Ami: Reconsideration of Axioms for The Real Number

Yuki Yoshino
yuki.yoshino.01@gmail.com

Abstract

The number 0 has no distinction between positive and negative as $-0 = +0$, it is a number with special properties. In this paper, we define a new concept of numbers that seems to be special, like 0, by the symbol ◎ and the name Ami.And I propose new axioms of real numbers extended by adding ◎ to Hilbert 's real axiom.

1 Discovery of Ami

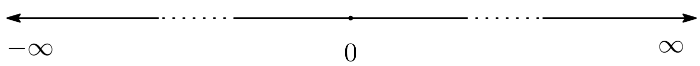
Although it is straightforward to imagine a number line, it is possible to dually capture numbers with positive and negative numbers for arbitrary real numbers other than 0.

$$-a \longleftrightarrow +a \quad \{a \in \mathbf{R} | a \neq 0\}$$

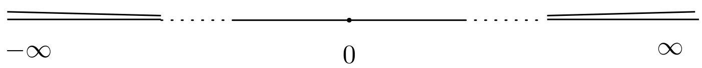
This means that we are specializing 0 as a real number. Here we introduce the following symbols and names.

$$◎ : \text{Ami}$$

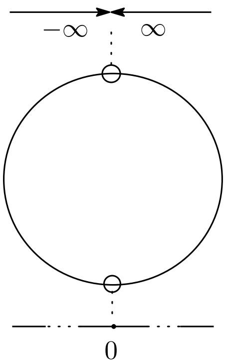
I thought about this as follows. First of all, please imagine that the number lines continue infinitely.



In recognizing real numbers based on Hilbert axioms, there are infinitely numerous possible numbers, and this number line continues forever. I imagined here that both ends of this number line have a slight inclination.



Then it seemed natural for the line to draw a circle.



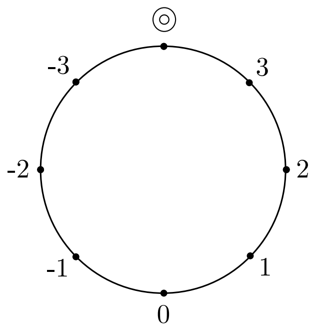
I thought that the point of the opposite position of 0 hit like this is worth characterizing with a symbol as a new concept. For the symbol, "∞" and From the combination with "$-\infty$", I first thought about "⊕", but because it is preferable to be simpler as a symbol, it was decided as "◎" which removed "+" from the middle.

2 Consideration on the sum of Ami

If ◎ is a number, what kind of number is it? By the way, as a special property of 0

$$-0 = +0 \tag{1}$$

There is no difference between the concept of positive and negative.From here it can also be assumed that ◎ is also a number with no difference between positive and negative.In considering the sum of ◎, for example, recognize it as '◎=±4' which is a mixture of -4 and 4 and think about the next circle.



This point is the integer point of 1, 2, 3, ◎,-3,-2,-1 exists counterclockwise from 0 at the point where ◎ is regarded as an integer and the circumference is divided into 8 equally.Let's call the circle determined as ◎=$\pm x\{x \in \mathbf{R} | x \neq 0\}$ as the number circle of ◎=$\pm x$.That is, the circle above is "the number circle of ◎=±4".For convenience, the following notation is defined.

$$◎_{+} = +4 \tag{2}$$
$$◎_{-} = -4 \tag{3}$$

Here, on this number circle, we recognize that there are no numbers larger than 4 or smaller than -4.Since this can actually be thought of as large numbers and small numbers as possible, we assumed this way for consideration. Let's think about naturally on this recognition.

$$1 + 1 = 2 \tag{4}$$
$$1 - 1 = 0 \tag{5}$$

There is no doubt as to the sum that ◎ does not involve, so recognize it about sum as follows.

$$+1 : \text{Add 1 counterclockwise on the number circle}$$
$$+(-1) = -1 : \text{Add 1 clockwise on the number circle}$$

Then, as a sum on this number circle, for example

$$3 + 1 = ◎ \tag{6}$$
$$-3 - 1 = ◎ \tag{7}$$

this is

$$-4 = ◎ = 4 \tag{8}$$

It seems strange in the sense of Hilbert's axiom. However, regarding the symbol of 0

$$1 - 1 = 0_{+} \tag{9}$$
$$-1 + 1 = 0_{-} \tag{10}$$

We regard as

$$0_{-} = 0 = 0_{+} \tag{11}$$

In this case also for ◎

$$-4 = ◎_{-} = ◎ = ◎_{+} = 4 \tag{12}$$

It seems natural even if imposing the character like.For example, the formula of 2+3=5 is as follows according to the above rule of sum

$$2 + 3 = -3 \tag{13}$$

This is not the Hilbert axiom.It is interesting to think as follows.

$$2 + 3 = ◎_{+} + 1 = ◎ + 1 = \pm 4 + 1 = -3 \tag{14}$$

The last equality thought that $+4 + 1 = 5$ does not exist, only $-4 + 1 = -3$ remained.You can calculate the sum for ◎ by thinking like this, for example, as follows.

$$◎ - 3 = \pm 4 - 3 = 1 \tag{15}$$
$$2 + ◎ = 2 \pm 4 = -2 \tag{16}$$
$$-1 + ◎ = -1 \pm 4 = 3 \tag{17}$$

Interestingly to take the sum of a corresponds to shifting the integer point to the opposite integer point.Also as for the sum of 0 and ◎, it becomes legitimate as an identity element about sum.

$$\circledcirc + 0 = \circledcirc \tag{18}$$

Even though the opposite position of 0 is $\circledcirc$, it is legitimate.Also, when adding integers at symmetrical positions with respect to a line connecting 0 and $\circledcirc$

$$1 + (-1) = 0 \tag{19}$$
$$2 + (-2) = 0 \tag{20}$$
$$3 + (-3) = 0 \tag{21}$$

And it seems legitimate to think that 0 and $\circledcirc$ are sums with themselves.

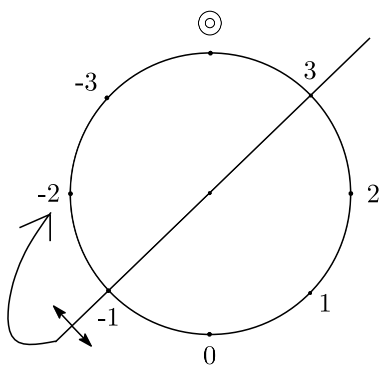
$$0 + 0 = 0 \tag{22}$$
$$\circledcirc + \circledcirc = 0 \tag{23}$$

Again it is definitely natural because the opposite position of $\circledcirc$ is 0.

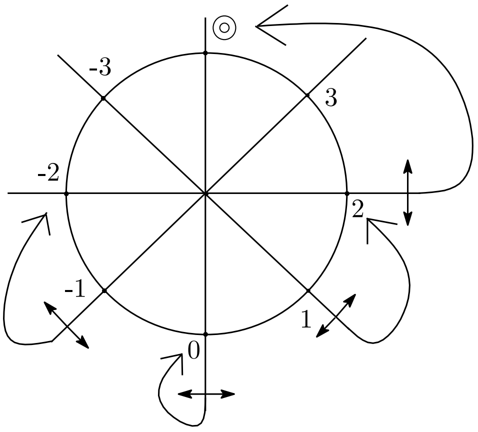
In such consideration, I thought that this sum rule is legitimate with the following relationship.Let us now consider a line equally dividing a circle across an integer point, adding integral points that are symmetrical with respect to that line, and adding a point on the line itself.For a straight line passing through -1 and 3, for example

$$-1 + (-1) = -2 \tag{24}$$
$$-2 + 0 = -2 \tag{25}$$
$$-3 + 1 = -2 \tag{26}$$
$$\circledcirc + 2 = -2 \tag{27}$$
$$3 + 3 = -2 \tag{28}$$

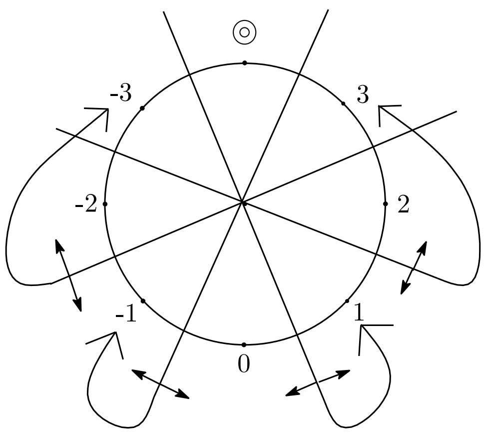
Then all sums will have a single value.



Now, the line connecting the integer points can be expressed in terms of the sum of symmetric points as follows.

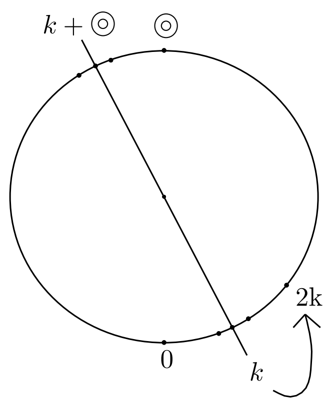


For a line equally dividing each integer point, it can be expressed as follows.



From this fact I thought it was justified as a recognition of the sum of ◎.

Consider whether the above argument holds even in the case of arbitrary positive real number other than ◎=±4.



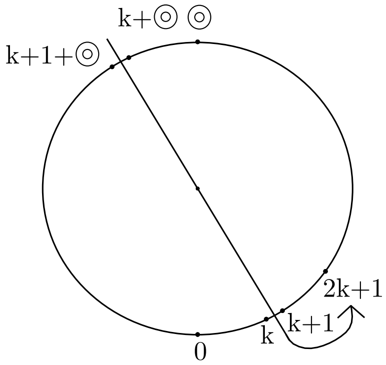
For example, considering a straight line that traverses arbitrary integers k and $k+$◎, add each integer point next to k

$$(k+1)+(k-1)=2k \tag{29}$$

Also, if you add each point of the opposite position on the circle

$$(k+1+◎)+(k-1+◎)=2k \tag{30}$$

Furthermore, when plus each of the points outside it, it is plus and minus, so they are the same distance. As a result, when k and $k+◎$ are connected by a straight line, the target points are summed up to be all $2k$. Similarly, when considering a line that crosses the midpoints of the integer points k and $k+1$ and consider the sum of the opposite integer point for that line

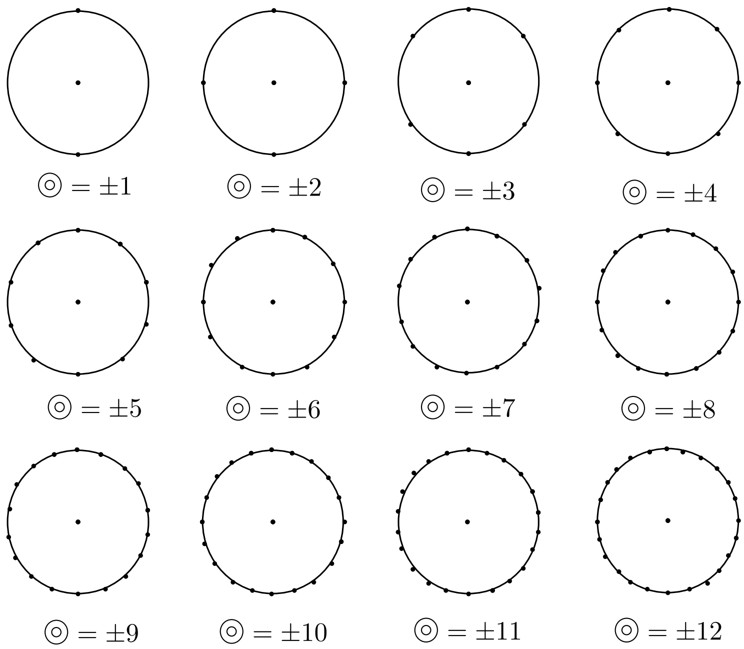


$$k+(k+1)=2k+1 \tag{31}$$
$$(k+◎)+(k+1+◎)=2k+1 \tag{32}$$

It is considered to be $2k+1$ all.I could generalize this.

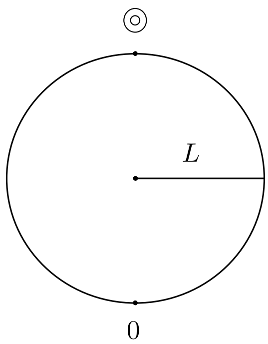
3 Ami's formula

In the second section I declared ◎=±4, but you can think of it in arbitrary pair of real numbers, for example as an integer indefinitely as shown below.



$\cdot \quad \cdot \quad \cdot \quad \infty$

For the number circle obtained by defining $\circledcirc$, we can see that the following equation is established by considering the circumference of a circle with a radius of L, with the length of the arc connecting adjacent integer points being 1 . Since $\circledcirc$ is a number including positive and negative and L is considered to be a positive real number, it expressed using absolute value.



$$|\circledcirc| = \pi L \tag{33}$$

If you express it without using absolute values.

$$\circledcirc = \pm \pi L \tag{34}$$

In this formula, as in the case discussed in Section 2, we catch the sign of pm. From these formulas, we have considered $\circledcirc$ as an integer without thinking deeply, but you can tell that $\circledcirc$ is an integer on the number circle when L is expressed as follows.

$$L = \frac{\text{(Natural number)}}{\pi} \tag{35}$$

4 Consideration on the product of Ami

In Hilbert's axiomatic system there is no inverse element of the product of 0. This means that if there was an inverse element for the product of 0 such as $1/0 = \infty$

$$0 + 0 = 0 \tag{36}$$

Multiply both sides by ∞

$$(0 + 0) \cdot \infty \stackrel{?}{=} 0 \cdot \infty \tag{37}$$

$$0 \cdot \infty + 0 \cdot \infty \stackrel{?}{=} 0 \cdot \infty \tag{38}$$

$$1 + 1 \stackrel{?}{=} 1 \tag{39}$$

Which leads to inconsistent expressions. For this reason there is no inverse element for the product of 0, $\tan(\pi/2)$ or $\tan(3\pi/2)$ etc are defined as undefined.From the discussion in Section 2, it is assumed that the following expression is satisfied.

$$\circledcirc + \circledcirc = 0 \tag{40}$$

From this expression, the inverse of the sum of $\circledcirc$ is taken as $-\circledcirc$

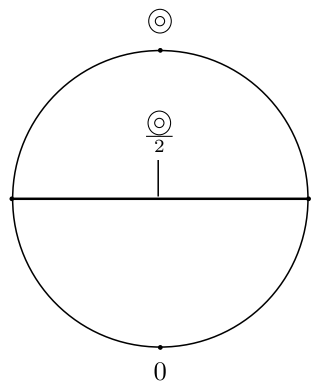
$$+\circledcirc = -\circledcirc \tag{41}$$

You can naturally convince this formula.It also shows that it satisfies the following formula.

$$\circledcirc \cdot (\text{Even number}) = 0 \tag{42}$$

$$\circledcirc \cdot (\text{Odd number}) = \circledcirc \tag{43}$$

(Even number) written here does not include 0.As for the natural number multiple of $\circledcirc$, it is decided neatly, but in other cases what will happen.For example, if a molecule is expressed as a fraction of $\circledcirc$ as $\frac{\circledcirc}{2}$ and so on, let us assume that you can recognize the line connecting the middle position just as $\frac{\circledcirc}{2}$ as shown in the figure below.Originally, the right of the number circle is regarded as a positive area, the left is a negative area, and although it is recognized that only 0 and $\circledcirc$ are located at the boundary, try brute interpretation.Also, even if you consider $\frac{\circledcirc}{2}$ (41) is established $-\frac{\circledcirc}{2} \stackrel{?}{=} +\frac{\circledcirc}{2}$.



Here, express (41) daringly and add as a sum of fractions represented by numerator of $\circledcirc$

$$\pm\frac{\circledcirc}{3}\pm\frac{\circledcirc}{4}\stackrel{?}{=}\frac{\pm 4\pm 3}{12}\circledcirc\stackrel{?}{=}\begin{cases}-\frac{7}{12}\circledcirc\stackrel{?}{=}\frac{7}{12}\circledcirc\\-\frac{1}{12}\circledcirc\stackrel{?}{=}\frac{1}{12}\circledcirc\end{cases}\tag{44}$$

I can not uniquely obtain a sum as. Therefore, we decide it as follows.

Real number x that satisfy $-x=+x$ are only 0 and $\circledcirc$.

Here we treat $\circledcirc$ as a real number and implicitly admit that it is $0\neq\circledcirc$. By doing this

$$-\frac{\circledcirc}{2}\stackrel{?}{=}+\frac{\circledcirc}{2}\tag{45}$$

I decided not to think about the notation of a fraction with $\circledcirc$ as a molecule as in the above formula.For $\tan(\pi/2)$ and $\tan(3\pi/2)$ which were undefined in Hilbert's axiom, it seems natural to decide as follows

$$\tan\left(\frac{\pi}{2}\right)=\frac{1}{0}=\circledcirc\tag{46}$$

$$\tan\left(\frac{3\pi}{2}\right)=\frac{-1}{0}=\circledcirc\tag{47}$$

Let's consider how to determine $0\cdot\circledcirc$ here.

$$0\cdot\circledcirc=x\tag{48}$$

And look at the following expression.

$$0+0=0\tag{49}$$

$$(0+0)\cdot\circledcirc=0\cdot\circledcirc\tag{50}$$

$$0\cdot\circledcirc+0\cdot\circledcirc=0\cdot\circledcirc\tag{51}$$

$$2x=x\tag{52}$$

From the expansion of this formula you can see that

$$\circledcirc\cdot 0=0\tag{53}$$

Also from the definition of Cauchy's extreme against the real x

$$\lim_{x\to\infty}\frac{1}{x}=0\tag{54}$$

From this equation it is natural to convince the following formula.

$$\frac{1}{\circledcirc}=0\tag{55}$$

Finally think about the product of $\circledcirc$ each other. As in the case of the product of $\circledcirc$ and 0, consider x as follows.

$$\circledcirc\cdot\circledcirc=x\tag{56}$$

It should satisfy the following expression

$$\circledcirc + \circledcirc = 0 \tag{57}$$
$$(\circledcirc + \circledcirc) \cdot \circledcirc = 0 \cdot \circledcirc \tag{58}$$
$$\circledcirc \cdot \circledcirc + \circledcirc \cdot \circledcirc = 0 \tag{59}$$
$$2x = 0 \tag{60}$$

Looking at the expression of (43), we should also satisfy the following expression.

$$\circledcirc + \circledcirc + \circledcirc = \circledcirc \tag{61}$$
$$(\circledcirc + \circledcirc + \circledcirc) \cdot \circledcirc = \circledcirc \cdot \circledcirc \tag{62}$$
$$\circledcirc \cdot \circledcirc + \circledcirc \cdot \circledcirc + \circledcirc \cdot \circledcirc = \circledcirc \cdot \circledcirc \tag{63}$$
$$3x = x \tag{64}$$

Furthermore, it should be expressed as follows from the expression of (46).

$$\circledcirc \cdot \circledcirc = \frac{1}{0} \cdot \frac{1}{0} \tag{65}$$

From the expressions of (60), (64) and (65), it is considered natural to define

$$\circledcirc \cdot \circledcirc = \circledcirc \tag{66}$$

5 Ami's axioms

Based on the above consideration, consider the following assumption in addition to Hilbert axioms with $\circledcirc$ as a real number.

$$\circledcirc + \circledcirc = 0 \tag{67}$$
$$\text{Real numbers } x \text{ that satisfy } -x = +x \text{ are only } 0 \text{ and } \circledcirc. \tag{68}$$
$$\circledcirc = \frac{1}{0} \tag{69}$$
$$\circledcirc \cdot 0 = 0 \tag{70}$$
$$0 = \frac{1}{\circledcirc} \tag{71}$$
$$\circledcirc \cdot \circledcirc = \circledcirc \tag{72}$$

(67) comes from consideration on sum. Moreover, although it seems that the following formula can be said naturally from this formula

$$+\circledcirc = -\circledcirc \tag{73}$$

Actually, since there is no description about the inverse about the sum of $\circledcirc$ (68) I think that it is necessary as an axiom. By doing so, it means that you do not think about products with $\circledcirc$ except 0, $\circledcirc$, and integer multiples. Likewise, from the (69) we may replace the denominator and numerator as a natural calculation feeling and the following expression may seem to be correct

$$\frac{1}{\circledcirc} \stackrel{?}{=} \frac{0}{1} = 0 \tag{74}$$

However, this is considered to be able to divide denominator and numerator by $0/0 \stackrel{?}{=} 1$ and in fact there is no guarantee that such calculation is allowed. So (71) recognized as axiom. As stated above, it is possible to give a unique interpretation to the following expressions as well.

$$\frac{0}{0} = 0 \cdot \frac{1}{0} = 0 \cdot \circledcirc = 0 \tag{75}$$

$$\frac{\circledcirc}{\circledcirc} = \circledcirc \cdot \frac{1}{\circledcirc} = \circledcirc \cdot 0 = 0 \tag{76}$$

Here we extend Hilbert's axiom by adding $\circledcirc$ and axioms are ultimately the philosophy of people who decided it, I do not think there are correct or incorrect answers.However, since it exists as the most fundamental assumption in discussing as mathematics, I think that it is an axiom that many people can convince and should be formed on the understanding of many people.Although I can not guarantee that this axiom is completely correct, I described what I felt was correct as far as I could think.

6 Consideration on the power of Ami

Here we take the position of $0^0 = 1$. Also. Here we consider the exponential calculation including $\circledcirc$, considering that the above axiom is also established in the power law.With respect to $\circledcirc^0$ it can be interpreted as follows.

$$\circledcirc^0 = \circledcirc^{-0} = \left(\frac{1}{\circledcirc}\right)^0 = 0^0 = 1 \tag{77}$$

If so determined, it seems natural even if you compare it with the above axiom as follows

$$\vdots$$

$$0^2 = 0 \tag{78}$$

$$0^1 = 0 \tag{79}$$

$$0^0 = 1 \tag{80}$$

$$0^{-1} = \circledcirc \tag{81}$$

$$0^{-2} = \circledcirc \tag{82}$$

$$\vdots$$

$$\vdots$$

$$\circledcirc^2 = \circledcirc \tag{83}$$

$$\circledcirc^1 = \circledcirc \tag{84}$$

$$\circledcirc^0 = 1 \tag{85}$$

$$\circledcirc^{-1} = 0 \tag{86}$$

$$\circledcirc^{-2} = 0 \tag{87}$$

$$\vdots$$

Next, consider the (-1) power of $\circledcirc$.When I consider the nature of the number $\circledcirc$, it seems like an odd number if $\circledcirc$ is either an even number or an odd number. There are two reasons for it.The first one is (72) This is because it is the same form as (43) for an odd multiple of $\circledcirc$.The second one can recognize 0 as an even number, so if you think of the number circle, it is symmetrical as the number of integer points on the upper and lower sides for the $\circledcirc$ which is at the opposite position of 0 .For the above reasons it seems like an odd number if it is said that $\circledcirc$ is either an even number or an odd number. However, if $\circledcirc$ is an odd number then the following expression holds

$$(-1)^{\circledcirc} \stackrel{?}{=} -1 \tag{88}$$

Here I think as follows

$$\circledcirc^{\circledcirc} = (-\circledcirc)^{\circledcirc} = (-1)^{\circledcirc}\circledcirc^{\circledcirc} \tag{89}$$

It is decided by this formula as follows

$$(-1)^{\circledcirc} = 1 \tag{90}$$

Regarding whether it is actually an even number or an odd number, I think that it can not be determined universally, as you can see from the expression of (33).Also regarding the power of the real number x excluding 0 and $\circledcirc$

$$x^{\circledcirc} \cdot x^{\circledcirc} = x^{(\circledcirc+\circledcirc)} = x^0 = 1 \tag{91}$$

From this equation

$$x^{\circledcirc} \stackrel{?}{=} \pm 1 \tag{92}$$

Here, ± 1 means $+1$ or -1, assuming $x^{\circledcirc} \stackrel{?}{=} -1$

$$x^{\circledcirc} = x^{\circledcirc \cdot \circledcirc} \stackrel{?}{=} (-1)^{\circledcirc} = 1 \tag{93}$$

This is contradictory.In other words it seems natural to decide as follows

$$x^{\circledcirc} = 1 \tag{94}$$

Moreover, it satisfies the following expression from the axiom set up above

$$\circledcirc^{\circledcirc} = \left(\frac{1}{0}\right)^{\circledcirc} = 0^{-\circledcirc} = 0^{\circledcirc} \tag{95}$$

By a flow of discussion similar to $x^{\circledcirc}$.

$$\circledcirc^{\circledcirc} \cdot \circledcirc^{\circledcirc} = \circledcirc^{\circledcirc+\circledcirc} = \circledcirc^{0} = 1 \tag{96}$$

From the above equation

$$\circledcirc^{\circledcirc} \stackrel{?}{=} \pm 1 \tag{97}$$

Here, assuming as $\circledcirc^{\circledcirc} \stackrel{?}{=} -1$

$$\circledcirc^{\circledcirc} = \circledcirc^{\circledcirc \cdot \circledcirc} \stackrel{?}{=} (-1)^{\circledcirc} = 1 \tag{98}$$

This expression is inconsistent. Therefore, finally, from (95) you can see that it can be determined as follows

$$\circledcirc^{\circledcirc} = 1 \tag{99}$$

$$0^{\circledcirc} = 1 \tag{100}$$

7 Conclusion

In this paper, we introduce $\circledcirc$ as the concept of a new number and discussed the operation rules and others.Although the idea of axiomism advocated by Duffit Hilbert is unshakable, I think that it was not mentioned much about the legitimacy of the axioms itself.In this paper I mentioned the form of the axioms that I felt was correct. Unlike natural science, axioms are concepts that people guarantee their legitimacy. So I think that the axioms should be made on the understanding of many people.