The wavefunction as an energy propagation mechanism

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Abstract :

Benefitting from valuable feedback, this article corrects some defects in the *physical* interpretation of the wavefunction that I had offered – and elaborated upon – in two previous pre-publication papers (see: http://vixra.org/abs/1709.0390 and http://vixra.org/abs/1712.0201). Most importantly, this paper incorporates *relativistically correct* formulas for the proposed interpretation of the energy of an electron as a two-dimensional oscillation of a pointlike charge in space.

The relativistic correction does not change any of the conclusions. For example, the interpretation of the wavefunction as an energy diffusion equation still holds. However, this paper defines the weaknesses in the approach (read: the agenda for my personal future research) much better. I have benefited a lot from comments on the previous papers and, therefore, I hope I will get the same enthusiastic reaction to this one.

Contents

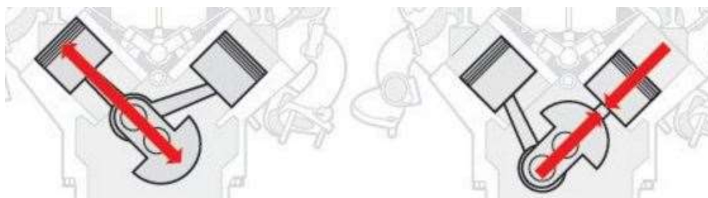
I. Energy as a two-dimensional oscillation

The mathematical structure of (1) Einstein's E = mc^2 postulate and the energy formulas for (2) a harmonic oscillator and (3) the kinetic energy of a moving body, are remarkably similar:

1. $E = mc^2$
2. $E = (½)ma^2\omega^2 = (½)mv_t^2$
3. $E = (½)mv^2$

The c, v_t and v are all velocities. There is the ½ factor, of course, and only the E = mc^2 is relativistically correct – but I will provide the relativistically correct modifications soon. Let us first see where we get with a classical analysis for an oscillator. Can we, somehow, combine two oscillators to get rid of the ½ factor? To focus the mind, we may think of a *perpetuum mobile* like the one below: an engine with the pistons at a 90-degree angle.

Figure 1: Oscillations in two dimensions



The assumption is that the equilibrium point for each piston is at the center of the cylinder: at that point, the pressure of the air inside of the piston will equal the pressure outside. We can ensure this equality, when we set up the machine, by opening the valves at that point. We then permanently close the valves. Hence, the air pressure in the cylinder will be higher than the pressure outside if the piston moves above the center of its motion, and the overpressure will, therefore, provide a restoring force. Conversely, if the piston moves below the center, then the pressure outside will be higher than the pressure inside, and will, therefore, provide a restoring force in the opposite direction. Combining two cylinders in a 90° angle (and assuming no friction, of course) should give us a *perpetuum mobile*.

The idea is inspired by the efficiency of a two-cylinder V-twin engine: the 90° angle makes it possible to perfectly balance the counterweight and the pistons, thereby ensuring smooth travel at all times. We might also think of connecting springs with the crankshaft. In fact, we should switch to a spring-based perpetuum mobile because it is easier to describe (but, therefore, it is also even more boring). In any case, the mechanical implementation is, obviously, irrelevant. The crux of the argument relies in (1) the assumption of a linear restoring force: $F = -kx$ and (2) the 90° angle between the two oscillators.

We have a great *metaphor* here. Somehow, in this beautiful interplay between linear and circular motion, energy is borrowed and returned from one place, cycle after cycle, and the system – as a whole – stores some *net* energy. Let us quickly review the math. If the magnitude of the oscillation is equal to a, then the motion of the piston (or the mass on a spring) will be described by $x = a\cdot\cos(\omega\cdot t + \Delta)$.[1] Needless to say, Δ is just a phase factor which defines our t = 0 point, and ω is the natural (angular) frequency of our oscillator. Because of the 90° angle between the two cylinders, Δ will be 0 for one oscillator, and –π/2 for the other. Hence, the motion of one piston is given by $x = a\cdot\cos(\omega\cdot t)$, while the motion of the other is given by $x = a\cdot\cos(\omega\cdot t - \pi/2) = a\cdot\sin(\omega\cdot t)$. The kinetic and potential energy of *one* oscillator (think of one piston or one spring only) can then be calculated as:

[1] Because of the sideways motion of the connecting rods, the sinusoidal function will describe the linear motion only *approximately,* but one can easily imagine the idealized limit situation.

1. K.E. = T = $(½)\cdot m\cdot v^2 = (½)\cdot m\cdot\omega^2\cdot a^2\cdot\sin^2(\omega\cdot t + \Delta)$
2. P.E. = U = $(½)\cdot k\cdot x^2 = (½)\cdot k\cdot a^2\cdot\cos^2(\omega\cdot t + \Delta)$

The coefficient k in the potential energy formula characterizes the restoring force: $F = -k\cdot x$. For a spring, k will be the *stiffness* of the spring, and we can write it as $k = m\cdot\omega^2$. The model with springs is easier because we would have to invoke the ideal gas law (PV = NkT) to prove the linearity of the restoring force for our V-twin engine, and the equivalent of the *stiffness* coefficient would be somewhat harder to describe. Adding these two, we find that the total energy of *one* oscillator is equal to:

$$E = T + U = (½)\cdot m\cdot\omega^2\cdot a^2\cdot[\sin^2(\omega\cdot t + \Delta) + \cos^2(\omega\cdot t + \Delta)] = (½)\cdot m\cdot a^2\cdot\omega^2/2$$

To facilitate the calculations, we will briefly assume $k = m\cdot\omega^2$ and a are equal to 1.[2] The motion of our first oscillator then simplifies to $\cos(\omega\cdot t) = \cos\theta$, and its kinetic energy will be equal to $(½)\cdot\sin^2\theta$. Hence, the (instantaneous) *change* in kinetic energy at any point in time will be equal to:

$$d[(½)\cdot\sin^2\theta]/d\theta = (½)\cdot 2\cdot\sin\theta\cdot d(\sin\theta)/d\theta = \sin\theta\cdot\cos\theta$$

The motion of the second second oscillator is given by the $\cos(\theta-\pi/2) = \sin(\omega\cdot t) = \sin\theta$ function, and its kinetic energy is equal to $(½)\cdot\sin^2(\theta-\pi/2) = (½)\cdot\cos^2\theta$. Its time rate of change is, therefore, equal to:

$$d[(½)\cdot\cos^2\theta]/d\theta = (½)\cdot 2\cdot\cos\theta\cdot d(\sin\theta)/d\theta = -\sin\theta\cdot\cos\theta$$

We have our *perpetuum mobile*! The kinetic energy that is *absorbed* by one piston always neatly matches the kinetic energy that is being *delivered* by the other. Hence, the crankshaft will rotate with a constant angular velocity: linear motion becomes circular motion, and vice versa.

What is the total energy that is being stored in the system? The answer to this question is less obvious than it may seem, because the *potential* energy in the oscillators changes all the time. Let us, therefore, calculate *averages*. The average of the $\sin^2\theta$ and $\cos^2\theta$ functions is ½. Hence, the average potential energy the *two* oscillators is equal to $2\cdot(½)\cdot k\cdot a^2\cdot(½) = (½)\cdot k\cdot a^2 = (½)\cdot m\cdot\omega^2\cdot a^2$. The sum of the (average) kinetic energies is, likewise, equal to $2\cdot(½)\cdot m\cdot\omega^2\cdot a^2\cdot(½) = (½)\cdot m\cdot\omega^2\cdot a^2$. Hence, the total energy in the system would be equal to what we had secretly hoped it would be:

$$E = ma^2\omega^2 = k\cdot a^2$$

We got rid of the ½ factor! But perhaps we should not take averages. Rather than adding averages, let us just add the potential and kinetic energy of both oscillators at *any* point in time:

1. For the kinetic energies, we get: $(½)\cdot m\cdot\omega^2\cdot a^2\cdot\sin^2(\omega\cdot t) + (½)\cdot m\cdot\omega^2\cdot a^2\cdot\cos^2(\omega\cdot t) =$ $(½)\cdot m\cdot\omega^2\cdot a^2\cdot[\sin^2(\omega\cdot t) + \cos^2(\omega\cdot t)] = (½)\cdot m\cdot\omega^2\cdot a^2 = (½)\cdot k\cdot a^2$.

2. For the potential energies, we get the same: $(½)\cdot k\cdot a^2\cdot\cos^2(\omega\cdot t) + (½)\cdot k\cdot a^2\cdot\sin^2(\omega\cdot t) =$ $(½)\cdot k\cdot a^2\cdot[\cos^2(\omega\cdot t) + \sin^2(\omega\cdot t)] = (½)\cdot k\cdot a^2$.

Adding both yields the same result: $E = ma^2\omega^2 = k\cdot a^2$.

What about the energy of the flywheel? Should we add it?

That is a great question. In *real life*, we should, obviously, add it, because the flywheel will have some mass of its own. To be precise, the kinetic energy of the flywheel would be equal to K.E. = $(½)\cdot I\cdot\omega^2$. I is the *angular* mass of the flywheel in this formula. The angular mass depends on the *distribution* of the mass of the flywheel. For example, if we imagine it to be a flat disc, then $I = m\cdot r^2$. If it's a hoop or a point mass at distance r (as measured from the axis of rotation), then it's equal to $I = (½)\cdot m\cdot r^2$.

[2] The various factors are the same for both oscillators and, hence, the simplification does not make any difference in terms of the analysis. We just didn't want to have any unnecessary clutter.

However, we are *not* imagining some actual *mass* going around here. We are thinking about the *fabric* of space, and how it could possibly sustain a two-dimensional oscillation of a pointlike charge which is – in the model that we are presenting here – might be represented by the elementary wavefunction[3]:

$$\psi(\theta) = a\cdot e^{-i\cdot\theta} = a\cdot e^{-i\cdot E\cdot t/\hbar} = a\cdot\cos[(E/\hbar)\cdot t] - i\cdot a\cdot\sin[(E/\hbar)\cdot t]$$

The goal is actually more ambitious: if an electromagnetic wave is a propagation mechanism for energy, can we interpret the wavefunction in a similar way? To assuage the first immediate fears of the reviewers, I should add two obvious remarks here:

1. An *actual* particle is always localized in space and can, therefore, *not* be represented by the elementary wavefunction. We must build a wave *packet* for that: a sum of wavefunctions, each with its own amplitude a_k and its own argument $\theta_k = (E_k\cdot t - \mathbf{p}_k\cdot\mathbf{x})/\hbar$. However, this is a paper about first principles only.

2. The elementary wavefunction above is the $\psi = a\cdot e^{-i[E\cdot t - \mathbf{p}\cdot\mathbf{x}]/\hbar} = a\cdot\cos(\mathbf{p}\cdot\mathbf{x}/\hbar - E\cdot t/\hbar) + i\cdot a\cdot\sin(\mathbf{p}\cdot\mathbf{x}/\hbar - E\cdot t/\hbar)$ function for $\mathbf{p} = \mathbf{0}$, or for an electron at rest – whatever that may be. Hence, the t in the argument is the *proper* time, which we should probably denote by t'. Indeed, the E and p in the argument of the wavefunction $\theta = \omega\cdot t - k\cdot x = (E/\hbar)\cdot t - (p/\hbar)\cdot x = (E\cdot t - p\cdot x)/\hbar$ are, of course, the energy and momentum as measured in *our* frame of reference. Hence, we will want to write these quantities as $E = E_v$ and $p = p_v = p_v\cdot v$. If we then use *natural* units (hence, the *numerical* value of *c* and ħ is equal to 1), we can relate the energy and momentum of a moving object to its energy and momentum when at rest using the following relativistic formulas:

$$E_v = E_0/\sqrt{(1-v^2)} = m_0/\sqrt{(1-v^2)} \text{ and } p_v = m_v\cdot v = m_0\cdot v/\sqrt{(1-v^2)} = E_0\cdot v/\sqrt{(1-v^2)}$$

Needless to say, *v* is the (relative) velocity here and, therefore, has a value between 0 and 1. The argument of the wavefunction can then be re-written as:

$$\theta = [E_0/\sqrt{(1-v^2)}]\cdot t - [E_0\cdot v/\sqrt{(1-v^2)}]\cdot x = E_0\cdot(t - v\cdot x)/\sqrt{(1-v^2)}$$

$$\Leftrightarrow \theta = E_0\cdot t' \text{ with } t' = (t - v\cdot x)/\sqrt{(1-v^2)}$$

3. We know we should not mix relativistic and non-relativistic equations. Hence, let us quickly look at that. The relativistally correct force equation for one oscillator is:

$$F = dp/dt = F = -kx \text{ with } p = m_v v = \gamma m_0 v$$

Multiplying both sides with $v = dx/dt$ yields the following *energy conservation* expression:

$$v\frac{d(\gamma m_0 v)}{dt} = -kxv \Leftrightarrow \frac{d(m_v c^2)}{dt} = -\frac{d}{dt}\left[\frac{1}{2}kx^2\right] \Leftrightarrow \frac{dE}{dt} = \frac{d}{dt}\left[\frac{1}{2}kx^2 + mc^2\right] = 0$$

We recognize the potential energy (it is the same $(½)\cdot k\cdot x^2$ formula). However, the $(½)\cdot m_0\cdot v^2$ term that we would get when using the non-relativistic formulation of Newton's Law is now replaced by the $m\cdot c^2 = m_0\cdot\gamma\cdot c^2$ term.

Hence, we must be doing *something* right here. What is that we are thinking of?

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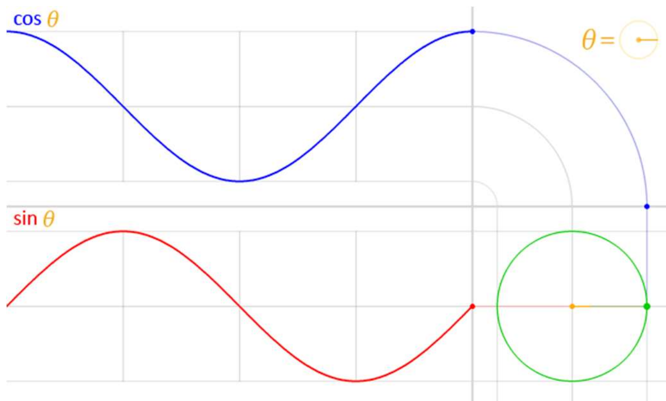
[3] This is the $\psi = a\cdot e^{-i[E\cdot t - \mathbf{p}\cdot\mathbf{x}]/\hbar} = a\cdot\cos(\mathbf{p}\cdot\mathbf{x}/\hbar - E\cdot t/\hbar) + i\cdot a\cdot\sin(\mathbf{p}\cdot\mathbf{x}/\hbar - E\cdot t/\hbar)$ function for an electron at rest ($\mathbf{p} = \mathbf{0}$), whatever that may be. We know that an *actual* particle is localized in space and can, therefore, *not* be represented by the elementary wavefunction. We must build a wave *packet* for that: a sum of wavefunctions, each with its own amplitude a_k and its own argument $\theta_k = (E_k\cdot t - \mathbf{p}_k\cdot\mathbf{x})/\hbar$. However, this is a paper about first principles only.

II. The wavefunction as a two-dimensional oscillation

The argument of the wavefunction rotates *clockwise* with time, while the mathematical convention for measuring the phase angle (φ) is *counter*-clockwise. The convention doesn't matter. From the Stern-Gerlach experiment, we know that the angular momentum of an electron is equal to ± ħ/2, and our model should accommodate both values. The illustration below imagines a pointlike charge (the green dot) to spin around some center in either of the two possible directions. The *cosine* keeps track of the oscillation in one dimension, while the *sine* (plus or minus) keeps track of the oscillation in a direction that is perpendicular to the first one. We will say more about the possible directions in the next section.

Figure 2: A pointlike charge in orbit



At this point, we would like to share the original calculations which got us thinking. If the mathematical similarity between the the $E = m \cdot a^2 \cdot \omega^2$ and $E = m \cdot c^2$ formulas would represent something real, then we need to give some meaning to the $c = a \cdot \omega$. Now, if we assume, just for fun, that E and m are the energy and mass of an electron, then the *de Broglie* relations suggest we should equate ω to E/ħ. As for *a,* the *Compton scattering radius* of the electron (ħ/(m·*c*)) would be a more likely candidate than, say, the Bohr radius, or the Lorentz radius. Why? Because we're not looking at an electron in orbit around a nucleus (Bohr radius), and we're also not looking at the size of the *charge* itself (classical electron radius). Let's see what we get:

$$a \cdot \omega = [\hbar/(m \cdot c)] \cdot [E/\hbar] = E/(m \cdot c) = m \cdot c^2/(m \cdot c) = c$$

Wow! Did we just *prove* something? No. We don't prove anything in this article. We only showed that our $E = m \cdot a^2 \cdot \omega^2 = m \cdot c^2$ equation *might* (note the emphasis: *might*) make sense.

Let me show you something else. If this *flywheel model* of an electron makes sense, then we can, obviously, also calculate a *tangential velocity* for our charge. The tangential velocity is the product of the *radius* and the *angular* velocity: $v = r \cdot \omega = a \cdot \omega = c$.

Wow! Did we just *prove* something? Is an electron nothing but a point charge that is spinning around some center at… Well… The speed of light?

Maybe. But probably not. We need to explain the *mass* of our electron here, and that's not so easy because we say it is basically a possibly massless point charge going up and down, and back and forth. So we need to calculate the *equivalent* mass of the energy of that oscillation. We are, of course, talking about the *electromagnetic* mass of a charge, but I am not aware of any model that does that what we want to there, and that is to calculate the electromagnetic mass of a charge that is simply moving up and down in a harmonic oscillation. There is also the added complication that an oscillating charge should radiate its energy away, so we would need an explanation of why that is *not* happening.

Hence, the situation is very complicated. We need a formula for the electromagnetic mass of a zero mass charge oscillating along one axis – let's denote that by m_{elec} – and that mass would be the *effective* mass for an oscillation that is perpendicular to the first one. Fortunately, the two motions are, effectively, independent (because of the 90° angle between them). Having said that, it is easy to see that the final calculation might be quite complicated.

Having said that, we believe the basic ideas might be valid:

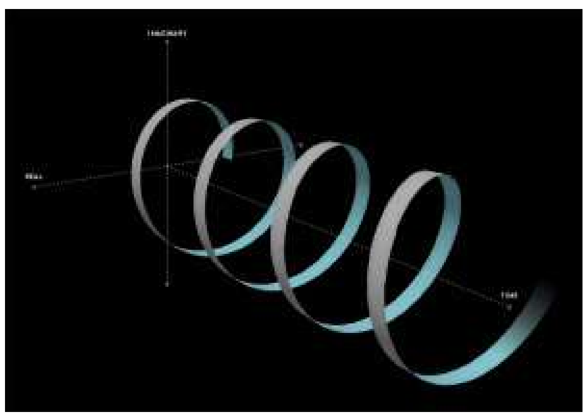
1. A charge with zero rest mass will acquire some *electromagnetic* mass when linearly oscillating.
2. This electromagnetic mass provides an *anchor* for a linear oscillation in a direction that is perpendicular to the original one.
3. Maxwell's propagation mechanism for an electromagnetic wave may ensure such two-dimensional oscillation is sustainable and *propagates* itself, so to speak.

Let us further explore the similarities – and differences – between Maxwell's equations and Schrödinger's equation.

III. The wavefunction as a propagation mechanism

The wavefunction is usually represented as shown below: the real and imaginary component are shown as being perpendicular to the direction of propagation of the wavefunction. Note how the phase difference between the cosine and the sine – the real and imaginary part of our wavefunction – appears to give some spin to the whole.

Figure 3: Customary geometric representation of the wavefunction



The basic intuition here might be correct: the real and imaginary component of the wavefunction may each carry half of the total energy of the particle, and the interplay between the real and the imaginary part of the wavefunction may describe how energy propagates through space over time. If so, we can build up a wave *packet* that might represent an *actual* particle (i.e. a particle that is *localized* in space): a sum of wavefunctions, each with their own amplitude a_k, and their own $\omega_i = -E_i/\hbar$. Each of these wavefunctions will *contribute* some energy to the total energy of the wave packet. To calculate the contribution of each wave to the total, both a_i as well as E_i will matter.

What is E_i? E_i varies around some average E, which we can associate with some *average mass* m: m = E/c^2. The Uncertainty Principle kicks in here. The analysis becomes more complicated, but a formula such as the one below might make sense:

$$\mathrm{E} = \sum \mathrm{m}_i \cdot a_i^2 \cdot \omega_i^2 = \sum \frac{\mathrm{E}_i}{c^2} \cdot a_i^2 \cdot \frac{\mathrm{E}_i{}^2}{\hbar^2}$$

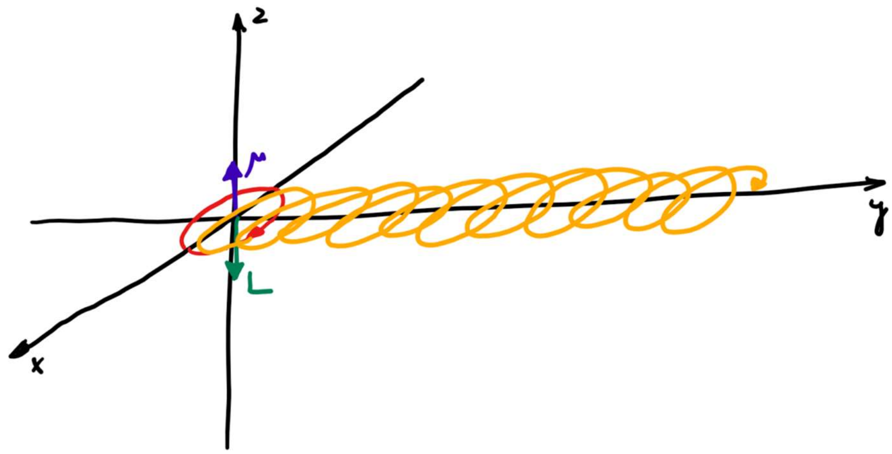
We can re-write this as:

$$c^2\hbar^2 = \frac{\sum a_i^2 \cdot \mathrm{E}_i^3}{\mathrm{E}} \Leftrightarrow c^2\hbar^2\mathrm{E} = \sum a_i^2 \cdot \mathrm{E}_i^3$$

What is the meaning of this equation? We may look at it as some sort of *physical* normalization condition when building up the *Fourier sum*. This can then easily be related to the *mathematical* normalization condition for the wavefunction.

But we first need to agree on the basics. In light of the Stern-Gerlach experiment (and the orientation of the magnetic moment it implies), we should not exclude an alternative model of propagation, such as the one suggested below. 😊

Figure 4: Alternative model of propagation



IV. Schrödinger's equation as an energy diffusion equation

The interpretation of Schrödinger's equation as a diffusion equation is straightforward. Feynman (*Lectures*, III-16-1) briefly summarizes it as follows:

> "We can think of Schrödinger's equation as describing the diffusion of the probability amplitude from one point to the next. […] But the imaginary coefficient in front of the derivative makes the behavior completely different from the ordinary diffusion such as you would have for a gas spreading out along a thin tube. Ordinary diffusion gives rise to real exponential solutions, whereas the solutions of Schrödinger's equation are complex waves."[4]

Let us review the basic math. For a particle moving in free space – with no external force fields acting on it – there is no potential (U = 0) and, therefore, the Uψ term disappears. Therefore, Schrödinger's equation reduces to:

[4] Feynman further formalizes this in his *Lecture on Superconductivity* (Feynman, III-21-2), in which he refers to Schrödinger's equation as the "equation for continuity of probabilities". The analysis is centered on the *local* conservation of energy, which confirms the interpretation of Schrödinger's equation as an energy diffusion equation.

$$\partial\psi(\mathbf{x}, t)/\partial t = i\cdot(1/2)\cdot(\hbar/m_{eff})\cdot\nabla^2\psi(\mathbf{x}, t)$$

The ubiquitous diffusion equation in physics is:

$$\partial\phi(\mathbf{x}, t)/\partial t = D\cdot\nabla^2\phi(\mathbf{x}, t)$$

The *structural* similarity is obvious. The key difference between both equations is that the wave equation gives us *two* equations for the price of one. Indeed, because ψ is a complex-valued function, with a *real* and an *imaginary* part, we get the following equations[5]:

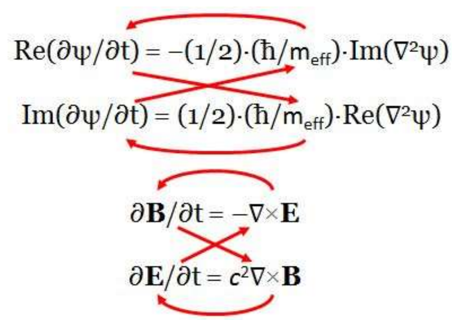
1. $Re(\partial\psi/\partial t) = -(1/2)\cdot(\hbar/m_{eff})\cdot Im(\nabla^2\psi)$
2. $Im(\partial\psi/\partial t) = (1/2)\cdot(\hbar/m_{eff})\cdot Re(\nabla^2\psi)$

These equations make us think of the equations for an electromagnetic wave in free space (no stationary charges or currents):

1. $\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}$
2. $\partial\mathbf{E}/\partial t = c^2\nabla\times\mathbf{B}$

The above equations effectively describe a *propagation* mechanism in spacetime, as illustrated below.

Figure 5: Propagation mechanisms



The Laplacian operator (∇^2), when operating on a *scalar* quantity, gives us a flux density, i.e. something expressed per square meter ($1/m^2$). In this case, it is operating on ψ(**x**, t), so what is the dimension of our wavefunction ψ(**x**, t)? To answer that question, we should analyze the diffusion constant in Schrödinger's equation, i.e. the $(1/2)\cdot(\hbar/m_{eff})$ factor:

1. As a *mathematical* constant of proportionality, it will *quantify* the relationship between both derivatives (i.e. the time derivative and the Laplacian);
2. As a *physical* constant, it will ensure the *physical dimensions* on both sides of the equation are compatible.

Now, the $\hbar/m_{eff}$ factor is expressed in $(N\cdot m\cdot s)/(N\cdot s^2/m) = m^2/s$. Hence, it does ensure the dimensions on both sides of the equation are, effectively, the same: ∂ψ/∂t is a time derivative and, therefore, its

[5] The m_{eff} is the *effective* mass of the particle, which depends on the medium. For example, an electron traveling in a solid (a transistor, for example) will have a different effective mass than in an atom. In free space, we can drop the subscript and just write $m_{eff} = m$. As for the equations, they are easily derived from noting that two complex numbers $a + i\cdot b$ and $c + i\cdot d$ are equal if, and only if, their real and imaginary parts are the same. Now, the $\partial\psi/\partial t = i\cdot(\hbar/m_{eff})\cdot\nabla^2\psi$ equation amounts to writing something like this: $a + i\cdot b = i\cdot(c + i\cdot d)$. Now, remembering that $i^2 = -1$, you can easily figure out that $i\cdot(c + i\cdot d) = i\cdot c + i^2\cdot d = -d + i\cdot c$.

dimension is s^{-1} while, as mentioned above, the dimension of $\nabla^2\psi$ is m^{-2}. However, this does not solve our basic question: what is the dimension of the real and imaginary part of our wavefunction?

At this point, mainstream physicists will say: it does not have a physical dimension, and there is no geometric interpretation of Schrödinger's equation. One may argue, effectively, that its argument, (**p**·**x** – E·*t*)/ħ, is just a number and, therefore, that the real and imaginary part of ψ is also just some number.

To this, we may object that ħ may be looked as a *mathematical* scaling constant only. If we do that, the argument of ψ will, effectively, be expressed in *action* units, i.e. in N·m·s. It then does make sense to also associate a physical dimension with the real and imaginary part of ψ. What could it be?

We would like to think it is just the same as **E** and **B** because, when everything is said and done, the force needs something to grab on, and because our charge has no (rest) mass, the charge is the only thing the force can grab onto.

V. Energy densities and energy flows

Pursuing the geometric equivalence between the equations for an electromagnetic wave and Schrödinger's equation, we can now, perhaps, see if there is an equivalent for the energy density. For an electromagnetic wave, we know that the energy density is given by the following formula:

$$u = \frac{\epsilon_0}{2}\mathbf{E}\cdot\mathbf{E} + \frac{\epsilon_0 \cdot c^2}{2}\mathbf{B}\cdot\mathbf{B}$$

E and **B** are the electric and magnetic field vector respectively. The Poynting vector will give us the directional energy flux, i.e. the energy flow per unit area per unit time. We write:

$$\frac{\partial u}{\partial t} = -\nabla\cdot\boldsymbol{S}$$

Needless to say, the ∇· operator is the divergence and, therefore, gives us the magnitude of a (vector) field's *source* or *sink* at a given point. To be precise, the divergence gives us the volume density of the outward *flux* of a vector field from an infinitesimal volume around a given point. In this case, it gives us the *volume density* of the flux of ***S***.

We can analyze the dimensions of the equation for the energy density as follows:

1. **E** is measured in *newton per coulomb*, so [**E**·**E**] = [E^2] = N^2/C^2.
2. **B** is measured in (N/C)/(m/s), so we get [**B**·**B**] = [B^2] = $(N^2/C^2)\cdot(s^2/m^2)$. However, the dimension of our c^2 factor is (m^2/s^2) and so we are also left with N^2/C^2.
3. The ϵ_0 is the electric constant, aka as the vacuum permittivity. As a *physical* constant, it should ensure the dimensions on both sides of the equation work out, and they do: [ε_0] = $C^2/(N\cdot m^2)$ and, therefore, if we multiply that with N^2/C^2, we find that *u* is expressed in J/m^3.[6]

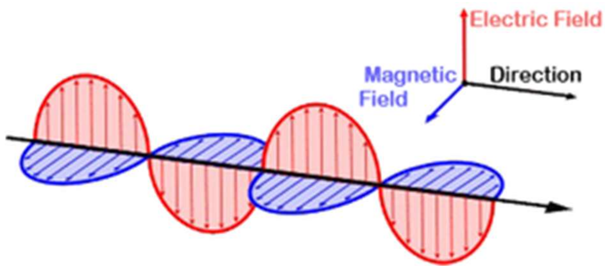
Let us see what we get for a photon, assuming the electromagnetic wave represents its wavefunction. Substituting **B** for (1/*c*)·*i*·**E** or for −(1/*c*)·*i*·**E** gives us the following result:

$$u = \frac{\epsilon_0}{2}\mathbf{E}\cdot\mathbf{E} + \frac{\epsilon_0 \cdot c^2}{2}\mathbf{B}\cdot\mathbf{B} = \frac{\epsilon_0}{2}\mathbf{E}\cdot\mathbf{E} + \frac{\epsilon_0 \cdot c^2}{2}\frac{i\cdot\mathbf{E}}{c}\frac{i\cdot\mathbf{E}}{c} = \frac{\epsilon_0}{2}\mathbf{E}\cdot\mathbf{E} - \frac{\epsilon_0}{2}\mathbf{E}\cdot\mathbf{E} = 0$$

[6] In fact, when multiplying $C^2/(N\cdot m^2)$ with N^2/C^2, we get N/m^2, but we can multiply this with 1 = m/m to get the desired result. It is significant that an energy density (*joule* per unit *volume*) can also be measured in *newton* (force per unit *area*.

Zero. An unexpected result? Perhaps not. We have no stationary charges and no currents: only an electromagnetic wave in free space. Hence, the local energy conservation principle needs to be respected at all points in space and in time. The geometry makes sense of the result: for an electromagnetic wave, the magnitudes of **E** and **B** reach their maximum, minimum and zero point *simultaneously*, as shown below.[7] This is because their *phase* is the same.

Figure 6: Electromagnetic wave: **E** and **B**



Should we expect a similar result for the energy densities that we would associate with the real and imaginary part of the matter-wave? For the matter-wave, we have a phase difference between $a \cdot cos\theta$ and $a \cdot sin\theta$, which gives a different picture of the *propagation* of the wave (see Figure 3).[8] In fact, the geometry of the suggestion suggests some inherent spin, which is interesting. I will come back to this. Let us first guess those densities. Making abstraction of any scaling constants, we may write:

$$u = a^2(cos\theta)^2 + a^2(-i \cdot sin\theta)^2 = a^2\ (cos^2\theta + sin^2\theta) = a^2$$

We get what we hoped to get: the absolute square of our amplitude is, effectively, an energy density !

$$|\psi|^2 = |a \cdot e^{-i \cdot E \cdot t/\hbar}|^2 = a^2 = u$$

This is very deep. A photon has no rest mass, so it borrows and returns energy from empty space as it travels through it. In contrast, a matter-wave carries energy and, therefore, has some (*rest*) mass. It is therefore associated with an energy density, and this energy density gives us the probabilities. Of course, we need to fine-tune the analysis to account for the fact that we have a wave packet rather than a single wave, but that should be feasible.

As mentioned, the phase difference between the real and imaginary part of our wavefunction (a cosine and a sine function) appears to give some spin to our particle. We do not have this particularity for a photon. Of course, photons are bosons, i.e. spin-zero particles, while elementary matter-particles are fermions with spin-1/2. Hence, our geometric interpretation of the wavefunction suggests that, after all, there may be some more intuitive explanation of the fundamental dichotomy between bosons and fermions, which puzzled even Feynman:

> "Why is it that particles with half-integral spin are Fermi particles, whereas particles with integral spin are Bose particles? We apologize for the fact that we cannot give you an elementary explanation. An explanation has been worked out by Pauli from complicated arguments of quantum field theory and relativity. He has shown that the two must necessarily go together, but we have not been able to find a way of reproducing his arguments on an elementary level. It appears to be one of the few places in physics where there is a rule which can be stated very simply, but for which no one has found a simple and easy explanation. The explanation is deep down in relativistic quantum mechanics. This probably means that we do

[7] The illustration shows a linearly polarized wave, but the obtained result is general.

[8] The sine and cosine are essentially the same functions, except for the difference in the phase: $\sin\theta = \cos(\theta - \pi/2)$.

not have a complete understanding of the fundamental principle involved." (Feynman, *Lectures*, III-4-1)

VI. Additional considerations

We talked about these in the two other pre-publication papers.[9] We will re-hash these over the coming months. The key area for research over the coming weeks is the concept of the electromagnetic mass: we will need to verify the relation – if any – between (1) the electromagnetic mass of a charge, moving up and down along a distance that is equal to the Compton radius of an electron and (2) the *total* rest mass of an electron. If there is no relation whatsoever, the hypotheses that have been offered in this paper will be irrelevant.

References

This paper discusses general principles in physics only. Hence, references can be limited to references to physics textbooks only. For ease of reading, any reference to additional material has been limited to a more popular undergrad textbook that can be consulted online: Feynman's Lectures on Physics (http://www.feynmanlectures.caltech.edu). References are per volume, per chapter and per section. For example, Feynman III-19-3 refers to Volume III, Chapter 19, Section 3.

All of the illustrations in this paper are open source or have been created by the author.

[9] See: http://vixra.org/author/jean_louis_van_belle_maec_baec_bphil.