The Holomorphic Quanta

Part 3: Reflection

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"A physicist is just an atom's way of looking at itself!" — *Neils Bohr*

"There are three classes of people:
Those who see,
Those who see when they are shown, and
Those who don't see."
– Leonardo da Vinci

Abstract

This is the third part of a four-part presentation. Part 1 introduced a relational model that allowed us to demonstrate the equivalence of space and time as $S = Tc^2$ and showed that S represents energy as the product of scalar space with spatial frequency and T represents energy as the product of time units with temporal frequency. In this part, I continue to develop the model by replacing a the inverse scale with a polar coordinate system to solve what I call the frequency problem. By analogizing vector spaces (velocity and acceleration domain) and scalar space (space-time domain) with computer windows, I "click" to refocus the visual model on the domain of interest and show how mathematical operations, like quantum operators, transform scales and coordinates in one domain to project, translate, reflect and rotate geometric symbols in other domains and produce more complex relations (including the classical wave equation and the Klein-Gordon equation).

Introduction

In part 1 of this presentation, I presented a simple geometric model called The Space-Time-Motion or STM diagram that represents the relationships between space, time and motion, to provide a visual aid to understanding quantum physics and relativity at an undergraduate level. I interposed the linear space-time domain with the inverse spatial-temporal *frequency* domain and showed that this inverse domain is a scaled representation of the quantum energy domain. In part 2, I began to include discussion of an underlying transformation process that is represented in the equations and relational diagrams.

Reflecting back on part 1, it was about the separation of a unified concept we call energy, first into two orthogonal yet equivalent measures of energy: space – quantified as scalar units (qbits) of displacement, and time – quantified as scalar units of "1 clock increment". Then in part 2, I explained how motion *projects* this energy (the second step in the transformation process) from the quantum domain into the relativistic domain producing a distortion that can be viewed as a curvature of space in time to produce a spherical unit of energy. I developed the visual model by referring to the quantum unit as a "DROPLET " to represent the fluidity and malleability of the optical property of energy. I used computer windows as a way of visualizing the coexistence of different perspectives, so rather than particles popping in and out of existence; I analogize different domains to computer popup windows, each having any number of hyperlinks to other domains that can be clicked in and out of view.

With that, I illustrated how the simultaneous view of two different domains on the same window has the effect of distorting the projected image, causing Lorentz magnification and parallax in the projection of a vector from the quantum domain into the relativistic domain. Then I briefly reflected on how this parallax played a role in the development of well-established equations for energy that, although mathematically correct, contain scaling factors that make them hard to interpret. However, they are projected onto the background domain, which serves as a basis for reflection.

In this section, I will continue to develop the STM model to show how it relates quantum physics and relativistic physics, including the statistical analysis currently used in quantum mechanics, the quantum wave function and the Klein-Gordon equation. In order to keep it at the undergraduate level, I will review and emphasize certain things about mapping and how vectors inherently separate one domain into two base domains and then project from one domain to the other creating a third domain whose scale is a ratio of the first two. These are things that are easily forgotten or ignored, especially when you learn the rules of math so well that you can perform them with your eyes closed, but can make a world of difference in the interpretation of results.

The theme of this part is reflection (the third step in the transformation process), which I use to explain and demonstrate reflection. I reflect upon other models and equations that are complementary to this model to convince you that they, just like space and time, are saying the same thing with different symbols. Transformation is an iterative process that produces multiple layers of information[i] and finding meaning requires you to look for complementarity and patterns in the events and information that have proven true or at least proven to work within the boundaries of their applicability.

The STM diagram revealed the equations for energy of a quantum particle in exactly the same geometric relation as the total energy relations that represent the dispersion of energy (another level of separation). It allowed us to visualize the particle-wave duality as a change in perspective, the same as you can visualize an object both at rest with respect to itself (the quantum perspective) yet in motion with respect to the "popup window" that is the background (the relativistic perspective), which means it has potential-energy-of-motion. It also allows us to visualize space (as a whole, $S = s^2$) and time (as a whole, $T = t^2$) as two equivalent yet different perspectives of motion, related by $S = Tc^2$.

The STM Transformation Model

Separation

Continuing the example from part 1, let's say that it takes 1 nsec from the flash event at t_0 for the light to reach the observer who's holding the bulb. A measurement at 1 nsec after the flash corresponds to Event 1 at t_1 (the Event Reference). From the light sphere or photon's perspective, the surface of the quantum sphere at Event 1 corresponds to "Here" on the S axis and "Now" on the T axis. A vector arrow is shown in Figure 1 referenced to t_1 and back-projected to t_0, to represent motion in the photon's inverse reference frame, which is at $t = \frac{1}{2}$ on the linear scale. The next measurement (in the future) will be at 2 nsec, shown as Event 2 at s_2 and t_2. The observer's relativistic perspective would be plotted at $s = 2$ and $t = 2$, which are farther out than what is shown as s_2 and t_2, but the vector the same magnitude as from t_0 to t_1. Therefore, both vectors (in the quantum domain) are half the length of the vector, which projects from 0 to 1 event in the relativistic domain[ii].

As it expands, the photon is still one light unit, moving at 1 light year/year so if it could see itself, it "sees" Event 2 as the new "Now" and Event 1 in its "past", which corresponds in the diagram to "inner space". Effectively, this measurement event separates the two perspectives. The relativistic observer at the center sees the sphere expand, but the photon *resets the world domain* to a new "Here" and "Now" pulling the *scale* of space and time into itself, back to the Event Reference.[iii]

This quantum perspective is the at-rest perspective, the photon's own center-of-mass frame – from the outside looking in at the unchanging surface of the sphere with the flashbulb at the origin. From here, an outside observer just sees an orb, a unit of illumination ("phot"). The photon could only detect motion if it could change its perspective and see, (or imagine - it would have "insight") the flash bulb at its center appearing to shrink. It would "remember" the bulb, the observer, and its former-self (qbits of Event 1) shrinking into its center, into the past. As a quantum computer, this would be in its memory.

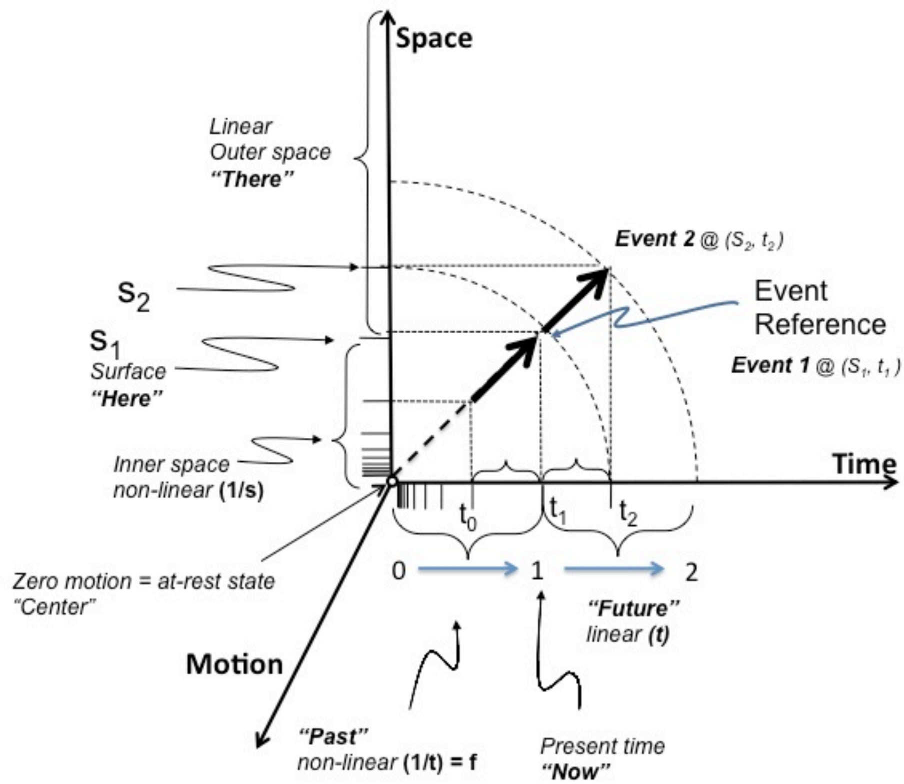

Figure 1 Two events plotted on the STM diagram. To the observer at the center, the spherical shell expands outward but the photon always sees itself as its own surface in the present time represented as the Event Reference.

Projection - Reflection

The Frequency Problem

In part 2, I addressed the scaling problem and showed how it is dealt with using a transform function – the Lorentz factor – as a correction. There is another problem with the STM model in Figure 1: a frequency problem. Since the inverse scale is interposed on the linear scale, as the clock ticks, two different sets of marks could be plotted on the axes: one that is projected *outward* (on a linear scale from 0 to 1 to 2…) and the other that moves *inward* as a reflection on a non-linear scale (at 1/t and 1/s), in fractionally smaller increments toward an infinitesimal point at the origin ($0 = \frac{1}{\infty}$ the singularity problem, one of the root causes for the problems listed by Smolin. It cannot use $t = 0$ either because $\frac{1}{0} = \infty$). Also, as time passed ($t = 1, 2, 3, \ldots$), the frequency would change ($f = 1,\ \frac{1}{2},\ \frac{1}{3}, \ldots$), which would not model a quantum particle. The problem is that the only point that actually represents the energy at the surface of the light shell is the boundary condition where $s = \frac{1}{s} = s^2 = 1$ and $t = \frac{1}{t} = t^2 = 1$.

The solution to the frequency problem is: use vectors for both regions and polar coordinates for the inner domain. This will solve the frequency problem and show that the scaling problem is actually an important part of the transformation process (an asymmetric reflection that shapes the holomorphic world to the Golden Ratio. But I'm getting ahead of myself.)

Solution to the Frequency Problem

First let's double-click the region between 0 and 1 into an icon, the tiny circle labeled "Zero motion" in Figure 1. We can write that into the DROPLET program code by simply integrating both regions so that

$$\text{One unit of space:}\quad s' = \int_1^s \frac{1}{s}\,ds = \ln(s) \rightarrow s = e^{s'} \tag{1}$$

$$\text{One unit of time:}\quad t' = \int_1^t \frac{1}{t}\,dt = \ln(t) \rightarrow t = e^{t'}. \tag{2}$$

So the icon would actually be two unit circles, superimposed as one. And since speed is a ratio of space over time, we have (dropping the prime marks)

$$s/t = e^s/e^t = e^{s-t}, \tag{3}$$

which provides a way of separating them. They can then be transformed into vectors by multiplying this by unity, $e^{2\pi i} = 1$ (Euler's identity) inserts the scale of 2π and the rotational component, i so that

$$s/t = e^{2\pi i(s-t)}. \tag{4}$$

Normalizing s and t (which just means scaling them to one unit: wavelength, λ, and period, T) with $k = \frac{2\pi}{\lambda}$ and $\omega = \frac{2\pi}{T}$, gives us a wave equation that represents a unit of motion

$$\mathcal{M} = e^{i(ks-\omega t)}. \tag{5}$$

Reflecting back on part 1 under Quantum Energy in the STM Diagram, where I used the inverse relation $f_t = \frac{1}{t} = c\frac{1}{\lambda}$, I can show the energy components E_s and E_t as eigenvalues of Equation (5). I simply replace ω with $\frac{2\pi}{T} = 2\pi\left(c\frac{1}{\lambda}\right)$ which is

$hc\frac{1}{\lambda} = E_s$ if $h = 2\pi$ (Planck's constant in natural units)[iv]. So the base component of the temporal axis $e^{\omega t} = e^{E_s t}$ and $\frac{d}{dt}(e^{E_s t}) = E_s e^{E_s t}$. The same can be done with k to get $\frac{d}{ds}(e^{E_t s}) = E_t e^{E_t s}$. In words, Equation (5) represents the integrated unit, the base vectors represent the unit projected onto the axes (so they would have the imaginary number i along to represent the domain, and then the second derivative would reflect them back to the quantum domain, but with a negative sign to indicate their new independence as distinct units.

We can click on the icon once and get a pop-up arrow pointing out of the screen, as in Figure 1. Or we can double click and open a polar-coordinates window to represent "inner space", as shown in Figure 2. Here, one increment on the clock is one cycle, which represents one event. This type of vector is called a phase vector a *phasor* (commonly used in electronic circuit theory), which the textbook explains as a "rotating vector" that represents a time-varying component, like the amplitude of a wave. The magnitude of a phasor is constant and the angle refers to phase with respect to a reference time.

The dashed arrow is the vector in the rectangular space-time plane.

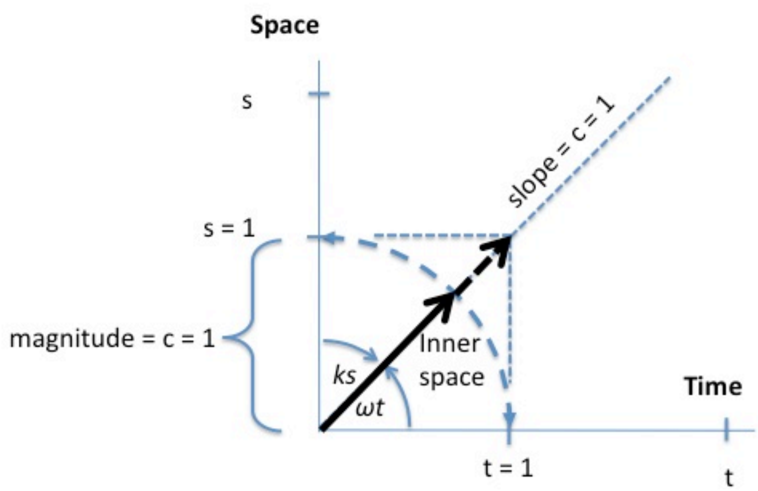

Figure 2 The magnitude of velocity (speed), represented by the slope of the diagonal line in the relativistic perspective, is represented in polar coordinates as a rotating vector called a Phasor, which also indicates outward direction at t=0.

In both coordinate systems the arrow ("arrow" refers to the "vector" in the rectangular coordinate system and the "phasor" in the polar system) represents the velocity of the expanding light sphere, with a magnitude of one unit of motion.

But there's still something wrong with this. The direction of the vector is *divergent* radially-outward, whereas the phasor rotates, so its "direction" is perpendicular to the direction of the arrow. For this reason, a phasor is not always considered to be a vector. Also, before we start a clock, both arrows should be positioned vertically on the Space axis to represent the *real* magnitude of motion outward in space. Then when we start a clock, the two models would split. They immediately become de-coherent. If we use the textbook procedures, the phasor would rotate to the left (increasing time) and the vector would immediately switch to the diagonal to indicate change with respect to time. Furthermore, only part of each arrow (the vector projection on the Space axis) would represent the real motion[v] (motion in space) so the two models would appear to represent different quantities.

Reintegration

This de-coherence of models can be avoided if we use *the event* as the reference rather than the coordinate frames. This means that rather than rotating the phasor or switching the vector, we can rotate the map as shown in Figure 3. By doing this we see that the two coordinate systems are 45° out of phase before the clock starts, and that the magnitude and direction of the phasor now corresponds to the vector (both are the solid arrow in the figure). Then when we start the clock, we use the phase-shifted *S-T* frame for relativistic motion and rotate the polar frame rather than the phasor. That way the magnitude and directions of both remain constant. The dashed vector just shows that a sliding vector can be moved to fit between events 1 and 2, but if we slide the divergent relativistic scale inward one unit with each event rather than sliding the vector, the vector and phasor become *reintegrated* as one icon and the *moving domains* represent motion instead of moving vectors. This solves the frequency problem and makes the scaling problem a non-issue, at least for the photon.

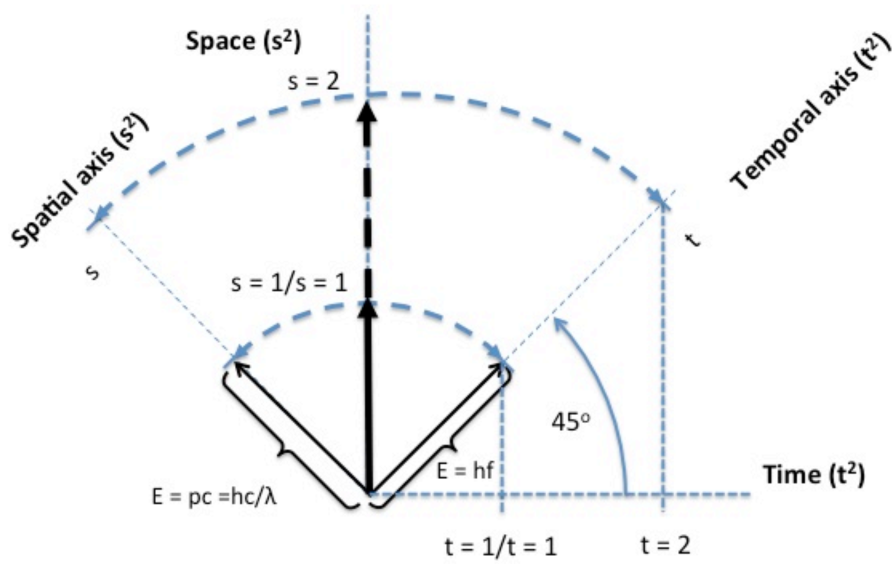

Figure 3 A phasor-type of vector models each event as one rotation of the phasor, solving the frequency problem. Rotating the polar coordinate system rather than the phasor and sliding the relativistic scale removes the scaling problem.

Unfortunately the observer at the center lives in the relativistic frame, so when the clock starts, his perspectives still separate. He can still use the polar wave function to represent motion, but if he measures one unit of space and one unit of time, and plots them on the scalar graph, he will get the un-phase-shifted vector and have to apply a Lorentz factor.

Reflection: Correspondence with tradition

This combination of coordinate systems will make it easier to interpret relationships when I discuss the transformation process, but physics evolved over the last century using traditional vectors in Hilbert space, so I have to show how I can arrive at the same equations. For example, consider the relativistic, "extra" energy discussed in part 1.

When we didn't move the coordinate systems and viewed the expanding sphere of light from the outside (as a quantum unit of energy represented by the magnitude of the vector $E = pc = mc^2$ as shown in Figure 4) we switched the vector to line up with the diagonal line. And according to the relativistic equation for total energy, $E_{Tot} = mc^2 + mc^2(\gamma - 1)$, it is a "Lorentz boosted" vector. The difference, shown in Figure 4 as v^2, looks just like, and could mistakenly be interpreted as the velocity, and thus kinetic energy, of a particle with respect to the background reference frame. A classical calculation of kinetic energy is $KE = \frac{1}{2}mv^2$, which is the area of the small shaded triangle (if the horizontal leg was scaled to represent a unit of mass, m). But this would not make sense because the photon in this example is an isolated, massless orb of light at rest with respect to itself. There is no relative velocity, except perhaps the velocity of the flash bulb at the center "moving" deeper into the photon's center. So it could represent that relative motion.

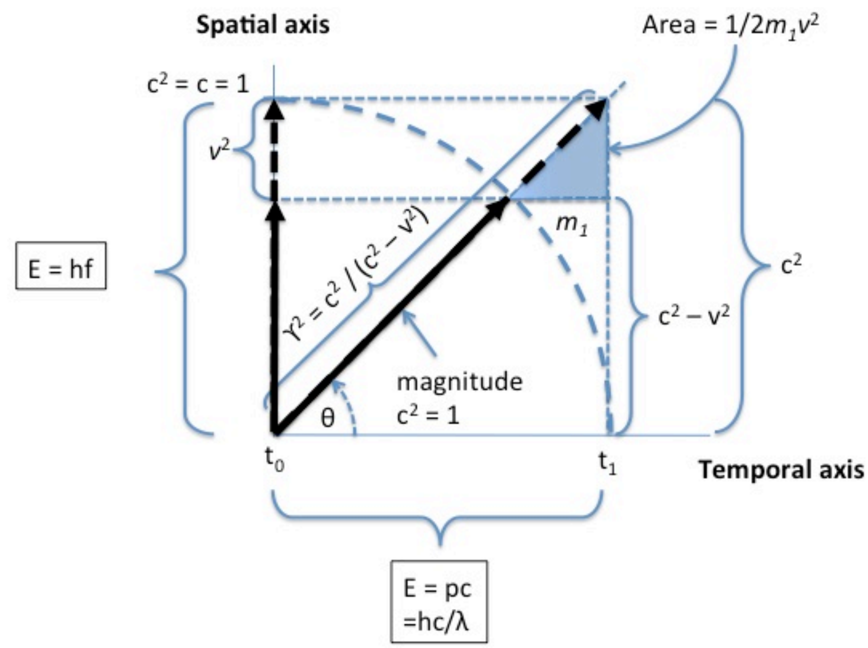

Figure 4 Lorentz magnification from part 1.

Alternatively we might say that the entire photon could *potentially* be a particle-antiparticle pair orbiting a central point and that the *extra* energy is the mass moving at a distance r from the center (reducing energy to a "thing"). Regardless of whether or not there really is mass in this massless orb, we can calculate a radius (if we froze the temporal component) or a momentum. But the STM diagram shows that these (position and momentum) are both measures of the spatial characteristic, only in different domains (position is scalar in the relativistic frame (spatial axis) and momentum is a vector in the quantum frame (inverse temporal axis)) related by Planck's constant[vi]. And since we are assuming that the total energy of the orb is contained within the mass, we could equate a unit of energy on the temporal axis ($E = \frac{hc}{\lambda}$) to a unit of mass-energy, ($E = mc^2$). To visualize this, imagine clicking on the shaded triangle as an icon so that it expands until it fits the vertical- and horizontal-scale units. We would get

$$mc^2 = \frac{hc}{\lambda} = \frac{hc}{r} \tag{6}$$

and, solving for $r = \frac{h}{mc}$, gives us the Compton wavelength times 2π, i.e. one cycle, and an angular momentum of

$$J = pr = (mc)\frac{h}{mc} = h, \tag{7}$$

which is the total angular momentum of an electron in ground state[vii] (also times 2π). We could then imagine double clicking on this large shaded triangle to hyperlink back to the rectangular domain and see the particle icon as a tiny sphere at a distance, r, from the center. Then click on *it* and get a pop-up of 3 mutually perpendicular axes that represent the total angular momentum, an orbital component and spin[viii]. And rather than *it* rotating around the original polar coordinate system, i.e. rather than orbiting, the coordinate system would rotate so that the particle maintains its location on both systems.

Now click on the icon at the origin of the S-T system. Since the polar coordinate system is scaled to λ as one complete rotation, then the energy of the particle $E = \frac{hc}{\lambda}$ would also be scaled by 2π. Figure 5 shows that there are 8 states in the cycle where the space-time relationships are the same. If that means that there are *potentially* 8 distinct units in one cycle of 2π, the "extra" energy would be the same magnitude for each but scaled by 2π. So each one would be at exactly one Compton wavelength, $r = \frac{1}{2\pi}\frac{h}{mc} = \frac{\hbar}{mc}$, (where, $\hbar = \frac{h}{2\pi}$), and have a potential angular momentum of $J = \frac{1}{2\pi}h = \hbar$, all pointed in different space-time directions. If there is one particle, it would potentially have 8 distinct states.

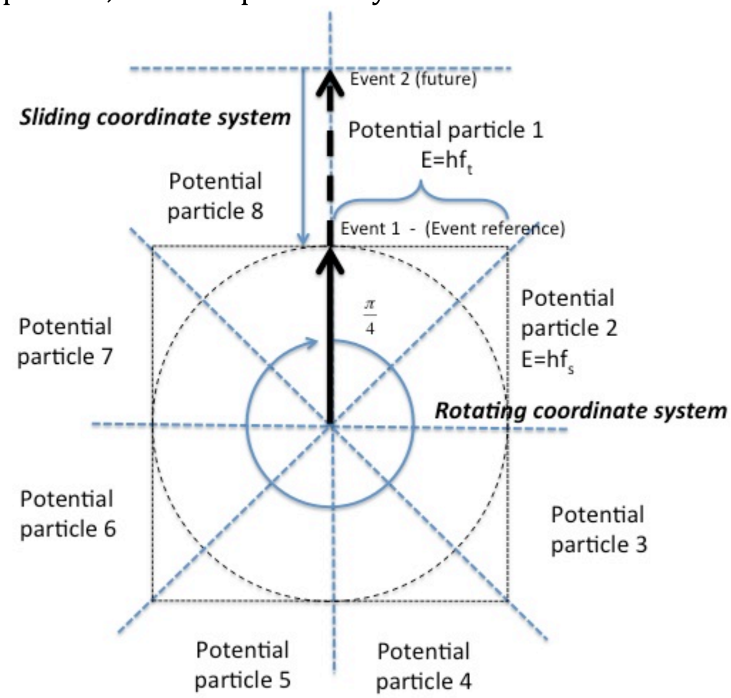

Figure 5 An Octet of potential particles, each having the same energy but different angular momentum (up, down or zero). The position in space relative to the center is constant, represented by the vector aligned with the space axis. The circle represents phase relations between 8 potential states.

So the STM model allows us to arrive at the same relations for energy, radial distance and angular momentum as we do with quantum mechanics. Note that there is no indication of *azimuthal* position in 3-D space, only their radial distance from the center to the potential shell. The orientation of these potential particles in the diagram is an indication of their relative *phase*, not position. The direction of each of the eight J vectors would be different, with magnitudes of 1. So each component vector (spin) would have a magnitude of $\frac{1}{\sqrt{2}}$ but pointing in different S-T "directions" for up spin, down spin, and zero (corresponding to a boson).

Interpretation

The fact that the actual position of a particle cannot be predicted is not a weakness in this model or quantum mechanics, but a characteristic of reality. Interpreting this characteristic is where the STM model differs, and hopefully improves on the interpretation of quantum physics. In the Copenhagen interpretation, we are told to consider the mysterious wave function ψ, whose amplitude is ϕ, as a *probability* amplitude. In other words, it is assumed that there already exists a position in space and time where a particle will arrive, guided by this probability wave, and that it can be measured when and where that happens. But you can only estimate where the particle is based on the probability wave. And according to the Heisenberg Uncertainty Principle, once you measure the position, you lose all hope of knowing its momentum, since the measurement changes the velocity of the particle.

But the STM interpretation does not assume that a "position" even exists. Instead, it assumes nothing, i.e. nothingness (darkness) is transformed into light through a continuous process, creating a "new" scalar position in space-time with each event. So the wave function represents the scale itself. Position and momentum are two different ways of expressing this spatial aspect, so an interaction is a transformation of temporal frequency into a spatial frequency resulting in the bit-wise expansion of the universe. It is tempting to call this a continuous "creation" process, but it is a transformation rather than a creation so energy is conserved. And this transformation is modeled by equations in relativistic quantum mechanics. In the next section, I will use the STM model to illustrate some of the most important relationships: the appropriate wave function, the Klein-Gordon equation, the Schrodinger equation and quantum operators.

The Wave Function

Equation (5) is a wave function that integrates the two scale units, *s* and *t*, but it does not solve Schrodinger's equation unless it is squared. However, it does solve the Klein-Gordon equation if the dispersion relation, $k^2 + \omega^2 = \frac{m^2}{\hbar^2}$ holds[ix]. The reason for this can be seen with the STM model by first seeing where the wave equation comes from. A classical wave equation is an important second-order partial differential equation, given by

$$\frac{\partial^2 \phi}{\partial s^2} = \frac{1}{v^2}\frac{\partial^2 \phi}{\partial t^2}. \tag{8}$$

It is a fairly simple equation that hints at the equivalence of space and time. And it is very easy to find it using the STM diagram. First, the magnitude of the spatial component of the energy vector, call it $\hat{s}$, is equal to the slope and therefore the *first derivative* of the equation for the diagonal line in rectangular coordinates, call it ϕ.

$$|\hat{s}| = \frac{\partial\phi}{\partial t} \tag{9}$$

Then, if you switch the axes, (the basis of the partial derivative) the magnitude of the temporal component of ϕ is also the slope of the same diagonal line – the first derivative with respect to s. So

$$|\hat{t}| = \frac{\partial\phi}{\partial s}. \tag{10}$$

Notice that the magnitude of the vector symbol represents an operation – the derivative. And the derivative is the tangent, which means the vector points in a direction tangent or perpendicular to the S-T plane. So the motion vector, with its base at $M = 0$, points in a direction perpendicular to the base of the phasor. This is shown in Figure 6 as $\hat{\boldsymbol{v}}$ pointing out of the page.

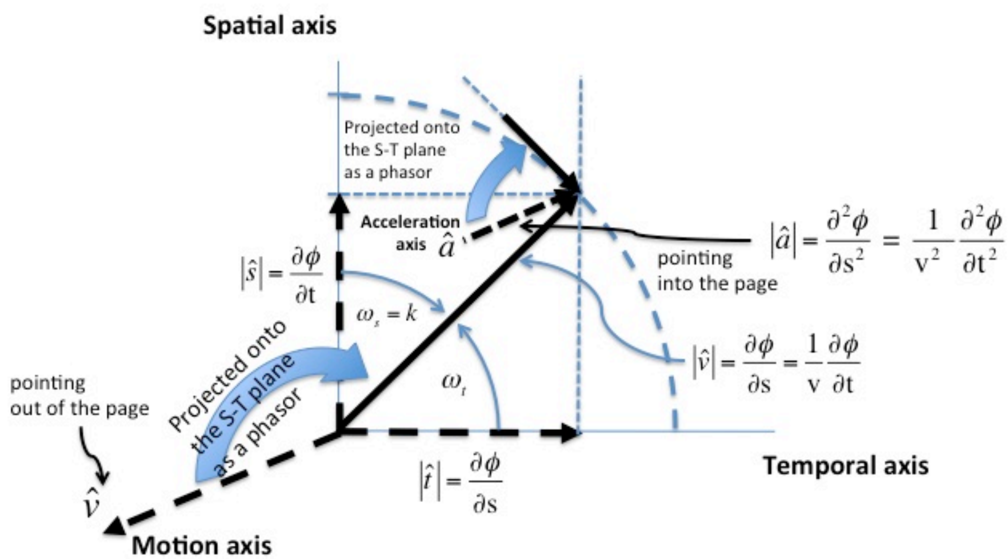

Figure 6 Vectors symbolize the derivative or slope of the scalar plot. The motion vector is shown perpendicular to the S-T plane (pointing out of the page) with *M=0* at the origin of the plot. Acceleration is also perpendicular to the S-T plane, and perpendicular to the motion phasor.

Click on M and it is projected back onto the S-T plane in polar coordinates as a phasor that is described by two complementary and equal angles (reflections of each other), one is measured with respect to the temporal axis and the other with respect to the spatial axis, giving it the initial 45 degree phase shift. As long as $v = 1$, the phasor stays oriented in line with the diagonal on the S-T plane, to represent the relationship of the two derivatives, related by $|\hat{s}| = v|\hat{t}|$. Inserting Equations (9) and (10) into this relation we get

$$\frac{\partial\phi}{\partial t} = v\frac{\partial\phi}{\partial s} \quad \text{or} \quad \frac{\partial\phi}{\partial s} = \frac{1}{v}\frac{\partial\phi}{\partial t} \tag{11}$$

A first-order differential equation such as this is *not* normally considered a wave equation (which is a second-order differential equation). But it is said to have "characteristics" that remain constant as the function progresses in time (Haberman, 1983, pp. 417-421). It has a general solution of the form $w(s,t) = P(s - vt)$ so at any value of t, the solution is the same shape, shifted a distance vt. In fluid mechanics, this is called an advective transport equation, which describes the transport of a substance such as a fluid, by bulk motion. The conserved properties of the substance, such as energy, are carried with it[x]. That is exactly what a photon is – the bulk motion of energy. In field theory, if we associate a direction with ϕ, in this case outward, Equation (11) is a continuity equation – the equation for a velocity vector field that governs the motion of a scalar field.

$$\frac{\partial\phi}{\partial t} = v\frac{\partial\phi}{\partial s} = \nabla\cdot v\boldsymbol{\phi} \tag{12}$$

Equation (11) may not be a standard wave equation, but it can be solved by a a wave function, that is Euler's formula, $e^{i\theta} = \cos(\theta) + isin(\theta)$, where θ is the angular position of the phasor in Figure 6. So $\theta = 2\pi f_s s + 2\pi f_t t = \omega_s s + \omega_t t$ or more commonly written as $\theta = ks + \omega t$. So Euler's formula, is the same as Equation (5)

$$\psi = e^{i(ks+\omega t)}, \tag{13}$$

and this solves Equation (11) since $\frac{\partial\psi}{\partial t} = \frac{\partial e^{i(ks+\omega t)}}{\partial t} = i\omega e^{i(ks+\omega t)}$ and $\frac{\partial\psi}{\partial s} = \frac{\partial e^{i(ks+\omega t)}}{\partial s} = ike^{i(ks+\omega t)}$. Therefore $i\omega\psi = v(ik\psi)$ if $v = \frac{\omega}{k} = \frac{2\pi f_t}{2\pi f_s} = \frac{1/t}{1/s} = \frac{s}{t} = v$.

Notice that $\phi = e^{i(ks)}$ represents the phasor at an angle of *ks* from the spatial axis, and $\phi = e^{i(\omega t)}$ represents exactly the same thing only referenced to the temporal axis. And the product of the two is $|\phi|^2 = e^{i(ks)}e^{i(\omega t)} = e^{i(ks+\omega t)} = \psi$. In this position, the phasor is in line with the vector, so once again it represents the integrated unit of energy. This is a graphical representation of Parseval's Theorem, which is used in electronic communications systems, for example, to calculate the energy delivered to a resistor by an electrical signal by integrating the square of the amplitude of the signal (Stremler, p. 85).

If you get ahead of me and try to use Equation (13) to solve Schrodinger's equation, you will find that it has to be squared again. I will explain why after presenting the Klein-Gordon equation below.

Both forms of ϕ (one in spatial units and the other in temporal units) are wave functions in the sense that they model cyclical characteristics[xi] and they are eigenfunctions, i.e. scaling functions that can be operated on by, say a product, derivative or integral and get a result that is the same function on a different scale (multiplied by an eigenvalue) or translated by a constant of integration.

This analysis must be extended to the second derivatives to get a classic wave equation. Since the second derivative of space with respect to time is acceleration and is orthogonal to space, time and motion (v), it is shown in Figure 6 as being perpendicular to the *S-T* plane *and* perpendicular to the phasor. It is parallel to the motion axis, but it points into the page, and is shifted to the tip of the diagonal vector (since the first derivative is the integral of the second, so the shift is the constant of integration). This, I submit, is the real meaning of parallel dimensions, unlike the notions presented in science fiction. Acceleration presents as a reflection of motion off of the *S-T* plane.

Click on the $\hat{a}$ axis to make it an icon. Then double click and the acceleration phasor projects onto the *S-T* plane at an angle, call it θ, written as either ${f_t}^a t$ or ${f_s}^a s$. Now we have to be careful since its frequency is different than the frequency associated with velocity, so time, t must treated as if were different from time in the first derivative (separation of variables). If it wasn't, then the time component would cancel, i.e. $f_t t = \frac{1}{t} t = 1$. Instead,

$$\frac{\partial^2 \theta}{\partial t^2} = \frac{\partial^2 (f_t^a)}{\partial t^2} = \frac{\partial f_t^a}{\partial t}\frac{\partial t}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{t}\right) = -\frac{1}{t^2}. \tag{14}$$

Similarly

$$\frac{\partial^2 \theta}{\partial s^2} = \frac{\partial^2 (f_s^a)}{\partial s^2} = \frac{\partial}{\partial s}\left(\frac{1}{s}\right) = -\frac{1}{s^2}. \tag{15}$$

Therefore

$$\frac{\partial^2 \theta}{\partial t^2}\left(\frac{t^2}{s^2}\right) = -\frac{1}{t^2}\left(\frac{t^2}{s^2}\right) = -\frac{1}{s^2} = \frac{\partial^2 \theta}{\partial s^2},$$

and

$$\frac{\partial^2 \theta}{\partial s^2} = \frac{t^2}{s^2}\frac{\partial^2 \theta}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 \theta}{\partial t^2} \tag{16}$$

which is the classical wave equation. It is also solved by ψ in Equation (13). Recall that the second derivative of space with respect to time (acceleration) is generated by a change in velocity and it, in reflection, generates an opposite change in velocity. It doesn't even exist until some external interaction attempts to change the existing pattern of energy (motion) we called a photon. It is simply a resistance to change, thus presents as a force.

An important take-away from this discussion is that this is a morphic process. Graphically, the first derivative transforms the motion icon into a phasor and projects it onto rectangular coordinates and the second derivative transforms it back to an icon, shifted out to where the phasor was projected. Its phasor is then projected again, by another derivative (another change) back onto the *S-T* plane as a centripetal component and a tangential component. Thus, any change in frequency, which is a change in velocity, will result in a non-zero acceleration that opposes the change (a characteristic that presents as angular momentum and inertia) giving the particle its form. So the morph is a two-step process: 1) projection dis-integrates the quantum (advection) and the reflection allows it to re-integrate (wave equation) and bring *closure* to the process (dispersion relation). Next I'll show that the spatial component and a temporal component of angular momentum show up re-integrated with the classic wave equation as the relativistic part of the *relativistic-quantum wave equation* known as Klein-Gordon equation.

The Klein-Gordon Equation

If we take the same approach as above and assume that the mass of a potential particle is located at a distance $r = \frac{\hbar}{mc}$ from the center, we can represent it on the STM diagram as shown in Figure 7.

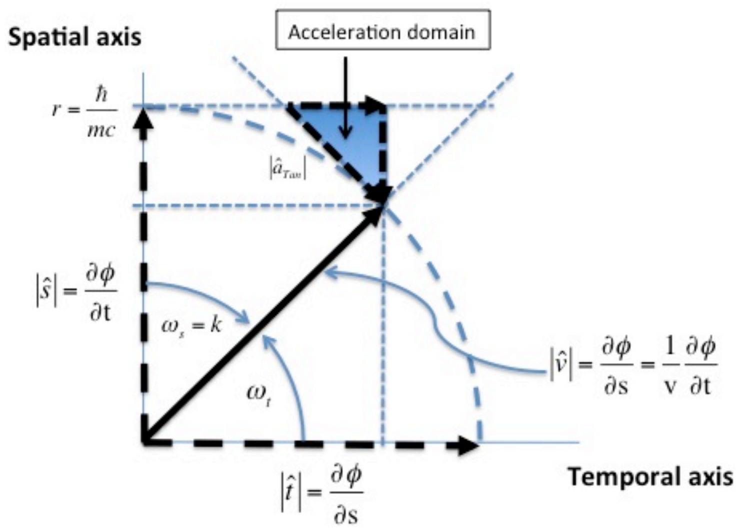

Figure 7 Acceleration domain representing a potential particle projected at the event reference

Now, we click on the acceleration domain so that the velocity vector collapses to an icon at the origin and the acceleration vector expands to use the inverse scale $(1/r)$ as shown in Figure 8.

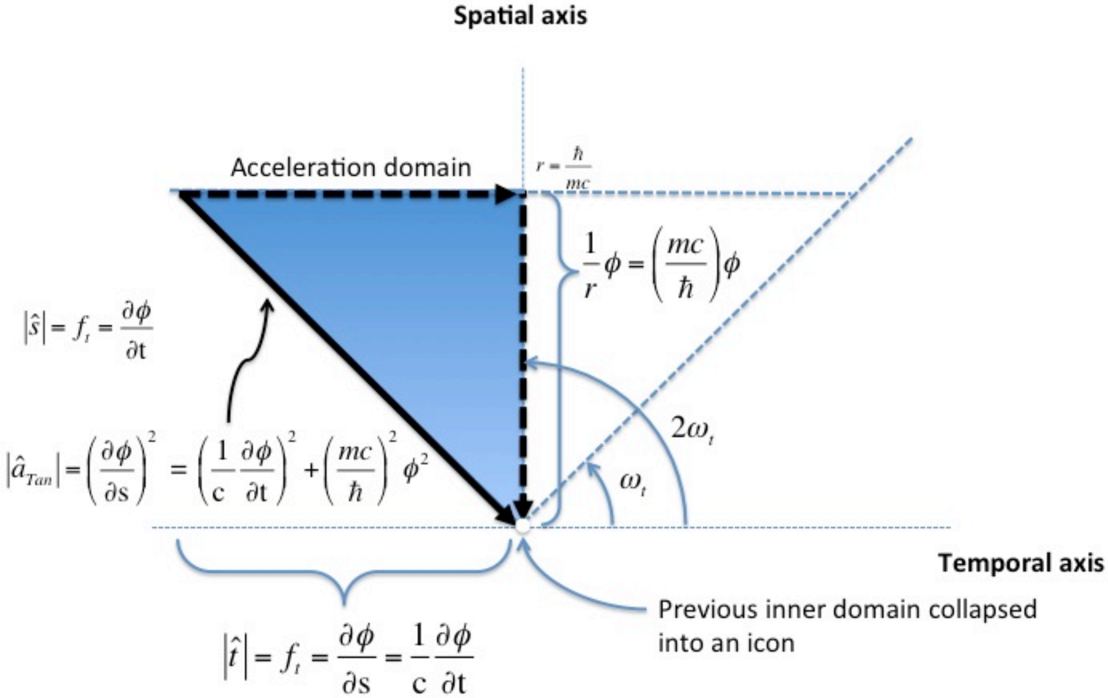

Figure 8 Click on the small shaded triangle in Figure 7 to expand the acceleration domain to reflect the scale of the *quantum* domain

We can see the phasor that represents this potential particle is the *hypotenuse* of the shaded triangle, whose magnitude is

$$\frac{1}{v^2}\left(\frac{\partial\phi}{\partial t}\right)^2 = \left(\frac{\partial\phi}{\partial s}\right)^2 + \left(\frac{mc}{\hbar}\right)^2\phi^2. \tag{17}$$

Notice that $\frac{1}{r}$ is multiplied by ϕ to account for the collapsed velocity domain (the icon). Equation (17) looks striking similar to the Klein-Gordon (KG) equation,

$$\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = \frac{\partial^2\phi}{\partial s^2} - \left(\frac{mc}{\hbar}\right)^2\phi^2, \tag{18}$$

except that it contains the square of first derivatives where KG has second derivatives and the negative sign, which should have been there anyway if these are vectors since acceleration opposes the change in velocity. This is actually a clue to some very special and important relationships that are responsible for certain patterns that show up in nature.

Transformation of domain

As I pointed out in the discussion for Figure 1 and Figure 4, the scale of the polar domain is half the size of the *S*-*T* domain, because the second increment on the inverse scale is at $\frac{1}{s} = \frac{1}{2}$ and $\frac{1}{t} = \frac{1}{2}$. This is a result of the scaling problem discussed in part 1 – the fact that speed, the magnitude of the velocity vector, is scaled by the measured values of space denominated by a standard unit of time.

Effectively, scaling energy (s^2), in the form of motion to match the measured values splits it in half, which is why $KE \propto \frac{1}{2}v^2$. A measurement of displacement (Δs) quantifies a unit of space as s, and therefore *sets the scale* for the unit s, which is half the scale of speed $\frac{d}{ds}(s^2) = 2s$. This relation is important because, just like the special condition defining the event reference where $s^2 = s = \frac{1}{s} = 1$, this is the only condition in which $s^2 = s + s = 2s$, i.e. *iff* $s = 2 \; or \; \frac{1}{2}$, meaning the *quantum scale* (s_q) is twice the size of the *relativistic scale* (s_r) (think about map scales: larger scale means smaller icons so it takes two quantum units to make one relativistic unit). In other words, s^2 represents space in the quantum domain (what I called S in Part 1) and $2s$ represents the same size unit in the relativistic domain. They are equivalent yet different, so we could say $s^2 \approx 2s$. But the difference in the domains is important and this doesn't help us reveal the important difference. Instead we could write the quantum scale (call it s_q) in the integrated form, $\phi = e^{s_q}$, then the derivatives just compare the magnitude of the quantum scale to itself, i.e. $1 = 1$ and doesn't affect the phasor $\phi = \frac{\partial \phi}{\partial s} = \frac{\partial^2 \phi}{\partial s^2} = e^{s_q}$. But the *square* of this function or the *square* of its derivative transforms the scale from the quantum domain (s_q) to the relativistic domain (s_r)

$$\left(\frac{\partial \phi}{\partial s}\right)^2 = e^{2s_q} = e^{s_r}. \tag{19}$$

$\phi = e^{s_r}$ is the domain of the relativistic scale and $\phi = e^{s_q}$ is the domain of the quantum scale. But we don't distinguish the scales or the domains in a wave equation. We use the same symbol for time in both domains and for space in both domains, which makes this a special case where the second derivative is equal to the square of the first derivative and Equation (17) can be written as

$$\frac{1}{v^2}\frac{\partial^2 \phi}{\partial t^2} = \frac{\partial^2 \phi}{\partial s^2} - \left(\frac{mc}{\hbar}\right)^2 \phi^2, \tag{20}$$

which is the Klein-Gordon equation[xii] with $c = v$.

Therefore, rather than being a probability amplitude, the symbol ϕ, *that mysterious quantum wave function*, represents the integral form of the domain itself, whose scale is transformed by "double clicking on the icon" (double differentiating) changing perspectives to another domain.

If we interpret Equation (20) as representing mass, the separation of domains comes across as representing two separate particles. In the term, $\left(\frac{mc}{\hbar}\right)^2 \phi^2$, the amplitude, ϕ, is squared. According to Parseval's Theorem (Stremler, p. 85), ϕ^2 represents the energy so this potential mass, scaled by $\left(\frac{mc}{\hbar}\right)^2$, can be written as $\frac{p^2}{\hbar^2}$.

This is twice the energy of a free particle, $E_{FP} = \frac{p^2}{2m}$ and this suggests that it represents 2 (dispersed) potential units of mass scaled by $\hbar$, each with energy E_{FP} also scaled by $\hbar$:

$$\left(\frac{mc}{\hbar}\right)^2 = \left(\frac{2m}{\hbar}\right)\frac{E_{FP}}{\hbar}. \tag{21}$$

Squaring ϕ also means that the angle to this component (ωt) with respect to the S-T frame is doubled. So it is the same function only rotated to indicate that the potential particle wave functions are phase-shifted, giving them a distinction as separate entities. Thus, the particle model works. It is also the reason that Equation (13) had to be squared in order to solve the free particle Schrodinger's equation,

$$\frac{\hbar^2}{2m}\frac{\partial^2(\Psi)}{\partial^2 s} + i\hbar\frac{\partial(\Psi)}{\partial t} = 0. \tag{22}$$

This is because Schrodinger's equation is lopsided – it is partially evaluated in that one temporal derivative has already been performed. Therefore, it already has the factor of 2 that comes from the split units. In the Klein-Gordon form, the "2" would cancel when all of the derivatives are evaluated, so it is not needed.

The Schrodinger equation can be found in the STM diagram as well, but since it contains a mixture of domains in a single equation it is a bit more complicated. So in the interest of flow, I have moved it to the endnotes.[xiii]

Even though the particle model works and has provided tremendous growth in physics and chemistry, it is wrong to conclude that the energy ***is*** a particle. Instead, we should say the energy can be perceived as the particulate form. The square of the wave function, ϕ^2, is simply an expression that describes formless energy that *has the potential to express* particulate *form* – as a quantum unit in polar coordinates, and the potential to express relative motion and acceleration of that form when projected onto the S -T plane and reflected back. Rather than reducing the particle to numerous sub-particles, I reduced the process to a few steps, each modeled by mathematical operations. The first derivative operation models separation of energy from the quantum domain and projection onto relative scales. The second derivative models separation from the translated quantum domain and reflection back onto the motion dimension with the translation expressed as a phase shift in polar coordinate system. So in essence, it has the potential of being perceived as a quantum unit having relative velocity (moving by advection), and thus kinetic energy, when it is expressed in rectangular coordinates (projected back onto a scale of 3D space) by some kind of interaction.

"Formation" is a transformation of energy from formless to form – a new level of complexity. How this transformation happens is the subject of part 3. As a preview, I will introduce another important number.

The Golden Ratio

Recall that the STM diagram, with motion vectors projected forward and back from the event reference as phasors, is an energy diagram, so it is a vector space. And the length of each of the two phasors is half of what it was when the origin was used for a reference. This splitting of the scale in half at the event reference has a profound effect on our perception of energy. It creates a special geometric relationship that has only one solution, the Golden Ratio, which shows up in natural patterns, like flowers, seashells, trees and even the shape of spiral galaxies[xiv]. See Figure 9. This is a copy of Figure 1 showing the inverse domain $\left(\frac{1}{\mathrm{t}}\right)$ between the origin and t_0. Since a unit of clock time is fixed and used in the vector speed[xv] domain, the temporal scale is held constant (labeled Δ1). Therefore, the scale that applies to speed in the quantum domain (the magnitude of each speed vector) would have to be "stretched" back out to account for the inverse relation, i.e. the sum of the first increment in the linear domain and the inverse domain,

$$1 + \frac{1}{\mathrm{t}} = \mathrm{t}. \tag{23}$$

This is well known to be the Golden Ratio Φ.

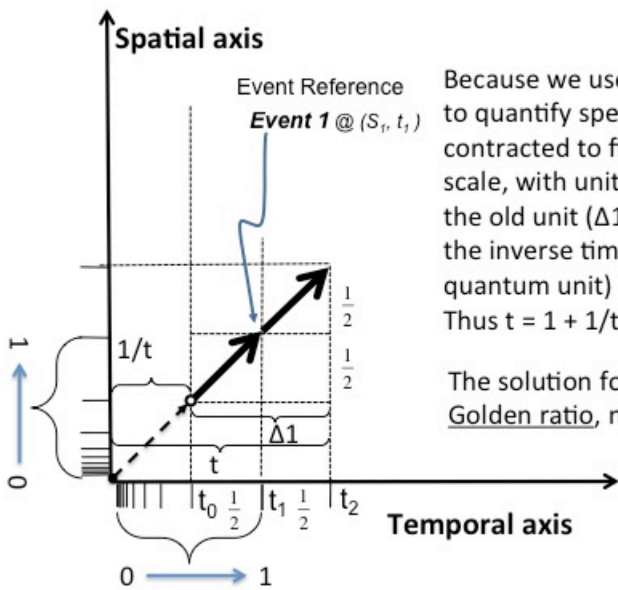

Figure 9 Using the same scale to quantify motion has the effect of splitting the unit in half. Rescaling the vector back to the clock scale changes it by an amount that is the Golden Ratio.

It may be hard to see this stretch in Figure 9 so we click on the event reference (tip of the first vector) and bring it into the quantum domain leaving the scale (½) on the right as shown in Figure 10. Then we determine the relation between the first increment (the quantum domain labeled "*a*") and second increment (relativistic domain, *a* + *b*) of the temporal scale. By placing a mark on the second vector using the spatial scale of ½ (drawing an arc of radius ½) and then rotating that onto the temporal axis, we get the stretched quantum scale unit (a). It is related to the relativistic scale (b) by the Golden Ratio in the form[xvi]

$$\Phi = \frac{a}{b} = \frac{a+b}{a}. \tag{24}$$

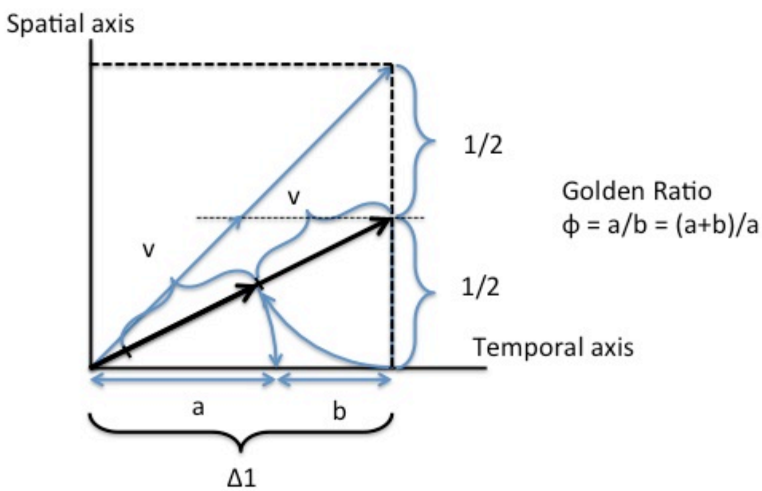

Figure 10 A standard procedure for dividing a line segment by the Golden ratio exactly like this is shown in https://en.wikipedia.org/wiki/Golden_ratio.

Thus, if we double click on the first vector and collapse it to an icon so a is $\left(\frac{1}{\mathrm{t}}\right)$, the value of b is set to 1 unit, and $\frac{1}{\mathrm{t}} + 1 = t = \Delta 1$ unit.

In Part 4 of this paper, I will show that this important relationship is responsible for contracting the spatial scale and transforming formless energy into form. I will also show how we get closure from multiple layers of simultaneous transformations, each of which apply the golden ratio.

Conclusion

The goal of this part of the paper was to further develop the STM model by integrating the polar coordinate system into the rectangular model and showing how operations such as the derivative and integrals have the effect of transforming our perspective from one system and scale to the other and back. Doing so allowed me to demonstrate how various important relationships can be gleaned from the model, such as the classical wave equation, the Klein-Gordon equation, the basic quantum wave function and the Golden Ratio. Reflecting on these well-known relationships will help us accept the change in the highly ingrained perspective on the meaning of time.

There are actually four different perspectives that the STM model allows us to visualize. 1) Clicking on the background, divergent domain inside the event reference, we can visualize the sphere of light from the light bulb's perspective projecting outward, and 2) clicking on the polar coordinate domain allows us to imagine an orb of constant radius but spinning in time with space projecting outward and collapsing inward with each event.

We can also imagine space or time as we normally do as the measurable and clock-able units. 3) Click a unit on the time axis (a measurement at a moment in time) and the spatial axis unfolds into a 3-D sphere. We see a motionless orb from the outside looking inward. We can only imagine this because time is not something we can actually be stopped. This is a sphere "*whose circumference is nowhere*."[xvii] 4) On the other hand, if we click on the space unit (a position in space), it also unfolds into 3-D space. But now we see the timeless universe from a point "*whose center is everywhere*" looking outward. Again, we can only imagine this because we cannot actually define a point in space.

Taken together, it is tempting to visualize the process like a sewing machine, with the linear vector as a needle popping a string in and out, and the polar system as the rotating bobbin, "lock stitching" together the fabric of reality. But this analogy is not exact either, because the fabric is 3-dimensional so each stitch is a spherical envelope, spun into a layered spiral web with proportions that reflect the special relationships like the Golden Ratio and scaling operations that draw space inward and project time forward.

So the model is not yet complete. So far, we have only considered a massless orb of light. In Part 4, I will find closure by integrating the outgoing with an incoming wave to show how this spherical standing wave transforms into a pair of holomorphic quanta.

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[i] Footnotes and endnotes represent another layer of information that is projected inward from the body of the text for more in-depth reflection.

[ii] In Geometric algebra, this is the geometric product of the two vectors, $st = \frac{1}{2}(s\hat{t} + t\hat{s}) + \frac{1}{2}(s\hat{t} - t\hat{s})$ with the second term (the "outer product") flipped outward. The hats are used here to distinguish which vectors are being used as bases.
[iii] For the next measurement, the first arrow would be shifted inward and contracted to fit between $\frac{1}{2}$ and $\frac{1}{3}$, creating the apparent curvature of space-time. If it were conscious, the photon would experience a psychological flow of time, yet it would see itself as just another stationary particle. Energy, which was *perceived* as being separated as space and time, is thus reintegrated as a whole.
[iv] This means that Planck's constant is simply 2π. It only becomes its special value (6.626070040(81)×10−34 Jsec) after converting space and time to their scalar values. $E = mc^2 = hf$ so $h = mc^2 t$ with units $kg({}^{m}/_{sec})^2 sec = Joules \times sec$
[v] It would then be a complex number with a real part to represent space and an imaginary part to represent time.
[vi] This is a statement of Heisenberg's uncertainty principle $\Delta x \Delta p = \frac{\hbar}{2} = \frac{h}{4\pi}$. 4π represents 2 cycles, one for each Δx and Δp.
[vii] Angular momentum of an electron is $\sqrt{l(l+1)}\frac{h}{2\pi}$
[viii] This is done in "the vector model of angular spin." (Goswami, 1992, pp. 220-221) See also https://en.wikipedia.org/wiki/Vector_model_of_the_atom
[ix] In fact Klein-Gordon equation is derived from this form of wave function. See https://en.wikipedia.org/wiki/Klein%E2%80%93Gordon_equation
[x] https://en.wikipedia.org/wiki/Advection
[xi] ψ would be an equation for a plane wave if s represented one dimension in space, but since s is the radius of a sphere, the equation represents an expanding spherical wave.
[xii] According to http://www.phy.ohiou.edu/~elster/lectures/advqm_2.pdf and https://en.wikipedia.org/wiki/Klein%E2%80%93Gordon_equation this equation is the quantized version of the relativistic energy-momentum relation and cannot be interpreted as a probability amplitude.
[xiii] I only include the Schrodinger equation because it, rather than the Klein-Gordon equation, is the one that is covered by introductory QM textbooks (Goswami, 1992) (Morrison, 1990) (Liboff, 1993). Quantum mechanics was invented because elementary particles were found to exhibit wave-like characteristics via the double-slit experiment. And Erwin Schrodinger found a way to express a particle in terms of a wave function by "creatively intuiting" what became known as the Schrodinger wave equation.

$$\frac{\hbar^2}{2m}\frac{\partial^2(\Psi)}{\partial^2 s} + i\hbar\frac{\partial(\Psi)}{\partial t} = \mathrm{V(r)}\Psi. \qquad \text{(EN-6)}$$

It is a textbook exercise (Morrison, 1990, p. 48) worked out in (St. John, 2014) to show that you can arrive at the Schrodinger wave equation (without including a potential field, $V(r) = 0$) from the classical wave equation

$$\frac{\partial^2(\Psi)}{\partial^2 t} = v^2 \frac{\partial^2(\Psi)}{\partial^2 s} \qquad \text{(EN-7)}$$

by squaring the wave equation, $\Psi = |\psi|^2 = e^{2i(ks+\omega t)}$, taking the first derivative with respect to time and using de Broglie relations to replace $\frac{\omega}{v^2}$ with $\frac{m}{\hbar}$. We can find the Schrodinger equation in the STM diagram by rearranging Equation (EN-6) and writing it as

$$\frac{\partial^2(\Psi)}{\partial^2 s} = -2i\frac{mc}{\hbar}\left(\frac{1}{c}\frac{\partial(\Psi)}{\partial t}\right). \qquad \text{(EN-8)}$$

Squaring the function, ψ, was necessary because it hyperlinked the function to the perpendicular map. And as long as ωt is negative, then $ks + \omega t = 0$, so doubling the argument in the exponent still represents the phasor at 45 degrees or ϕ. The first term in Equation (EN-8) is the acceleration vector $\hat{a}_T$ in Figure 6, which is on the acceleration map scale. Then from the advective transport equation (Equation (11)) the term in parentheses $\left(\frac{1}{c}\frac{\partial\phi}{\partial t}\right)$, is shown in Figure 6 as the magnitude of the diagonal phasor. It is scaled by the inverse Compton wavelength, $\left(\frac{mc}{\hbar}\right)$, just as the Klein-Gordon equation, and by $2i$, which is the eigenvalue of the "missing derivative" in the Schrodinger equation.

$$\frac{\partial}{\partial t}e^{2i(ks+\omega t)} = 2i\omega e^{2i(ks+\omega t)} = 2i(\omega\Psi) \qquad \text{(EN-9)}$$

The value of ω is 1 and it serves here as a unit vector to represent the acceleration map. So this term is one of the legs of the small shaded triangle, which represents energy. The other leg is $\frac{1}{r}\Psi = \mathrm{V(r)}\Psi$. So the Schrodinger equation is a mixture of functions from different maps, which makes it more difficult to interpret.

[xiv] See https://io9.gizmodo.com/5985588/15-uncanny-examples-of-the-golden-ratio-in-nature

[xv] Notice that I use the term "vector speed" domain rather than velocity domain because I am referring specifically to the scale.

[xvi] This procedure is used as an illustration in https://en.wikipedia.org/wiki/Golden_ratio and it applies here only because the vertical leg of the triangle is ½ the length of the horizontal leg.

[xvii] Blaise Pascal once said, "God is an infinite sphere whose center is everywhere and circumference is nowhere."