# Grand Unified Theory by the Oktoquintenfield

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Abstract

I show an extension of the Standard Model and the General Relativity by the symmetries of the E9 Coxeter Group.

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*Contents*



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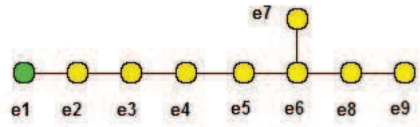
*Introduction :*

*This symmetries arises by the symmetries of the coxeterelement of the affine group E9 which is the affine one point extension of the well known exceptional group E8.*
*Why do we consider the E9 group (more specifically the Coxeter element of this group)?*
1) *E9 is an affine group and thus has something to do with extension.*
2) *The extension is flat as the universe.*
3) *The action of the Coxeter elements of the group produces symmetries involving our current standard model.*
*The fundamentals here :*
*https : //en.wikipedia.org/wiki/Coxetergroup*
*https : //de.wikipedia.org/wiki/Wurzelsystem*
*http : //home.mathematik.uni−freiburg.de/soergel/Skripten/XXSPIEG.pdf*

*Dynkin Diagram E9 (affine one point extension of group E8) :*




$\tilde{\mathbf{E}}_8$

*Derivative of the symmetries of ESM from the invariants of the Coxeter elements E9.*
*A Coxeter element is a product of the generating reflections of E9.*
*For example :* *Coxeterelement* $=$ e1.e2.e3.e4.e5.e6.e7.e8.e9
*The Coxeterpolynom is the characteristic polynomial of Coxeter elements and has the form :*

$$E_9(x) = \frac{x^5-1}{x-1}.\frac{x^3-1}{x-1}.\frac{x^2-1}{x-1}.(x-1)^2$$

$$E9CS = SU(5) \times SU(3) \times SU(2) \times U(1)^2$$

$E_9(x)$ ... *characteristical polynom of the coxeterelement of E9*

*E9(x) is a polynom with terms of cyclotomic factors* $Z_n = \dfrac{x^n-1}{x-1}$
*for* $n > 1$ *and* $(x-1)$ *for* $n = 1$.
*The cyclotomic factors are the characteristical polynom of the*
$A_{n-1}$ *which is the Dynkin diagram for the* $SU(n)$ *Liegroup.*
*See more here :* *https : //en.wikipedia.org/wiki/Specialunitarygroup.*
*So finally the symmetry space by the actions of the Coxeterelements is*

$$SU(5) \times SU(3) \times SU(2) \times U(1) \times U(1)$$

*symmetries by orbital or horizontal root system* — $SU(5) \times SU(3) \times SU(2)$
*symmetrie by vertical root* — $U(1)$
*symmetrie by Translation − Axe root* — $U(1)$

*details see* *https : //arxiv.org/abs/1312.7781* *and Appendix I*

*Compact :*

| *Lie − group* | | *Coxeter − Weyl* | | *name* | *count bosons* |
|---|---|---|---|---|---|
| $SU(5)$ | $\simeq$ | $A_4$ | ●—●—●—● | *Repelions* | $5^2-1 = 20+4 = 24$ |
| x | | | | | |
| $SU(3)$ | $\simeq$ | $A_2$ | ●—● | *Gluons* | $3^2-1 = 6+2 \; = 8$ |
| x | | | | | |
| $SU(2)$ | $\simeq$ | $A_1$ | ● | $W^+, W^-, Z,$ | $2^2-1 = 2+1 \; = 3$ |
| x | | | | | |
| $U(1)$ | $\simeq$ | $A_0$ | ○ | *Photon* | 1 |
| x | | | | | |
| $U(1)$ | $\simeq$ | $A_0$ | ◎ | *Graviton* | 1 |

*See also https : //arxiv.org/pdf/1808.05090.pdf*
*The action of a Coxeter element on an affine root system.*

*Hint :*
*Our symmetry is a special case of the models given by*

$$SU(n_1) \times SU(n_2) \times ... \times SU(n_k) \times U(1)^{k-1}$$

*If we set* $n_1 = 5, n_2 = 3, n_3 = 2$ *and* $k = 3$ *then we get our ESM.*

*What bring us the additional symmetries?*
(1) *These have the potential to describe new particles.*
(2) *These have the potential to describe the space and time.*
(3) *These have the potential to describe gravity.*

## < 1 > The Idea

*Light and gravitation just like photon and graviton have something in common.*
*Both are massless and propagate with the speed of light.*
*We know that light by the*
*Symmetry breaking 1 :* $SU(2) \times U(1) \rightarrow U(1)e$ *is described as a mixture.*
*So light is a part of the electro – weak interactions.*
*we consider analog gravity as a result of a further symmetry breaking*
*Symmetry breaking 2 :* $SU(5) \times U(1) \rightarrow U(1)g$
*Our extended standard model allows us this.*

*We show later by the Golden Potential short GP that this symmetriebreak is responsible for the speed of light.It generates the fourvelocity c.*

*We will now like to assign our relevant* $SU(n)$*'s to division algebras (real numbers, complex numbers, ...).*

$SU(1) \longleftrightarrow \mathbb{R}$
$SU(2) \longleftrightarrow \mathbb{C}$
$SU(3) \longleftrightarrow \mathbb{H}$
$SU(5) \longleftrightarrow \mathbb{O}$

*This 4 divison algebras can be generated by the doubling process*
*(see more at https : //de.wikipedia.org/wiki/Verdopplungsverfahren).*
*Considering the rank of the* $SU(2) = 1, SU(3) = 2, SU(5) = 4$ *then this is double as well.*
*There appears to be a connection between the division algebras and the* $SU(n)$ $(n = 2, 3, 5)$.

*The connections are the rank (maximal torus) of the* $SU(n)$ *and the orthogonal complex subspaces of the divison algebra.*
*For example the quaternions have two orthogonal complex subspaces*
$a + b.i1$ *and* $c.i2 + d.i3$ $(a, b, c, d$ *real).*
*The* $SU(3)$ *has also 2 neutral elements (toris).*

*With this assignment we can create backgroundfields to the* $SU(3)$ *and* $SU(5)$ *like it is the higgsfield for* $SU(2)$.

$SU(1) \leftrightarrow \mathbb{R}^1$ *real singulet*
$SU(2) \leftrightarrow \mathbb{C}^2$ *complex dublet higgsfield*
$SU(3) \leftrightarrow \mathbb{H}^3$ *quaternionic triplet*
$SU(5) \leftrightarrow \mathbb{O}^5$ *octonionic quintet Octoquintenfield*

*Therefore, we rely analogously on the Higgsfield (2 × complex = doublet).*

$$\phi = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} = \begin{bmatrix} \phi_1^+ + i.\phi_2^+ \\ \phi_1^0 + i.\phi_2^0 \end{bmatrix}$$

## < 2 > the Octoquintenfield (5 × Octonions = Quintet).

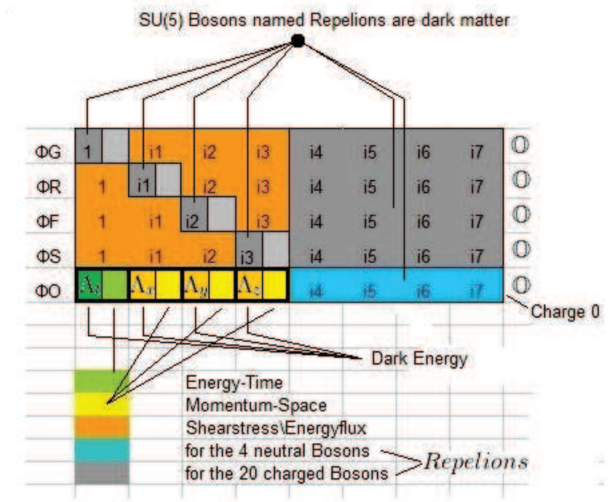
$$\phi = \begin{bmatrix} \phi^G \\ \phi^R \\ \phi^F \\ \phi^S \\ \phi^O \end{bmatrix} = \begin{bmatrix} \phi_0^G + i_1.\phi_1^G + i_2.\phi_2^G + i_3.\phi_3^G + i_4.\phi_4^G + i_5.\phi_5^G + i_6.\phi_6^G + i_7.\phi_7^G \\ \phi_0^R + i_1.\phi_1^R + i_2.\phi_2^R + i_3.\phi_3^R + i_4.\phi_4^R + i_5.\phi_5^R + i_6.\phi_6^R + i_7.\phi_7^R \\ \phi_0^F + i_1.\phi_1^F + i_2.\phi_2^F + i_3.\phi_3^F + i_4.\phi_4^F + i_5.\phi_5^F + i_6.\phi_6^F + i_7.\phi_7^F \\ \phi_0^S + i_1.\phi_1^S + i_2.\phi_2^S + i_3.\phi_3^S + i_4.\phi_4^S + i_5.\phi_5^S + i_6.\phi_6^S + i_7.\phi_7^S \\ \phi_0^O + i_1.\phi_1^O + i_2.\phi_2^O + i_3.\phi_3^O + i_4.\phi_4^O + i_5.\phi_5^O + i_6.\phi_6^O + i_7.\phi_7^O \end{bmatrix}$$

$$\phi_i^O = \phi_{i_1}^O + i.\phi_{i_2}^O \;\; \epsilon\, \mathbb{C} \quad for\ i = 0, 1, 2, 3$$

$$\phi_0^G, \phi_1^R, \phi_2^F, \phi_3^S \;\; \epsilon\, \mathbb{C} \qquad else \qquad \phi \;\; \epsilon\, \mathbb{R}$$

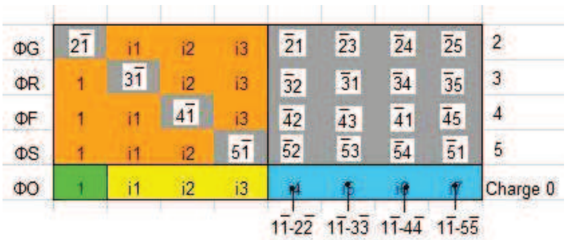
*This provides 48 degrees of freedom.*
*24 of which will be "spent" for our* $SU(5)$ *tensor bosons for the 5th longitudinal spin degree of freedom*
*(24 Goldstone bosons swallowed over gauge transformation) thus remain 16 + 8 left.*
*The S, F, R, G and H charges are the 5 charges of the SU (5) analogous to the 3 color charges of* $SU(3)$ *and the 2 charges (+ .–) of* $SU(2)$.
*The letters stand for S = See, F = Touch , R = Smell , G = Taste and H = Hear*
*Calling therefore the charges of the* $SU(5)$ *sense charges.*
*Note : These charges have (such as the color charges of quarks with color) nothing to do with the senses, but to give a name to the child for reference only.*

*Make the following division for the 48 field components of the Octoquinten field as a physical approach :*




*Similar to the Higgsfield we assign our Repelions to the Oktoquintenfield by the following scheme.*
*The numbers are the sense charges (see* $<2>$*).*

$5 = See$
$4 = Touch$
$3 = Smell$
$2 = Taste$
$1 = Hear$




## $< 2.1 >$ *Octoquintenfield OQF and the symmetric group (permutationgroup)* $S_{48}$

*We will assign every degree of freedome of the OQF (48) to a position in the permutation sorted by the charge lines* $S, F, R, G, O$
$(S = See, F = Touch, R = Smell, G = Taste, O = neutral)$

$$\left.\begin{pmatrix} \phi_0^S & \phi_1^S & ... & ... & \phi_7^O \\ 1 & 2 & ... & ... & 48 \\ \alpha(1) & \alpha(2) & ... & ... & \alpha(48) \end{pmatrix}\right\} permutation$$

*Then we have 4 blocks of 9* $\phi$*'s (chargelines) and one block of 12* $\phi$*'s (charge = zero) in sum* $4 \times 9 + 1 \times 12 = 48$ $\phi$*'s.*

*In our divison of the OQF we have assigned every boson to a fix place which means to a fix degree of freedome* $\phi$*.*
*For example the boson of type* $\bar{5}2$ *is on the place* $\phi_4^S$*.*
*The idea now is that every boson is a fixpoint (an invariant) of a permutation of* $S_{48}$*.*

*The question now is how many bosons do we have?*
*For example of type* $\bar{5}1$*?*
*The answer is very simple because every permutation with a fixpoint on* $\phi_4^S$ *is a boson of type* $\bar{5}1$*.*
*Then we have 47! bosons of type* $\bar{5}1$ *because we can permutate all other* $\phi$*'s.*
*Doing this with all types of bosons we get* $24 \times 47! = \frac{48!}{2}$ *bosons.*
*Let us say the mass of the bosons is the planckmass then we get A total mass of :*

$$M_T = \frac{48!}{2}.m_{planck} \approx \ 6,2 \times 10^{52}\ kg$$

*Hint : This is near to the calculated total mass of the visible universe.*

*Analogeous to the Higgspotential we declare a Potential on the Octoquintenfield*

# $< 3 >$ *Potential over the Octoquintenfield*

$$V(\phi) = \frac{\mu^2}{2} \mid \phi \mid^2 + \frac{\lambda^2}{4} \mid \phi \mid^4 + \frac{\gamma^2}{8} \mid \phi \mid^8 \quad with\ \phi \ \epsilon\ OQF$$

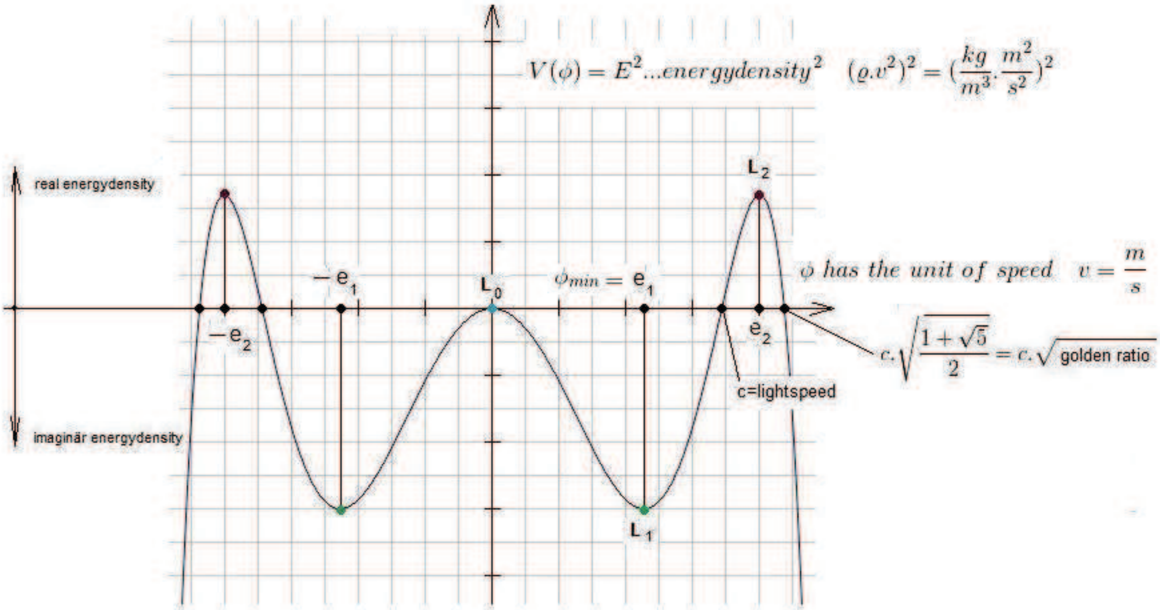
$$\mid \phi \mid^2 = \phi^\dagger . \phi$$

$\gamma, \mu\ \epsilon\ i.\mathbb{R}\ (imaginaer)\ and\ \ \lambda\ \epsilon\ \mathbb{R}$

$\frac{\mu^2}{2}$ ...$momentumdensity^2 \quad (\varrho.v)^2 = (\frac{kg}{m^3}.\frac{m}{s})^2$

$\frac{\lambda^2}{4}$ ...$massdensity^2 \quad (\varrho)^2 = (\frac{kg}{m^3})^2$

$\frac{\gamma^2}{8}$ ...$spindensity^2 \quad (\frac{\varrho}{v^2})^2 = (\frac{\frac{kg}{m^3}}{\frac{m^2}{s^2}})^2$ $\quad$ *Spin here is used not in the sense of Spin = action*




*The coefficients of the potential comes from selfinteractions.*
*Therefore we make the assumption that we have the following relation :*

$C := \frac{-\lambda^2}{\gamma^2} = (4.\frac{\mu^2}{\lambda^2})^2 \qquad \gamma^2, \mu^2 < 0 \qquad \lambda^2 > 0$

*Then it follows by exact calculation that*

$C = c^4.\varphi^2$

*c...speed of light*
$\varphi$*...golden ratio* $= 1,6180...$

*Because of the appearence of the golden ratio and some nice properties of it we call the potential the* <u>*Golden − Potential*</u> *short GP.*

*The first angle which comes from the minimum of the Oktoquintenpotential is appr. equal to the* $WEINBERG - ANGLE \approx 28,89^o$
*see* <8>

*Comparing the Golden − Potential GP with the standard relativistic energy (density) equation :*

$E^2 = p^2.c^2 + m^2.c^4 \quad p = momentum; m = mass$

*Our GP has a third term and expand the equation to*

$E^2 = p^2.c^2 + m^2.c^4 + s^2.c^8$

*we call the third term the spin − term.*

<u>< 3.1 > *Einstein − Form*</u>

$We\ want\ that\ the\ second\ part\ of\ the\ Golden-Potential$
$is\ our\ quadratic\ vacuumenergydensity.$

$$\frac{\lambda^2}{4}\mid c\mid^4=(\frac{\Lambda.c^4}{8\pi G})^2=(\varrho_{vacuum}.c^2)^2$$

$\Lambda...cosmological\ constant$
$\varrho_{vacuum}...vacuum\ massdensity$
$then\ with\ the\ relation\ c^4.\varphi^2\ =\frac{-\lambda^2}{\gamma^2}=(4.\frac{\mu^2}{\lambda^2})^2\ we\ get\ the\ potential\ as$

$EINSTEIN-FORM$

$$V(\phi)=(\frac{\Lambda.c^4}{8\pi G})^2.\left(-\frac{\varphi}{2}.(\frac{\mid\phi\mid}{c})^2+(\frac{\mid\phi\mid}{c})^4-\frac{1}{2.\varphi^2}.(\frac{\mid\phi\mid}{c})^8\right)$$

$We\ have\ 3\ "spheres"\ where\ the\ potential\ vanishes.$
$The\ first\ in\ the\ center\ which\ is\ a\ point. We\ name\ it\ S_0.$
$Then\ one\ with\ |\phi|=c=1\ which\ we\ name\ S_c\ and\ one\ with$
$|\phi|=\sqrt{\varphi}.c=\sqrt{\varphi}\ which\ we\ name\ S_{\sqrt{\varphi}}.$

$Compact\ (S_0,S_c,S_{\sqrt{\varphi}})\ for\ the\ zero\ subspace.$

## $<3.3>\ Planck-Form$

$With\ the\ relation$

$$P_p.l_p^2=\frac{c^4}{G}$$

$we\ get\ from\ the\ Einstein-Form\ the\ Planck-Form\ of\ the\ GP.$

$PLANCK-FORM$

$$V(\phi)=\frac{1}{2}.(\frac{P_p}{2\pi})^2.(\frac{\Lambda.l_p^2.\varphi}{4})^2.\left(-\left|\frac{\phi}{c\sqrt{\varphi}}\right|^2+2.\left|\frac{\phi}{c\sqrt{\varphi}}\right|^4-\left|\frac{\phi}{c\sqrt{\varphi}}\right|^8\right)$$

$where$

$P_p=\frac{c^7}{\hbar.G^2}\quad Planckpressure$

$l_p^2=\frac{\hbar.G}{c^3}\quad Plancklength^2$

$\Lambda.l_p^2\approx\frac{2,6}{10^{122}}\approx\frac{4}{48!^2}\quad dimensionless$

$This\ assumption\ \approx\longrightarrow=\ is\ backcalculated\ from\ the$
$Combinatorial-Form\ of\ the\ Golden-Potential\ GP$
$which\ you\ can\ see\ later\ in\ this\ document.$
$On\ the\ combinatorial\ form\ the\ normalization\ factor\ N\ is\ very\ easy\ to\ explain$
$and\ it\ leads\ to\ this\ assumption\ by\ going\ backward\ to\ the$
$Planck-Form\ of\ the\ Golden-Potential\ GP$

$if\ \ \Lambda.l_p^2\overset{\Downarrow}{=}\frac{4}{48!^2}\quad then\ we\ can\ interprete$

$N=\frac{\sqrt{\varphi}}{2}.\frac{1}{48!}\quad as\ normalization\ factor$

$then\ our\ potential\ has\ the\ form$

$$V(\phi)=(\frac{P_p}{2\pi})^2.N^4.\left(-8\cdot\left|\frac{\phi}{c\sqrt{\varphi}}\right|^2+16\cdot\left|\frac{\phi}{c\sqrt{\varphi}}\right|^4-8\cdot\left|\frac{\phi}{c\sqrt{\varphi}}\right|^8\right)$$

*In* $< 3.1 >$ *we have defined the potential in a way so that the second term of the potential is the quadratic vacuumenergydensity on* $|\phi| = c$.

*Here we can see clearly that the vacuumenergydensity is :*

$$\frac{P_p}{2\pi.48!^2} \quad P_p ... Planckpressure$$

*So we can say that we have such a low vacuumenergydensity because we have a lot of coordinates (48 counted) and the vacuumenergydensity comes from selfinteractions of the permutations.*

*For simplification we want to set* $c = h = G = 1$.

*then the potentialpolynomial can be written in determinantform :*

$$V(\phi) = 8.\left(\boxed{\frac{\sqrt{\varphi}}{2}}\,\boxed{\frac{1}{48!}}\right)^4 . \begin{vmatrix} 1^2 & 1^2 & 0 & \left|\frac{\phi}{\sqrt{\varphi}}\right|^2 \\ 0 & \left|\frac{\phi}{\sqrt{\varphi}}\right|^2 & 0 & 1^2 \\ 0 & 0 & \left|\frac{\phi}{\sqrt{\varphi}}\right|^2 & 0 \\ \left|\frac{\phi}{\sqrt{\varphi}}\right|^2 & 1^2 & 0 & 1^2 \end{vmatrix}$$

*with* $\phi \in \mathbb{O}^6$

*On this form we can see why the normalizationfactor has fourth potenz* $(4 \times 4\ matrix)$.

*why* $\boxed{\frac{1}{48!}}$

*the Octoquintenfield has 48* ($\phi$'s) *degrees of freedome.*

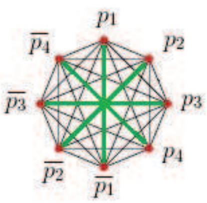
*For every value of* $|\phi|^2$ *we can write*

$$|\phi|^2 = \sum_{i=1}^{48} \phi_i^2 \quad with\ \phi_i \in \mathbb{R}$$

*then for every permutation of the 48 degrees of freedome (coordinates) we get the same value for* $|\phi|^2$.
*That is why we have the factor* $\frac{1}{48!}$ *in the normalizationfactor N.*

*where*
$\phi_1 ... \phi_8 = \phi_0^O ... \phi_7^O$ *and*
$\phi_9 ... \phi_{16} = \phi_0^S ... \phi_7^S$ *and*
$\phi_{17} ... \phi_{24} = \phi_0^F ... \phi_7^F$ *and*
$\phi_{25} ... \phi_{32} = \phi_0^R ... \phi_7^R$ *and*
$\phi_{33} ... \phi_{40} = \phi_0^G ... \phi_7^G$ *and*
$\phi_{41} ... \phi_{48} = \phi_0^H ... \phi_7^H$
*S, F, R, G, H are the five senses − charges and O is no charge.*

## < 3.4 > *Golden − Potential and the 16 − Cell (Coxeter group B4, D4) :*




*As seen above we can write the potential very simple as a product of*
$V = Normfactor \times Determinant$
*The Determinant is like a higher dimensional "Volume".*
*In our case the dimension is 4.*
*Don't misunderstood "Volume" as real spacetimevolume.*
*It is similar to the Determinant in the Einstein Hilbert action.*

*Now we want to go TopDown and take a look on the* $16-cell\ C_{16}$ .
*More details see here https : //en.wikipedia.org/wiki/16 − cell.*

*Some important known facts about the* $C_{16}$ .

1)*count of cells* = 16 *tetrahedra*
2)*count of faces* = 32 *triangle*
3)*count of edges* = 24
4)*count of vertices* = 8
5)*every vertices has* 6 *edges*

6)*The Euler Characteristic of the* 16−*Cell is zero* :

$$\chi = k_0 - k_1 + k_2 - k_3 = \#vertices - \#edges + \#faces - \#cells =$$
$$8 - 24 + 32 - 16 \quad = 0$$

7) *The order of the automorphism group* $S_2^4 \wr S_4$ *is* :
$|Aut(C_{16})| = 2^4.4! = 384$

*How can we compare the* 16 − *Cell with our Potential?*

*Es mentioned before our potential is like a* 4 − *dimensional "volume".*
*More exact it is a sum of* 3 (4) *volumes.*

$$V(z) = \frac{N^4}{2}.(0.z^0 - 16.z^1 + 32.z^2 - 16.z^4)$$

*where* $z = \left|\frac{\phi}{\sqrt{\varphi}}\right|^2$ *and*
$c = G = h = 1$

*To make the potential zero we set* $z = 1$ *or aequivalent* $\phi = \sqrt{\varphi}$

$$V(1) = \frac{N^4}{2}.(0.1^0 - 16.1^1 + 32.1^2 - 16.1^4) = 0 \text{ and compare it with}$$
*vertices edges faces cells*

$$\chi = k_0 - k_1 + k_2 - k_3 = 8 \quad -24 \quad +32 \quad -16 \ = 0 \text{ Euler} - \text{Characteristic}$$

*we can see that both formulars alternate*
*and if we draw out* 16 *we get for the potential*

$$V(1) = N^4.\frac{1}{2}.(-16.1^1 + 32.1^2 - 16.1^4)$$

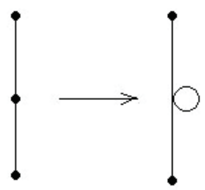
(8 − 24) + 32 − 16
*vertices edges faces cells*

*We have two disagreements*

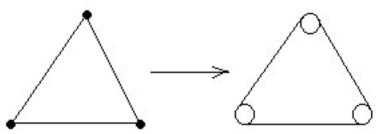
1)*we have no vertices in the potential.*
*The vertices are added to the edges!*

2)*The power of the* 1 *which should be the dimension of the object does fit for the edges and the faces but not for the cells!*

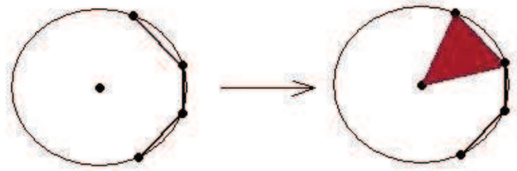
*to* 1)
*How can we annihilate the vertices?*
*Answer : we replace it by a loop (string) and get a closed edge.*
*We can interpret this loops as particles.*



or



*to* 2)
*the cells can be easy extended to fourth dimension on a convex graph.*
2 − *dimension example* :



*So finally the potential shows* 1 − *dimensional* , 2 − *dimensional and* 4 − *dimensional stimulated objects.*

*Interpretation* :
1 − *dimension objects are particles*
*We have two types of* 1 − *dimensional object.*
*Closed and open ones.*

(8 − 24) + 32 − 16
*edges* *faces* *cells*

$$V(1) = N^4.\frac{1}{2}.(-16\,.1^1 + 32\,.1^2 - 16\,.1^4)$$

*With the other zeropoint* ($\phi = c$) *we can create the same particlegraph* $C_{16}$.
*z then is* $\frac{1}{\varphi}$

$$V(z) = V(\frac{1}{\varphi}) = N^4.\frac{1}{2}.(-16\,.(\frac{1}{\varphi})^1 + 32\,.(\frac{1}{\varphi})^2 - 16\,.(\frac{1}{\varphi})^4)$$

<u>< 3.5 > *Combinatorial − Form*</u>

*We have the strange factor* $\frac{1}{2}$ *in front of the bracket.*
*How can we interprete this factor?*

*We can draw this factor inside the brackets and set finally*

$N = \boxed{\frac{\sqrt{\varphi}}{2}} \cdot \boxed{\frac{1}{48!}}$ *then we get finally the*

*COMBINATORIAL − FORM*

$$\boxed{V(\phi) = N^4.\left(-8\cdot\left|\frac{\phi}{\sqrt{\varphi}}\right|^2 + 16\cdot\left|\frac{\phi}{\sqrt{\varphi}}\right|^4 - 8\cdot\left|\frac{\phi}{\sqrt{\varphi}}\right|^8\right)}$$

*for the potential.*

*In mathematics this is called a generating function.*

2) the red $-8$ (edges).

As seen above the 8 comes from $-8 = 4 - 12$.

4 particles in the diagonal ↘ zero curvaturetensor
12 particles ↗

3) the blue 16 (faces).

This is the count of the $16 = 4 + 2.6$ fields of the first curvaturetensor ($\approx$ metrictensor). (4: in the diagonal)

4) the green 8 (cells).

This is the count of the $8 = 2 + 2.3$ fields of the second curvaturetensor ($\approx$ spinmetrictensor). (2: in the diagonal)
Details to this curvaturtensors see later.

Some analytics on the Golden – Potential

without absolute values and $\varphi \in \mathbb{C}$ see < 8 >

$$V(\varphi) = N^4 \cdot \left(-8 \cdot \left(\frac{\varphi}{\sqrt{\varphi}}\right)^2 + 16 \cdot \left(\frac{\varphi}{\sqrt{\varphi}}\right)^4 - 8 \cdot \left(\frac{\varphi}{\sqrt{\varphi}}\right)^8\right) \qquad N = \boxed{\frac{\sqrt{\varphi}}{2}} \cdot \boxed{\frac{1}{48!}}$$

| Zeropoints | momentum density² | mass density² | spin density² | $\times \frac{1}{48!^4}$ |
|---|---|---|---|---|
| $0$ | $0$ | $0$ | $0$ | $\sum = 0$ |
| $\pm 1$ | $-\frac{\varphi}{2}$ | $1$ | $-\frac{1}{2\varphi^2}$ | $\sum = 0$ |
| $\pm\sqrt{\varphi}$ | $-\frac{\varphi^2}{2}$ | $\varphi^2$ | $-\frac{\varphi^2}{2}$ | $\sum = 0$ |
| $\pm i.\varphi$ | $\frac{\varphi^3}{2}$ | $\varphi^4$ | $-\frac{\varphi^6}{2}$ | $\sum = 0$ |
| quadratic sum $= 0$ $-\varphi^2 + \varphi + 1$ | $\sum = 0$ | $\sum = 4\varphi^2$ | $\sum = -4\varphi^2$ | $\sum = 0$ |

Some possible deductions by $\Lambda . l_p^2 = \frac{4}{48!^2}$

We want write it this way $(\frac{48!}{2})^2 = (\frac{l_\Lambda}{2.l_p})^2 = \frac{l_\Lambda^2}{4.l_p^2}$ with $l_\Lambda = \frac{2}{\sqrt{\Lambda}}$

1) Entropy in the universe $= 48!^2$ UOE

We know from Bekenstein Hawking that $4.l_p^2$ is one unit of entropy short UOE.
Setting $c = \hbar = G = 1$ then $l_p = 1$ then

$$\frac{48!^2}{4} = \frac{l_\Lambda^2}{4} \quad \text{then} \quad S_{universe} = l_\Lambda^2 = 48!^2 \approx 1,541 \times 10^{122}$$

is the count of Entropy in the universe.

Later in < 6.1 > we will see that $l_\Lambda$ belongs to a double Clifford – Torus.
We can use this 2 – dimensional flat object as the Entropyobject for the universe.
This object can give an answer to the holographical – principle.

$2) Mass\ in\ the\ universe = \frac{48!}{2}.m_p$

*thinking that the universe is like a black hole we get by the Bekenstein Hawking Entropy and* $c = \hbar = G = k = 1$ :

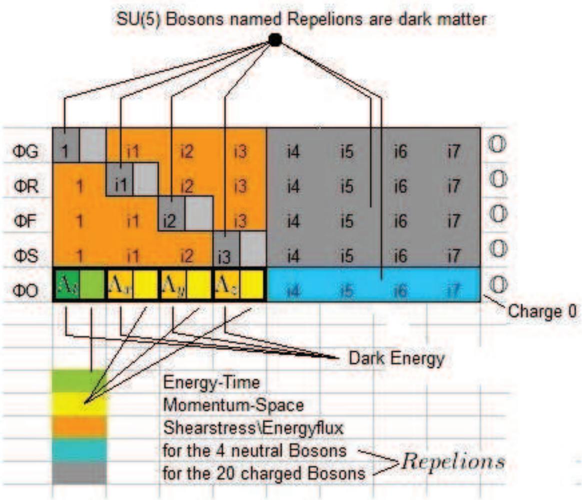
$S_{universe} = (2.M_{universe})^2 \quad S....Entropy, M...Mass$

*Then with the result above it follows that* :

$M_{universe} = \frac{48!}{2}.m_p \approx 6,2 \times 10^{52}\ kg \qquad with\ m_p...Planckmass$

## < 4 > Lagrangedensity of the Octoquintenfield/Golden − Potential

*Hint* :
*I do the same steps as shown in this cooking recipe for the Higgsfield.*
*https : //www.lsw.uni − heidelberg.de/users/mcamenzi/HD*$_H$*iggs.pdf*




*similar to the the higgsfield where the vacuumexpectation is*

$$\phi_{vac} = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

*the vacuumexpectation of the Oktoquintenfield is (green, yellow, orange, lightgray)*

$$\phi_{vac} = v. \begin{pmatrix} 1 + i_1 + i_2 + i_3 \\ 1 + i_1 + i_2 + i_3 \\ 1 + i_1 + i_2 + i_3 \\ 1 + i_1 + i_2 + i_3 \\ 1 + i_1 + i_2 + i_3 \end{pmatrix} \qquad \begin{matrix} v = c...speed\ of\ light \\ not\ the\ minimum\ of\ the\ potential! \end{matrix}$$

$i_1, i_2$ *and* $i_3$ *the imaginaer quaternions.*

*Different to the Higgsfield where the expectationvalue is on the minimum of the potential the expectationvalue for the fourvelocity is on the zeropoints (zerospheres) of the potential.*

*How can we motivate this?*

*Let us say that the expectationvalue is located where the absolut of the potential vanishes.*

*Formal* :

$|V(\phi)| = minimal = 0$

*This is the case if*

$$|\phi| = \begin{cases} 0 \\ c \\ c.\sqrt{\varphi} \end{cases}$$

*case if* $|\phi| = 0$

*Trivial because in this case we have no action and no dynamics.*
*Simply nothing.*

*case if* $|\phi| = c$

*This is the universe we observe.*
*Every particle, quant have fourvelocity* $= c$.
*Our observed Universe* $U_c$.

*case if* $|\phi| = c.\sqrt{\varphi}$

*This is an open question because actually we don't know anything about particles, quants with fourvelocity* $c.\sqrt{\varphi}$ *where* $\varphi$*...golden ratio.*
*Therefore it is an open question if there is an Universe* $U_{c.\sqrt{\varphi}}$.

<u>*STEP 1 : Lorentzinvariant Lagrangedensity for the Octoquintenfield*</u>

$$\mathcal{L}_\phi = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi^\dagger \phi)$$

| $\mathcal{T}_{ij}$ | j -----> | *Generatores of the* $SU(5)$ | | | |
|---|---|---|---|---|---|
| i ↓ | $\begin{pmatrix} 0&1&0&0&0\\ 1&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{pmatrix}$ | $\begin{pmatrix} 0&-i&0&0&0\\ i&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{pmatrix}$ | $\begin{pmatrix} 0&0&0&0&0\\ 0&0&1&0&0\\ 0&1&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{pmatrix}$ | $\begin{pmatrix} 0&0&0&0&0\\ 0&0&0&1&0\\ 0&0&0&0&0\\ 0&1&0&0&0\\ 0&0&0&0&0 \end{pmatrix}$ | $\begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&1\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&1&0&0&0 \end{pmatrix}$ |
| | $\begin{pmatrix} 0&0&1&0&0\\ 0&0&0&0&0\\ 1&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{pmatrix}$ | $\begin{pmatrix} 0&0&0&0&0\\ 0&0&-i&0&0\\ 0&i&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{pmatrix}$ | $\begin{pmatrix} 0&0&-i&0&0\\ 0&0&0&0&0\\ i&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{pmatrix}$ | $\begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&1&0\\ 0&0&1&0&0\\ 0&0&0&0&0 \end{pmatrix}$ | $\begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&1\\ 0&0&0&0&0\\ 0&0&1&0&0 \end{pmatrix}$ |
| | $\begin{pmatrix} 0&0&0&1&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 1&0&0&0&0\\ 0&0&0&0&0 \end{pmatrix}$ | $\begin{pmatrix} 0&0&0&0&0\\ 0&0&0&-i&0\\ 0&0&0&0&0\\ 0&i&0&0&0\\ 0&0&0&0&0 \end{pmatrix}$ | $\begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&-i&0\\ 0&0&i&0&0\\ 0&0&0&0&0 \end{pmatrix}$ | $\begin{pmatrix} 0&0&0&-i&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ i&0&0&0&0\\ 0&0&0&0&0 \end{pmatrix}$ | $\begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&1\\ 0&0&0&1&0 \end{pmatrix}$ |
| | $\begin{pmatrix} 0&0&0&0&1\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 1&0&0&0&0 \end{pmatrix}$ | $\begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&-i\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&i&0&0&0 \end{pmatrix}$ | $\begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&-i\\ 0&0&0&0&0\\ 0&0&i&0&0 \end{pmatrix}$ | $\begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&-i\\ 0&0&0&i&0 \end{pmatrix}$ | $\begin{pmatrix} 0&0&0&0&-i\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ i&0&0&0&0 \end{pmatrix}$ |
| | $\begin{pmatrix} 1&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&-1 \end{pmatrix}$ | $\begin{pmatrix} 0&0&0&0&0\\ 0&1&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&-1 \end{pmatrix}$ | $\begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&1&0&0\\ 0&0&0&0&0\\ 0&0&0&0&-1 \end{pmatrix}$ | $\begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&1&0\\ 0&0&0&0&-1 \end{pmatrix}$ | |

*W − Boson scheme*

$$\begin{pmatrix} W^{11} & W^{12} & W^{13} & W^{14} & W^{15} \\ W^{21} & W^{22} & W^{23} & W^{24} & W^{25} \\ W^{31} & W^{32} & W^{33} & W^{34} & W^{35} \\ W^{41} & W^{42} & W^{43} & W^{44} & W^{45} \\ W^{51} & W^{52} & W^{53} & W^{54} & \end{pmatrix}$$

*hint :* $W^{ij} = W^{ij}_\mu$

*We take a look on the symmetry*

$SU(5) \times U(1)$

$W^{ij}$ $\quad B^0$

*calculate covariant derivation*

$$D_\mu \phi = (\partial_\mu + \frac{i.g}{2}.\tau_{ij}.W^{ij}_\mu + \frac{i.g'}{2}.Id.^0 B^0_\mu).\phi$$

$$\tau_{ij}.W^{ij}_\mu = \begin{pmatrix} W^{51} & W^{11}-i.W^{12} & W^{21}-i.W^{23} & W^{31}-i.W^{34} & W^{41}-i.W^{45} \\ W^{11}+i.W^{12} & W^{52} & W^{22}-i.W^{13} & W^{14}-i.W^{32} & W^{15}-i.W^{42} \\ W^{21}+i.W^{23} & W^{22}+i.W^{13} & W^{53} & W^{24}-i.W^{33} & W^{25}-i.W^{43} \\ W^{31}+i.W^{34} & W^{14}+i.W^{32} & W^{24}+i.W^{33} & W^{54} & W^{35}-i.W^{44} \\ W^{41}+i.W^{45} & W^{15}+i.W^{42} & W^{25}+i.W^{43} & W^{35}+i.W^{44} & -(W^{51}+W^{52}+W^{53}+W^{54}) \end{pmatrix}$$

$$Id^0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

*and for example*

$$W_{\bar{1}2} = \frac{W^{11} - i.W^{12}}{\sqrt{2}}$$

*The boson which changes the charge from* 1 (*hear*) *to* 2 (*taste*).

*Then*

$$D_\mu\phi_{vac} = \frac{v}{2}\cdot i\cdot g\cdot \begin{pmatrix} W^{51} & \sqrt{2}\,W_{\bar{1}2} & \sqrt{2}\,W_{\bar{1}3} & \sqrt{2}\,W_{\bar{1}4} & \sqrt{2}\,W_{\bar{1}5} \\ \sqrt{2}\,W_{\bar{2}1} & W^{52} & \sqrt{2}\,W_{\bar{2}3} & \sqrt{2}\,W_{\bar{2}4} & \sqrt{2}\,W_{\bar{2}5} \\ \sqrt{2}\,W_{\bar{3}1} & \sqrt{2}\,W_{\bar{3}2} & W^{53} & \sqrt{2}\,W_{\bar{3}4} & \sqrt{2}\,W_{\bar{3}5} \\ \sqrt{2}\,W_{\bar{4}1} & \sqrt{2}\,W_{\bar{4}2} & \sqrt{2}\,W_{\bar{4}3} & W^{54} & \sqrt{2}\,W_{\bar{4}5} \\ \sqrt{2}\,W_{\bar{5}1} & \sqrt{2}\,W_{\bar{5}2} & \sqrt{2}\,W_{\bar{5}3} & \sqrt{2}\,W_{\bar{5}4} & -(W^{51}+W^{52}+W^{53}+W^{54}) \end{pmatrix} . \begin{pmatrix} 1+i_1+i_2+i_3 \\ 1+i_1+i_2+i_3 \\ 1+i_1+i_2+i_3 \\ 1+i_1+i_2+i_3 \\ 1+i_1+i_2+i_3 \end{pmatrix}$$

$$+ \frac{v}{2}\cdot i\cdot \begin{pmatrix} g'.B^0 & 0 & 0 & 0 & 0 \\ 0 & g'.B^0 & 0 & 0 & 0 \\ 0 & 0 & g'.B^0 & 0 & 0 \\ 0 & 0 & 0 & g'.B^0 & 0 \\ 0 & 0 & 0 & 0 & g'.B^0 \end{pmatrix} . \begin{pmatrix} 1+i_1+i_2+i_3 \\ 1+i_1+i_2+i_3 \\ 1+i_1+i_2+i_3 \\ 1+i_1+i_2+i_3 \\ 1+i_1+i_2+i_3 \end{pmatrix}$$

*then*

$$\begin{aligned}(D^\mu\phi_{vac})^\dagger(D_\mu\phi_{vac}) = \frac{v^2}{4}.[\; & 4.(gW^{51}+g'B^0)^2 + 8g^2.(W_{\bar{1}2}W_{\bar{2}1} + W_{\bar{1}3}W_{\bar{3}1} + W_{\bar{1}4}W_{\bar{4}1} + W_{\bar{1}5}W_{\bar{5}1}) + something + \\ & 4.(gW^{52}+g'B^0)^2 + 8g^2.(W_{\bar{1}2}W_{\bar{2}1} + W_{\bar{2}3}W_{\bar{3}2} + W_{\bar{2}4}W_{\bar{4}2} + W_{\bar{2}5}W_{\bar{5}2}) + something + \\ & 4.(gW^{53}+g'B^0)^2 + 8g^2.(W_{\bar{1}3}W_{\bar{3}1} + W_{\bar{2}3}W_{\bar{3}2} + W_{\bar{3}4}W_{\bar{4}3} + W_{\bar{3}5}W_{\bar{5}3}) + something + \\ & 4.(gW^{54}+g'B^0)^2 + 8g^2.(W_{\bar{1}4}W_{\bar{4}1} + W_{\bar{2}4}W_{\bar{4}2} + W_{\bar{3}4}W_{\bar{4}3} + W_{\bar{4}5}W_{\bar{5}4}) + something + \\ & 4.(-g(W^{51}+W^{52}+W^{53}+W^{54})+g'B^0)^2 + 8g^2.(W_{\bar{1}5}W_{\bar{5}1} + W_{\bar{2}5}W_{\bar{5}2} + W_{\bar{3}5}W_{\bar{5}3} + W_{\bar{4}5}W_{\bar{5}4})\end{aligned}$$

*hint* : $W^{ij} = W^{ij}_\mu$ *and* $B^0 = B^0_\mu$

*like the result of the Higgsfield we expect something like that* :

$$(D^\mu\phi_{vac})^\dagger(D_\mu\phi_{vac}) = \frac{v^2}{8}.(g^2.(W^+)^2 + g^2.(W^-)^2 + (g^2+g'^2).Z_\mu.Z^\mu + 0.A_\mu.A^\mu)$$

*We have a lot of summands so we first want to take a look on the diagonal elements of the covariant derivation.*
*In the Higgsfieldtheory we get as result the massive Z − Bosons and the Photon as a mixing of neutral W and B bosons.*

*We calculate the expression which is a symmetric bilinearform* :

*Momentumdensity − Matrix*

$$\begin{pmatrix} W^{51} & W^{52} & W^{53} & W^{54} & B^0 \end{pmatrix} . \begin{pmatrix} 8g^2 & 4g^2 & 4g^2 & 4g^2 & 0 \\ 4g^2 & 8g^2 & 4g^2 & 4g^2 & 0 \\ 4g^2 & 4g^2 & 8g^2 & 4g^2 & 0 \\ 4g^2 & 4g^2 & 4g^2 & 8g^2 & 0 \\ 0 & 0 & 0 & 0 & 20g'^2 \end{pmatrix} . \begin{pmatrix} W^{51} \\ W^{52} \\ W^{53} \\ W^{54} \\ B^0 \end{pmatrix}$$

*linearly independent*

*and compare it with the red area of the dynamic lagrangepart.*

*Someone can easy proof that is identical.*

*Then with diagonalizing the Momentumdensity – Matrix*
*we get the following result :*

$$(D^{\mu}\phi_{vac})^{\dagger}(D_{\mu}\phi_{vac}) = \frac{v^2}{4}.[\underline{green\ area} + \underline{red\ area} + something] \quad something = 0$$

$$(D^{\mu}\phi_{vac})^{\dagger}(D_{\mu}\phi_{vac}) = \frac{c^2}{2}.[4.g^2.(|W_{\bar{1}2}|^2 + |W_{\bar{2}1}|^2 + |W_{\bar{1}3}|^2 + |W_{\bar{3}1}|^2 + |W_{\bar{1}4}|^2 + |W_{\bar{4}1}|^2 + |W_{\bar{2}3}|^2 + |W_{\bar{3}2}|^2 + |W_{\bar{2}4}|^2 + |W_{\bar{4}2}|^2 + |W_{\bar{3}4}|^2 + |W_{\bar{4}3}|^2 + |W_{\bar{1}5}|^2 + |W_{\bar{5}1}|^2 + |W_{\bar{3}5}|^2 + |W_{\bar{5}3}|^2 + |W_{\bar{2}5}|^2 + |W_{\bar{5}2}|^2 + |W_{\bar{4}5}|^2 + |W_{\bar{5}4}|^2) + 4.g^2.(|Z^0|^2 + |Z^1|^2 + |Z^2|^2 + |Z^3|^2) + 10.g'^2.|G|^2 \;]$$

*c...speed of light*

*U(1) Gravitation*

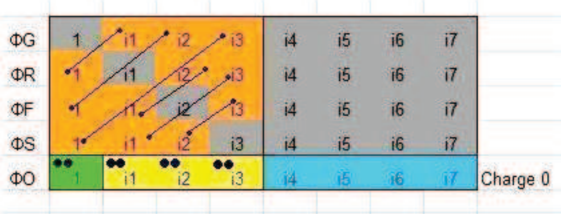
*eigenvalues of the Momentumdensity matrix*

*Momentumdensity Quadrats*

## $<5>$ *Curvaturetensors by the Octoquintenfield*

*The construction comes from multiplications (symmetric to the diagonal)*
*by 2 degrees of freedome (complex subspaces).*
*With this construction the tensor is symmetric in the diagonal.*

$$\phi = \begin{bmatrix} \phi^G \\ \phi^R \\ \phi^F \\ \phi^S \\ \phi^O \end{bmatrix} = \begin{bmatrix} \phi_0^G + i_1.\phi_1^G + i_2.\phi_2^G + i_3.\phi_3^G + i_4.\phi_4^G + i_5.\phi_5^G + i_6.\phi_6^G + i_7.\phi_7^G \\ \phi_0^R + i_1.\phi_1^R + i_2.\phi_2^R + i_3.\phi_3^R + i_4.\phi_4^R + i_5.\phi_5^R + i_6.\phi_6^R + i_7.\phi_7^R \\ \phi_0^F + i_1.\phi_1^F + i_2.\phi_2^F + i_3.\phi_3^F + i_4.\phi_4^F + i_5.\phi_5^F + i_6.\phi_6^F + i_7.\phi_7^F \\ \phi_0^S + i_1.\phi_1^S + i_2.\phi_2^S + i_3.\phi_3^S + i_4.\phi_4^S + i_5.\phi_5^S + i_6.\phi_6^S + i_7.\phi_7^S \\ \phi_0^O + i_1.\phi_1^O + i_2.\phi_2^O + i_3.\phi_3^O + i_4.\phi_4^O + i_5.\phi_5^O + i_6.\phi_6^O + i_7.\phi_7^O \end{bmatrix}$$




*c...speed of light*
*G...Gravitationconstant*
*ℏ...Planckconstant*

**symmetric Curvature Tensor**

$$C_{em} = \frac{c}{G.\hbar}\cdot$$

| | | | |
|---|---|---|---|
| $\phi_0^O \cdot \phi_0^O$ | $i_1.\ \phi_0^R.\phi_1^G$ | $i_2.\ \phi_0^F.\phi_2^G$ | $i_3.\ \phi_0^S.\phi_3^G$ |
| $i_1.\ \phi_0^R.\phi_1^G$ | $-\ \phi_1^O \cdot \phi_1^O$ | $i_3.\ \phi_1^F.\phi_2^R$ | $-i_2.\ \phi_1^S.\phi_3^R$ |
| $i_2.\ \phi_0^F.\phi_2^G$ | $i_3.\ \phi_1^F.\phi_2^R$ | $-\ \phi_2^O \cdot \phi_2^O$ | $i_1.\ \phi_2^S.\phi_3^F$ |
| $i_3.\ \phi_0^S.\phi_3^G$ | $-i_2.\ \phi_1^S.\phi_3^R$ | $i_1.\ \phi_2^S.\phi_3^F$ | $-\ \phi_3^O \cdot \phi_3^O$ |

10 *independent fields.*

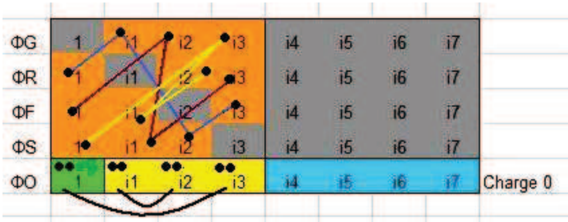
*remark :*
*for $\phi = c$ we get as curvature the plank curvature*
*which is the reciprocal of the planck area.*
*The value of the curvature is :*

$$0,34 \times 10^{70} \quad \frac{1}{m^2}$$

<u>*Second CURVATURE TENSOR from the Octoquintenfield (generates a spinpotential)*</u>

*The construction comes from multiplications by 4 degrees of freedome (quaternionic subspaces).*
*With this construction the tensor is symmetric in both diagonals.*

$$\phi = \begin{bmatrix} \phi^G \\ \phi^R \\ \phi^F \\ \phi^S \\ \phi^O \end{bmatrix} = \begin{bmatrix} \phi_0^G + i_1.\phi_1^G + i_2.\phi_2^G + i_3.\phi_3^G + i_4.\phi_4^G + i_5.\phi_5^G + i_6.\phi_6^G + i_7.\phi_7^G \\ \phi_0^R + i_1.\phi_1^R + i_2.\phi_2^R + i_3.\phi_3^R + i_4.\phi_4^R + i_5.\phi_5^R + i_6.\phi_6^R + i_7.\phi_7^R \\ \phi_0^F + i_1.\phi_1^F + i_2.\phi_2^F + i_3.\phi_3^F + i_4.\phi_4^F + i_5.\phi_5^F + i_6.\phi_6^F + i_7.\phi_7^F \\ \phi_0^S + i_1.\phi_1^S + i_2.\phi_2^S + i_3.\phi_3^S + i_4.\phi_4^S + i_5.\phi_5^S + i_6.\phi_6^S + i_7.\phi_7^S \\ \phi_0^O + i_1.\phi_1^O + i_2.\phi_2^O + i_3.\phi_3^O + i_4.\phi_4^O + i_5.\phi_5^O + i_6.\phi_6^O + i_7.\phi_7^O \end{bmatrix}$$




$$C_{spin} = \left(\frac{c}{G.\hbar}\right)^2 . \begin{bmatrix} -\phi_0^O.\phi_0^O.\phi_3^O.\phi_3^O & -C & B & -A \\ -C & \phi_1^O.\phi_1^O.\phi_2^O.\phi_2^O & -A & B \\ B & -A & \phi_1^O.\phi_1^O.\phi_2^O.\phi_2^O & -C \\ -A & B & -C & -\phi_0^O.\phi_0^O.\phi_3^O.\phi_3^O \end{bmatrix}$$

$$A = \phi_0^S.\phi_3^G.\phi_1^F.\phi_2^R \quad B = \phi_0^F.\phi_2^G.\phi_1^S.\phi_3^R \quad C = \phi_0^R.\phi_1^G.\phi_2^S.\phi_3^F$$

*5 independent fields A, B, C and two in the diagonal (blue and yellow).*

<u>*So finally we get three derivation – or curvaturetensors of the*</u>
<u>*Golden – Potential for twisted spacetime excitation*</u>

0-th Curvaturetensor

| | | | |
|---|---|---|---|
| $\phi_0^O$ | $i_1.\phi_1^G$ | $i_2.\phi_2^G$ | $i_3.\phi_3^G$ |
| $\phi_0^R$ | $i_1.\phi_1^O$ | $i_2.\phi_2^R$ | $i_3.\phi_3^R$ |
| $\phi_0^F$ | $i_1.\phi_1^F$ | $i_2.\phi_2^O$ | $i_3.\phi_3^F$ |
| $\phi_0^S$ | $i_1.\phi_1^S$ | $i_2.\phi_2^S$ | $i_3.\phi_3^O$ |

$\phi_i$ unit is speed m/s

1-Curvaturetensor Cem em = energy-momentum

| | | | |
|---|---|---|---|
| $\phi_0^O\cdot\phi_0^O$ | $i_1.\phi_0^R.\phi_1^G$ | $i_2.\phi_0^F.\phi_2^G$ | $i_3.\phi_0^S.\phi_3^G$ |
| $i_1.\phi_0^R.\phi_1^G$ | $-\phi_1^O\cdot\phi_1^O$ | $i_3.\phi_1^F.\phi_2^R$ | $-i_2.\phi_1^S.\phi_3^R$ |
| $i_2.\phi_0^F.\phi_2^G$ | $i_3.\phi_1^F.\phi_2^R$ | $-\phi_2^O\cdot\phi_2^O$ | $i_1.\phi_2^S.\phi_3^F$ |
| $i_3.\phi_0^S.\phi_3^G$ | $-i_2.\phi_1^S.\phi_3^R$ | $i_1.\phi_2^S.\phi_3^F$ | $-\phi_3^O\cdot\phi_3^O$ |

This tensor is up to a constant equal to the energy-momentum tensor

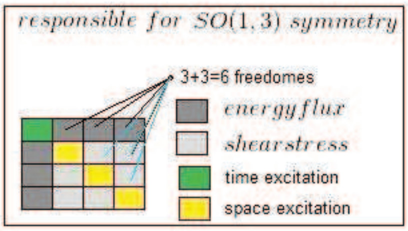
*Energydensity :*

$$\frac{E_{i,j}}{m^3} = \phi_i.\phi_j.\frac{c}{G.\hbar}.\frac{c^4}{8.\pi.G} = \phi_i.\phi_j.\frac{c^2}{K.l_p^2}$$

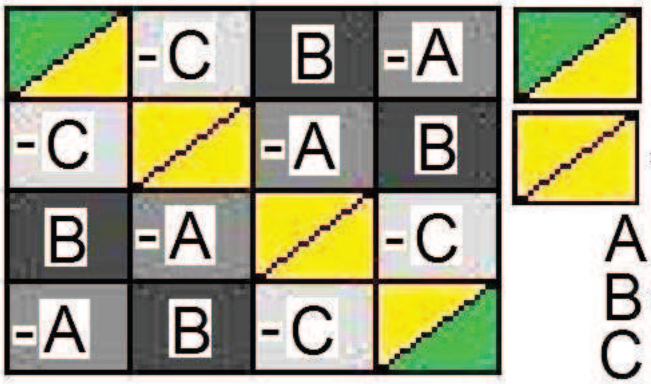
*c...speed of light*
*G...Gravitationconstant*
*$\hbar$...Planckconstant*
*K..Einsteinconstant*
*$l_p^2$...Planckarea*

*the vacuumexcitation :*

$$\phi_i^{o2} = \Lambda.\frac{G.\hbar}{c} \quad i = 0,1,2,3$$




2-Curvaturetensor Cspin



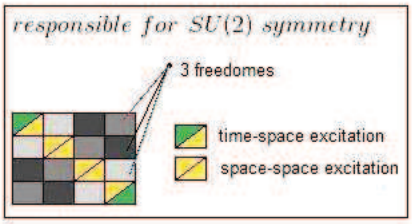

$= -\phi_0^O.\phi_0^O.\phi_3^O.\phi_3^O$

$= \phi_1^O.\phi_1^O.\phi_2^O.\phi_2^O$

$A = \phi_0^S.\phi_3^G\cdot\phi_1^F.\phi_2^R$

$B = \phi_0^F.\phi_2^G\cdot\phi_1^S.\phi_3^R$

$C = \phi_0^R.\phi_1^G.\phi_2^S.\phi_3^F$




## < 6 > *Extension of the General Relativity GR by the second curvaturetensor*

*The Golden − Potential GP has two symmetric curvaturetensors.*
*This motivates us to extend the Einstein Equation.*
*I think this shows that the GR (General Relativity) has to be extended by an imaginry part (spinpart) to be a consistent quantumtheorie.*
*So finally we expect something like GR+i.GR° where GR° is the spinpart.*

*with the two curvaturetensors* $C_{em}$ *and* $C_{spin}$ *we can define following equation :*

**EGR Extended General Relativity**

$$g \cdot Real(C_{em}) + g^2 . \frac{i}{\varphi\sqrt{2}} . \frac{1}{\Lambda} . C_{spin} = \frac{8.\pi.G}{c^4} . (T_{\mu\nu} + \frac{i}{\varphi\sqrt{2}} . S_{\mu\nu})$$

*where the real part is the GR and the imaginaer part is* $GR^O$

*GR...General Relativity*
$GR^O$*...Spinextension of GR*
$\varphi$*...golden ratio*
$\Lambda$*...cosmological constant*
$l_p$*...Plancklength*

$$g = \begin{cases} \Lambda.l_p^2 = \dfrac{4}{48!^2} \approx \dfrac{2,6}{10^{122}} & \text{for the vacuum} \\ 1 & \text{else} \end{cases}$$

*Hint :*
*The small value of g shows the cosmological constant problem or vacuum catastrophe.*

$$g = \Lambda.l_p^2 = \frac{4}{48!^2} \approx \frac{2,6}{10^{122}} \quad \text{for the vacuum}$$

*the operator Real(A) is defined by*

$$Real(\begin{pmatrix} a_{0,0} & i_1.a_{0,1} & i_2.a_{0,2} & i_3.a_{0,3} \\ i_1.a_{1,0} & a_{1,1} & i_3.a_{1,2} & i_2.a_{1,3} \\ i_2.a_{2,0} & i_3.a_{2,1} & a_{2,2} & i_1.a_{2,3} \\ i_3.a_{3,0} & i_2.a_{3,1} & i_12.a_{3,2} & a_{3,3} \end{pmatrix}) = \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix}$$

*the reversing* $Real^{-1}$ *is :*

$$Real^{-1}(\begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix}) = \begin{pmatrix} a_{0,0} & i_1.a_{0,1} & i_2.a_{0,2} & i_3.a_{0,3} \\ i_1.a_{1,0} & a_{1,1} & i_3.a_{1,2} & i_2.a_{1,3} \\ i_2.a_{2,0} & i_3.a_{2,1} & a_{2,2} & i_1.a_{2,3} \\ i_3.a_{3,0} & i_2.a_{3,1} & i_12.a_{3,2} & a_{3,3} \end{pmatrix}$$

$i_1, i_2, i_3$*...imaginaer quaternions*

*more detailed with the two curvaturetensors of the Octoquintenfield :*

$$\frac{8.\pi.G}{c^4} . (T_{\mu\nu} + \frac{i}{\varphi\sqrt{2}} . S_{\mu\nu}) = g \cdot Real(C_{em}) + g^2 . \frac{i}{\varphi\sqrt{2}} . \frac{1}{\Lambda} . C_{spin} =$$

$$= \frac{g.c}{G.\hbar} . \left[ \begin{pmatrix} \phi_0^0.\phi_0^0 & \phi_0^R.\phi_1^G & \phi_0^F.\phi_2^G & \phi_0^S.\phi_3^G \\ sym. & -\phi_1^0.\phi_1^0 & \phi_1^F.\phi_2^R & -\phi_1^S.\phi_3^R \\ sym. & sym. & -\phi_2^0.\phi_2^0 & \phi_2^S.\phi_3^F \\ sym. & sym. & sym. & -\phi_3^0.\phi_3^0 \end{pmatrix} + \frac{i}{\varphi\sqrt{2}} . \frac{g}{\Lambda} . \frac{c}{G.\hbar} . \begin{pmatrix} -\phi_0^0.\phi_0^0.\phi_3^0.\phi_3^0 & -\phi_0^R.\phi_1^G.\phi_2^S.\phi_3^F & \phi_0^F.\phi_2^G.\phi_1^S.\phi_3^R & -\phi_0^S.\phi_3^G.\phi_1^F.\phi_2^R \\ sym. & \phi_1^0.\phi_1^0.\phi_2^0.\phi_2^0 & -\phi_0^S.\phi_3^G.\phi_1^F.\phi_2^R & \phi_0^F.\phi_2^G.\phi_1^S.\phi_3^R \\ sym. & sym. & \phi_1^0.\phi_1^0.\phi_2^0.\phi_2^0 & -\phi_0^R.\phi_1^G.\phi_2^S.\phi_3^F \\ sym. & sym. & sym. & -\phi_0^0.\phi_0^0.\phi_3^0.\phi_3^0 \end{pmatrix} \right]$$

10 different products — *generates Poincare group* $\mathbb{R}^{1,3} \rtimes O(1,3)$

5 different products
$sym.$ and the red products are redundant — $2 \times S^1 \times S^1 \times SU(2) = 2 \times \mathbb{T}^2 \times SU(2)$ *with* $\mathbb{T}^2 = Torus$

$\mathbb{T}^2$*...Clifford − Torus*
*This flat torus is a subset of the unit 3 − sphere* $S^3$.
*The Clifford torus divides the 3 − sphere into two congruent solid tori.*
*The Clifford − Torus embedded in* $S^3$ *becomes a minimal surface.*

*The second curvaturetensor Cspin is determinded by the first curevaturetensor Cem because its components are a mix of the components of Cem.*

< 6.1 > *The vacuumpart of the extended Einstein equation then is :*

*with*

$\varphi_0^0 = \varphi_1^0 = \varphi_2^0 = \varphi_3^0 = c$ *speed of light and other* $\varphi$*'s are zero and*

$g = \Lambda . l_p^2$ *for the vacuum*

*then*

hyperbolic (1,3) ultrahyperbolic (2,2)

$$\underline{Vacuum\ Energydensity} = \frac{c^4}{8\pi G}.[\Lambda.\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + \frac{i}{\varphi\sqrt{2}}.\Lambda.\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}] \quad \mathbb{T}^2_{up},\ \mathbb{T}^2_{down}$$

*generates flat expanding spacetime*

*generates* $2 \times$ *spinning Torus* $\mathbb{T}^2_{\Lambda^\varphi} = S^1 \times S^1$

*flat Clifford − Torus*

$$\mathbb{T}^2_{\Lambda^\varphi} = \begin{pmatrix} (-1) \\ (+1) \end{pmatrix} = \begin{pmatrix} C_- \\ C_+ \end{pmatrix}$$

Spinor

*generates flat expanding and twisting vacuum*

*This is in accordance with the Golden − Potential GP on* $\varphi = c$

$$V(c) = (\frac{\Lambda.c^4}{8\pi G})^2.\frac{1}{2.\varphi^2}.\left(-\varphi^3 + 2.\varphi^2 - 1\right) = Energydensity^2 = 0$$

$\sqrt{\ }$ $\sqrt{\ }$

$$\frac{8.\pi.G}{c^4}.(T_{\mu\nu} + \frac{i}{\sqrt{2}\varphi}.S_{\mu\nu}) = \Lambda.\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + \frac{i\Lambda}{\varphi\sqrt{2}}.\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

## < 6.2 > *Scalefactor for the accelerated expanding Universe by our assumption*

*To make it simple we are thinking about an universe without radiation and mass.*
*This means only the vacuumenergydensity is acting.*
*Then the Hubbleconstant is really constant.*

$$H = \sqrt{\frac{c^2.\Lambda}{3}} = \frac{a'(t)}{a(t)} \quad then$$

$$a(t) \propto e^{H.t} = e^{\sqrt{\frac{c^2.\Lambda}{3}}.t}$$

*then with assumption* $\Lambda.l_p^2 = \dfrac{4}{48!^2}$

*we get finally*

$$\boxed{a(t) \propto e^{H.t} = e^{\sqrt{\frac{4}{3}}.\frac{t}{48!\,t_p}}}$$

*a...scalefactor*
$\Lambda$*...cosmological constant*
*c...speed of light*
$l_p$*...Plancklength*
$t_p$*...Plancktime*

## < 7 > *Getting a closed form for the Extended General Relativity EGR.*

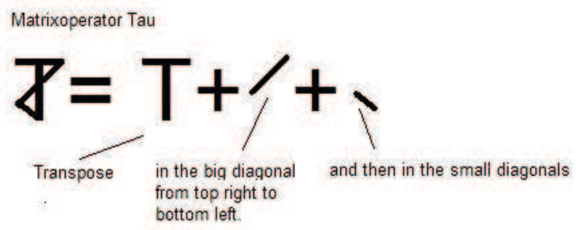
*we know that our energy − momentum curvaturetensor*

$$Real(C_{em}) = R_{\mu\nu} - \frac{R}{2}.g_{\mu\nu} + \Lambda.g_{\mu\nu} \quad and\ that$$

*$C_{spin}$ is defined by multiplication of tensorelements of $C_{em}$*

*The question now is how can we express $C_{spin}$ analogeous to $C_{em}$ above as terms of Riemann − Geometrie?*

*For that we define the operator for $4 \times 4$ matrices or tensors :*




$$\begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix} -> \begin{pmatrix} a_{3,3} & a_{2,3} & a_{1,3} & a_{0,3} \\ a_{3,2} & a_{2,2} & a_{1,2} & a_{0,2} \\ a_{3,1} & a_{2,1} & a_{1,1} & a_{0,1} \\ a_{3,0} & a_{2,0} & a_{1,0} & a_{0,0} \end{pmatrix} -> \begin{pmatrix} a_{3,3} & a_{2,3} & a_{1,3} & a_{1,2} \\ a_{3,2} & a_{2,2} & a_{0,3} & a_{0,2} \\ a_{3,1} & a_{3,0} & a_{1,1} & a_{0,1} \\ a_{2,1} & a_{2,0} & a_{1,0} & a_{0,0} \end{pmatrix}$$

A $\qquad\qquad$ $A^{\tau}$

*and a*

*special simple matricesmultiplication*
$C = A.B$
*with*

$$c_{i,j} = \begin{cases} +a_{i,j}.b_{i,j} & \text{if } i = j \\ -a_{i,j}.b_{i,j} & \text{if } i \neq j \end{cases}$$

*then it is easy to see that*

$$(A+B)^{\tau} = A^{\tau} + B^{\tau}$$

$$(A^{\tau})^{\tau} = A$$

*with*

$$C_{em} = \frac{c}{G.\hbar}.\begin{pmatrix} \phi_0^0.\phi_0^0 & i_1.\phi_0^R.\phi_1^G & i_2.\phi_0^F.\phi_2^G & i_3.\phi_0^S.\phi_3^G \\ sym. & -\phi_1^0.\phi_1^0 & i_3.\phi_1^F.\phi_2^R & -i_2.\phi_1^S.\phi_3^R \\ sym. & sym. & -\phi_2^0.\phi_2^0 & i_1.\phi_2^S.\phi_3^F \\ sym. & sym. & sym. & -\phi_3^0.\phi_3^0 \end{pmatrix} = \frac{c}{G.\hbar}.V_{em}$$

*and*

$$C_{spin} = (\frac{c}{G.\hbar})^2.\begin{pmatrix} -\phi_0^0.\phi_0^0.\phi_3^0.\phi_3^0 & -\phi_0^R.\phi_1^G.\phi_2^S.\phi_3^F & \phi_0^F.\phi_2^G.\phi_1^S.\phi_3^R & -\phi_0^S.\phi_3^G.\phi_1^F.\phi_2^R \\ sym. & \phi_1^0.\phi_1^0.\phi_2^0.\phi_2^0 & -\phi_0^S.\phi_3^G.\phi_1^F.\phi_2^R & \phi_0^F.\phi_2^G.\phi_1^S.\phi_3^R \\ sym. & sym. & \phi_1^0.\phi_1^0.\phi_2^0.\phi_2^0 & -\phi_0^R.\phi_1^G.\phi_2^S.\phi_3^F \\ sym. & sym. & sym. & -\phi_0^0.\phi_0^0.\phi_3^0.\phi_3^0 \end{pmatrix} = (\frac{c}{G.\hbar})^2.V_{spin}$$

*it follows that*

$$\boxed{C_{spin} = Real(C_{em}).Real(C_{em})^{\tau}}$$

*with*

$$Real\,(C_{em}) = R_{\mu\nu} - \frac{R}{2}.g_{\mu\nu} + \Lambda.g_{\mu\nu} = G_{\mu\nu} + \Lambda.g_{\mu\nu} = K_{\mu\nu}$$

*and*

$$Real\,(C_{em}) + \frac{i}{\varphi\sqrt{2}}.\frac{1}{\Lambda}.C_{spin} = \frac{8.\pi.G}{c^4}.\,(T_{\mu\nu} + \frac{i}{\varphi\sqrt{2}}.\,S_{\mu\nu})$$

*we get the final compact result for the extension of General Relativity by*

**EGR**

$$\boxed{K_{\mu\nu} + \frac{i}{\varphi\sqrt{2}}.\frac{1}{\Lambda}.K_{\mu\nu}.K_{\mu\nu}^{\text{ব}} = \frac{8.\pi.G}{c^4}.\,(T_{\mu\nu} + \frac{i}{\varphi\sqrt{2}}.\,S_{\mu\nu})}$$

*with*

$$S_{\mu\nu} = \frac{1}{\Lambda}.T_{\mu\nu}.T_{\mu\nu}^{\text{ব}}...Spintensor$$

$$K_{\mu\nu} = R_{\mu\nu} - \frac{R}{2}.g_{\mu\nu} + \Lambda.g_{\mu\nu}$$

$$K_{\mu\nu}^{\text{ব}} = R_{\mu\nu}^{\text{ব}} - \frac{R}{2}.g_{\mu\nu}^{\text{ব}} + \Lambda.g_{\mu\nu}^{\text{ব}}$$

$\varphi$*...golden ratio*

*The real part is the known General Relativity.*
*The imaginaer part is the Spinextension of GR.*

*Hint : The Energy−Stress tensor is still symmetric with or without Spin!*

*< 7.1 > Showing a combinatorial dimensionless form of the EGR*

*With the relation shown in < 3.5 > and Appendix III*

$$\Lambda.l_p^2 = \frac{4}{48!^2}$$

*we can write :*

$$\frac{1}{\Lambda} = l_p^2.\frac{48!^2}{4}$$

*Then our formular in < 6 > can be written to :*

$$\frac{8.\pi.G}{c^4}.\,(T_{\mu\nu} + \frac{i}{\varphi\sqrt{2}}.\,S_{\mu\nu}) = g\cdot Real\,(C_{em}) + g^2.\frac{i}{\varphi\sqrt{2}}.\frac{1}{\Lambda}.C_{spin} \qquad g = \begin{cases} \Lambda.l_p^2 \ \ for\ the\ vacuum \\ 1\ else \end{cases}$$

*Hint : instead of* $Real(C_{em})$ *we write short* $C_{em}$ *and keep it in mind!*

$$\frac{8.\pi.G}{c^4}.\,(T_{\mu\nu} + \frac{i}{\varphi\sqrt{2}}.\,S_{\mu\nu}) = g\cdot C_{em} + g^2.\frac{i}{\varphi\sqrt{2}}.\,l_p^2.\frac{48!^2}{4}.C_{spin}$$

$$\frac{8.\pi.G}{c^4}.(T_{\mu\nu}+\frac{i}{\varphi\sqrt{2}}.S_{\mu\nu})= g\cdot\frac{c}{G.\hbar}.V_{em}+g^2.\frac{i}{\varphi\sqrt{2}}.l_p^2.\frac{48!^2}{4}.(\frac{c}{G.\hbar})^2.V_{spin}$$

*then with*

$$\frac{c}{G.\hbar}=\frac{1}{c^2.l_p^2}$$

$$\frac{8.\pi.G}{c^4}.(T_{\mu\nu}+\frac{i}{\varphi\sqrt{2}}.S_{\mu\nu})= g\cdot\frac{1}{c^2.l_p^2}.V_{em}+g^2.\frac{i}{\varphi\sqrt{2}}.\frac{48!^2}{4}.\frac{1}{c^4.l_p^2}.V_{spin}\quad \Big|.l_p^2$$

$$\frac{8.\pi.G}{c^4}.l_p^2.(T_{\mu\nu}+\frac{i}{\varphi\sqrt{2}}.S_{\mu\nu})= g\cdot\frac{1}{c^2}.V_{em}+g^2\,\frac{i}{\varphi\sqrt{2}}\,\frac{48!^2}{4}.\frac{1}{c^4}.V_{spin}$$

*then with*

$P_p=\frac{c^7}{G^2.\hbar}$ *Planckpressure and* $g=1$ *for not vacuum*

*we get finally*

$$\boxed{\frac{8.\pi}{P_p}.(T_{\mu\nu}+\frac{i}{\varphi\sqrt{2}}.S_{\mu\nu})=\frac{1}{c^2}.V_{em}+\frac{i}{\sqrt{2}}.\frac{1}{8N^2}.\frac{1}{c^4}.V_{spin}}$$

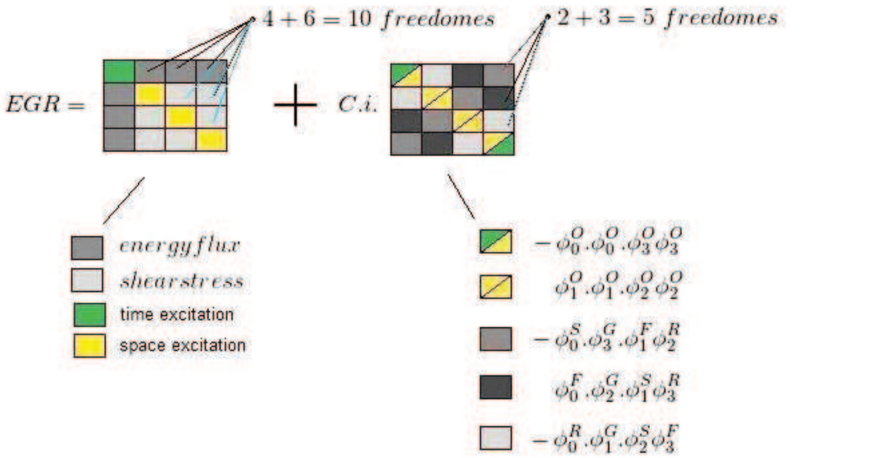
$V_{em}, V_{spin}$ *see* <7>
$N=\frac{\sqrt{\varphi}}{2.48!}$

*dimensionless Combinatorial – Form of the EGR*

## <7.2> *Spin and possible proofing of the Extended General Relativity EGR.*

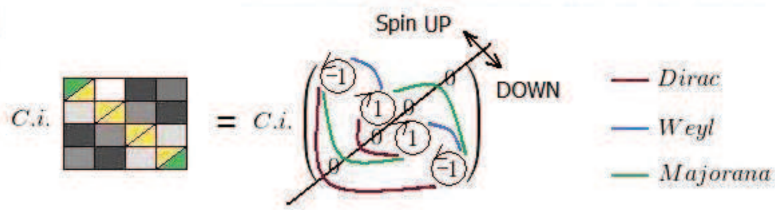
*As noted in* <5> *the second curvaturetensor* $C_{spin}$ *is responsible for the Spin.*
*Obviously the* $C_{spin}$ *is directly connected to the energy – momentum curvaturetensor* $C_{em}$.
*So in principle the Extended General Relativity short EGR could be proofed because the spin of a particle changes the Energy – Momentum – Tensor.*

*First we have to show how the spin of a particle acts on the* $C_{spin}$.
*For that we have to take a closer look on the* $C_{spin}$.
*As seen in* <5> *the structure is :*



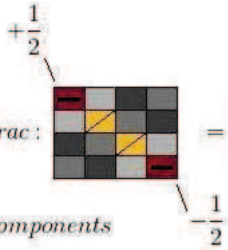

*Now how can we embed dirac – fermions with spin* $\frac{1}{2}$ *into the EGR?*

*For that we want to assign the structure of the* $C_{spin}$ *in the following way to fermions :*




*Then a resting electron excites the* $C_{spin}$ *and the* $C_{em}$ *as follow :*

$Spin = \frac{1}{2}$ $Dirac:$

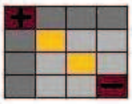


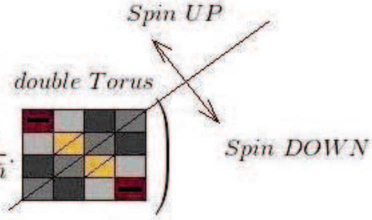

$$= \begin{pmatrix} -\phi_0^{0^2}.\phi_3^{0^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\phi_0^{0^2}.\phi_3^{0^2} \end{pmatrix} \qquad \text{Spinor } \psi_D^G = \begin{pmatrix} (-1) \\ \bullet \\ \bullet \\ (-1) \end{pmatrix}$$

*excited components*

*Then complete in the EGR :*

$$\frac{8.\pi.G}{c^4}.(T_{\mu\nu} + \frac{i}{\varphi\sqrt{2}}.S_{\mu\nu}) = \frac{c}{G.\hbar}.\Bigg( \text{Minkowski} + \frac{i}{\varphi\sqrt{2}}.\frac{1}{\Lambda}.\frac{c}{G.\hbar}.\text{double Torus} \Bigg)$$




$$= \frac{c}{G.\hbar}.\left[ \begin{pmatrix} \phi_0^{0^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\phi_3^{0^2} \end{pmatrix} + \frac{i}{\varphi\sqrt{2}}.\frac{1}{\Lambda}.\frac{c}{G.\hbar}.\begin{pmatrix} -\phi_0^{0^2}.\phi_3^{0^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\phi_0^{0^2}.\phi_3^{0^2} \end{pmatrix} \right]$$

*excited*

$\varphi$*...golden ratio*

*pressure on a random spacedirection z*

*This pressure to a spacedirecton should be proofable in principle!*

*We allow* $\phi \in \mathbb{C}$

## < 8 > *Some important points of the Golden − Potential*

*To get the maxima, minima and the zeropoints of the potential we have to substitute* $z = \phi^2$ *it is enough (because of symmetry) to take a look on the positive* $\phi'$*s.* *and solve the cubic equations in the bracket*

$$V(\sqrt{z}) = z.(\frac{\mu^2}{2} + \frac{\lambda^2}{4}z + \frac{\gamma^2}{8}z^3) \text{ and}$$

$$V'(\sqrt{z}) = \sqrt{z}.(\mu^2 + \lambda^2.z + \gamma^2.z^3)$$

*We will make it short and write the results.*
*First the Zeropoints :*

$$z_1 = u + v = -\sqrt{\frac{2C}{3}}(\sqrt[3]{\frac{\sqrt{27} - i.\sqrt{5}}{\sqrt{32}}} + \sqrt[3]{\frac{\sqrt{27} + i.\sqrt{5}}{\sqrt{32}}})$$

$$z_2 = \epsilon_1.u + \epsilon_2.v$$

$$z_3 = \epsilon_2.u + \epsilon_1.v$$

$C = c^4.\varphi^2$
*c...speed of light*
$\varphi$*...golden ratio*

*Where* $\epsilon_1 = -\frac{1}{2} + i.\frac{\sqrt{3}}{2}$ *and* $\epsilon_2 = -\frac{1}{2} - i.\frac{\sqrt{3}}{2}$
*then*

$$z_1 = -0,990839414 \times 2.\sqrt{\frac{2C}{3}}$$

$$z_2 = 0,378466979 \times 2.\sqrt{\frac{2C}{3}}$$

$$z_3 = 0,612372435 \times 2.\sqrt{\frac{2C}{3}}$$

*then the zeropoints are*

$$\phi_1 = i.0,995409169 \times \sqrt[4]{\frac{8.C}{3}}$$

$$\phi_2 = 0,615196699 \times \sqrt[4]{\frac{8.C}{3}}$$

$$\phi_3 = 0,782542290 \times \sqrt[4]{\frac{8.C}{3}}$$

$\boxed{\phi_1 = i.c.\varphi} = i.0,995409169 \times \sqrt[4]{\frac{8.C}{3}} = i.sin(84,507759190) \times \sqrt[4]{\frac{8.C}{3}}$

$\alpha_{c.\varphi} = 84,507759190^o$

$\boxed{\phi_2 = c} = 0,615196699 \times \sqrt[4]{\frac{8.C}{3}} = sin(37,966214178) \times \sqrt[4]{\frac{8.C}{3}}$

$\alpha_c = 37,966214178^o$

$\boxed{\phi_3 = c.\sqrt{\varphi}} = 0,78254229 \times \sqrt[4]{\frac{8.C}{3}} = sin(128,506061932) \times \sqrt[4]{\frac{8.C}{3}}$

$\alpha_{c.\sqrt{\varphi}} = 128,506061932^o$

*Then the Maxima and the Minima :*

$z_1 = u + v = -\sqrt{\frac{C}{3}}.(\sqrt[3]{\frac{\sqrt{37} - i.\sqrt{27}}{\sqrt{64}}} + \sqrt[3]{\frac{\sqrt{37} + i.\sqrt{27}}{\sqrt{64}}})$

$z_2 = \epsilon_1.u + \epsilon_2.v$

$z_3 = \epsilon_2.u + \epsilon_1.v$

*Finally we have two positiv results :*

$z_{min} = 0,233475630 \times 2.\sqrt{\frac{C}{3}}$ *and*

$z_{max} = 0,725352944 \times 2.\sqrt{\frac{C}{3}}$

*and one negative*

$z_3 = -(z_{max} + z_{min})$

*Then because of* $z = \phi^2$

$\phi_{min} = 0,483193160 \times \sqrt[4]{\frac{4.C}{3}}$ *and*

$\phi_{max} = 0,8516765489 \times \sqrt[4]{\frac{4.C}{3}}$

*In cubic equations the real zeropoints comes from the $cos(\alpha)$ or from $sin(90-\alpha)$ of angles (see https : //en.wikipedia.org/wiki/Cubic_function).*

*Then for* $\phi_{min}$ *we get an angle* $\alpha_{min}$ :

$\phi_{min} = 0,483193160 \times \sqrt[4]{\frac{4.C}{3}} = sin(28,894160846) \times \sqrt[4]{\frac{4.C}{3}}$

$\alpha_{min} = 28,894160846$ *degrees is very near to the Weinbergangle*

$sin^2(\alpha_{min}) = sin^2(28,894160846) = 0,233475630$

*with Cardanic formular and so on we can express* $\alpha_{min}$ *by :*

$$\boxed{\alpha_{min} = arcsin(\sqrt{-cos(\frac{arccos(\frac{-\sqrt{27}}{8}) + \pi}{3})}) \approx 28,9^o}$$

*and for* $\phi_{max}$ *we get an angle*

$\phi_{max} = 0,8516765489 \times \sqrt[4]{\frac{4.C}{3}} = sin(121,605508985) \times \sqrt[4]{\frac{4.C}{3}}$

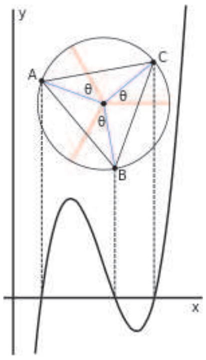
$\alpha_{max} = 121,605508985$ *degrees*

$\phi_{min} = 0,483193160 \times \sqrt[4]{\frac{4.C}{3}} \approx 0,660464.c$ *c...speed of light*

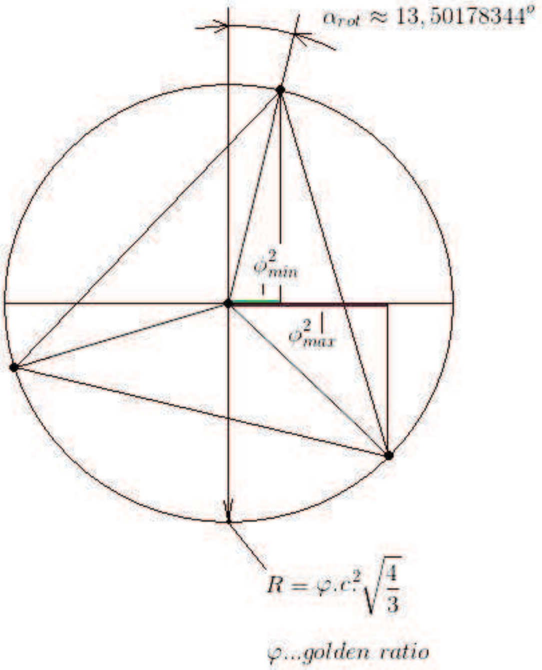
$\phi_{max} = 0,851676548 \times \sqrt[4]{\frac{4.C}{3}} \approx 1,164134.c$

*In cubic equations the real zeropoints comes from the $cos(\alpha)$ or from $sin(90-\alpha)$ of angles (see https : //en.wikipedia.org/wiki/Cubic_function).*

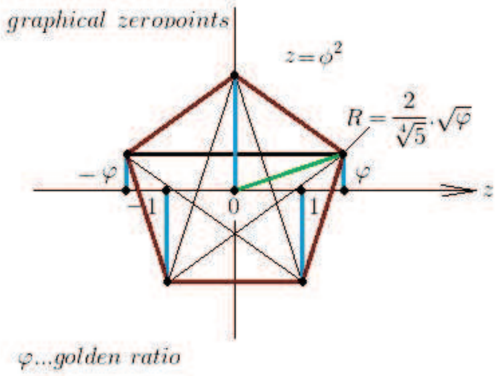
<u>*Geometric interpretation of the roots (zeropoints) in cubic equations with 3 real zeropoints*</u>




*graphical zeropoints of the derivation of the*
*(radicaled $\varphi^2 = z$) Golden − Potential GP*




*Hint : On our special Golden − Potential GP the zeropoints (spheres) comes from a pentagon.*




## < 9 > *Conclusions*

*Dark Energy comes by definition from the Golden − Potential (the second term in the potential).*

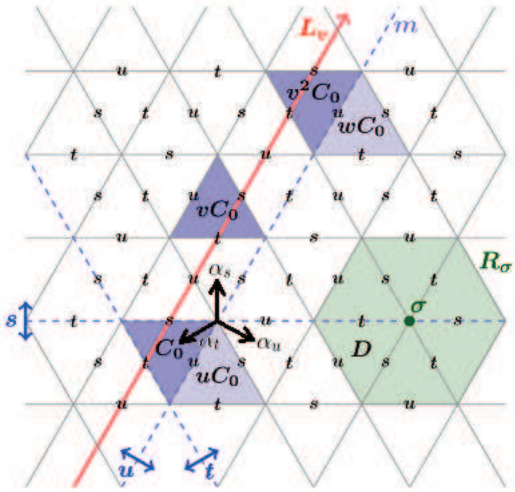
*DarkMatter could be the W , Z Bosons and the particles by the SU(5) Symmetry*
*(adjoint and fundamental presentation).*

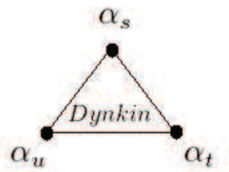
## *APPENDIX I*

*Unterstanding the action of the coxeterelement.*
*As mentioned on the beginning of the paper a coxeterelement is a composition of the generating reflections of the reflectiongroup.*

*In our case the affine group E9 (the one point extension of E8) has 9 such generating reflections* $e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9$.
*A reflection* $e_i$ *is a reflection on the hyperspace of the root* $\alpha_i$.
*So the coxeterelement is the composition of the rootreflections.*
*In our case the roots are vectors in the euclidean − space* $\mathbb{R}^9$ *and the coxeterelement is a map on this space.*

*We want to visualize it by the simple example* $\tilde{A}_2$




*action of* $v = s.u.t$ *the coxeterelement on chamber* $C_0$ $\quad C_0 \to v.C_0$

*This shows that the action of the coxeterelement in this case moves the chamber along the red Line* $L_v$ *and then reflect it on* $L_v$.
*Doing the action twice then we move the chamber the double way.*

*This symmetries can be descriped by the coxeterpolynomial which is the characteristical polynomial of the action (map) of the coxeterelement which is an affine map.*
*This affine maps are well studied so i will write only the results.*

*In our special example* $\tilde{A}_2$ *the coxeterpolynomial is*

$$f_{\tilde{A}_2}(x) = (x+1).(x-1)^2 = \frac{x^2-1}{x-1}.(x-1)^2 \qquad \text{eigenvalues } \lambda_h = -1, \lambda_v = 1, \lambda_t = 1$$
$$\text{eigenvectors } v_h \quad v_v \quad v_t$$

$\lambda_h = -1$ *is the eigenvalue by this cyclotomic factor and is the eigenvalue of the so called horizontal root(system).This produces the reflection on the red line.*

$\lambda_v = 1$ *is one eigenvalue of this factor and is the eigenvalue of the so called vertical root(system).Vertical because the root is orthogonal to the horizontal root(system).*

$\lambda_t = 1$ *is one eigenvalue of this factor and produces the translation on the red line.*

*With the eigenvector* $v_h$ *which will be reflected by the coxeterelement action we have a simple root for the Liealgebra* $\mathfrak{su}(2)$.

*And with the eigenvectors* $v_v$ *and* $v_t$ *we have roots for* $\mathfrak{u}(1)$.

*So in summary the coxeterelement generates the Symmetrie*

$SU(2) \times U(1) \times U(1)$

*Analogeous for E9 which is $\tilde{E}_8$ the coxeterelement generates the symmetrie composition* $SU(5) \times SU(3) \times SU(2) \times U(1) \times U(1)$.

## APPENDIX II

*Our target is to show that*

$$\Lambda.l_p^2 = \frac{16}{\varphi}.N^2 = \frac{16}{\varphi}.(\frac{\sqrt{\varphi}}{2}.\frac{1}{48!})^2 = \frac{4}{48!^2} \approx \frac{2,6}{10^{122}} \quad \varphi...\text{golden ratio}$$

*For that we start with the Combinatorial − Form and going back to the Einstein − Form of the Golden − Potential GP*

*On the combinatorial form we have set* $c = h = G = 1$.

$$V(\phi) = N^4.\left(-8.\left|\frac{\phi}{\sqrt{\varphi}}\right|^2 + 16.\left|\frac{\phi}{\sqrt{\varphi}}\right|^4 - 8.\left|\frac{\phi}{\sqrt{\varphi}}\right|^8\right) \qquad \boxed{N = \frac{\sqrt{\varphi}}{2}.\frac{1}{48!}}$$

*We will take this back and get*

$$V(\phi) = (\frac{P_p}{2\pi})^2.N^4.\left(-8\cdot\left|\frac{\phi}{c\sqrt{\varphi}}\right|^2 + 16\cdot\left|\frac{\phi}{c\sqrt{\varphi}}\right|^4 - 8\cdot\left|\frac{\phi}{c\sqrt{\varphi}}\right|^8\right)$$

*With the relation*

$$P_p.l_p^2 = \frac{c^4}{G}$$

*we get*

$$V(\phi) = (\frac{c^4}{2\pi G}\cdot\frac{1}{l_p^2})^2.N^4.\left(-8\cdot\left|\frac{\phi}{c\sqrt{\varphi}}\right|^2 + 16\cdot\left|\frac{\phi}{c\sqrt{\varphi}}\right|^4 - 8\cdot\left|\frac{\phi}{c\sqrt{\varphi}}\right|^8\right) \quad \left|\cdot\frac{\Lambda^2}{\Lambda^2}\right.$$

*Then*

$$V(\phi) = (\frac{\Lambda c^4}{2\pi G}\cdot\frac{1}{\Lambda l_p^2})^2.N^4.\left(-8\cdot\left|\frac{\phi}{c\sqrt{\varphi}}\right|^2 + 16\cdot\left|\frac{\phi}{c\sqrt{\varphi}}\right|^4 - 8\cdot\left|\frac{\phi}{c\sqrt{\varphi}}\right|^8\right) \quad \left|\cdot\frac{\varphi^2}{\varphi^2}\cdot\frac{4^2}{4^2}\right.$$

*Then*

$$V(\phi) = (\frac{\Lambda c^4}{8\pi G})^2\cdot(\frac{4^2}{\Lambda l_p^2\varphi})^2.N^4\left(-\frac{\varphi}{2}\cdot\left|\frac{\phi}{c}\right|^2 + \left|\frac{\phi}{c}\right|^4 - \frac{1}{2.\varphi^2}\cdot\left|\frac{\phi}{c}\right|^8\right)$$

*Comparing with the Einstein − Form this must be* 1.
*Then*

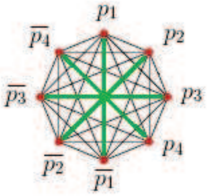
$$\boxed{\Lambda.l_p^2 = \frac{4}{48!^2}}$$

*QED.*

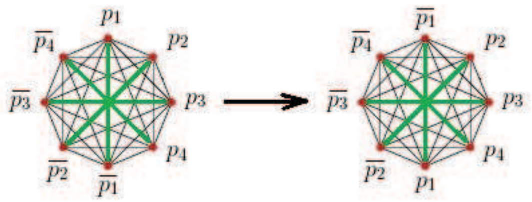
## APPENDIX III

1) *The* 16 − *Cell*

*On the* 16 − *Cell each vertizes is connected by an edge to all other vertizes except the opposite one!*
*We have* 4 *disjunct such pairs which are not connected by an edge.*



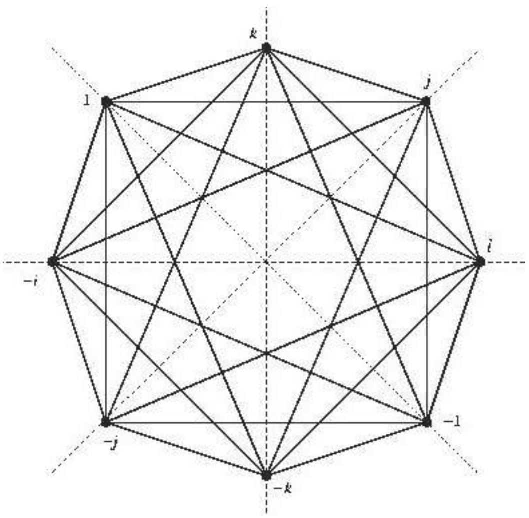

*Now exchanging this points which are not connected is an automorphism (for example p1 with p1−line) because p1 has the same connections as p1 line.*
*So at all we generate* $2^4 = 16$ *automorphisms by this actions because we can say for all 4 pairs 0 means pair IS NOT exchanged and 1 for pair IS exchanged.*
*So every binary code like* $(0, 1, 0, 0)$ *is an automorphism.*




*But this are not all automophism. Independend from that we can permutate* $p1, p2, p3, p4$ *when we simultaneously permutate their opposite points.*
*For example :*

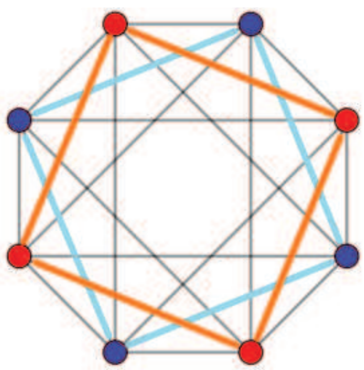
$p_1 \longrightarrow p_2$
$\overline{p_1} \longrightarrow \overline{p_2}$

*This give us* $4! = 24$ *automorphisms independend from the* 16 *automorphisms before.*
*So at all we get* $16.24 = 384$ *automorphisms.*
*With this we can divide the* $Aut(C_{16})$ *into* $AutOpposite(C_{16})$ *and* $AutFakt(C_{16})$ *so that* $Aut(C_{16}) = AutOpposite(C_{16}) \times AutFakt(C_{16}) = 16 \times 4!$.
$AutOpposite(C_{16})$ *are the Automorphisms of the* 16*−Cell* $C_{16}$ *which comes from exchanging the opposite vertizes (points).*
$AutFakt(C_{16})$ *are the Automorphisms of the* 16*−Cell* $C_{16}$ *which comes from permutating* $p1, p2, p3, p4$ *vertizes (points) as described above.*
$AutFakt(C16)$ *is simply the permutation − group* $Sym(4) = S_4$.
*Embedding (projection) the* 16*−Cell into the quartenionic subgroups of the Octoquintenfield :*
*See also Quaterniongroup* $Q8$!

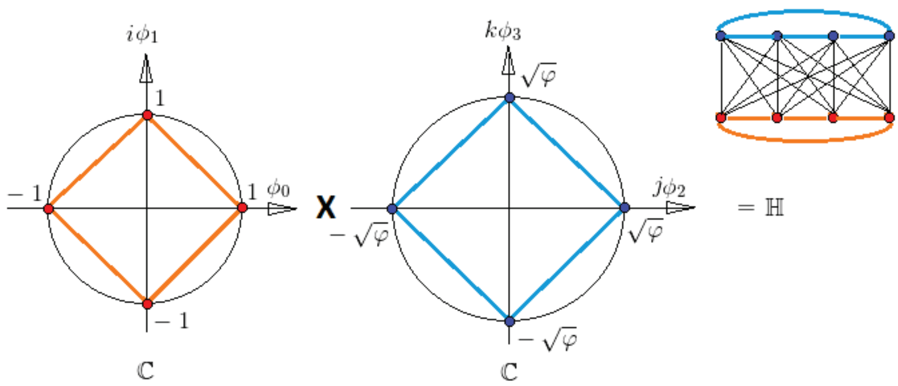



*The* $16 - Cell$ $C_{16}$ *can be seen as a so called* <u>$4 - 4$ *duopyramid*</u>

*more here https : //en.wikipedia.org/wiki/DuopyramidExample_16 − cell*



$16 - Cell$ *as a duopyramid with special embedding in the quaternions.*

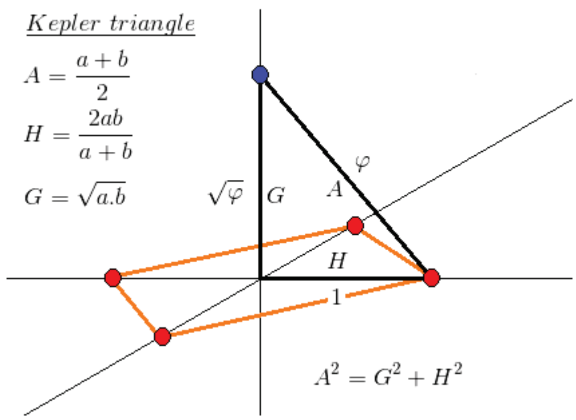



*The orange base of the duopyramid is on the sphere* $S_c$ *with* $c = 1$ *and the blue base on the sphere* $S_{\sqrt{\varphi}.c}$

*How long are the edges of this special* $16 - Cell$?

*The coordinates of the* 8 *vertices are* :
$(\pm 1, 0, 0, 0), (0, \pm 1, 0, 0)$ *and*
$(0, 0, \pm\sqrt{\varphi}, 0), (0, 0, 0, \pm\sqrt{\varphi})$

*Length of the orange edges are* : $\sqrt{2}$
*Length of the blue edges are* : $\sqrt{2\varphi}$
*Length of the black edges are* : $\varphi$




*An interesting point is* :
*For positive real numbers* $a$ *and* $b$, *their arithmetic mean* $A$, *geometric mean* $G$, *and harmonic mean* $H$ *are the lengths of the sides of a right triangle* $\Leftrightarrow$ *that triangle is a Kepler triangle.*
$a = b.\varphi^3$ $\varphi$*...golden ratio.*