

question 470: Two Integrals

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abstract

This note presents two integrals.

1. Introduction

If $s = \sqrt[5/2]{1 + \sqrt[5/2]{1 + \sqrt[5/2]{1 + \dots}}}$, then

$$s = \sqrt[3/2]{1 + \frac{1}{\sqrt[3/2]{1 + \frac{1}{\sqrt[3/2]{1 + \dots}}}}} \quad (1)$$

$$s^5 - s^2 - 2s - 1 = 0 \quad (2)$$

$$s^{5/2} = 1 + s \quad (3)$$

$$s^{3/2} = 1 + \frac{1}{s} \quad (4)$$

In this note we give some integrals involving s .

2. Two Integrals

If $s = \sqrt[5/2]{1 + \sqrt[5/2]{1 + \sqrt[5/2]{1 + \dots}}}$, then

$$\int_0^{1/s} \left(\sqrt[3]{\sqrt{\frac{1}{27} + \frac{1}{4x^2} + \frac{1}{2x}} - \sqrt[3]{\sqrt{\frac{1}{27} + \frac{1}{4x^2} - \frac{1}{2x}}}} \right)^2 dx = \frac{\pi}{2} + s^{-2} + 2 \tan^{-1} \left(\frac{\sqrt{s} - 1}{\sqrt{s} + 1} \right) \quad (5)$$

$$\int_0^{1/s} \left\{ \left(\sqrt{\frac{1}{27} + \frac{1}{4x^2} + \frac{1}{2x}} \right)^{2/3} + \left(\sqrt{\frac{1}{27} + \frac{1}{4x^2} - \frac{1}{2x}} \right)^{2/3} \right\} dx = \frac{\pi}{2} + \frac{2}{3} s^{-1} + s^{-2} + 2 \tan^{-1} \left(\frac{\sqrt{s} - 1}{\sqrt{s} + 1} \right) \quad (6)$$

Related integrals :

Let $u = \tan^{-1} \left(\frac{2}{3\sqrt{3}s} \right)$, then

$$\int_0^u \left(\sqrt[3]{\tan\left(\frac{x}{2}\right)} - \sqrt[3]{\cot\left(\frac{x}{2}\right)} \right)^2 (\sec x)^2 dx = \frac{1}{\sqrt[3]{3\sqrt{3}}} \left(\pi + 2s^{-2} + 4 \tan^{-1} \left(\frac{\sqrt{s}-1}{\sqrt{s}+1} \right) \right) \quad (7)$$

$$\int_0^u \left\{ \left(\tan\left(\frac{x}{2}\right) \right)^{2/3} + \left(\cot\left(\frac{x}{2}\right) \right)^{2/3} \right\} (\sec x)^2 dx = \frac{1}{\sqrt[3]{3\sqrt{3}}} \left(\pi + 2s^{-2} + 4 \tan^{-1} \left(\frac{\sqrt{s}-1}{\sqrt{s}+1} \right) \right) + \frac{4}{3\sqrt{3}s} \quad (8)$$

Let $v = \sinh^{-1} \left(\frac{2}{3\sqrt{3}s} \right)$, then

$$\int_0^v \left(\sqrt[3]{\coth\left(\frac{x}{2}\right)} - \sqrt[3]{\tanh\left(\frac{x}{2}\right)} \right)^2 \cosh x dx = \frac{1}{\sqrt[3]{3\sqrt{3}}} \left(\pi + 2s^{-2} + 4 \tan^{-1} \left(\frac{\sqrt{s}-1}{\sqrt{s}+1} \right) \right) \quad (9)$$

$$\int_0^v \left\{ \left(\coth\left(\frac{x}{2}\right) \right)^{2/3} + \left(\tanh\left(\frac{x}{2}\right) \right)^{2/3} \right\} \cosh x dx = \frac{1}{\sqrt[3]{3\sqrt{3}}} \left(\pi + 2s^{-2} + 4 \tan^{-1} \left(\frac{\sqrt{s}-1}{\sqrt{s}+1} \right) \right) + \frac{4}{3\sqrt{3}s} \quad (10)$$

References

- A. Boros, G. and Moll, V.H.: Irresistible Integrals. Cambridge University Press, 2004.
- B. Chaudhuri, J.: Some special integrals. Amer.Math.Monthly,74, 1967.
- C. Kaspar, T.: Integration in finite terms: The Liouville theory. Math.Mag.,53,1980.
- D. Moll, V.: The evaluation of integrals: a personal story. Notices Amer. Math. Soc.,2002.