The shadow of the smile of the "Cheshire cat"[1)]

(*topological aspects*)

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According to [1], let us consider a pair of photons in the "entanglement" state, moving in opposite directions. Each of the photons is in a state with circular polarization. According to the method of "preparation" of the wave function of a two-particle system, it is clear that each photon can have a full set of quantum numbers, including polarization. Let's call a photon a "Cheshire cat" and its linear polarization (one of two possible) as a "smile". Thus, we have two cats, each of which may have its own expression (smile or vice versa), that we can see as a result of measurement (in the photon version).

The essence of the results of the experiment A. Aspect is the experimental establishment of a close correlation between the smiles of two cats (with the appropriate orientation of the analyzers in the original formulation of the problem). With an unknown facial expression of the first cat's facial expression second cat is also unknown. The situation changes dramatically if we pay attention to the neighbor cat: if we see a smiling cat near us, the expression of the second cat automatically becomes smiling and vice versa, if we are looking at a disgruntled cat, the second cat will also get angry. The situation is that our view (measurement of photon polarization in the near) triggers the message faces the distant cat (certainty of polarization of the distant photon). Is it not "forwarding" intangible qualities through the distance (and even unknown way)?

So, the *Cheshire cat* and his *smile*. It is clear that in a normal physical language we are talking about the object and some of its inherent property, which is not separable from the object.

Therefore, if the object changes, the property must "follow" it. In other words, we need to be sure that when an object is changed, its integrity is preserved: that is, in the process of change – before and after the change, we are dealing with the same object. Let's start with the formulation of the physical formulation of the problem of the relationship (integrity) of the object and any of its properties. The result of the decision should be something to motivate the transfer of properties at a distance ...

Preliminary comments. We will talk about preserving the integrity of the changing system as a *genetic identity* of the system in the process of change. The genetic identity of the physical system is fixed with the help of the state function and the equations of evolution of this state as a way to describe the change of the integral system

The role of the state functions play in classical mechanics the Lagrangian L, the Hamiltonian H or the action S. The solution of the equations of evolution for systems described by the above functions for dot physical systems (equations of motion) yield results in the form of continuous functions of the coordinates and functions of time describing the trajectory, since the functions L,H,S contains only the coordinates, their time derivatives and the associated parameters. In this case, the evolution of the point system will be reduced to a change in the position of the point in space, and the ultimate goal of solutions of the evolution equations (for the point system – dynamic equations) is to determine the coordinates of the point as functions of time. The genetic identity of the moving point itself is determined by the continuity of movement. It is the continuity of mechanical motion that allows us to "shift" the very

1) **I beg your pardon for my not very good English!** The original text on Russian: http://vixra.org/pdf/1804.0401v1.pdf

concept of the genetic identity of a point object into a topological hypostasis, that is, to track the genetic identity of a point object in mechanics by its continuous motion. This is what we see in everyday life, that is, at the macro level of physical description.

Thus, *in the topology of classical mechanics, the genetic identity of the object is given by the continuity of motion*. This topology will be called point-metric classical topology (*PMC-topology*).

However, point systems and their structures can be endowed with such characteristics as charge, impulse, shape, "color" and other "smiles", including quantum mechanical, which gives our imagination the illusion that for all systems it is possible to establish a genetic identity by means of continuous motion, described in *PMC-topology*. These illusions disappear when considering the problems of quantum mechanics.

First. In quantum mechanics there are no point prototypes for the application of the *PMC-topology* concept.

Second. In the quantum theory, the wave functions or vectors of the linear Hilbert space play the role of the functions of the state of the quantum system; the role of the evolution equations are the wave equations.

If the functions of state in classical mechanics - Lagrangian and Hamiltonian-have quite clear meaning associated with such measurable characteristics as energy, momentum, coordinates, time and are therefore quite suitable [2)] for the role of unambiguously and clearly defined functions of the state of the system, then in quantum mechanics the wave functions and state vectors are simply immeasurable, that is, unobservable. In classical mechanics, there is an example of such a state function ─ this action S. This function is too "abstract" in comparison with Lagrangian and Hamiltonian and is physically immeasurable. The appearance of the "shadow" of the action of S in the phase of the wave function of the quantum system aggravates its immeasurability as a function of the state: the wave function has a fundamentally immeasurable parameter ─ phase, and the determination of the state vector itself is possible only with accuracy to this phase.

The physical non-observability of wave functions and the limited applicability of *PMC-topology* make it impossible to use continuity as a tool for determining the genetic identity of quantum objects. Thus, **in the quantum theory, unambiguous identification of the object and tracking its evolution along a continuous trajectory become impossible.**

Third. It should be noted that in non-relativistic quantum mechanics a peculiar possibility of a one-particle description is still preserved. However, in the relativistic theory, this certainty disappears: with the birth of the relativistic quantum theory, the era of one-particle description of quantum mechanics is over. In addition, the principle of identity of particles in quantum theory does not allow to count the number of particles of the quantum system. The last point concerns both non-relativistic and relativistic quantum theories.

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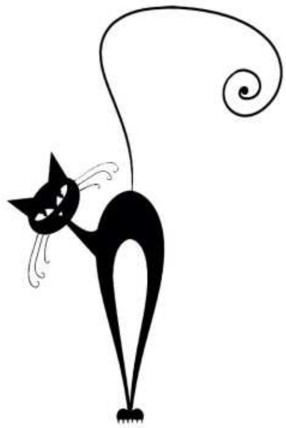
What can be said about the paradox, the appearance of which was demonstrated by two Cheshire cats using the results of the Aspect's experiments? Recall that the need to comply with the principle of genetic identity involves maintaining the integrity of the system when it changes.

There are two possible solutions to the photon version of the problem:

[2)] "tangible and smelling", that is, quite observable and measurable - directly or indirectly.

1. To consider a two-photon system consisting of two integral subsystems (photons).
2. To consider it as a single integrated system. The linearity of the wave equation allows us to work with the aggregate wave function obtained as superpositions of solutions for the first and second photons



In the first case, it is necessary to open the mechanism of transmission of the property at a distance. Here we should note the contradiction about the speed of propagation of perturbations (a smile) between the first and second cats. After all, the distance between the cats is not regulated by anything, and therefore the transmission rate of the disturbance can be arbitrarily large. But this contradicts the fundamental principle of STR, which states the existence of the maximum velocity invariant and the consequent Lorentz transformations for point objects.

In the second case it is necessary to state the fact of existence of non-point quantum objects. Lorentz transformations for 4-coordinates of point events are simply not applicable for non-point objects, due to the inapplicability of *PMC-topology* to the described events.

The cause of the paradox is the separation of the system described by one wave function into parts described by two wave functions. ***And here, in the end, we must either describe the mechanism of property transfer or state the existence of non-point systems.***

One way or another, but the results of the Aspect experiments indicate the limited applicability of the Lorentz point transformations. There is a need to turn to the origins of the concept of *PMC-topology*.

PMC-topology [3)]

In the mathematical description of physical phenomena is used mainly the point-metric classical topology (*PMC*-topology), embodied in the methods of mathematical analysis. It is necessary to note the important features of the application of *PMC*-topology to the solution of the problems of space-time relations, which will give us an unambiguous hint at the limitations of its applicability. To understand the reasons for limiting its application in space-time relations in physics, it is necessary to return to the origins of the concept of continuity of the classical topology of a point metric.

The carriers of conceptual spatial relations in *PMC*-topology are dimensionless points. Accordingly, it is necessary to confirm the possibility of the existence of physical objects that do not have sizes that may be represented by points. In the physics by the carriers space-time relations are point *events*. Let us examine several theorems of mathematical analysis and find out how the points of conceptual space correlate with the point events of space-time relations in physics.

An important property of the conceptual space is its *completeness*: *the space R is called complete if any Cauchy sequence is fundamental there.* The concept of completeness of space – the basic for mathematical analysis and ensures the applicability of a powerful apparatus of mathematical analysis. The property of completeness of the space allows to introduce the concept of proximity elements, *the genetic identity of the points* in the motion, to clearly define the limiting properties of the sets of convergence, limit and other logically related moments and elements of differential and integral calculus. The

[3)] To avoid confusion here, the term space will be refer to the conceptual space with which mathematics works. In the physical context, we will talk about space-time relations. The necessity of distinction between these concepts is obvious.

convergence of sequences, limit transitions, that is, the topological properties of space is formalized by the introduction of a space metric to determination the proximity of elements.

The following theorems of mathematical analysis can be used as the basic criteria of completeness of metric space:

1. *In order for the metric space R to be complete, it is necessary and sufficient that in it any sequence of nested closed balls whose radii tend to zero, had a non-empty intersection.* This statement is known as the nested balls theorem.

   In other words lets say this: by reducing the dimensions of the balls (using the property of metricity of space), can be infinitely close to the center of the ball (topology of "tightness"). The center point of the ball exists by the completeness of space. From a physical point of view, the concepts of distances, their measurements and the proximity of point elements of space became very clear.
2. *Every metric space R has replenishment, and this replenishment is unique up to isometry leaving fixed points from R.* This theorem suggests the possibility of replenishing the metric space, but with the possible occurrence of extraneous (fictitious) events due to the completion of the space. A priori endowment of real space-time relations with the completeness property leads us to the Newtonian concept of continuous space-time.
3. *The complete metric space R cannot be represented as a union of countable number of nowhere dense sets.* This statement (Baer's theorem) correlates our conceptual reasoning with the possibility of comparing them with experience and states the impossibility of representing a dense space by a limited number of dimensions (and even their infinitely countable number).

Thus, the main features of *PMC*-topology application to the solution of space-time problems are the following:

When considering problems in which the size of physical objects is much smaller than the characteristic size of the systems under consideration, *PMC*-topology is an acceptable approximation. However, there are problems associated with the "birth" of fictitious events in the replenishment of space (for example, in the STR when considering the Twin Paradox).

- ☝ The occurrence of fictitious events in force 2° and the inability to empirically confirm the completeness of the spatial-temporal relations of physical events in force 3°.
- ☝ In addition, in the microworld there are no objects which can be provided by points of conceptual space.
- ☝ When considering problems in which the size of physical objects is much smaller than the characteristic size of the systems under consideration, *PMC*-topology is an acceptable approximation. However, there are problems associated with the" birth " of fictitious events in the replenishment of space (for example, in the CTR when considering the Twin Paradox).

Tasks with superluminal speeds

The formalization of the concepts of proximity of elements, continuity of their transformations is the subject of topology. Of course, the EPR paradox and related issues of locality and causality have topological content. However, there are other models that raise the issues of locality and point localization in a more visual form. Let's consider some of them on the example of three simple tasks.

The fact that the consideration of physical objects, non-localized point, leads to paradoxes and contradictions with the principles of SRT (the principles of relativity and causality, Lorentz transformations) can be seen on the solutions of simple *tasks*, the discussion of which is presented in [1], [2].

Task 1. Let the inertial frame of reference Σ' move along the common axis X at a velocity V with respect to the stationary laboratory system Σ. Along the entire *positive* axis X''placed light bulbs. At the time $t' = 0$, all the light bulbs flare up. What picture "will see" the observer of the system Σ on its axis X? To consider the continuation of the scenario: the light bulbs flare up and go out after a finite period of time τ'. **Note:** at the time $t = t' = 0$, the origin of the systems Σ and Σ' coincided: $x = x' = 0$.

From the point of view of the observer of the system Σ at the point $x' = x = 0$ at the time $t' = t = 0$ there will arise a "running **light** bunny" (front edge of flashes) along the axis X. The speed of this "bunny" is determined by the formula:

$$U = c\,\frac{c}{V}, \text{here } c \text{ is the speed of light.} \tag{1}$$

This speed will be the greater the value, the smaller the value will be the speed V of the system Σ' relative to Σ . In the limit $V = 0$, this speed will be converted to ∞. Next, we will assume $c = 1$.

Let the bulbs relative to system Σ' go off at time τ' the synchronous. For the same reasons, at the time τ there will be a “running **dark** bunny” (front edge of disappearance of light) switching off the bulbs along the axis X. The speed of this "bunny" will also be equal to U. The front of the "light bunny", which arose in the system Σ at the time of turning on the bulbs, at the time τ will removed at a distance R:

$$R = \tau U = \frac{\tau'}{\sqrt{1-V^2}}\frac{1}{V} \tag{2}$$

and will continue to its movement with the same speed U.

Thus, after the simultaneous switch-off in the system Σ' all of the lights, an observer of system Σ will fix the appearance of extending to the *right* along the axis X with velocity U zug glowing light bulbs length l: bulbs of the right front consistently include, and the bulb of the left front is consistently off.

Summarizing, we can say that from the point of view of an observer of system Σ , at time $t = 0$ begins to form a zug of a light bulbs, right front which propagates with speed U along the axis X; at time τ the formation of zug of light bulbs ends , and as a whole, zug starts to move along the axis X. The length of this zug l is determined by the formula:

$$l = \frac{\tau'}{\sqrt{1-V^2}}\left(\frac{1}{V} - V\right) \approx \frac{\tau'}{\sqrt{1-V^2}}\,\frac{1}{V}. \tag{3}$$

The latter equality implies the fulfillment of the condition $V \ll 1$.

The first paradox arises when considering the entire axis $X = (-\infty, +\infty)$, filled with bulbs, turning on synchronously in the system Σ'.

According to the solution of the problem with the semi-infinite axis $X = (0, +\infty)$, the light zug in the system Σ arises at $x = 0$ and then continues to move along the positive direction of the axis X: there is a starting point $x = 0$ in the case of the problem with the semi-infinite axis. In the formulation of the problem with an infinite axis $X = (-\infty, +\infty)$, such a point has the coordinate $x = -\infty$, and the Lorentz transformations lead to the solution, according to which the light zug for the possibility of its observation must make a "journey" from the point $x = -\infty$ to the point with a finite coordinate and with a finite speed $U = c\,(c/V)$. It is obvious that under such conditions it is impossible appearance a zug of the points of the axis X, remote at a finite distance from the beginning. That is, the solution with the help of the Lorentz transformation says that in coordinate system Σ cannot be observation of a luminous zug of lights.

Another result is a solution using the principle of relativity, according to which the observed physical phenomena should "look the same" in all inertial reference systems. If we consider luminosity as an attribute or an integral characteristic of a non-local object, then according to the principle of relativity, the observer of the system Σ must necessarily fix the manifestation of this attribute.

By comparing the patterns of events in both reference systems, we can see that the reference system Σ and Σ' are unequal: in the system Σ' bulbs of positive half-axis are lit all at once, and in the system Σ they will never be all, because the spreading bunny final speed, and the length of the axis is infinite. *Thus, we observe an obvious contradiction between the point Lorentz transformations and the principle of relativity.*

Task 2. Along the segment $O'A'$ of the inertial frame of reference Σ', moving at a speed V with respect to the laboratory frame of reference Σ along the common axis X, there are light bulbs (see Fig.1). The length of $|O'A'|$ equals L'. Due to the synchronous switching of the lamps at the time $t' = 0$ in the system Σ' all the light bulbs flare up of the segment $O'A'$, and when switching off at the time $t' = \tau'$ all the lamps synchronously go out. What picture "will see" the observer of the system Σ on its axis X?

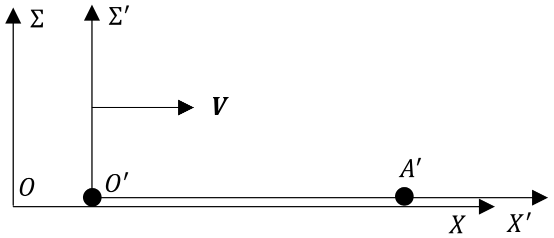


Fig. 1

The observer of the system Σ will see the following picture: at the time $t_{initial} = 0$, a light zug is born at the point O', which begins to propagate along the axis X. Reached the point A', it begins to "be absorbed" by it. The process ends with a complete "absorption".

The rear of the zug reaches the moving point A' at the time t_{final}, by this point the point A' will acquire the coordinate x_{final}; t_{final} and x_{final} are determined by the formulas:

$$t_{final} = \frac{\tau' + VL'}{\sqrt{1-V^2}}\ ; \quad x_{final} = \frac{L' + V\tau'}{\sqrt{1-V^2}} \tag{4}$$

so the effective[4)] speed w of its propagation is determined according to the formula

$$w = \frac{L' + V\tau'}{\tau' + VL'}. \tag{5}$$

When $\tau' \sim 0$ we get

$$w = \frac{1}{V}\ , \tag{6}$$

which is quite consistent with the result of the previous task.

It follows from (5) that $w > 1$ under the condition $\tau' < L'$. In the metric system of units, this means that $w > c$ at $\tau' < L'/c$; here c is the speed of light propagation.

The process of signal propagation in the system Σ can be interpreted as the transmission of information from a moving point O' to a moving point A'. At the same time, the beginning of the information transmission will be fixed by the observer of the system Σ at the beginning of the coordinate system at $x_{initial} = 0$ at the time $t_{initial} = 0$; the end of the transmission will be fixed by another observer of the same system at the point x_{final} at the time t_{final}, determined according to (4). The

4) the effective speed of information transmission is determined by the signs of the beginning and end of information transmission.

essential fact here is that the information is transmitted at a superluminal speed, which seems to be contrary to the STR principles.

The way out of this ”paradox" is simple. It can be noted that neither the first nor the second observers of the Σ system participate in the procedure of information transmission. The observers of the system $\Sigma^{'}$, who lit each of their light bulbs, acted completely independently abd on their synchronized clocks, and on a pre-prepared command, that is, they (all) possessed complete information even before its transmission, including those located on the edges of the segment $O^{'}A^{'}$ and between which the observers of the system Σ and recorded the superluminal transmission of information.

This remark takes off the arisen paradox STR. However, by changing the problem condition a bit, we return to the paradox of superluminal information transmission.

Task 3. In addition to the condition of task 2, we introduce one more observer at the point $B^{'}$ (see Fig. 2), which is spaced from the point $A^{'}$ ' along the axis $X^{'}$ at a finite distance. As in the previous problem, due to the synchronous switching on and off of the lamps along the segment $O^{'}A^{'}$, the systems $\Sigma^{'}$ all the light bulbs flare up at the moment $t^{'} = 0$, and at the moment $t^{'} = \tau^{'}$ all the lamps synchronously go out. What will be the picture of information transmission in the system Σ from the observer at the point $O^{'}$ to the observer at the point $B^{'}$?

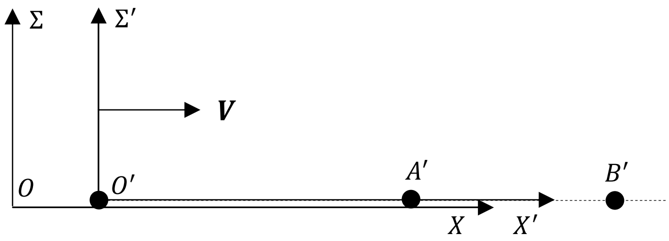


Fig. 2

Note that the input of the observer $B'$is necessary for the real procedure of information transfer from the point O' to the point B'. This is exactly what will happen, since the observer B' is not connected to observers who switch on and off the lights according to a predetermined prescription.

According to the solution of the previous problem, the observer of the reference system Σ will see the following picture: from the point $O^{'}$ the front of the zug begins to spread with a superluminal speed $U = 1/V$. The propagation of the zug stops at the moving point $A^{'}$, in which it is "absorbed". The rear front of the zug will propagate with a velocity of light, because in the direction of the point $\boldsymbol{B'}$ will continue to spread is a real light signal from the point $A^{'}$ (as the light of a distant star from the point O'). The resulting picture for the observer of the system Σ will be as follows: from the point $O^{'}$ to the point $B^{'}$ comes the light signal (*the beginning of the connection*). The time spent on the passage of a signal path $O^{'}B^{'}$, there will be less time required by the passage of this path by light. Thus, the "information transfer" will occurr at superluminal speed.

However, in the above reasoning, we have depicted only the beginning of the process of information transfer. To finish the transfer of this portion of information, that is, to transfer a sign of the end of the connection, we need to stop the transfer process. To do this, you need to send next the “dark bunny”, which will correspond to the simultaneous shutdown of all the lights in the system $\Sigma^{'}$. But then the time of transmission of information will be determined by the time of propagation of the back front of the light signal. And this time is determined by the time of overcoming by the real light signal in the system $\Sigma^{'}$ distance O'b ' from the most remote light bulb $O^{'}$. Since the speed of light propagation is an invariant of Lorentz transformations, all contradictions are removed for all inertial reference systems. However, it is here that a paradoxical nuance arises: before you send a “dark bunny”, you can send another sign of the end of the bit – a signal of a different color. Alternating sending "dark" and "multicolored bunnies", we can hope that it is still possible to establish a channel of information transmission with superluminal speed.

Thus, modulation of the properties of "color" of light bulbs zug carrying bits of information, giving hope to establish the transmission of information faster than the speed of propagation of a light signal, and phase modulation of the photon pair singlet state for a similar situation in the experiments of aspect.

Analysis of the results of solving problems about light bulbs

It is noteworthy that with the existence of formal kinematic solutions of tasks 1, 2, 3, based on the use of point Lorentz's transformations, it is possible to formulate a variants of the task 1 with an infinite axis, for which a formal kinematic solution contradicts the solution based on the use of the principle of relativity, what causes the appearance of an irreparable contradiction between of the point representation of events and the principle of relativity, and leads to the need to pay attention to a number of nuances, concerning the topology of space-time relations in the solutions of the presented problems.

The kinematic solution of the problem 1 for the semi-infinite axis $[0,+\infty)$ describes by means of point Lorentz transformations the transformation of the spatial infinity and the point moment of time (system Σ') into the temporal infinity and the spatial point spreading in the process of motion (system Σ).

The topological parameter of this transformation is speed. Similar features are typical for the other two tasks. The limited speed of the signal propagation leads to different patterns of light bulbs switching events, which, of course, means the unequal reference systems and is perceived as a contradiction with the principle of relativity: in one inertial system, the property of luminosity appears simultaneously, in another frame of reference - "in parts". The impossibility of implementing the initial conditions of the problem removes this contradiction, but technically. The topological parameter of this transformation is speed. Similar features are typical for the other two tasks. The limited speed of the signal propagation leads to different patterns of light bulbs switching events, which, of course, means the unequality of reference systems and is perceived as a contradiction with the principle of relativity: in one inertial system, the property of luminosity appears one-stage, in another frame of reference - "in parts". The impossibility of implementing the initial conditions of the problem removes this contradiction, but technically.

There is no kinematic solution to the problem for the infinite axis $(-\infty,+\infty)$. However, the application of the principle of relativity allows us to obtain a conceptual solution in which the spatial infinity and the point time are preserved in the transition from one inertial frame of reference Σ' to another Σ.

If the segment of bulbs in task 2 is considered in the system Σ' as a single non-point object, the transition to the system Σ with the help of Lorentz's transformations raises a natural contradiction: there is this integral object does not arise immediately, but "in parts". Spatial non discreteness of object is transformed under Lorentz transformation into the time non discreteness, and the one-time dynamic time (synchronized non local clock) becomes evolutionary (ordering the formation of an integral object) – a phenomenon entirely new to physics.

A *non-connected* object allows the selection of a reference system to carry out the transmission of information at superluminal speed. This is demonstrated by the solution of task 3. In contrast to the previous cases, here the observer of the system Σ will fix the real data transmission procedure from the observer O' to the observer B' at a rate exceeding the speed of propagation of the standard reference signal.

It is easy to see that a pair of "tangled" photons in the mental experiment of Aspect is also an example of a disjointed object. Indeed, although the symmetrized state vector describes a pair of photons, they are indistinguishable, therefore it is not clear how to count indistinguishable objects. The vector of state $|\Psi(\nu_1,\nu_2)\rangle$ when $\nu_1 = \nu_2$ describes a single quantum mechanical object. Which photon (first or second) of these two will be on the left polarizer, and which of them will be on the right polarizer? Even it becomes possible to say that a single quantum mechanical object is located in two spatial points.

On the relativistic task of pencil and pencil case

Pencil case and pencil at rest have the same linear dimensions. Consider the situation when the pencil flies towards the pencil case (Fig. 3). According to the STR length of the pencil is reduced, which makes it possible to fully immerse in the pencil case. At the moment when the pencil is fully included in the pencil case, it is closing. The picture changes when considering the situation in the frame of reference associated with the pencil (Fig. 4). Pictures represent moment of time $t = t' = 0$.

The frame of reference S associated with the pencil case

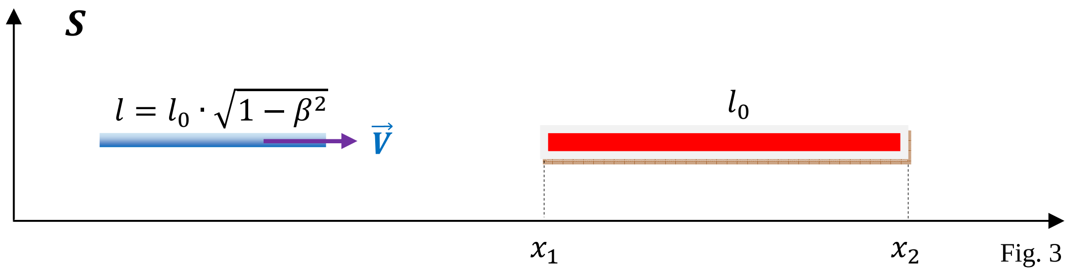


Fig. 3

The reference system associated with the pencil S'

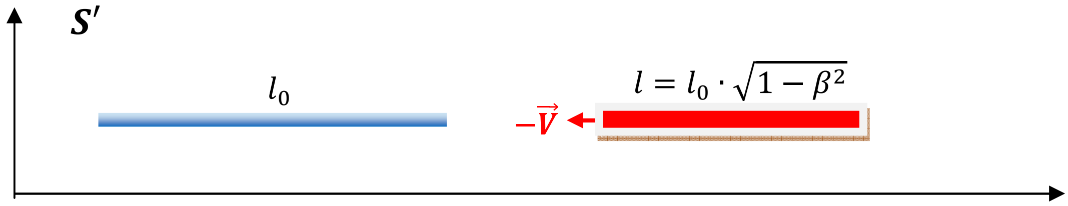


Fig. 4

According to the principle of relativity on the equality of inertial reference systems, if the events occurred in one of them, they will be noticed by observers of any of the inertial reference systems. The events themselves have the status of absoluteness, and only their space-time coordinates can change. We will not use words like "reduction" of sizes and time intervals, we use only Lorentz transformations for the coordinates of events.

Let's consider the movement of the pencil with respect to the pencil case with a velocity $\vec{V}$ along the common axis X. With the pencil case we associate the inertial frame of reference S (pencil box in this frame is at rest), and with the pencil – a system S' (in this frame of reference at rest is a pencil). The initial conditions of the problem: the clocks of reference systems synchronized on time $t = t' = 0$; pencil case at this time is at rest with the coordinates of their ends x_1 and x_2; $x_2 - x_1 = l_0$.

We are interested in three events:

- meeting the right end of the pencil with the left end of the pencil case – the beginning of the process (zero event);
- meeting the left end of the pencil with the left end of the pencil case – the moment of the possibility of closing the pencil case (the first event);
- meeting the right end of the pencil with the right end of the pencil case – the moment of possible breaking the bottom of the pencil case with a pencil (the second event).

We introduce the coordinates of these events.

For reference frame *S*:

x_0, t_0 – the coordinate and the moment of the meeting of the right end of the pencil and the left end of the pencil case;
x_1, t_1 – the coordinate and the moment of the meeting of the left ends of the pencil and the pencil case;
x_2, t_2 – the coordinate and the moment of the meeting of the right pencil and pencil ends.

For reference frame *S'*:

x_0', t_0' – the coordinate and the moment of the meeting of the right end of the pencil and the left end of the pencil case;
x_1', t_1' – the coordinate and the moment of the meeting of the left ends of the pencil and the pencil case;
x_2', t_2' – the coordinate and the moment of the meeting of the right pencil and pencil ends.

The connection between the coordinates in the $S \leftrightarrow S'$ transition is established by Lorentz's transformations:

(direct)

$$x = \frac{x' + V t'}{\sqrt{1 - V^2/c^2}};\quad y = y';\quad z = z';\quad t = \frac{t' + \frac{V}{c^2}x'}{\sqrt{1 - V^2/c^2}}.$$

(reverse)

$$x' = \frac{x - V t}{\sqrt{1 - V^2/c^2}};\quad y' = y;\quad z' = z;\quad t' = \frac{t - \frac{V}{c^2}x}{\sqrt{1 - V^2/c^2}}.$$

The solution of the problem is to apply Lorentz's transformations to the coordinates of the same event - direct and reverse. In this case, they take the form:

$$x_0 = x_1, \quad t_0 \qquad \text{(Event 0)}$$

$$x_0' = \frac{x_1 - V t_0}{\sqrt{1-\beta^2}}, \quad t_0' = \frac{t_0 - \frac{V}{c^2}x_1}{\sqrt{1-\beta^2}}. \qquad \text{(Event 0')}$$

$$x_1' = \frac{x_1 - V t_0 - l_0\sqrt{1-\beta^2}}{\sqrt{1-\beta^2}}, \qquad t_1' = \frac{l_0}{V} + \frac{(t_0 - \frac{V}{c^2}x_1)}{\sqrt{1-\beta^2}}, \qquad \text{(Event 1')}$$

because

$$x_1' = x_0' - l_0, \quad t_1' = t_0' + \frac{l_0}{V},$$

$$x_1' = x_0' - l_0 = \frac{x_1 - Vt_0}{\sqrt{1-\beta^2}} - l_0 = \frac{x_1 - Vt_0 - l_0\sqrt{1-\beta^2}}{\sqrt{1-\beta^2}},$$

$$t_1' = t_0' + \frac{l_0}{V} = \frac{l_0}{V} + \frac{t_0 - \frac{V}{c^2}x_1}{\sqrt{1-\beta^2}}.$$

$$x_1, \qquad t_1 = t_0 + \frac{l_0\sqrt{1-\beta^2}}{V}, \qquad \text{(Event 1)}$$

because

$$t_1 = \frac{t_1' + \frac{V}{c^2}x_1'}{\sqrt{1-\beta^2}} = \frac{Vt_0 - \frac{V^2}{c^2}x_1 + l_0\sqrt{1-\beta^2} + \beta^2(x_1 - Vt_0 - l_0\sqrt{1-\beta^2})}{V(1-\beta^2)} = t_0 + \frac{l_0\sqrt{1-\beta^2}}{V}.$$

$$x_2 = x_1 + l_0, \quad t_2 = t_0 + \frac{l_0}{V}. \qquad \text{(Event 2)}$$

$$x_2' = \frac{x_1 - Vt_0}{\sqrt{1-\beta^2}}; \quad t_2' = \frac{l_0}{V}\sqrt{1-\beta^2} + \frac{t_0 - \frac{V}{c^2}x_1}{\sqrt{1-\beta^2}}, \qquad \text{(Event } 2')$$

because

$$x_2' = \frac{x_2 - Vt_2}{\sqrt{1-\beta^2}} = \frac{x_1 + l_0 - V(t_0 + \frac{l_0}{V})}{\sqrt{1-\beta^2}} = \frac{x_1 - Vt_0}{\sqrt{1-\beta^2}},$$

$$t_2' = \frac{t_2 - \frac{V}{c^2}x_2}{\sqrt{1-\beta^2}} = \frac{t_0 + \frac{l_0}{V} - \frac{V}{c^2}(x_1 + l_0)}{\sqrt{1-\beta^2}} = \frac{l_0}{V}\sqrt{1-\beta^2} + \frac{t_0 - \frac{V}{c^2}x_1}{\sqrt{1-\beta^2}}.$$

Analysis of the results of solving the task of pencil and pencil case

As expected, the order of occurrence of events in the system of reference S associated with the pensile case is next: zero, first, second. This is not the case for the system of observer S'. For it, the order of occurrence of the first and second will change, and the order of events will be as follows: zero, second, first.

However, for point events, the STR paradigm is not violated and the paradox, about we saying,, does not occur [5)]. In STR, with respect to any pair of events, one can talk about whether it is a pair of spatially-like or time-like events. And this relation is invariant under the Lorentz's transformations.

[5)] Except that the interval (4-"distance") between such events is assumed to be imaginary.

Time-like events have the possibility to consist in a causal relationship with each other, because they can be related signal, the speed of propagation of which does not exceed the speed of light. The set of these events is called the one STR *light cone*. Due to the invariance of the relation of belonging to the light cone, the possibility of causation is preserved at transitions between inertial reference systems. There is a frame of reference in which such events occur at one point in space.

Space-like events cannot be causal, since the speed of propagation of a signal capable of relating such events must be greater than the speed of light. There is a frame of reference in which such space-like events occur simultaneously. In this task, such a frame of reference is the one in which the pencil and the pencil case approach each other with equal velocities. Space-like events are related to the imaginary interval.

The paradox arises when considering the topological moments of the problem.

The observer of system S has an opportunity to stop the pencil movement in the time interval from t_1 to t_2, since the second event after the first one will occur in the time interval $\Delta t = t_2 - t_1 = (l_0/V)\sqrt{1-\beta^2}$. In this case, the event of the meeting of the right pencil ends and the right pencil case will not occur. However, the observer of the system S'such a possibility is absent: the event of breaking the bottom of the pencil case for him is inevitable when the event of the meeting of the right end of the pencil and the left end of the pencil case − just because of the order of their occurrence. This is the manifestation of the paradox, namely − topological: the relationship of input is violated.

Thus, in this task *there is a topological paradox associated with the violation of the ratio of inputting for non-localized objects*. The connection of the first and second events is carried out by means of a pencil, that is, non-localized point object. Meanwhile, the coordinate transformations used are point transformations.

The experiment, the idea of which is described above, can give an answer about the the possibility of existence of point-non-localized physical objects and the don't use of point transformations for them.

And more. ..

Perfectly rigid body

Suppose that there are topologically integral objects that have finite dimensions in the metric topology. *The results of the Aspect's experiments allow us to talk about it, at least.*

It is also known statement about the impossibility of existence of absolutely rigid bodies. This conclusion is based on a STR, claiming the impossibility of exceeding the speed of propagation of light. However, such conclusions are methodologically wrong, since it is logically impossible to prove "non-existence", it is only possible to lead any reasoning to a contradiction.

But from this can not follow the fact of non-existence, but can only fix the fact of a contradiction. Therefore, the conclusion is not a conceptual, but a purely technical moment of fixing the contradiction. But what is the contradiction? The answer is obvious: in the illegality of the application of point Lorentz transformations to non-point objects. Let us consider this in a more illustrative example. But from this cannot follow the fact of non-existence, but can only fix the fact of a contradiction. Therefore, the conclusion is not a conceptual, but a purely technical moment of fixing the contradiction. But what is the contradiction? The answer is obvious: in the illegality of the application of point Lorentz transformations to non-point objects. Let us consider this in a more illustrative example.

The task of light bulbs

In its own frame of reference, the object remains identical to itself from the moment of turning on and to the moment of turning off the bulbs. In the laboratory frame of reference, the object evolves, that is, it changes and ceases to be identical to itself in the period between turning on and off the bulbs by the clock of its own frame of reference. Actually, this is a violation of the principle of relativity for point-non-localized objects: in two inertial reference systems, the same non-point object behaves differently.

The twin paradox

Similar arguments for the non-legitimacy of the application of point Lorentz transformations can be expressed about the so-called "twin paradox", operating with non-point temporal objects (age of brothers). However, there is clearly another effect. This effect is discussed in more detail in [2]. The resolution of the twin paradox within the framework of the principle of relativity becomes possible with the discretization (quantization) of evolutionary time, that is, with the change of the topology of the time axis. And here we note that if the age of the twin brothers is measured by the number of revolutions of the Earth around the Sun (that is, in discrete years), then upon return from the trip of the cosmonaut brother, the age of the twins will be the same, since the number of events occurred (the Earth's revolutions around the Sun) during the cosmonaut's journey is an absolute invariant for both brothers.

Macroapproach

When considering problems in which the size of physical objects is much smaller than the characteristic size of the systems under consideration, *PMC*-topology is an acceptable approximation. However, there are also problems related to the "birth" of fictitious events in space replenishment, for example in STR when discussing the effects of deformation of lengths, time intervals, etc.[6]

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V.A.Kasimov. The shadow of the smile of the "Cheshire cat" (English version)

Abstract
Offers a discussion of some topological paradoxes arising in the theory of relativity.

[6] Theorem of mathematical analysis, which implies the completion of the metric space, but with the possible occurrence of foreign (fictitious) events to space. For more information, see T2 in the "*PMC*-topology".