The Pauli Objection Addressed in a Logical Way

Espen Gaarder Haug
Norwegian University of Life Sciences

June 10, 2018

Abstract

One of the greatest unsolved problems in quantum mechanics is related to time operators. Since the Pauli objection was first raised in 1933, time has only been considered a parameter in quantum mechanics and not as an operator. The Pauli objection basically asserts that a time operator must be Hermitian and self-adjoint, something the Pauli objection points out is actually not possible. Some theorists have gone so far as to claim that time between events does not exist in the quantum world. Others have explored various ideas to establish an acceptable type of time operator, such as a dynamic time operator, or an external clock that stands just outside the framework of the Pauli objection. However, none of these methods seem to be completely sound. We think that a better approach is to develop a deeper understanding of how elementary particles can be seen, themselves, as ticking clocks, and to examine more broadly how they relate to time.

Key words: Time operator, Pauli objection, Heisenberg uncertainty principle.

1 A New Consistent Time Operator

Time operators have not been commonly used in quantum mechanics. Time in quantum mecahnics has, therefore, typically been considered only a parameter, but not an operator. The main resistance against time operators can be traced back to Wolfgang Pauli's strong objection [1] regarding the existence of a self-adjoint time operator. As we understand it, the Pauli objection is closely related to the concept that energy and time will not have the same spectrum.

Pauli's objections have encountered several counterexamples, criticisms, and discussions; see, for example, [2–16]. Some have taken the Pauli objection to the extreme, and argued that time between two events is meaningless in quantum mechanics, [17], "I prove that quantum theory rules out the possibility of any quantity that one might call 'the time interval between two events.'". Others have tried to come up with creative, yet acceptable time operators by introducing dynamic time operators, or clocks that are outside the quantum system and therefore may be able to bypass the Pauli objection. Here we will suggest a logical, new time operator. Modern physics, despite enormous progress in understanding time (in particular through the work of Larmor [18] and Einstein's special relativity theory [19]), does not have an in-depth understanding of what time is or is not at the deepest quantum level. Haug has suggested a model where elementary masses are closely related to the tick of time; see [20, 21]. In this view, elementary particles may also be seen as subatomic quantum clocks. However, this new perspective does not mean we claim to have all the answers and further work is to be done.

In this paper, we will here suggest a new way to look at particles that is related to Schrödinger's [22] hypothesis in 1930 of a ("trembling motion" in German) in the electron. Schrödinger indicated that the electron was in a sort of trembling motion $\frac{2mc^2}{\hbar} \approx 1.55269 \times 10^{21}$ per second. We will suggest that the electron is in a Planck mass state $\frac{c}{\bar{\lambda}_e} \approx 7.76344 \times 10^{20}$ per second (exactly half of that of Schrödinger's "Zitterbewegung" frequency). However, each Planck mass state only lasts for one Planck second and we therefore get the normal electron mass from

$$\frac{c}{\bar{\lambda}_e} m_p \frac{l_p}{c} = \frac{\hbar}{\bar{\lambda}_e} \frac{1}{c} \approx 9.10938 \times 10^{-31} \text{ kg} \tag{1}$$

We can also look at the same idea from a slightly different angle. It is well-known that the mass of any elementary particle can be expressed as

$$m = \frac{\hbar}{\bar{\lambda}} \frac{1}{c} \tag{2}$$

This can be rewritten as

$$
\begin{aligned}
m &= \frac{\hbar}{\bar{\lambda}}\frac{1}{c} \\
m &= \frac{\frac{\hbar}{c^2}}{\frac{\bar{\lambda}}{c^2}}\frac{1}{c} \\
m &= \frac{\hbar}{c^2}\frac{1}{\frac{\bar{\lambda}}{c}}
\end{aligned} \tag{3}
$$

The part $\frac{\bar{\lambda}}{c}$ we can call the reduced Compton time t, and we then have

$$
m = \frac{\hbar}{c^2}\frac{1}{t} \tag{4}
$$

Be aware that $\frac{\hbar}{c^2}$ is indeed identical to the Planck mass times one Planck second. Further, the plane wave function of the Klein–Gordon equation can be written as

$$
\Psi = e^{\frac{i}{\hbar}(px-Et)} \tag{5}
$$

Replacing the momentum, p, and the energy E, with their relativistic formulas gives

$$
\begin{aligned}
\Psi &= e^{\frac{i}{\hbar}\left(\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}x-\left(\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}-mc^2\right)t\right)} \\
\Psi &= e^{\frac{i}{\hbar}\left(\frac{\frac{\hbar}{c^2}\frac{1}{t}v}{\sqrt{1-\frac{v^2}{c^2}}}x-\frac{\frac{\hbar}{c^2}\frac{1}{t}c^2}{\sqrt{1-\frac{v^2}{c^2}}}t+\frac{\hbar}{c^2}\frac{1}{t}c^2t\right)} \\
\Psi &= e^{\frac{i}{\hbar}\left(\frac{\frac{\hbar}{c^2}\frac{1}{t}v}{\sqrt{1-\frac{v^2}{c^2}}}x-\frac{\hbar}{\sqrt{1-\frac{v^2}{c^2}}}+\hbar\right)}
\end{aligned} \tag{6}
$$

Now, taking the partial derivative with respect to the plane wave function with respect to time we get

$$
\begin{aligned}
\frac{\partial\Psi}{\partial t} &= -\frac{ix}{\hbar t}\frac{\frac{\hbar}{c^2}\frac{1}{t}v}{\sqrt{1-\frac{v^2}{c^2}}}e^{\frac{i}{\hbar}\left(\frac{\frac{\hbar}{c^2}\frac{1}{t}v}{\sqrt{1-\frac{v^2}{c^2}}}x-\frac{\hbar}{\sqrt{1-\frac{v^2}{c^2}}}+\hbar\right)} \\
\frac{\partial\Psi}{\partial t} &= -\frac{ix}{\hbar t}\frac{\frac{\hbar}{c^2}\frac{1}{t}v}{\sqrt{1-\frac{v^2}{c^2}}}\Psi \\
\frac{\partial\Psi}{\partial t} &= -\frac{ix}{\hbar t}p\Psi
\end{aligned} \tag{7}
$$

remember $t = \frac{\bar{\lambda}}{c}$ and if we set x equal to the reduced Compton wavelength, that is $t = \bar{\lambda}$, and we get

$$
i\frac{\hbar}{c}\frac{\partial\Psi}{\partial t} = p\Psi \tag{8}
$$

This means that the time momentum operator is

$$
\hat{p} = i\frac{\hbar}{c}\frac{\partial}{\partial t} \tag{9}
$$

and the time operator we suggest is simply $\hat{t} = t$. These two operators are time operators: the momentum time operator and the time operator. Based on its construction, this time operator we are quite sure must be Hermitian and self-adjoint. In other words, the Pauli objection likely does not hold in this instance. What quantum mechanics seems to have been missing is that elementary particles are functions of time; they are quantum clocks that tick in every reduced Compton time period. Each tick is the Planck mass that lasts for one Planck second $m_p t_p = \frac{\hbar}{c^2}$.

Next we will check to see whether the momentum operator and time operator commute or not

$$
\begin{aligned}
[\hat{p}, \hat{t}]\Psi &= [\hat{p}\hat{t} - \hat{t}\hat{p}]\Psi \\
&= \left(i\frac{\hbar}{c}\frac{\partial}{\partial t}\right)(t)\Psi - (t)\left(i\frac{\hbar}{c}\frac{\partial}{\partial t}\right)\Psi \\
&= i\frac{\hbar}{c}\left(\Psi + t\frac{\partial \Psi}{\partial (t)}\right) - i\hbar t\frac{\partial \Psi}{\partial (t)} \\
&= i\frac{\hbar}{c}\left(\Psi + t\frac{\partial \Psi}{\partial (t)} - \frac{\partial \Psi}{\partial (t)}\right) \\
&= i\frac{\hbar}{c}\Psi
\end{aligned} \tag{10}
$$

As we can see they do not commute. Further, we get the following uncertainty relation

$$
\begin{aligned}
\sigma_p\sigma_t &\geq \frac{1}{2}|\int \Psi^*[\hat{p}, \hat{t}]\Psi dt| \\
&\geq \frac{1}{2}|\int \Psi^*(i\frac{\hbar}{c})\Psi dt| \\
&\geq \frac{1}{2}|i\frac{\hbar}{c}\int \Psi^*\Psi dt|
\end{aligned} \tag{11}
$$

and since $\int \Psi^*\Psi dt$ must sum to 1, we are left with

$$
\begin{aligned}
\sigma_p\sigma_t &\geq \frac{1}{2}|i\frac{\hbar}{c}| \\
\sigma_p\sigma_t &\geq \frac{\hbar}{2}\frac{1}{c}
\end{aligned} \tag{12}
$$

That is, we have a new momentum time uncertainty principle in addition to the known momentum position uncertainty principle.

One may then wonder if multiplying each side with the speed of light c one will obtain the energy time uncertainty relationship given by Heisenberg

$$
\begin{aligned}
\sigma_p c\sigma_t &\geq \frac{\hbar}{2}\frac{1}{c}c \\
\sigma_E\sigma_t &\geq \frac{\hbar}{2}
\end{aligned} \tag{13}
$$

For many years, researchers in the field have questioned if the energy time uncertainty principle is valid, as it has been assumed that one cannot have a time operator. Here we have suggested a new perspective on elementary particles that indicates they are clocks ticking discretely through each reduced Compton time interval. Each elementary particle is a Planck mass for one Planck second during every reduced Compton time period. This would mean the elementary particles actually have a time tick quantization that correlates perfectly with the energy spectra. We need no dynamic time operator or no external clocks to try to get around the Pauli objection; we simply do so by understanding that elementary particles can be seen as quantum clocks. This means energy must come in quanta of $f\hbar$, where f is the frequency. This also means mass comes in quanta of $\frac{\hbar}{c^2}$, which is the mass gap.

2 Uncertainty Collapses at the Planck Scale

Haug [20, 23, 24] has recently published a paper showing that the maximum velocity for anything with rest-mass is given by

$$
v_{max} = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}} \tag{14}
$$

Based on this we have

$$\Psi = e^{\frac{i}{\hbar}\left(\frac{\frac{\hbar}{c^2}\frac{1}{t}v_{max}}{\sqrt{1-\frac{v_{max}^2}{c^2}}}x-\frac{\hbar}{\sqrt{1-\frac{v_{max}^2}{c^2}}}+\hbar\right)}$$

$$\Psi = e^{\frac{i}{\hbar}\left(\frac{\frac{\hbar}{c^2}\frac{1}{t}c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}{\sqrt{1-\frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}}x-\frac{\hbar}{\sqrt{1-\frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}}+\hbar\right)} \tag{15}$$

in the special case of a Planck mass particle, we have $\bar{\lambda} = l_p$, and thereby a maximum velocity $v_{max} = c\sqrt{1-\frac{l_p^2}{l_p^2}} = 0$, that is zero velocity. This means the wave function for the Planck mass particle is

$$\Psi_p = e^{\frac{i}{\hbar}\left(\frac{\frac{\hbar}{c^2}\frac{1}{t}\times 0}{\sqrt{1-\frac{0^2}{c^2}}}x-\frac{\hbar}{\sqrt{1-\frac{0^2}{c^2}}}+\hbar\right)} = e^0 = 1 \tag{16}$$

This means the square of the wave function is always one, $\Psi^2 = 1$, for a Planck particle, indicating that there is no uncertainty in the Planck mass particle. Further, taking the partial derivative of the wave function with respect to time, we get

$$\frac{\partial \Psi_p}{\partial t} = 0 \tag{17}$$

This means the momentum time operator is zero in the special case of the Planck mass particle. In all other cases, it is $\hat{p} = i\frac{\hbar}{c}\frac{\partial}{\partial t}$. This means the time operators are always non-commuting for all non-Planck mass particles, $[\hat{p}, \hat{t}] = i\frac{\hbar}{c}$. But they do commute for Planck mass particles, $[\hat{p}, \hat{t}] = 0$. This means the uncertainty principle for momentum, energy, and time collapses at the Planck scale and become the certainty principle. Again, since we predict the Planck mass particle only lasts for one Planck second, this means the certainty only lasts for one Planck second. This is based on the idea that Haug's newly introduced maximum velocity holds, something that should be investigated more thoroughly. It can be seen as a new and alternative extension of quantum mechanics that should be examined in-depth in the future.

3 Conclusion

We have suggested that all elementary particles are time dependent at the quantum scale (at time periods linked to the reduced Compton time). This idea seems to give us a time operator that is sound and does not conflict with the Pauli objection. That is to say, we have a new momentum time uncertainty principle that may lead towards new directions in the study of time and physics. Establishing a consistent time operator could be important in making fresh progress in quantum gravity [25], for example. Clearly, more work is needed on this area of quantum physics.

References

[1] W. Pauli. Die allgemeinen prinzipien der wellenmechanik. *Springer, Berlin, p.84*, 190.

[2] J. Kijowski. On the time operator in quantum mechanics and the Heisenberg uncertainty relation for energy and time. *Reports on Mathematical Physics*, 6(3), 1974.

[3] V. S. Olkkovsky, E. Recami, and A. J. Gerasimchuk. Time operator in quantum mechanics. *IL Nuovo Cimento*, 22(2), 1974.

[4] P. Busch, M. Grabowski, and P. J. Lahtic. Time observables in quantum theory. *Physics Letters A*.

[5] M. Bauer. A time operator in quantum mechanics. *Annals of Physics*, 150, 1983.

[6] N. Grot, C. Rovelli, and R. S. Tate. Time of arrival in quantum mechanics. *Phys. Rev. A*, 54(6), 1996.

[7] J. Kijowski. Comment on the "arrival time" in quantum mechanics. *Phys. Rev. A*, 59, 1999.

[8] E. A. Galapon. Self-adjoint time operator is the rule for discrete semibounded Hamiltonians. *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 458(2027), 2002.

[9] E. A. Galapon. Pauli's theorem and quantum canonical pairs: The consistency of a bounded, self-adjoint time operator canonically conjugate to a Hamiltonian with non-empty point spectrum. *Philosophical Transactions of the Royal Society of London,*, 458(2018), 2002.

[10] L. Nanni. A new derivation of the time-dependent Schrödinger equation from wave and matrix mechanics. *Advances in Physics Theories and Applications*, 43, 2005.

[11] Z. Y. Wang and C. Xiong. How to introduce time operator. *Annals of Physics*, 322(10), 2007.

[12] M. Bauer. A dynamical time operator in Dirac's relativistic quantum mechanics. *International Journal of Modern Physics A*, 54(6), 2014.

[13] C. Kinyanjui and D. S. Wamalwa. On the existence of a non-relativistic hermitian time operator. *International Journal of Pure and Applied Mathematics*, 110(3), 2016.

[14] S. Khorasani. Time operator in relativistic quantum mechanics. *Communications in Theoretical Physics*, 68(1), 2017.

[15] M. Bauer. On the problem of time in quantum mechanics. *European Journal of Physics*, 38(3), 2017.

[16] J. Leon and L. Maccone. The Pauli objection. *The Foundations of Physics*, 47(12), 2017.

[17] H. Halvorsen. Does quantum theory kill time? *Working paper, Princeton University*, 2010.

[18] J. J. Larmor. *Aether and Matter: A Development of the Dynamical Relations of the Aether to Material Systems*. Cambridge University Press, 1900.

[19] A. Einstein. On the electrodynamics of moving bodies. *Annalen der Physik, English translation by George Barker Jeffery 1923*, (17), 1905.

[20] E. G. Haug. *Unified Revolution: New Fundamental Physics*. Oslo, E.G.H. Publishing, 2014.

[21] E. G. Haug. The mystery of mass as understood from atomism. *Working paper Norwegian University of Life Sciences*, 2018.

[22] E. Schrödinger. Über die kräftefreie bewegung in der relativistischen quantenmechanik. *Sitzungsberichte der Preuischen Akademie der Wissenschaften. Physikalisch-mathematische Klasse*, 1930.

[23] E. G. Haug. The gravitational constant and the Planck units. A simplification of the quantum realm. *Physics Essays Vol 29, No 4*, 2016.

[24] E. G. Haug. The ultimate limits of the relativistic rocket equation. The Planck photon rocket. *Acta Astronautica*, 136, 2017.

[25] M. Bauer. Quantum gravity and a time operator in relativistic quantum mechanics. *arXiv:1605.01659*, 2016.