix} 1 \\ \mathbf{v}_2\end{bmatrix}\begin{bmatrix} 1 \\ -\mathbf{v}_1\end{bmatrix} = \frac{1}{\sqrt{a_1^2+c_1^2}}\frac{1}{\sqrt{a_2^2+c_2^2}}\begin{bmatrix} a_2 \\ -i\mathbf{c}_2\end{bmatrix}\begin{bmatrix} a_1 \\ i\mathbf{c}_1\end{bmatrix}$

Since on the right we get the imaginary vector, so the $\mathbf{v}_1$ vector must be equal to $\mathbf{v}_2$.

Analogously for the second case.

□

Theorem 2.6. Each unitary paravector can be represented as a composition of speed paravectors.

Proof.

Niech $\Lambda = \frac{1}{\sqrt{a^2-b^2+c^2-d^2}}\begin{bmatrix} a+id \\ \mathbf{b}+i\mathbf{c}\end{bmatrix} = \frac{1}{\sqrt{1-v_1^2}}\begin{bmatrix} 1 \\ \mathbf{v}_1\end{bmatrix}\frac{1}{\sqrt{1-v_2^2}}\begin{bmatrix} 1 \\ \mathbf{v}_2\end{bmatrix}\frac{1}{\sqrt{1-v_3^2}}\begin{bmatrix} 1 \\ \mathbf{v}_3\end{bmatrix}$

From Theorem 2.4.5 it follows that

$$\frac{1}{\sqrt{a^2+c^2}}\begin{bmatrix} a \\ i\mathbf{c}\end{bmatrix}\underline{\Lambda|} = \frac{1}{\sqrt{1-v_1^2}}\begin{bmatrix} 1 \\ \mathbf{v}_1\end{bmatrix}\frac{1}{\sqrt{1-v_2^2}}\begin{bmatrix} 1 \\ \mathbf{v}_2\end{bmatrix}\frac{1}{\sqrt{1-v_3^2}}\begin{bmatrix} 1 \\ \mathbf{v}_3\end{bmatrix}$$

hence $\frac{1}{\sqrt{a^2+c^2}}\begin{bmatrix} a \\ i\mathbf{c}\end{bmatrix} = \frac{1}{\sqrt{1-v_1^2}}\begin{bmatrix} 1 \\ \mathbf{v}_1\end{bmatrix}\frac{1}{\sqrt{1-v_2^2}}\begin{bmatrix} 1 \\ \mathbf{v}_2\end{bmatrix}\frac{1}{\sqrt{1-v_3^2}}\begin{bmatrix} 1 \\ \mathbf{v}_3\end{bmatrix}\underline{\Lambda|}^{-}$

□

When examining the properties of realisation, it is worth noticing that:

1. The velocity paravector is realised to itself $\underline{|\mathrm{V}} = \underline{\mathrm{V}|} = \mathrm{V}$

2. The realization of the unitary paravector is equal to 1.

$$\left|\underline{\frac{1}{\sqrt{a^2+c^2}}\begin{bmatrix} a \\ i\mathbf{c}\end{bmatrix}} = \underline{\frac{1}{\sqrt{a^2+c^2}}\begin{bmatrix} a \\ i\mathbf{c}\end{bmatrix}}\right| = 1$$

or otherwise $\underline{\left(\Lambda, \underline{\Lambda|}\right\rangle|} = \underline{|\left(\Lambda, \underline{\Lambda|}\right\rangle} = 1$

3. We can formulate the Euclidean turn:

$\left(\Lambda, \underline{\Lambda|}\right\rangle X \left(\underline{\Lambda|}, \Lambda\right\rangle$

4. Writing $\begin{bmatrix} id \\ \mathbf{b}\end{bmatrix}\begin{bmatrix} a \\ i\mathbf{c}\end{bmatrix}^{-1} = \begin{bmatrix} 0 \\ \frac{a\mathbf{b}+d\mathbf{c}+\mathbf{b}\times\mathbf{c}}{a^2+c^2}\end{bmatrix}$,

we get in the scalar part the condition that the paravector be the proper one, and in the vector part we get the vector of realized velocity.

As can be seen, the realization of the complex orthogonal paravector has very interesting properties.

3 Examples of application of the realisation

We introduced the concept of realisation of state paravectors because we didn't find the interpretation of the imaginary scalar, and primarily the imaginary time. For this reason, only the possibility of complex multiplication of two real state paravectors has been allowed at the phase in the observer's frame. On the other hand, it is obvious that energy, as the product of mutually conjugated paravectors, must be a real quantity. Therefore, the state of each object described by the observer should be able to be written in real coordinates, but this does not necessarily mean that the mutual relations between these objects must be described by real paravectors.

Below it is shown how the phase changes when we realise the complex paravector. In the article [5] phase was defined as a product

$$\Lambda^- X = \frac{1}{\sqrt{a^2 - b^2 + c^2 - d^2}}\begin{bmatrix} a + id \\ -\mathbf{b} - i\mathbf{c}\end{bmatrix}\begin{pmatrix} t \\ \mathbf{x}\end{pmatrix} \tag{18}$$

Based on the Theorem 2.5 we have

$$\Lambda^- X = V^- R^- X = V^- X', \tag{19}$$

where $V = \underline{|\Lambda}$ and $\begin{pmatrix} t' \\ \mathbf{x}'\end{pmatrix} = \frac{1}{\sqrt{a^2+c^2}}\begin{bmatrix} a \\ -i\mathbf{c}\end{bmatrix}\begin{pmatrix} t \\ \mathbf{x}\end{pmatrix}$.

If we realise the phase interval $\Lambda^- \mathbb{X} = \Delta t^0 \longrightarrow V^- \mathbb{X}' = \Delta t^0$ then 4-vector $\mathbb{X}'$ must be a real one. Realisation changes the domain onto a real space-time.

However, the thorough examinations indicate that the concept of realisation of the speed paravector must be limited to a single physical object or to objects whose states describe parallel paravectors. To describe in this way many objects being in motion to each other and to the observer is impossible because realisation is not an orthogonal transformation and it introduces deformations but, what is most important for use, realisation preserves the scalar products of the vigors of paravectors (Theorem 2.4.7) i.e. energy paravectors.

We hypothesize that in general the space-time is complex, and only the local space-time of the observer is real and that the observer does not examine relativistic phenomena directly, but its projections on his real space-time. Such a projection introduces distortions. The question arises: how and what is distorted? Or is it still different? Perhaps the observer, by examining only one (or many objects, but each separately), can describe them with real math, and does not see that the real mathematics is not enough to describe its mutual relations? We do not know this yet and that is why we emphasize that everything we write is a hypothesis, mathematically consistent, but requiring further theoretical research and practical verification.

The aim of our work is to show that, contrary to the opinion promoted by the mainstream, it is possible to build an alternative theory to the official STR, which fulfils its postulates. Such a possibility was the finding of a transformation invariant for the wave equation, but not belonging to the Lorentz group. Although we agree with the postulates of the official STR, the extended domain causes the need for some problems of physics to be modified, as was the case of Maxwell's equations [6]. That is why every detail has to be checked many times.

3.1 Two receding bodies.

We start the examination of the realization from the simplest case: the description of two free physical objects. We assume that realisation is a form of projection of phenomena from complex space-time onto the real space-time of the observer. The subject of the analysis will be the simple experiment performed in complex space-time, described in the article [5] section 3.2, „Growing vector”:

In the rest frame the observer describes two mutually receding points, one of which (B) is immobile with respect to the observer, and the other (A) moves away at the speed of $\mathbf{w}$. The movement of point A in time (t_0, t_1) is described by the equation

$$\begin{bmatrix} 1 \\ -\mathbf{w} \end{bmatrix} \begin{pmatrix} \Delta t \\ \Delta \mathbf{x} \end{pmatrix} = \begin{pmatrix} p_0 \\ 0 \end{pmatrix}, \tag{20}$$

where the 4-vector $\begin{pmatrix} \Delta t \\ \Delta \mathbf{x} \end{pmatrix}$ describes the point A in the interval (t_0, t_1), and p_0 is a parameter that does not interest us. We are interested in the vector part of the above equation. For this reason, and because the velocity paravector is orthogonal, the above equation can be saved

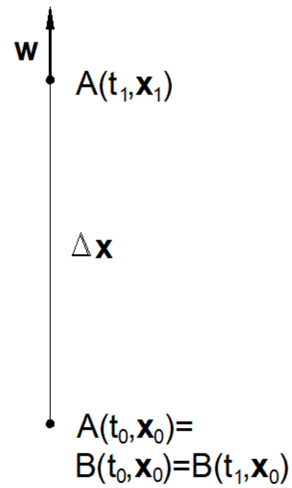

Figure 1: Vector $\Delta\mathbf{x}$ increasing at the velocity of **w** in a rest frame.

as

$$\frac{1}{\sqrt{1-w^2}}\begin{bmatrix}1\\ -\mathbf{w}\end{bmatrix}\begin{pmatrix}\Delta t\\ \Delta\mathbf{x}\end{pmatrix}=\begin{pmatrix}p\\ 0\end{pmatrix}. \tag{21}$$

At this same time the point B stands in place, so the spatial component of a 4-vector of the position of point B is equal to zero. Vector $\overrightarrow{BA}$ coincides with the spatial component of 4-vector describing point A, and its temporal component is equal to 0 (the four-vector is improper).

After relocation to the frame moving at the speed **-v** the observer measures the projections of points A and B in his real space-time. The projection (B') of the point B, made real by the measuring instruments, does not differ from the one we have in the complex space-time, but since point A' moves with compound velocity (velocity vector is complex), the instruments will measure the real speed and deform the image. The same is with the vector $\overrightarrow{B'A'}$. Measured coordinates of point A' meet the dependence:

$$\underline{\frac{1}{\sqrt{1-w^2}}\begin{bmatrix}1\\ -\mathbf{w}\end{bmatrix}\frac{1}{\sqrt{1-v^2}}\begin{bmatrix}1\\ -\mathbf{v}\end{bmatrix}}\Bigg|\begin{pmatrix}\Delta t'\\ \Delta\mathbf{x}'\end{pmatrix}=\begin{pmatrix}p\\ 0\end{pmatrix}, \tag{22}$$

The realised velocity **w**' is:

$$\mathbf{w}'=\frac{\mathbf{v}(1+2\mathbf{vw}+w^2)+\mathbf{w}(1-v^2)}{1+2\mathbf{vw}+v^2w^2} \tag{23}$$

The images of points A 'and B' are shown in Figure 2.

If the vector **w** is perpendicular to **v** then a scalar product

$$\mathbf{vw}'=\frac{v^2(1+2\mathbf{vw}+w^2)}{1+2\mathbf{vw}+v^2w^2}>v^2, \tag{24}$$

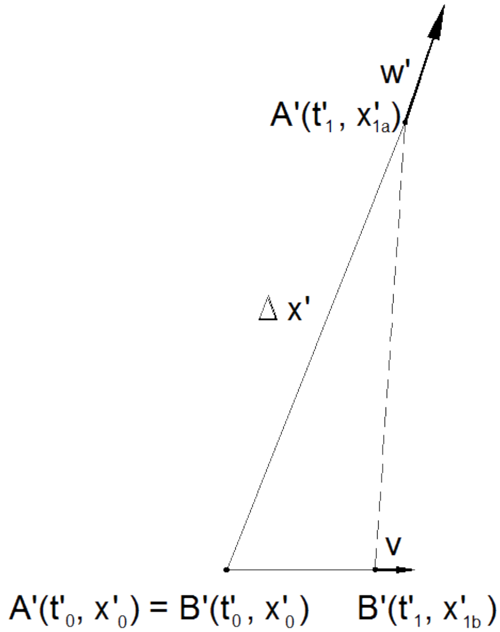

Figure 2: Vector $\Delta\mathbf{x}$ increasing at velocity of **w** in a frame that moves at velocity of **v**.

which means that in the moving frame the object A is ahead of the object B. If we invert the vector **w** then the case looks similar. We conclude that if we describe the experiment by realisation which be to the elastic reflection of object A (experiment described in [5]) and its return to point B, after passing to the moving system we get a nonsense result, because both objects will miss each other. We could expect this because the phase intervals of each object are transformed according to their own parameters. Moreover, realisation introduces deformation which we know from the property (Th2.4.6). From the above we conclude that the coordinates of one object only can be brought into the real with the help of the transformation (7).

Four-vector B'A' cannot be measured because it is improper but we can count its coordinates.

3.2 Spherical explosion.

Below images of an expandable sphere are compared seen by an observer moving at a relativistic speed **-w** in the complex space (realised image) and in accordance with the classic Lorentz transformation.

3.2.1 Realised image - a misinterpretation!

Now we check what the image of the spherical explosion looks like, viewed from a moving vehicle in complex space-time after realisation according to the formula (7). Please note that this is only a mathematical calculation that confirms that the realisation cannot be applied simultaneously to many objects!

As a result of the explosion of a material point, its particles are evenly distributed in all directions with the relativistic speed w. In the experimenter's frame the motion equation of a single particle has the form

$$\frac{1}{\sqrt{1-w^2}}\begin{bmatrix}1\\-\mathbf{w}\end{bmatrix}\begin{pmatrix}\Delta t\\\Delta\mathbf{x}\end{pmatrix}=\begin{pmatrix}p\\0\end{pmatrix} \tag{25}$$

This same equation in the frame of moving observer:

$$\frac{1}{\sqrt{1-w^2}}\frac{1}{\sqrt{1-v^2}}\begin{bmatrix}1\\-\mathbf{w}\end{bmatrix}\begin{bmatrix}1\\-\mathbf{v}\end{bmatrix}\begin{pmatrix}\Delta t'\\\Delta\mathbf{x}'\end{pmatrix}=\begin{pmatrix}p\\0\end{pmatrix} \tag{26}$$

The paravector of the resultant speed is

$$W'=\frac{1}{\sqrt{1-w^2}}\frac{1}{\sqrt{1-v^2}}\begin{bmatrix}1+\mathbf{vw}\\\mathbf{v}+\mathbf{w}+i\mathbf{v}\times\mathbf{w}\end{bmatrix} \tag{27}$$

Vector of the velocity after realisation:

$$\mathbf{w}'=\frac{(1+\mathbf{vw})(\mathbf{v}+\mathbf{w})+(\mathbf{v}+\mathbf{w})\times(\mathbf{v}\times\mathbf{w})}{(1+\mathbf{vw})^2+(\mathbf{v}\times\mathbf{w})^2} \tag{28}$$

If the parameter is the direction of the particle's motion (an angle between vector **w** and **v**) one, then on the XOY plane the vector **w**' coordinates are:

$$\begin{aligned}w'_x &= \frac{v(1+w^2)+(1+v^2)w\cos\alpha}{1+2vw\cos\alpha+v^2w^2}\\ w'_y &= \frac{(1-v^2)w\sin\alpha}{1+2vw\cos\alpha+v^2w^2}\end{aligned}$$

where α is the angle between the vector **w** and **v**.

For the calculations we assume two cases :

$w=0{,}7$ and $v=0{,}7$

$w=0{,}4$ and $v=0{,}8$

The results are graphically presented in the figure 3, but it does not show(!) the image of the particles' movement in space-time, because the image is deformed. Deformation consists in the concentration of particles on the side of the direction of sphere motion.

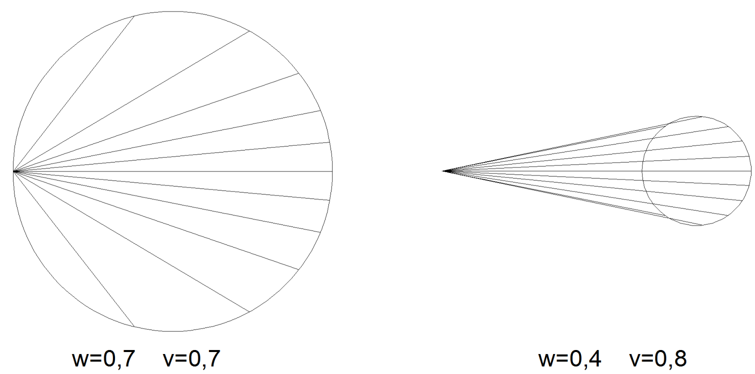

Figure 3: The realized view of the explosion front seen from a frame moving at velocity of v

3.2.2 Image of an explosion in the classic STR

The composition of velocities in the classic STR [8] is described by the formula:

$$\mathbf{w}' = \frac{1}{1+\mathbf{vw}}\left\{\mathbf{w}\sqrt{1-v^2}+\mathbf{v}\left[1-\frac{\mathbf{vw}}{v^2}\left(\sqrt{1-v^2}-1\right)\right]\right\} \tag{29}$$

which on the XOY plane means

$$w'_x = \frac{1}{1+v\,w\cos\alpha}\left\{w\cos\alpha\sqrt{1-v^2}+v\left[1-\frac{w\cos\alpha}{v}\left(\sqrt{1-v^2}-1\right)\right]\right\} \tag{30}$$

$$w'_y = \frac{w\sqrt{1-v^2}\sin\alpha}{1+v\,w\cos\alpha} \tag{31}$$

We assume that the observer moves at the speed $-\mathbf{v}$ ($v = 0.7$) along the OX axis, and the particles, created as a result of the explosion, are spreading spherically at the speed $\mathbf{w}$ ($w = 0.7$). In the moving frame of the observer, the ball is flattened. Below we show a comparison of the results obtained from classic STR formulas and realised speed in complex space-time (Fig.4).

The figure shows the theoretical images that an observer moving at the speed $-\mathbf{v}$ along the horizontal axis should "see". The image of the explosion front calculated from the classical dependence STR (29) shows an ellipsoid, whereas the black circle results from the realised complex transformation formulas.

Both images are deformed. In the picture an image of a vector $\mathbf{w}_\perp$ perpendicular to the $\mathbf{v}$ in the resting frame is marked. In the case of real velocity, the particle moving at the velocity $\mathbf{w}'_\perp$ is ahead of the moving center of explosion. Both images are deformed in contrast to the image obtained in the work [5] section 3.4, "Movement of point with the elastic collision" Fig. 17, which is not deformed (Fig. 5). We draw the conclusion that the realization in the form of (7) can be applied only to a single object or to objects that are in mutual rest.

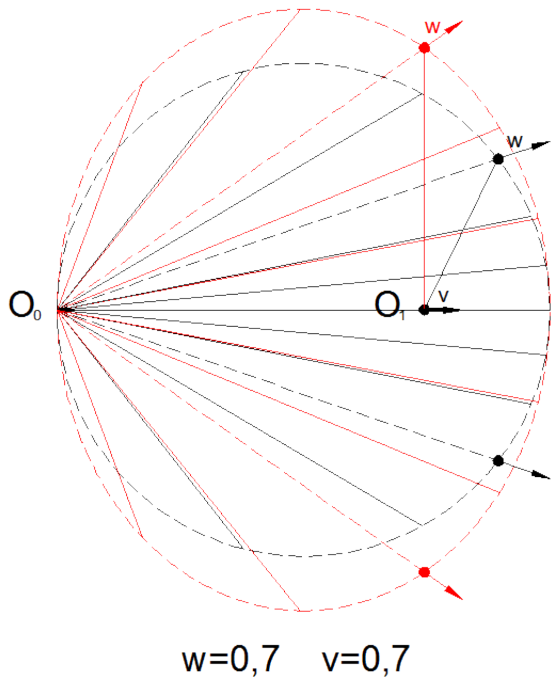

Figure 4: A comparison of images of the explosion front calculated in accordance with the official STR (red) and in accordance with the formula (7) (black)

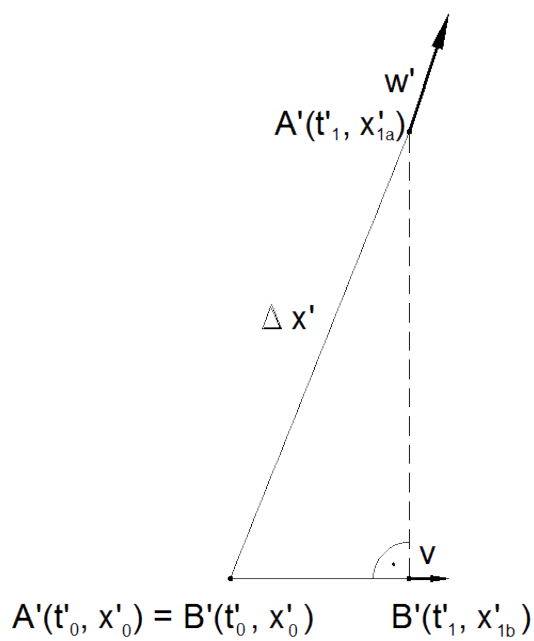

Figure 5: Real components of the road and velocities of points A and B in complex model ([5] pt. 3.4)

4 Realisation of time.

Spatio-temporal phenomena are inextricably linked with movement that is with change of location in time. Time, although it has been added to the 3-dimensional Euclidean space as the fourth dimension, does not fit into the geometrical concepts because of one basic cause: Time is not standing, it means that it has no points. The sense of time is its one-way passage, and so always mentioning time we mean the interval. The beginning and end of this interval are points on the time axis and we call them moments. The properties of time are fundamentally different from those of geometrical space. In order not to lose the basic property of time which is dynamics, we assume that the time axis consists of no points (moments), but episodes (intervals). Such space-time is not an affine space but it is a vector space. Such a general approach means that we come to the conclusion that space-time phenomena should not be organized either on the spot or on time. Having problems with fitting a complex theory in the real space-time of the observer, we compromise and modernize our hypothesis. We assume that the motion space is complex, and only the time in the observer's system must be real.

Now, we will repeat the reasoning at the beginning of this article, but with the assumption that in complex space the time is real and the velocity represents a complex vector. Then we have

$$\frac{1}{1-v^2+w^2}\begin{bmatrix}1\\ \mathbf{v}+i\mathbf{w}\end{bmatrix}\begin{bmatrix}1\\ \mathbf{v}-i\mathbf{w}\end{bmatrix}=\frac{1}{a^2-b^2+c^2-d^2}\begin{bmatrix}a+id\\ \mathbf{b}+i\mathbf{c}\end{bmatrix}\begin{bmatrix}a-id\\ \mathbf{b}-i\mathbf{c}\end{bmatrix} \tag{32}$$

From the above, we get a system of equations:

$$\begin{cases}\frac{1+v^2+w^2}{1-v^2+w^2} & = & \frac{a^2+b^2+c^2+d^2}{a^2-b^2+c^2-d^2}\\ \frac{\mathbf{v}+\mathbf{v}\times\mathbf{w}}{1-v^2+w^2} & = & \frac{a\mathbf{b}+d\mathbf{c}+\mathbf{b}\times\mathbf{c}}{a^2-b^2+c^2-d^2}\end{cases} \tag{33}$$

We add the third condition to the above: because the new paravector of complex speed $\mathbf{v}+i\mathbf{w}$ also has to be orthogonal, so the vectors $\mathbf{v}$ and $\mathbf{w}$ must satisfy the condition $\mathbf{vw}=0$.

From the scalar equation (33) we calculate

$$v^2=\left(1+w^2\right)\frac{b^2+d^2}{a^2+c^2} \tag{34}$$

After substituting in the vector equation (33) we get

$$\frac{\mathbf{v}+\mathbf{v}\times\mathbf{w}}{1+w^2}=\frac{a\mathbf{b}+d\mathbf{c}+\mathbf{b}\times\mathbf{c}}{a^2+c^2} \tag{35}$$

Together with the condition $\mathbf{vw}=0$ (since the velocity paravector is proper) the above equation can be presented in the form of a paravector equation

$$\frac{1}{1+w^2}\begin{bmatrix}1\\ i\mathbf{w}\end{bmatrix}\begin{bmatrix}0\\ \mathbf{v}\end{bmatrix}=\begin{bmatrix}0\\ \frac{a\mathbf{b}+d\mathbf{c}+\mathbf{b}\times\mathbf{c}}{a^2+c^2}\end{bmatrix} \tag{36}$$

From the above it is easy to calculate the vector $\mathbf{v}$

$$\mathbf{v} = \frac{a\mathbf{b} + d\mathbf{c} + \mathbf{b} \times \mathbf{c}}{a^2 + c^2} + \mathbf{w} \times \frac{a\mathbf{b} + d\mathbf{c} + \mathbf{b} \times \mathbf{c}}{a^2 + c^2} \quad \text{and} \quad \mathbf{w} \perp \frac{a\mathbf{b} + d\mathbf{c} + \mathbf{b} \times \mathbf{c}}{a^2 + c^2} \tag{37}$$

We received an ambiguous solution of the equation (32). Let consider the vector $\mathbf{w}$ as a parameter. We can see that if we select the vector $\mathbf{w} = 0$, the problem becomes realisation considered at the beginning of the article. However, we have a certain freedom here and we can choose $\mathbf{w}$ so that the real vector of the velocity $\mathbf{v}$ takes the values satisfying us.

Although the vector $\mathbf{v}$ may have a length greater than 1 because $\mathbf{v}^2 = (\mathbf{u} + \mathbf{w} \times \mathbf{u})^2 = u^2(1 + w^2)$, where for simplicity there was replaced $\mathbf{u} = \frac{a\mathbf{b} + d\mathbf{c} + \mathbf{b} \times \mathbf{c}}{a^2 + c^2}$. Since it is only a real component so in the complex space there is no contradiction with the assumption of the constant speed of light. Of course, the vector of the speed of light can be complex, too.

5 Mutual relationship between moving systems.

At the same time we observe two objects moving at relativistic velocity in relation to us and to each other. We are able to determine the velocity of each of them in our frame (origin of the observer - O), where the object A moves at velocity $\mathbf{v}_{AO}$. The object B moves at velocity $\mathbf{v}_{BO}$ (Indexes should be read from left to right: the first index means the object, and the second one means the system in which the velocity is measured). The object A in the frame B has the velocity $\mathbf{v}_{AB}$, and the object B in the frame A has the velocity $\mathbf{v}_{BA} = -\mathbf{v}_{AB}$. The velocity $\mathbf{v}_{AB}$ cannot be measured, because it is the velocity of the object A in the B frame, but we can calculate it with the velocities $\mathbf{v}_{AO}$ i $\mathbf{v}_{BO}$ measured in frame O.

The velocity $\mathbf{v}_{AO}$ is the realisation of the compound velocities $\mathbf{v}_{AB}$ and $\mathbf{v}_{BO}$, which we note using the velocity paravectors

$$V_{AO} = \underline{\lfloor V_{AB} V_{BO}}$$

Based on the definition of the right realisation the above equation is equivalent to

$$V_{AO} V_{AO} = V_{AB} V_{BO} (V_{AB} V_{BO})^* = V_{AB} V_{BO} V_{BO} V_{AB}, \tag{38}$$

because the measurement in space-time is real.

The same is true the other way. We measured velocity of the object B, which we save as V_{BO}. The object B in the system A moves with an unknown velocity V_{BA} (an orthogonal paravector!). We measured the velocity of the object A which is V_{AO}. We get from this

$$V_{BO} = \underline{\lfloor V_{BA} V_{AO}}$$

Daje to

Hence

$$V_{BO} V_{BO} = V_{BA} V_{AO} V_{AO} V_{BA} = V_{AB}^{-} V_{AO} V_{AO} V_{AB}^{-} \tag{39}$$

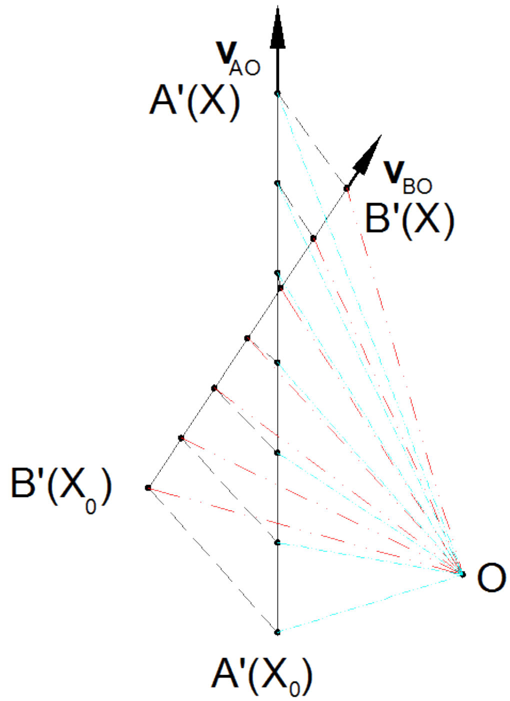

Figure 6: Two objects moving in relation to each other and to the observer.

As expected, the equations (38) and (39) are equivalent.

If we have paravectors V_{AO}, V_{BO} then the velocity V_{AB} can be calculated from the above equation.

The square of proper paravectors is also a proper paravector. We know from [3] that each paravector can be represented as a 4x4 matrix whose determinant is the square of the determinant of this valve. Each paravector can also be expressed as a four-dimensional vector.

$$W_{AO} = V_{AB} W_{BO} V_{AB} \tag{40}$$

where

$W_{AO} = V_{AO} V_{AO} = \frac{1}{1-v_{AO}^2} \begin{bmatrix} 1+v_{AO}^2 \\ 2v_{AOx} \\ 2v_{AOy} \\ 2v_{AOz} \end{bmatrix}$ - vigor of the velocity paravector V_{AO} in a vector form,

$W_{BO} = V_{BO} V_{BO} =$

$= \frac{1}{1-v_{BO}^2} \begin{bmatrix} 1+v_{BO}^2 & 2v_{BOx} & 2v_{BOy} & 2v_{BOz} \\ 2v_{BOx} & 1+v_{BO}^2 & -i2v_{BOz} & 2v_{BOy} \\ 2v_{BOy} & 2i\,v_{BOz} & 1+v_{BO}^2 & -i2v_{BOx} \\ 2v_{BOz} & -2i\,v_{BOy} & 2i\,v_{BOx} & 1+v_{BO}^2 \end{bmatrix}$ - vigor of the velocity paravector V_{BO} in a matrix form,

on the left side W_{BO}

$$V_{AB}=\frac{1}{\sqrt{1-v_{AB}^2}}\begin{bmatrix}1 & v_{ABx} & v_{ABy} & v_{ABz}\\ v_{ABx} & 1 & -i\,v_{ABz} & v_{ABy}\\ v_{ABy} & i\,v_{ABz} & 1 & -i\,v_{ABx}\\ v_{ABz} & -i\,v_{ABy} & i\,v_{ABx} & 1\end{bmatrix}$$ - parawektor of velocity V_{AB} in a matrix form,

at the right side W_{BO}

$$V_{AB}=\frac{1}{\sqrt{1-v_{AB}^2}}\begin{bmatrix}1\\ v_{ABx}\\ v_{ABy}\\ v_{ABz}\end{bmatrix}$$ - this same velocity paravector in a vector form.

It follows that in the equation (38), for the scalar $(W_{AO})_s$, we have a quadratic form positively defined. For spatial components we have positively defined bilinear forms.

We'll calculate the velocity of the object a in frame A (velocity vector $(\mathbf{v}_{AB})_V$) using the paravector calculus because it is much simpler. To do this, in the equation (40) we move the velocity paravector V_{AB} from left to the other side

$$V_{AB}^{-}W_{AO}=W_{BO}V_{AB} \tag{41}$$

that is

$$\frac{1}{\sqrt{1-v_{AB}^2}}\begin{bmatrix}1\\ -\mathbf{v}_{AB}\end{bmatrix}\frac{1}{1-v_{AO}^2}\begin{bmatrix}1+v_{AO}^2\\ 2\mathbf{v}_{AO}\end{bmatrix}=\frac{1}{1-v_{BO}^2}\begin{bmatrix}1+v_{BO}^2\\ 2\mathbf{v}_{BO}\end{bmatrix}\frac{1}{\sqrt{1-v_{AB}^2}}\begin{bmatrix}1\\ \mathbf{v}_{AB}\end{bmatrix} \tag{42}$$

After multiplying the paravectors on both sides of the equality and after reducing the module $\sqrt{1-v_{AB}^2}$ we obtain

$$\frac{1}{1-v_{AO}^2}\begin{bmatrix}1+v_{AO}^2-2\mathbf{v}_{AB}\mathbf{v}_{AO}\\ 2\mathbf{v}_{AO}-(1+v_{AO}^2)\mathbf{v}_{AB}-2i\mathbf{v}_{AB}\times\mathbf{v}_{AO}\end{bmatrix}=\frac{1}{1-v_{BO}^2}\begin{bmatrix}1+v_{BO}^2+2\mathbf{v}_{AB}\mathbf{v}_{BO}\\ 2\mathbf{v}_{BO}+(1+v_{BO}^2)\mathbf{v}_{AB}+2i\mathbf{v}_{BO}\times\mathbf{v}_{AB}\end{bmatrix} \tag{43}$$

$$\text{or}\qquad\left\{\begin{array}{ccc}\frac{1+v_{AO}^2-2\mathbf{v}_{AB}\mathbf{v}_{AO}}{1-v_{AO}^2} & = & \frac{1+v_{BO}^2+2\mathbf{v}_{AB}\mathbf{v}_{BO}}{1-v_{BO}^2}\\ \frac{2\mathbf{v}_{AO}-(1+v_{AO}^2)\mathbf{v}_{AB}}{1-v_{AO}^2} & = & \frac{2\mathbf{v}_{BO}+(1+v_{BO}^2)\mathbf{v}_{AB}}{1-v_{BO}^2}\\ \frac{\mathbf{v}_{AO}\times\mathbf{v}_{AB}}{1-v_{AO}^2} & = & \frac{\mathbf{v}_{BO}\times\mathbf{v}_{AB}}{1-v_{BO}^2}\end{array}\right. \tag{44}$$

$$\text{hence}\qquad\left\{\begin{array}{ccc}\mathbf{v}_{AB}\left(\frac{2\mathbf{v}_{AO}}{1-v_{AO}^2}+\frac{2\mathbf{v}_{BO}}{1-v_{BO}^2}\right) & = & \frac{1+v_{AO}^2}{1-v_{AO}^2}-\frac{1+v_{BO}^2}{1-v_{BO}^2}\\ \mathbf{v}_{AB}\left(\frac{1+v_{AO}^2}{1-v_{AO}^2}+\frac{1+v_{BO}^2}{1-v_{BO}^2}\right) & = & \frac{2\mathbf{v}_{AO}}{1-v_{AO}^2}-\frac{2\mathbf{v}_{BO}}{1-v_{BO}^2}\\ \left(\frac{\mathbf{v}_{AO}}{1-v_{AO}^2}-\frac{\mathbf{v}_{BO}}{1-v_{BO}^2}\right)\times\mathbf{v}_{AB} & = & 0\end{array}\right. \tag{45}$$

The second equation results in the third one, so to calculate the vector $\mathbf{V}_{AB}$ it is necessary to solve the system of the first two equations of the above system. In these equations we replace

$$\mathbf{w}_A=\frac{2\mathbf{v}_{AO}}{1-v_{AO}^2},\qquad \mathbf{w}_B=\frac{2\mathbf{v}_{BO}}{1-v_{BO}^2},\qquad a=\frac{1+v_{AO}^2}{1-v_{AO}^2},\qquad b=\frac{1+v_{BO}^2}{1-v_{BO}^2} \tag{46}$$

and we get

$$\begin{aligned}\mathbf{v}_{AB}(\mathbf{w}_A+\mathbf{w}_B) &= a-b \qquad (47)\\ \mathbf{v}_{AB}(a+b) &= \mathbf{w}_A-\mathbf{w}_B \qquad (48)\end{aligned}$$

From the second equation we get

$$\mathbf{v}_{AB}=\frac{\mathbf{w}_A-\mathbf{w}_B}{a+b} \qquad (49)$$

The velocity of the object A calculated from the above equation in the frame of the object B is

$$\mathbf{v}_{AB}=\frac{\mathbf{w}_A-\mathbf{w}_B}{a+b}=\frac{\mathbf{v}_{AO}(1-v_{BO}^2)-\mathbf{v}_{BO}(1-v_{AO}^2)}{(1-v_{AO}^2v_{BO}^2)} \qquad (50)$$

It only remains to verify that the result is always less than 1, which is left to the reader.

The movement of object A can be determined by the observer in various ways. The basic way is to refer the object A only to the observer, for which the real coordinates are sufficient

$$V_{AO}^{-}\mathbb{X}=\Delta t^0, \qquad (51)$$

but it can be referred to other objects, eg to the object B, then

$$V_{AO}'^{-}\mathbb{X}'=(V_{AB}V_{BO})^{-}\mathbb{X}'=\Delta t^0. \qquad (52)$$

In this case the velocity paravector V'_{AO} has the form:

$$V'_{AO}=\frac{1}{\sqrt{1-v_{AB}^2}}\begin{bmatrix}1\\ \mathbf{v}_{AB}\end{bmatrix}\frac{1}{\sqrt{1-v_{BO}^2}}\begin{bmatrix}1\\ \mathbf{v}_{BO}\end{bmatrix}= \qquad (53)$$

$$=\frac{1}{\sqrt{(1+\mathbf{v}_{AB}\mathbf{v}_{BO})^2-(\mathbf{v}_{AB}+\mathbf{v}_{BO})^2+(\mathbf{v}_{AB}\times\mathbf{v}_{BO})^2}}\begin{bmatrix}1+\mathbf{v}_{AB}\mathbf{v}_{BO}\\ \mathbf{v}_{AB}+\mathbf{v}_{BO}+i\mathbf{v}_{AB}\times\mathbf{v}_{BO}\end{bmatrix}$$

Here the coordinates of the vectors are already complex, but the time interval is real, although it is different than before. By referring the movement description to other objects we change the coordinates. Vectors do not have universal coordinates, but the observer matches them according to the situation that interests him. However, it should be noted that the determinants of the paravectors state and the vigor of the velocity paravector are invariant. From the formulas (53) and (50) it can be check that $V'_{AO}V'^{*}_{AO}=V_{AO}V_{AO}$.

Returning to the example from point 3.1 (Fig. 2), we can see that by measuring the movement of the point A', we apply the real axis to this point. The same happens when we measure the point B', but these measurements do not reflect directly the relationship between these points. This relationship must be calculated only as we did above. In the observer's system it is complex because, in order to keep the scalar products, the point A has been replaced, and this is said about the spatial imaginary component. In the frame of the object B, the state of the object A is described by real paravectors.

Equations of state with the V'_{AO} (53) and V_{AO} (52) paravectors describe the movement of the same object in the same frame, but in a different time relationship. There is another time difference between the objects A and O in both cases. Not only that the object A is not simultaneously in both cases, that difference is changes constantly. Hence the additional imaginary component of the velocity vector V'_{AO}.

5.1 Compound inertial motion

The object moves at a compound velocity $V' = V_2 V_1$.

$$V' = \frac{1}{\sqrt{1-v_2^2}}\frac{1}{\sqrt{1-v_1^2}}\begin{bmatrix}1\\ \mathbf{v}_2\end{bmatrix}\begin{bmatrix}1\\ \mathbf{v}_1\end{bmatrix} = \frac{1}{\sqrt{1-v_2^2}}\frac{1}{\sqrt{1-v_1^2}}\begin{bmatrix}1+\mathbf{v}_2\mathbf{v}_1\\ \mathbf{v}_2+\mathbf{v}_1+i\mathbf{v}_2\times\mathbf{v}_1\end{bmatrix} \tag{54}$$

Hence the equation of motion is $V'^{-}X' = \Delta t^0$

$$\frac{1}{\sqrt{1-v_{re}'^2+v_{im}'^2}}\begin{bmatrix}1\\ -\mathbf{v}'_{re}-i\mathbf{v}'_{im}\end{bmatrix}\begin{pmatrix}\Delta t'\\ \Delta\mathbf{x}'\end{pmatrix} = \begin{pmatrix}\Delta t^0\\ 0\end{pmatrix}, \tag{55}$$

$$\text{where} \qquad \mathbf{v}'_{re} = \frac{\mathbf{v}_2+\mathbf{v}_1}{1+\mathbf{v}_1\mathbf{v}_2} \qquad \text{and} \qquad \mathbf{v}'_{im} = \frac{\mathbf{v}_2\times\mathbf{v}_1}{1+\mathbf{v}_1\mathbf{v}_2}.$$

Since the own time of object Δt^0 is a real size, and we want the time in the primed system be real, too, so after extracting $\Delta t'$ we get

$$\frac{\Delta t'}{\sqrt{1-v_{re}'^2+v_{im}'^2}}\begin{bmatrix}1\\ -\mathbf{v}'_{re}-i\mathbf{v}'_{im}\end{bmatrix}\begin{bmatrix}1\\ \mathbf{v}'\end{bmatrix} = \Delta t^0\begin{bmatrix}1\\ 0\end{bmatrix} \tag{56}$$

Hence, after moving the velocity paravector V' to the other side of equality we receive

$$\Delta t'\begin{bmatrix}1\\ \mathbf{v}'\end{bmatrix} = \frac{\Delta t^0}{\sqrt{1-v_{re}'^2+v_{im}'^2}}\begin{bmatrix}1\\ \mathbf{v}'_{re}+i\mathbf{v}'_{im}\end{bmatrix} \tag{57}$$

In the primed frame the observer describes the object in primed coordinates at primed time, so we need to convert the equation (54) so that traditionally the road be a function of time. From the equation (54) is seen that $\Delta\mathbf{x}'$ must be a complex vector.

$$\Delta\mathbf{x}' = (\mathbf{v}'_{re}+i\mathbf{v}'_{im})\Delta t'$$

When we pass to the system moving at the speed $-\mathbf{v}_3$ and we want to realise only the time, then the resultant speed will be determined by the paravector

$$\frac{1}{\sqrt{1-v_3^2}}\begin{bmatrix}1\\ \mathbf{v}_3\end{bmatrix}\underline{\left|\frac{1}{\sqrt{1-v_2^2}}\frac{1}{\sqrt{1-v_1^2}}\begin{bmatrix}1\\ \mathbf{v}_2\end{bmatrix}\begin{bmatrix}1\\ \mathbf{v}_1\end{bmatrix}\right.} = \frac{1}{\sqrt{1-v_3^2}}\begin{bmatrix}1\\ \mathbf{v}_3\end{bmatrix}\frac{1}{\sqrt{1-v^2}}\begin{bmatrix}1\\ \mathbf{v}\end{bmatrix} = \tag{58}$$

$$= \frac{1}{\sqrt{1-v_{re}'^2+v_{im}'^2}}\begin{bmatrix}1\\ \mathbf{v}'_{re}+i\mathbf{v}'_{im}\end{bmatrix}$$

We have shown that a complex orthogonal paravector characterizing the velocity of an object in a complex space-time can always be reduced to the form of a real velocity paravector in the observer's space-time. We can also present it in such a form that the scalar component is one and the vector is complex. This is not a disadvantage; on the contrary, it gives us great interpretive possibilities.

In this place there are seen analogies with the description of movement in the Euclidean space. If we describe a straight line motion of a single object, the easiest way is to choose a coordinate system that is consistent with the direction of motion. Then, we have a motion in a 1-dimensional space. In the case of two objects, if we choose a system assigned to one of the them, then the movement of the second one is divided into a parallel component and perpendicular to the movement of first object. In the complex world, the axis to which we compare is the real space-time of the observer. Here we can select the real coordinate system of the observer to describe the movement of one of object. If we add a second object and we want to describe their mutual relations, then we can choose only the coordinate system in which time is real. For the description of three and more objects moving in different directions, it is no longer possible - the time must also have an imaginary component. It does not matter for paravectors of energy (which are vigors) because regardless of the choice of the reference system the vigors are always real and as shown in Theorem 5.2 [3] the dot product of these paravectors is preserved. The realisation, despite the disadvantage of non-orthogonality, has a great advantage: it gives a positive time, which is consistent with the obvious fact: the time of physical objects never goes back.

6 Summary

On the mathematical side the official special relativity is a coherent and thoroughly tested theory, so finding an error in it is very unlikely, and yet intuition tells that something is not right. Einstein's postulates are treated by his followers as the axioms but these are not axioms, because one follows from the other. The postulate about the speed of light results from a much more general postulate about the universal validity of the laws of physics, because the speed of the light wave is contained in a homogeneous wave equation, which is the law of physics. Both postulates are contained in one statement: The wave equation is invariant due to the boost. The problem of special relativity lies in finding all the transformations that maintain the invariance of the wave equation, and this issue was investigated by great theorists at the beginning of the 20th century. They collected these transformations in the so-called Poincare group, and modern physicists are sure that nothing more can be discovered. In this regard they are right: **In the field of real space-time, all transformations that maintain the invariance of the wave equation have been studied and the theory built on them is complete.** But here we touch the heart of the problem of special relativity. Its creators assumed that space-time is a real structure in the sense of real numbers. Space-time research using the paravector calculus shows that it is not. Relativistic phenomena are much better described in the complex domain. What's more, the paravector calculus, as opposed to the tensor calculus, is simple and intuitive because the properties of the paravectors are similar to vectors'. The only problem is how to imagine a complex space and time? While complex space can be dealt with, it seems impossible to accept the complex time. For everyone it is obvious that time passes linearly in one direction and events can be ordered chronologically, while complex numbers cannot be ordered. Therefore, at the beginning, we suggest to the reader that the imaginary components should be treated as additional conditions imposed on intuitively acceptable real quantities, and the fact that these conditions behave like imaginary components matching the real quantities they relate to, over time will make the reader get accystomed to the fact that space-time has a complex structure.

At the moment I can say with full confidence that **space-time has a complex structure**. It is difficult to imagine an imaginary direction, because our direct experiences relates to stationary phenomena in the sense of the speed of light, and those happening at high speeds reach us through the transmitted energy, which is always real by its nature. However, there are the well-known phenomena of classical physics that they testify to the complex space-time. These are, for example, the magnetic field existing despite the lack of magnetic charge, and in mechanics - a gyroscope that seems to contradict the laws of gravity.

Looking at the special theory of relativity from the perspective of the complex space-time it can be seen that the situation is similar to the one which took place in the Middle Ages, when there was a belief that the Earth is flat. Copernicus discovered that the Earth is a huge sphere orbiting another sphere - the Sun, although he could not yet explain how it is held in space and how people can walk "upside down" on the other side. He did not know anything about the gravitational field yet. To travel from Torun to Krakow, or even to Rome, a flat map was enough for him to map a flat Earth. Similarly with relativistic speeds - the world around the stationary observer is real, just as the world around the medieval man was flat.

With the paravector calculus we can see that the world is more complex than we think. However, it is not as complicated and illogical as modern physicists show us. On the contrary! From the considerations it can be seen that the method does not consist in complicating mathematics, but in the exercise of a four-dimensional imagination. Just as an engineer exercises his spatial imagination analysing the permeability of solids on descriptive geometry courses, a physicist should analyse simple complex formulas of phase difference describing relations between objects to develop his four-dimensional imagination. *Space-time*, as the name suggests, is a four-dimensional structure and without the fourth dimension imagination a researcher cannot move in it. Expanding the imagination is hindered by the habit of photographic thinking. What a person sees is a film consisting of photographic frames. Relativistic imagination consists in sensing the phase difference, which describes the connections between objects (or the same object) being in a different place and at another time. It should always be borne in mind that this relationship is not arbitrary, because it can only be positively defined, i.e. one that does not describe improper paravectors.

We will show what the mathematical structure of a complex space-time looks like when we present more circumstantial evidence in favour of our hypothesis.

References

[1] Windsor University Collection of articles written by professor William Baylis

[2] Arizona State University. Collection of articles on Geometric Algebra written by professor David Hestenes

[3] Radomański J.: *Algebra of paravectors* arXiv:1601.02965

[4] Radomański J.: *Four-divergence as a paravector operator. Invariance of the wave equation under orthogonal paravector transformation.* arXiv:1605.04499

[5] Radomański J.: *Special relativity in complex space-time. Part 1. A choice of the domain and transformation preserving the invariance of wave equation.* viXra:1703.0157

[6] Radomański J.: *Special Relativity in Complex Space-Time. Part 2. Basic Problems of Electrodynamics.* http://vixra.org/abs/1708.0008

[7] Radomański J.: *Does Thomas-Wigner rotation show the fallacy of „Lorentz rotation"?* viXra:1702.0314

[8] Wikipedia *Relativistic velocity addition formula*