First Newtonian Analyzes of the Michelson-Morley Experiment Using the Fixed Elastic Collision Law

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Abstract: In these papers, physicists will find the latest mathematical analysis of the results of the Michelson-Morley experiment, which was the source of all papers for Fitzgerald, Lorentz, Poincare, Minkowski and Einstein. This analyzes are the result of a study of the elastic collision theory, and are quite different from those of early physicists who have paved the way for the emergence of relativity theory. This work will discuss Fitzgerald-Lorentz's explanations and prove that they are not correct.

Keyword: The elastic collision, axis vector, tangent vector, [F.E.C], Lorentz–FitzGerald contraction, Michelson experiment.

1. INTRODUCTION AND OVERVIEW

Despite trying to provide definitive evidence of the invalidity of relativity theory, I will face many problems that arise as a result of a conflict over the theory's validity in the content of this letter. All physicists and mathematicians are called upon to be courageous in order to accept the bitter truth if the corruption of the theory of relativity is proved in this article, but indirectly. And back again to celebrate Newtonian physics triumphs, which was formulated by Newton in *Principia* **[13],** and in *Opticks* **[14],** after questioning its principles for more than a century.

In this article, I again demonstrate that Newton's laws are perfectly valid for the interpretation of many outstanding questions in physics, including the famous *Michelson-Morley* experiment **[16], [17].**

Unlike Newtonian physics, Einstein postulated that the velocity of light was constant in all Galileans references or independent of the relative motion of the source **.** Through this postulate, the *special relativity theory* [**23**], and *general relativity theory* [**24**] were founded. It is well known that his papers were another attempt to address the historical problem that resulted from the null results of the famous *Michelson-Morley* experiment, using the Lorentz transformations [**22**]**.** In principle, all specialists agree on Einstein's relativity arising from the echo generated by the null results of the *Michelson-Morley* experiment as detailed in this article **[18],[19].** And any radical change in the interpretation of its terms will change all the foundations that have resulted previously, including the transformations of Lorentz**;** the platform where Einstein's historical works were founded.

If many physicists and mathematicians such as William de Sitter **[39],[40]** were convinced that the Michelson-Morley experiment demonstrated that the speed of light was constant in all inertial reference frames, despite the null interpretation of their experiment by Michelson and Morley, Was repeated several times before the publication of Einstein's papers in 1905 and after its publication, before its adoption in 1919 and after its adoption, without a radical change on its theoretical framework.

We can say that relativity has prevented any attempt to interpret it again in a different direction. With a few exceptions that confirmed that the work of Lorentz and Einstein can't be excluded from criticism, and the famous physicist Kaufmann strongly criticized their work, and demanded the rejection of the "basic assumption of Lorentz and Einstein" (the relativity principle), after his experiments on the electron to investigate the validity of the principle or not **[37],** despite Max Blanc criticism of its accuracy **[38].**

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Only with technical improvements has the experiment been repeated in the present century as with Müller *et al.* **[42].** And only to strengthen the applied branch of Lorentz's analyzes and Einstein's interpretations of light. But this did not prevent the emergence of criticisms of the theory of relativity because of the ambiguity that justified many scientific questions in physics, and we can see one of the criticisms of the theory in this article **[43]**.

To these lines, I can say that I am in the right line to finish the argument around. This study, which examines the theoretical framework of the experiment, demonstrates the corruption of most of the assumptions adopted by Michelson and Morley, which caused the emergence of controversy over the validity of the laws of classical physics.

In these papers, we present a new and different interpretation of *Fitzgerald-Lorentz's* interpretation **[20], [21]** where we present the most recent analysis of the experiment using the laws of classical mechanics.

the content of the research of four chapters where we offer in the second chapter to study the elastic collision in order to obtain the algebraic formulas of the velocity vectors after collision between two spheres, and the third chapter we derive the law of the fixed elastic collision **[F.E.C],** and then we are theoretically simulation to the generalization the law on optics.

In Chapter 4, we review and discuss *Michelson-Morley* experiment with its results, and we try to find errors in its assumptions. After that, we theoretically apply the law of the fixed elastic collision **[F.E.C]** to the work of the Michelson's interferometer, and try to extract the information, and compare it with its assumptions.

2. DESIGNATION THE ALGEBRAIC FORMULAS OF THE VELOCITY VECTORS RESULTING FROM THE ELASTIC COLLISION BETWEEN TWO OBJECTS

As I mentioned in the introduction, I try in this article to explain the results of the experiment by Newton's classical laws, and it is known that the experiment built the movement of light in absolute space; I have to start this step, which is the cornerstone of this research.

Although it is important to design algebraic formulas of the velocity vectors on any subject that addresses the elastic collision between any two bodies, whether an atom or electron, it has been neglected in several important research, including the two articles **[11]and [12].** Where the first dealt with the subject of the collision between the hydrogen atom and the electron without the use of velocity vectors of the two bodies, and as the mass of the electron is neglected in front of the mass of the atom of hydrogen, the velocities after the collision will take other formulas and will be addressed in the next chapter. The second analyzed the Collision dynamics of granular particles with adhesion. Although the study is extensive, it did not address the relationship of velocity vectors after collision with adhesion.

There is an exception in articles **[8],[9]** and **[10]** where Kosinski studied a subject related to the elastic collision in two-dimensional, and concluded this relationship through his research.

$$\begin{cases} \boldsymbol{v_1 = v_1^{(0)} - \dfrac{m_2}{m_1+m_2}(1+e_m)n.G^{(0)}(n+ft)} \\ \qquad\qquad\qquad \boldsymbol{and} \\ \boldsymbol{v_2 = v_2^{(0)} + \dfrac{m_1}{m_1+m_2}(1+e_m)n.G^{(0)}(n+ft)} \end{cases}$$

In this chapter, we offer a more accurate analysis, which we can circulate to all studies that dealt with the elastic collision between two bodies.

In my opinion, the reflection of light on the hard surface is only an accident of elastic collision. This does not contradict the wave nature of the light, and the evidence that the electron is of both a wave-particle nature is subject to the laws of collision like other objects in classical physics **[12], [41].** So I have to set algebraic velocity formulas after colliding between two objects

For a detailed exposition, we refer to reference textbooks **[1],[2],[3]** and **[4].** In the ***Chapter 15: Collision Theory* [4],** physicist ***Lisa Randall*** as many physicists analyzed the subject of collision between two objects in one-dimension and two-dimensions but a method known in the lessons of physics. Alongside, she did not study analytically collision in space of three-dimensional. So, we conduct an analytical study using a new mathematical method in order to obtain algebraic formulas of the velocity vectors after collision in all dimensions.

We will use the ***Hilbert Space*** lessons from two textbooks **[5]** and **[6]** in physics**.** It will be clear at the beginning of ***the proposition 2.5*** when to demonstrate it, and to the following. The algebraic formulas of the velocity vectors will be the result of the study of elastic collisions in an isolated system.
Again for clarity, the law of Conserved Energy will be essential for progress in the study. So, in the elastic collision field, assuming two hard-spheres $(\boldsymbol{P})$ and $(\boldsymbol{\Delta})$ respectively, with two centers ***P*** and ***Δ*** respectively, and two mass $\boldsymbol{m}_{(\boldsymbol{P})}$ and $\boldsymbol{M}_{(\boldsymbol{\Delta})}$ respectively are colliding with the latter, and from this collision we get a set of information.

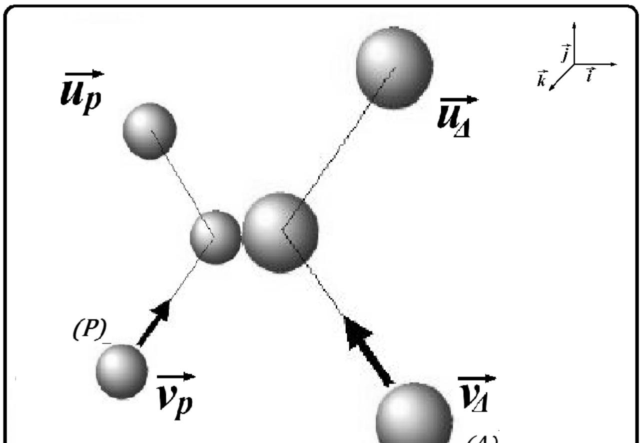


Figure 01

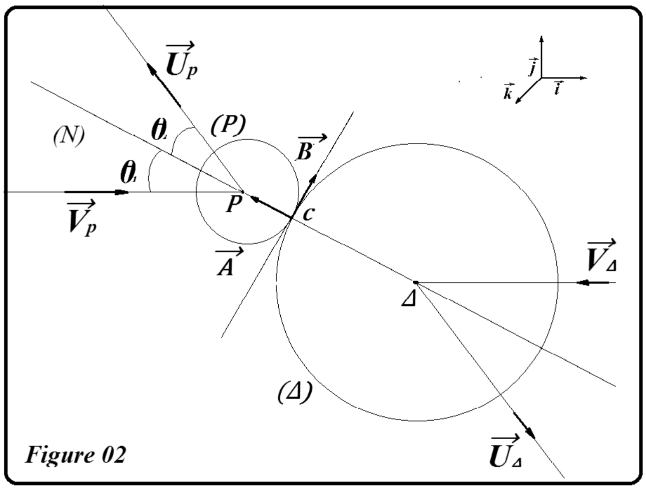


Figure 02

Let us consider two inertial reference frames $\boldsymbol{\Sigma_1}$ and $\boldsymbol{\Sigma_2}$ which distinguished by the center ***O*** and ***C*** respectively. The reference frame $\boldsymbol{\Sigma_2}$ moves relative to $\boldsymbol{\Sigma_1}$ with velocity $\overrightarrow{\boldsymbol{w_C}}$. According to the information in ***Fig №01*** and ***Fig №02***, we give the following definitions and propositions: The most famous law in physics is the conservation of kinetic energy i.e. conserved before and after collision, and is written as follows.

The conservation of kinetic energy 2.1:

In the elastic collision field; the conservation of kinetic energy remains valid in all Galileans references i.e.

$$\sum E_{Before} = \sum E_{After}$$

As mentioned in the introduction, the two hard-spheres move towards colliding, which means that they have velocity vectors before and after collisions in two inertial reference frames $\boldsymbol{\Sigma_1}$ and $\boldsymbol{\Sigma_2}$. Therefore, we put the basic definitions of velocity vectors.

Definition 2.1:

In the Galilean reference $\boldsymbol{\mathcal{R}}(\boldsymbol{O};\vec{\boldsymbol{\imath}};\vec{\boldsymbol{\jmath}};\vec{\boldsymbol{k}})$*; we define that* $\overrightarrow{\boldsymbol{v_p}}$ $(\overrightarrow{\boldsymbol{u_p}} respectively)$ *is the velocity vector of the hard-sphere* ***(P)*** *before the collision* $(\,after\ the\ collision\ respectively)$ *with the hard-sphere* ***(Δ).***

Definition 2.2:

In the Galilean reference $\boldsymbol{\mathcal{R}}(\boldsymbol{O};\vec{\boldsymbol{\imath}};\vec{\boldsymbol{\jmath}};\vec{\boldsymbol{k}})$*; we define that* $\overrightarrow{\boldsymbol{v_\Delta}}$ $(\overrightarrow{\boldsymbol{u_\Delta}} respectively)$ *is the velocity vector of the hard-sphere* ***(Δ)*** *before the collision* $(\,after\ the\ collision\ respectively)$ *with the hard-sphere* ***(P).***

In the collision field, physicists such as ***Randall*** in her textbook **[5]** and ***Raymond*** **[4]** that point (***C***) is the centre of mass reference, which is not a fixed point but is a virtual point that is relatively moving for the colliders, so we put this definition of it.

Definition 2.3:

We define that the point ***C*** *is the center of mass reference relative to two hard- spheres* $(\boldsymbol{P})$ *and* $(\boldsymbol{\Delta})$ *i.e.*

$$\boldsymbol{m_P}.\overrightarrow{\boldsymbol{CP}} + \boldsymbol{M_\Delta}.\overrightarrow{\boldsymbol{C\Delta}} = \vec{\boldsymbol{0}}$$

And in the Galilean reference $\boldsymbol{\mathcal{R}}(\boldsymbol{O};\vec{\boldsymbol{\imath}};\vec{\boldsymbol{\jmath}};\vec{\boldsymbol{k}})$*; we define that* $\overrightarrow{\boldsymbol{w_C}}$ *is the velocity vector of point* ***C.***

In the inertial reference frame $\boldsymbol{\Sigma_2}$; where the point (***C***) is centre, the velocity vector values of the objectors change in form and content before and after collisions. For this reason, we set the definitions of velocity vectors with the new Galilean reference.

Definition 2.4:

*In the Galilean reference $\boldsymbol{\mathcal{R}(C;\vec{\imath};\vec{\jmath};\vec{k})}$; we define that $\overrightarrow{\boldsymbol{V_p}}$ $\left(\overrightarrow{\boldsymbol{U_p}} respectively\right)$ is the velocity vector of the hard-sphere **(P)** before the collision $(\,after\ the\ collision\ respectively)$ with the hard-sphere **(Δ).***

Definition 2.5:

*In the Galilean reference $\boldsymbol{\mathcal{R}(C;\vec{\imath};\vec{\jmath};\vec{k})}$; we define that $\overrightarrow{\boldsymbol{V_\Delta}}$ $\left(\overrightarrow{\boldsymbol{U_\Delta}} respectively\right)$ is the velocity vector of the hard-sphere **(Δ)** before the collision $(\,after\ the\ collision\ respectively)$ with the hard-sphere **(P).***

Now, we may put the first proposition in which we determine the values of the velocity vector relative to Galilean reference $\boldsymbol{\mathcal{R}(C;\vec{\imath};\vec{\jmath};\vec{k})}$ in terms of values of the velocity vector relative to the Galilean reference $\boldsymbol{\mathcal{R}(O;\vec{\imath};\vec{\jmath};\vec{k})}$ and his proof.

Proposition 2.1:

With the use of the definitions mentioned above, the followings relationships are deduced that:

$$\begin{cases}\overrightarrow{\boldsymbol{V_p}} = \overrightarrow{\boldsymbol{v_p}} - \overrightarrow{\boldsymbol{w_C}} \\ \quad and \\ \overrightarrow{\boldsymbol{V_\Delta}} = \overrightarrow{\boldsymbol{v_\Delta}} - \overrightarrow{\boldsymbol{w_C}}\end{cases} before\ the\ collision, \left[\begin{cases}\overrightarrow{\boldsymbol{U_p}} = \overrightarrow{\boldsymbol{u_p}} - \overrightarrow{\boldsymbol{w_C}} \\ \quad and \\ \overrightarrow{\boldsymbol{U_\Delta}} = \overrightarrow{\boldsymbol{u_\Delta}} - \overrightarrow{\boldsymbol{w_C}}\end{cases} after\ the\ collision\ respectevely\right]$$

Proof

01-Before the collision:

With using ***Chasles relation***, we have that:

$$\overrightarrow{\boldsymbol{OP}} = \overrightarrow{\boldsymbol{OC}} + \overrightarrow{\boldsymbol{CP}} \Rightarrow \overrightarrow{\boldsymbol{CP}} = \overrightarrow{\boldsymbol{OP}} - \overrightarrow{\boldsymbol{OC}}$$

$$\Rightarrow \frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}\overrightarrow{\boldsymbol{CP}}_{(t)} = \frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}\overrightarrow{\boldsymbol{OP}}_{(t)} - \frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}\overrightarrow{\boldsymbol{OC}}_{(t)}$$

$$\Rightarrow \overrightarrow{\boldsymbol{V_p}} = \overrightarrow{\boldsymbol{v_p}} - \overrightarrow{\boldsymbol{w_C}}$$

Again, with using ***Chasles relation***, we have that:

$$\overrightarrow{\boldsymbol{O\Delta}} = \overrightarrow{\boldsymbol{OC}} + \overrightarrow{\boldsymbol{C\Delta}} \Rightarrow \overrightarrow{\boldsymbol{C\Delta}} = \overrightarrow{\boldsymbol{O\Delta}} - \overrightarrow{\boldsymbol{OC}}$$

$$\Rightarrow \frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}\overrightarrow{\boldsymbol{C\Delta}}_{(t)} = \frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}\overrightarrow{\boldsymbol{O\Delta}}_{(t)} - \frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}\overrightarrow{\boldsymbol{O\Delta}}_{(t)}$$

$$\Rightarrow \overrightarrow{\boldsymbol{V_\Delta}} = \overrightarrow{\boldsymbol{v_\Delta}} - \overrightarrow{\boldsymbol{w_C}}$$

02-After the collision:

With using ***Chasles relation***, we have that:

$$\overrightarrow{\boldsymbol{OP}} = \overrightarrow{\boldsymbol{OC}} + \overrightarrow{\boldsymbol{CP}} \Rightarrow \overrightarrow{\boldsymbol{CP}} = \overrightarrow{\boldsymbol{OP}} - \overrightarrow{\boldsymbol{OC}}$$

$$\Rightarrow \frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}\overrightarrow{\boldsymbol{CP}}_{(t)} = \frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}\overrightarrow{\boldsymbol{OP}}_{(t)} - \frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}\overrightarrow{\boldsymbol{OC}}_{(t)}$$

$$\Rightarrow \overrightarrow{\boldsymbol{U_p}} = \overrightarrow{\boldsymbol{u_p}} - \overrightarrow{\boldsymbol{w_C}}$$

Again, with using ***Chasles relation***, we have that:

$$\overrightarrow{\boldsymbol{O\Delta}} = \overrightarrow{\boldsymbol{OC}} + \overrightarrow{\boldsymbol{C\Delta}} \Rightarrow \overrightarrow{\boldsymbol{C\Delta}} = \overrightarrow{\boldsymbol{O\Delta}} - \overrightarrow{\boldsymbol{OC}}$$

$$\Rightarrow \frac{\partial}{\partial t}\overrightarrow{C\Delta}_{(t)} = \frac{\partial}{\partial t}\overrightarrow{O\Delta}_{(t)} - \frac{\partial}{\partial t}\overrightarrow{O\Delta}_{(t)}$$

$$\Rightarrow \overrightarrow{U_\Delta} = \overrightarrow{u_\Delta} - \overrightarrow{w_C}$$

Then, the point C that is the center of mass reference has the velocity vector $\overrightarrow{w_C}$ and its value is written in the flowing proposition:

Proposition 2.2:

In the Galilean reference $\mathcal{R}(O;\vec{\imath};\vec{\jmath};\vec{k})$*; the velocity vector of the point* C *is written as follows:*

$$\overrightarrow{w_C} = \frac{1}{m_P + M_\Delta}\left(m_P.\overrightarrow{v_p} + M_\Delta.\overrightarrow{v_\Delta}\right)$$

Proof

With using ***Chasles relations*,** we have that:

$$\overrightarrow{OP} = \overrightarrow{OC} + \overrightarrow{CP} \quad \Rightarrow \quad \overrightarrow{OC} = \overrightarrow{OP} - \overrightarrow{CP}$$

$$\Rightarrow m_P.\overrightarrow{OC} = m_P.\overrightarrow{OP} - m_P.\overrightarrow{CP}$$

Again, with using ***Chasles relation***, we have that:

$$\overrightarrow{O\Delta} = \overrightarrow{OC} + \overrightarrow{C\Delta} \quad \Rightarrow \quad \overrightarrow{OC} = \overrightarrow{O\Delta} - \overrightarrow{C\Delta}$$

$$\Rightarrow M_\Delta.\overrightarrow{OC} = M_\Delta.\overrightarrow{O\Delta} - M_\Delta.\overrightarrow{C\Delta}$$

$$\Rightarrow \; m_P.\overrightarrow{OC} + M_\Delta.\overrightarrow{OC} = \left(m_P.\overrightarrow{OP} - m_P.\overrightarrow{CP}\right) + \left(M_\Delta.\overrightarrow{O\Delta} - M_\Delta.\overrightarrow{C\Delta}\right)$$

$$\Rightarrow (m_P + M_\Delta).\overrightarrow{OC} = \left(m_P.\overrightarrow{OP} + M_\Delta.\overrightarrow{O\Delta}\right) - \left(\underbrace{m_P.\overrightarrow{CP} + M_\Delta.\overrightarrow{C\Delta}}_{=\vec{0}}\right)$$

According to ***the definition 2.3;*** we deduce:

$$(m_P + M_\Delta).\overrightarrow{OC} = m_P.\overrightarrow{OP} + M_\Delta.\overrightarrow{O\Delta}$$

$$\Rightarrow \overrightarrow{OC} = \frac{1}{m_P + M_\Delta}\left(m_P.\overrightarrow{OP} + M_\Delta.\overrightarrow{O\Delta}\right)$$

$$\Rightarrow \frac{\partial}{\partial t}\overrightarrow{OC}_{(t)} = \frac{1}{m_P + M_\Delta}\left(m_P.\frac{\partial}{\partial t}\overrightarrow{OP}_{(t)} + M_\Delta.\frac{\partial}{\partial t}\overrightarrow{O\Delta}_{(t)}\right)$$

$$\Rightarrow \overrightarrow{w_C} = \frac{1}{m_P + M_\Delta}\left(m_P.\overrightarrow{v_p} + M_\Delta.\overrightarrow{v_\Delta}\right)$$

In any inertial reference frames, the momentum quantity are conserved (just like in any other collision) and given in the following relationship:

$$\vec{\mathcal{P}}_{Before} = \vec{\mathcal{P}}_{After}$$

That is, the momentum vector of the objects just after the collision is the same as it was just before the collision i.e.

$$\vec{\mathcal{P}} = m_P.\overrightarrow{V_P} + M_\Delta.\overrightarrow{V_\Delta} = m_P.\overrightarrow{U_P} + M_\Delta.\overrightarrow{U_\Delta}$$

And in the inertial reference frames $\mathbf{\Sigma_2}$ they are equal to zero-vector. Therefore, we present the following proposition and his proof.

Proposition 2.3:

In the Galilean reference $\mathcal{R}(C;\vec{\imath};\vec{\jmath};\vec{k})$ *, the value of the momentum quantity* $\vec{\mathcal{P}}$ *equal zero-vector i.e.*

$$\begin{cases} m_P.\overrightarrow{V_P} + M_\Delta.\overrightarrow{V_\Delta} = \vec{0} \qquad before\ the\ collision \\ \qquad\qquad\qquad\qquad\quad \textbf{and} \\ m_P.\overrightarrow{U_P} + M_\Delta.\overrightarrow{U_\Delta} = \vec{0} \qquad after\ the\ collision \end{cases}$$

Proof

01-Before the collision:

According to ***the definition 2.3;*** we have that:

$$\boldsymbol{m_P}.\overrightarrow{\boldsymbol{CP}} + \boldsymbol{M}_\Delta.\overrightarrow{\boldsymbol{C\Delta}} = \vec{\boldsymbol{0}}$$

$$\Rightarrow \boldsymbol{m_P}.\frac{\partial}{\partial t}\overrightarrow{\boldsymbol{CP}}_{(t)} + \boldsymbol{M}_\Delta.\frac{\partial}{\partial t}\overrightarrow{\boldsymbol{C\Delta}}_{(t)} = \vec{\boldsymbol{0}}$$

$$\Rightarrow \boldsymbol{m_P}.\overrightarrow{\boldsymbol{V_P}} + \boldsymbol{M}_\Delta.\overrightarrow{\boldsymbol{V}_\Delta} = \vec{\boldsymbol{0}}$$

02-After the collision:

According to ***the definition 2.3;*** we have that:

$$\boldsymbol{m_P}.\overrightarrow{\boldsymbol{CP}} + \boldsymbol{M}_\Delta.\overrightarrow{\boldsymbol{C\Delta}} = \vec{\boldsymbol{0}}$$

$$\Rightarrow \boldsymbol{m_P}.\frac{\partial}{\partial t}\overrightarrow{\boldsymbol{CP}}_{(t)} + \boldsymbol{M}_\Delta.\frac{\partial}{\partial t}\overrightarrow{\boldsymbol{C\Delta}}_{(t)} = \vec{\boldsymbol{0}}$$

$$\Rightarrow \boldsymbol{m_P}.\overrightarrow{\boldsymbol{U_P}} + \boldsymbol{M}_\Delta.\overrightarrow{\boldsymbol{U}_\Delta} = \vec{\boldsymbol{0}}$$

Again, the equalities $(\mathbf{2.1})$ and $(\mathbf{2.2})$ in the next proposition are very important for progress in the study.

Proposition 2.4:

In the Galilean reference $\mathcal{R}(\boldsymbol{C};\vec{\boldsymbol{\imath}};\vec{\boldsymbol{\jmath}};\vec{\boldsymbol{k}})$ *, the following relationships are deduced that:*

$$\begin{cases} \|\overrightarrow{\boldsymbol{V_P}}\| = \|\overrightarrow{\boldsymbol{U_P}}\| & (\mathbf{2.1}) \\ \quad \boldsymbol{and} \\ \|\overrightarrow{\boldsymbol{V}_\Delta}\| = \|\overrightarrow{\boldsymbol{U}_\Delta}\| & (\mathbf{2.2}) \end{cases}$$

Proof

1-According to ***The law of Conserved Energy 2.1*****;** we have that:

$$\sum \boldsymbol{E_{Before}} = \sum \boldsymbol{E_{After}}$$

$$\Rightarrow \frac{\mathbf{1}}{\mathbf{2}}.\boldsymbol{m_P}.\|\overrightarrow{\boldsymbol{V_p}}\|^2 + \frac{\mathbf{1}}{\mathbf{2}}.\boldsymbol{M_\Delta}.\|\overrightarrow{\boldsymbol{V}_\Delta}\|^2 = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{m_P}.\|\overrightarrow{\boldsymbol{U_p}}\|^2 + \frac{\mathbf{1}}{\mathbf{2}}.\boldsymbol{M_\Delta}.\|\overrightarrow{\boldsymbol{U}_\Delta}\|^2$$

$$\Rightarrow \boldsymbol{m_P}.\|\overrightarrow{\boldsymbol{V_p}}\|^2 + \boldsymbol{M_\Delta}.\|\overrightarrow{\boldsymbol{V}_\Delta}\|^2 = \boldsymbol{m_P}.\|\overrightarrow{\boldsymbol{U_p}}\|^2 + \boldsymbol{M_\Delta}.\|\overrightarrow{\boldsymbol{U}_\Delta}\|^2 \qquad (\mathbf{2.3})$$

And, according to ***the proposition 2.3,*** we have that:

$$\begin{cases} \boldsymbol{m_P}.\overrightarrow{\boldsymbol{V_P}} + \boldsymbol{M}_\Delta.\overrightarrow{\boldsymbol{V}_\Delta} = \vec{\boldsymbol{0}} \\ \quad \boldsymbol{and} \\ \boldsymbol{m_P}.\overrightarrow{\boldsymbol{U_P}} + \boldsymbol{M}_\Delta.\overrightarrow{\boldsymbol{U}_\Delta} = \vec{\boldsymbol{0}} \end{cases} \Leftrightarrow \begin{cases} \frac{-\boldsymbol{m_P}}{\boldsymbol{M}_\Delta}.\overrightarrow{\boldsymbol{V_P}} = \overrightarrow{\boldsymbol{V}_\Delta} \\ \quad \boldsymbol{and} \\ \frac{-\boldsymbol{m_P}}{\boldsymbol{M}_\Delta}.\overrightarrow{\boldsymbol{U_P}} = \overrightarrow{\boldsymbol{U}_\Delta} \end{cases} \qquad (\mathbf{2.4})$$

Applying last result **(2.4)** in relation **(2.3),** we deduce the following result:

$$\boldsymbol{m_P}.\|\overrightarrow{\boldsymbol{V_p}}\|^2 + \boldsymbol{M_\Delta}.\left\|\frac{-\boldsymbol{m_P}}{\boldsymbol{M_\Delta}}.\overrightarrow{\boldsymbol{V_P}}\right\|^2 = \boldsymbol{m_P}.\|\overrightarrow{\boldsymbol{U_p}}\|^2 + \boldsymbol{M_\Delta}.\left\|\frac{-\boldsymbol{m_P}}{\boldsymbol{M_\Delta}}.\overrightarrow{\boldsymbol{U_P}}\right\|^2$$

$$\Rightarrow \left[\boldsymbol{m_P} + \frac{\boldsymbol{m_P}^2}{\boldsymbol{M_\Delta}}\right].\|\overrightarrow{\boldsymbol{V_p}}\|^2 = \left[\boldsymbol{m_P} + \frac{\boldsymbol{m_P}^2}{\boldsymbol{M_\Delta}}\right].\|\overrightarrow{\boldsymbol{U_p}}\|^2$$

$$\Rightarrow \|\overrightarrow{\boldsymbol{V_p}}\|^2 = \|\overrightarrow{\boldsymbol{U_p}}\|^2$$

$$\Rightarrow \|\overrightarrow{\boldsymbol{V_p}}\| = \|\overrightarrow{\boldsymbol{U_p}}\| \qquad (\mathbf{2.1})$$

2- With using last result **(2.4),** we deduce the following result:

$$\begin{cases} \dfrac{-m_P}{M_\Delta}.\overrightarrow{V_P} = \overrightarrow{V_\Delta} \\ \quad and \\ \dfrac{-m_P}{M_\Delta}.\overrightarrow{U_P} = \overrightarrow{U_\Delta} \end{cases} \Rightarrow \begin{cases} \left\|\dfrac{-m_P}{M_\Delta}.\overrightarrow{V_P}\right\| = \left\|\overrightarrow{V_\Delta}\right\| \\ \quad and \\ \left\|\dfrac{-m_P}{M_\Delta}.\overrightarrow{U_P}\right\| = \left\|\overrightarrow{U_\Delta}\right\| \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{m_P}{M_\Delta}.\left\|\overrightarrow{V_P}\right\| = \left\|\overrightarrow{V_\Delta}\right\| \\ \quad and \\ \dfrac{m_P}{M_\Delta}.\left\|\overrightarrow{U_P}\right\| = \left\|\overrightarrow{U_\Delta}\right\| \end{cases}$$

But, we have the equality $(\mathbf{2.1})$ that:

$$\left\|\overrightarrow{V_p}\right\| = \left\|\overrightarrow{U_p}\right\|$$

So, we deduce:

$$\begin{cases} \dfrac{m_P}{M_\Delta}.\left\|\overrightarrow{V_P}\right\| = \left\|\overrightarrow{V_\Delta}\right\| \\ \quad and \\ \dfrac{m_P}{M_\Delta}.\left\|\overrightarrow{U_P}\right\| = \left\|\overrightarrow{U_\Delta}\right\| \end{cases} \Rightarrow \begin{cases} \dfrac{m_P}{M_\Delta}.\left\|\overrightarrow{V_P}\right\| = \left\|\overrightarrow{V_\Delta}\right\| \\ \quad and \\ \dfrac{m_P}{M_\Delta}.\left\|\overrightarrow{V_P}\right\| = \left\|\overrightarrow{U_\Delta}\right\| \end{cases}$$

$$\Rightarrow \left\|\overrightarrow{V_\Delta}\right\| = \left\|\overrightarrow{U_\Delta}\right\| \qquad (\mathbf{2.2})$$

Moreover, if we look at ***Fig №02***, we see that the collision field is divided into two halves by the normal line **(N),** that its direction vector is $\overrightarrow{A}$, so we put the definition of the vector $\overrightarrow{A}$ and the vector $\overrightarrow{B}$ **.**

Definition 2.6:

We define that $\overrightarrow{A}$ is the axis vector of the collision **[**$\overrightarrow{B}$ *is the tangent vector respectively* **]** *such that* $\left\|\overrightarrow{A}\right\| = \mathbf{1}$ *,and* $\overrightarrow{A}$ *orthogonal with* $\left(\overrightarrow{V_P} + \overrightarrow{U_P}\right)$ **[**$\overrightarrow{B}$ *parallel to* $\left(\overrightarrow{V_P} + \overrightarrow{U_P}\right)$ *respectively* **].**

Remark 2.1:

In section of Hilbert space, common in the mathematical notations that $\overrightarrow{A}$ is a unit vector as the direction vector, and we must not differentiate between them. And in addition, I chose the operation $\langle . | . \rangle$ as the relation of the scalar product.

Lemma 2.1

*As we define the vector $\overrightarrow{A}$ and $\overrightarrow{B}$ in the **definition 2.6**, we concluded the flowing equalities:*

$$\left\langle \overrightarrow{A} \middle| \overrightarrow{A} \right\rangle = \mathbf{1} \quad and \quad \left\langle \overrightarrow{B} \middle| \overrightarrow{A} \right\rangle = \mathbf{0}$$

Proof:

We have

$$\left\langle \overrightarrow{A} \middle| \overrightarrow{A} \right\rangle = \left\|\overrightarrow{A}\right\|^2$$

From the ***definition 2.6***, where we define:

$$\left\|\overrightarrow{A}\right\| = \mathbf{1} \Rightarrow \left(\overrightarrow{A}\right)^2 = \mathbf{1} \Rightarrow \left\langle \overrightarrow{A} \middle| \overrightarrow{A} \right\rangle = \mathbf{1}$$

Again, we have

$$\overrightarrow{A} \perp \left(\overrightarrow{V_P} + \overrightarrow{U_P}\right) \quad and \quad \overrightarrow{B} \parallel \left(\overrightarrow{V_P} + \overrightarrow{U_P}\right)$$

$$\Rightarrow \overrightarrow{A} \perp \overrightarrow{B} \Rightarrow \left\langle \overrightarrow{B} \middle| \overrightarrow{A} \right\rangle = \mathbf{0}$$

With all the data we mentioned and the results we obtained, we will write the algebraic formulas of velocity vector after the collision in this ***proposition 2.5***, but in the Galilean reference $\boldsymbol{\mathcal{R}}(\boldsymbol{C}; \vec{\imath}; \vec{\jmath}; \vec{k})$ **,** which is centered at ($\boldsymbol{C}$) with his proof.

Proposition 2.5:

In the elastic collision field relative to the Galilean reference $(\boldsymbol{C};\vec{\boldsymbol{\imath}};\vec{\boldsymbol{\jmath}};\vec{\boldsymbol{k}})$ *, with used the latest notations, the velocity vectors after the collision between two objects are deduced that:*

$$\begin{cases} \overrightarrow{U_P} = \overrightarrow{V_P} - 2.\langle \overrightarrow{V_P} | \vec{A} \rangle.\vec{A} & (2.5) \\ \quad and \\ \overrightarrow{U_\Delta} = \overrightarrow{V_\Delta} - 2.\langle \overrightarrow{V_\Delta} | \vec{A} \rangle.\vec{A} & (2.6) \end{cases}$$

Proof

1- We use the latest notations and definitions.

So, according to ***the proposition 2.4,*** we have the equality $(\mathbf{2.1})$ that:

$$\|\overrightarrow{V_P}\| = \|\overrightarrow{U_P}\|$$

$$\Rightarrow \|\overrightarrow{V_P}\|^2 = \|\overrightarrow{U_P}\|^2$$

$$\Rightarrow (\overrightarrow{V_P})^2 = (\overrightarrow{U_P})^2$$

$$\Rightarrow (\overrightarrow{V_P})^2 - (\overrightarrow{U_P})^2 = 0$$

$$\Rightarrow (\overrightarrow{V_P} - \overrightarrow{U_P})(\overrightarrow{V_P} + \overrightarrow{U_P}) = 0$$

$$\Rightarrow (\overrightarrow{V_P} - \overrightarrow{U_P}) \perp (\overrightarrow{V_P} + \overrightarrow{U_P})$$

But, according to ***last definition 2.6,*** we have that:

$$\vec{A} \perp (\overrightarrow{V_P} + \overrightarrow{U_P})$$

So, we deduce

$$(\overrightarrow{V_P} - \overrightarrow{U_P}) \parallel \vec{A}$$

$$\exists \lambda \in \mathbb{R} \quad / \quad \overrightarrow{V_P} - \overrightarrow{U_P} = \lambda.\vec{A} \qquad (2.7)$$

Again, we have that:

$$\vec{B} \parallel (\overrightarrow{V_P} + \overrightarrow{U_P})$$

$$\Rightarrow \exists \rho \in \mathbb{R} \quad / \quad \overrightarrow{V_P} + \overrightarrow{U_P} = \rho.\vec{B} \qquad (2.8)$$

According to the two relations **(2.7)** and **(2.8),** we deduce the following result:

$$\exists \lambda, \rho \in \mathbb{R} \, / \begin{cases} \overrightarrow{V_P} - \overrightarrow{U_P} = \lambda.\vec{A} \\ \quad and \\ \overrightarrow{V_P} + \overrightarrow{U_P} = \rho.\vec{B} \end{cases}$$

$$\Rightarrow (\overrightarrow{V_P} - \overrightarrow{U_P}) + (\overrightarrow{V_P} + \overrightarrow{U_P}) = \lambda.\vec{A} + \rho.\vec{B}$$

$$\Rightarrow 2.\overrightarrow{V_P} = \rho.\vec{B} + \lambda.\vec{A}$$

$$\Rightarrow \overrightarrow{V_P} = \frac{1}{2}\rho.\vec{B} + \frac{1}{2}\lambda.\vec{A}$$

$$\Rightarrow \langle \overrightarrow{V_P} | \vec{A} \rangle = \left\langle \frac{1}{2}\rho.\vec{B} + \frac{1}{2}\lambda.\vec{A} \middle| \vec{A} \right\rangle$$

$$\Rightarrow \langle \overrightarrow{V_P} | \vec{A} \rangle = \left\langle \frac{1}{2}\rho.\vec{B} \middle| \vec{A} \right\rangle + \left\langle \frac{1}{2}\lambda.\vec{A} \middle| \vec{A} \right\rangle$$

$$\Rightarrow \langle \overrightarrow{V_P} | \vec{A} \rangle = \frac{1}{2}\rho.\underbrace{\langle \vec{B} | \vec{A} \rangle}_{=0} + \frac{1}{2}\lambda.\underbrace{\langle \vec{A} | \vec{A} \rangle}_{=1}$$

According to the ***lemma 2.1,*** we deduce the following result:

$$\langle \overrightarrow{V_P} | \overrightarrow{A} \rangle = \frac{1}{2}\lambda \Rightarrow \lambda = 2.\langle \overrightarrow{V_P} | \overrightarrow{A} \rangle$$

Lastly, we apply the value of $\boldsymbol{\lambda}$ in relation **(2.7),** and we deduce the following result:

$$\overrightarrow{V_P} - \overrightarrow{U_P} = \lambda.\overrightarrow{A}$$
$$\Rightarrow \overrightarrow{V_P} - \overrightarrow{U_P} = 2.\langle \overrightarrow{V_P} | \overrightarrow{A} \rangle.\overrightarrow{A}$$
$$\Rightarrow \overrightarrow{U_P} = \overrightarrow{V_P} - 2.\langle \overrightarrow{V_P} | \overrightarrow{A} \rangle.\overrightarrow{A} \qquad (2.5)$$

2- According to ***the proposition 2.3,*** we have that:

$$\begin{cases} m_P.\overrightarrow{V_P} + M_\Delta.\overrightarrow{V_\Delta} = \vec{0} \\ \wedge \\ m_P.\overrightarrow{U_P} + M_\Delta.\overrightarrow{U_\Delta} = \vec{0} \end{cases} \Rightarrow \begin{cases} \overrightarrow{V_P} = \dfrac{-M_\Delta}{m_P}.\overrightarrow{V_\Delta} \\ \wedge \\ \overrightarrow{U_P} = \dfrac{-M_\Delta}{m_P}.\overrightarrow{U_\Delta} \end{cases} \qquad (2.4)$$

Applying the result (2.4) in last equality **(2.5),** we deduce the following result **(2.6)**:

$$\overrightarrow{U_P} = \overrightarrow{V_P} - 2.\langle \overrightarrow{V_P} | \overrightarrow{A} \rangle.\overrightarrow{A}$$
$$\Rightarrow \frac{-M_\Delta}{m_P}.\overrightarrow{U_\Delta} = \frac{-M_\Delta}{m_P}.\overrightarrow{V_\Delta} - 2.\left\langle \frac{-M_\Delta}{m_P}.\overrightarrow{V_\Delta} \middle| \overrightarrow{A} \right\rangle.\overrightarrow{A}$$
$$\Rightarrow \frac{-M_\Delta}{m_P}.\overrightarrow{U_\Delta} = \frac{-M_\Delta}{m_P}.\overrightarrow{V_\Delta} - 2.\left[\frac{-M_\Delta}{m_P}.\langle \overrightarrow{V_\Delta} | \overrightarrow{A} \rangle\right].\overrightarrow{A}$$
$$\Rightarrow \frac{-M_\Delta}{m_P}.\overrightarrow{U_\Delta} = \frac{-M_\Delta}{m_P}.\overrightarrow{V_\Delta} - 2.\frac{-M_\Delta}{m_P}.\langle \overrightarrow{V_\Delta} | \overrightarrow{A} \rangle.\overrightarrow{A}$$
$$\Rightarrow \frac{-M_\Delta}{m_P}.\overrightarrow{U_\Delta} = \frac{-M_\Delta}{m_P}.\left[\overrightarrow{V_\Delta} - 2.\langle \overrightarrow{V_\Delta} | \overrightarrow{A} \rangle.\overrightarrow{A}\right]$$
$$\Rightarrow \overrightarrow{U_\Delta} = \overrightarrow{V_\Delta} - 2.\langle \overrightarrow{V_\Delta} | \overrightarrow{A} \rangle.\overrightarrow{A} \qquad (2.6)$$

Finally, we may put the text of the theory in which the algebraic formulas of vector velocities after the collision in the Galilean reference $\mathcal{R}(O;\vec{i};\vec{j};\vec{k})$, and the proof will be simple.

Theorem 2.1:

In the elastic collision field relative to the Galilean reference $\mathcal{R}(O;\vec{i};\vec{j};\vec{k})$; the velocity vectors after the collision between two objects are written in the following form:

$$\begin{cases} \overrightarrow{u_p} = \overrightarrow{v_p} - 2\frac{M_\Delta}{M_\Delta + m_P}\langle \overrightarrow{v_p} - \overrightarrow{v_\Delta} | \overrightarrow{A} \rangle.\overrightarrow{A} & (2.9) \\ \text{and} \\ \overrightarrow{u_\Delta} = \overrightarrow{v_\Delta} - 2\frac{m_P}{M_\Delta + m_P}\langle \overrightarrow{v_\Delta} - \overrightarrow{v_p} | \overrightarrow{A} \rangle.\overrightarrow{A} & (2.10) \end{cases}$$

Proof

1- According to ***the proposition 2.5,*** we have the equality **(2.5)** that:

$$\overrightarrow{U_P} = \overrightarrow{V_P} - 2.\langle \overrightarrow{V_P} | \overrightarrow{A} \rangle.\overrightarrow{A}$$

Again, with applying the equality of ***the proposition 2.1,*** we deduce the following result:

$$(\overrightarrow{u_p} - \overrightarrow{w_C}) = (\overrightarrow{v_p} - \overrightarrow{w_C}) - 2.\langle \overrightarrow{v_p} - \overrightarrow{w_C} | \overrightarrow{A} \rangle.\overrightarrow{A}$$
$$\Rightarrow \overrightarrow{u_p} = \overrightarrow{v_p} - 2.\langle \overrightarrow{v_p} - \overrightarrow{w_C} | \overrightarrow{A} \rangle.\overrightarrow{A}$$

Then, we use the value of the vector $\overrightarrow{w_C}$ to ***the proposition 2.2.***

So, we deduce the following result:

$$\overrightarrow{u_p} = \overrightarrow{v_p} - 2.\langle \overrightarrow{v_p} - \overrightarrow{w_C} | \overrightarrow{A} \rangle . \overrightarrow{A}$$

$$\Rightarrow \overrightarrow{u_p} = \overrightarrow{v_p} - 2.\left\langle \overrightarrow{v_p} - \frac{1}{m_P + M_\Delta}\left(m_P.\overrightarrow{v_p} + M_\Delta.\overrightarrow{v_\Delta}\right) \middle| \overrightarrow{A} \right\rangle . \overrightarrow{A}$$

$$\Rightarrow \overrightarrow{u_p} = \overrightarrow{v_p} - 2.\left\langle \frac{m_P + M_\Delta}{m_P + M_\Delta}.\overrightarrow{v_p} - \frac{m_P}{m_P + M_\Delta}\overrightarrow{v_p} - \frac{M_\Delta}{m_P + M_\Delta}\overrightarrow{v_\Delta} \middle| \overrightarrow{A} \right\rangle . \overrightarrow{A}$$

$$\Rightarrow \overrightarrow{u_p} = \overrightarrow{v_p} - 2.\left\langle \frac{M_\Delta}{m_P + M_\Delta}.\overrightarrow{v_p} - \frac{M_\Delta}{m_P + M_\Delta}\overrightarrow{v_\Delta} \middle| \overrightarrow{A} \right\rangle . \overrightarrow{A}$$

$$\Rightarrow \overrightarrow{u_p} = \overrightarrow{v_p} - 2.\left\langle \frac{M_\Delta}{m_P + M_\Delta}.\left(\overrightarrow{v_p} - \overrightarrow{v_\Delta}\right) \middle| \overrightarrow{A} \right\rangle . \overrightarrow{A}$$

$$\Rightarrow \overrightarrow{u_p} = \overrightarrow{v_p} - 2.\frac{M_\Delta}{m_P + M_\Delta}.\langle \overrightarrow{v_p} - \overrightarrow{v_\Delta} | \overrightarrow{A} \rangle . \overrightarrow{A}$$

2- According to ***the proposition 2.5*,** we have the equality **(2.6)** that:

$$\overrightarrow{U_\Delta} = \overrightarrow{V_\Delta} - 2.\langle \overrightarrow{V_\Delta} | \overrightarrow{A} \rangle . \overrightarrow{A}$$

Again, with applying ***the proposition 2.1*,** we deduce the following result:

$$(\overrightarrow{u_\Delta} - \overrightarrow{w_C}) = (\overrightarrow{v_\Delta} - \overrightarrow{w_C}) - 2.\langle \overrightarrow{v_\Delta} - \overrightarrow{w_C} | \overrightarrow{A} \rangle . \overrightarrow{A}$$

$$\Rightarrow \overrightarrow{u_\Delta} = \overrightarrow{v_\Delta} - 2.\langle \overrightarrow{v_\Delta} - \overrightarrow{w_C} | \overrightarrow{A} \rangle . \overrightarrow{A}$$

Then, we use the value of the vector $\overrightarrow{w_C}$ to ***the proposition 2.2***.

So, we deduce the following result:

$$\Rightarrow \overrightarrow{u_\Delta} = \overrightarrow{v_\Delta} - 2.\left\langle \overrightarrow{v_\Delta} - \frac{1}{m_P + M_\Delta}\left(m_P.\overrightarrow{v_p} + M_\Delta.\overrightarrow{v_\Delta}\right) \middle| \overrightarrow{A} \right\rangle . \overrightarrow{A}$$

$$\Rightarrow \overrightarrow{u_\Delta} = \overrightarrow{v_\Delta} - 2.\left\langle \frac{m_P + M_\Delta}{m_P + M_\Delta}\overrightarrow{v_\Delta} - \frac{1}{m_P + M_\Delta}\left(m_P.\overrightarrow{v_p} + M_\Delta.\overrightarrow{v_\Delta}\right) \middle| \overrightarrow{A} \right\rangle . \overrightarrow{A}$$

$$\Rightarrow \overrightarrow{u_\Delta} = \overrightarrow{v_\Delta} - 2.\left\langle \frac{m_P + M_\Delta}{m_P + M_\Delta}\overrightarrow{v_\Delta} - \frac{m_P}{m_P + M_\Delta}\overrightarrow{v_p} - \frac{M_\Delta}{m_P + M_\Delta}\overrightarrow{v_\Delta} \middle| \overrightarrow{A} \right\rangle . \overrightarrow{A}$$

$$\Rightarrow \overrightarrow{u_\Delta} = \overrightarrow{v_\Delta} - 2.\left\langle \frac{m_P}{m_P + M_\Delta}\overrightarrow{v_\Delta} - \frac{m_P}{m_P + M_\Delta}\overrightarrow{v_p} \middle| \overrightarrow{A} \right\rangle . \overrightarrow{A}$$

$$\Rightarrow \overrightarrow{u_\Delta} = \overrightarrow{v_\Delta} - 2.\left\langle \frac{m_P}{m_P + M_\Delta}\left(\overrightarrow{v_\Delta} - \overrightarrow{v_p}\right) \middle| \overrightarrow{A} \right\rangle . \overrightarrow{A}$$

$$\Rightarrow \overrightarrow{u_\Delta} = \overrightarrow{v_\Delta} - 2.\frac{m_P}{m_P + M_\Delta}.\langle \overrightarrow{v_\Delta} - \overrightarrow{v_p} | \overrightarrow{A} \rangle . \overrightarrow{A}$$

Remark 2.2:

I would like to remind you that the theorem are important and can be used for studying in the field of billiards and others.

3. CONVERTING THE THEORETICAL RESULTS OF THE COLLISION

In this chapter, we continue to study the results we have obtained from the second chapter. This is in order to obtain the algebraic formulas of the velocity vector after colliding between a material point and a solid flat plate in the absolute space, and considering the mass of the material point completely neglected by definition in front of the solid flat plate. However, the transition of the analytical study from collision between two hard-spheres to colliding between a material point and a solid flat plate should be done in two steps to maintain the logical serial correlation between the chapters.

Therefore, we divide the study in this chapter into two steps. The first step is to study the results after colliding between a material point with a hard-sphere and then in the second step to study the results after colliding between a material point and a solid flat plate.

3.1 First step: From two different spheres to a collision between a material point and a hard-sphere.

In the second chapter (**2**), the most important part of the study of collisions was to convert the study from a normal Galilean reference to a Galilean reference $\mathcal{R}(C;\vec{i};\vec{j};\vec{k})$ centered at the point (C) center of mass reference so that the momentum quantity is equal to zero-vector. Therefore, in this chapter, we conduct another analytical study from a collision between two hard-spheres to a collision between a material point and a hard-sphere.

If we consider that the study was done between two hard-spheres, it is necessarily distinguished by two different radius and we define them as ($r_{(P)}$) and ($R_{(\Delta)}$) respectively.
And also because they have two different mass, they may have ***two different volumetric mass***, and we define them respectively ($\mathcal{C}_{(P)}$) and ($\mathcal{C}_{(\Delta)}$).
So we now have to study it in the Galilean Reference $\mathcal{R}(C;\vec{i};\vec{j};\vec{k})$; which is centered on point (C).
In the moment of collision, the two spheres are in contact point (C), and from ***Fig №03*** and ***Fig №04***, we see that they have the ***same tangent plane*** S_Δ, which the normalized vector is $\vec{A}$, and the normal line **(N)** that is perpendicular to the tangent plane S_Δ at the point of tangency (C).

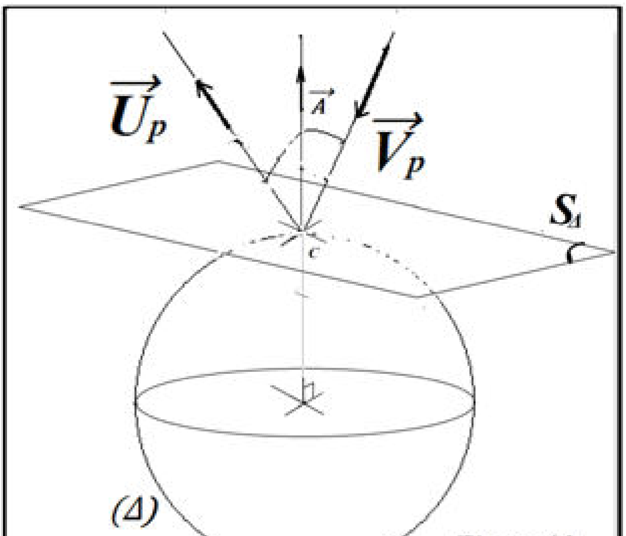


Figure 03

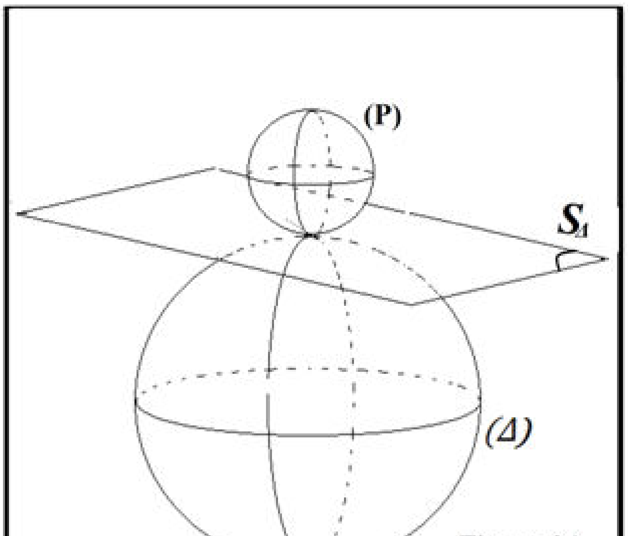


Figure 04

Now we are trying to find important relationships that relate to the elastic collision between a material point and a hard-sphere. So, if we consider that the mass of the sphere ***(P)*** is ***neglected*** for the sphere (***Δ***), that is to say:

$$\frac{m_{(P)}}{M_{(\Delta)}} \cong 0 \qquad (3.11)$$

Hence, we define $\mathcal{V}_{(P)}$ and $\mathcal{V}_{(\Delta)}$ respectively that the volumes of the hard-spheres ***(P)*** *and* (***Δ***) respectively**.**
So, we have:

$$\mathcal{V}_{(P)} = \frac{4}{3}.\pi.r_{(P)}{}^3 \quad and \quad \mathcal{V}_{(\Delta)} = \frac{4}{3}.\pi.R_{(\Delta)}{}^3$$

Again, we have:

$$m_{(P)} = \mathcal{C}_{(P)}.\mathcal{V}_{(P)} \quad and \quad M_{(\Delta)} = \mathcal{C}_{(\Delta)}.\mathcal{V}_{(\Delta)}$$

So, we obtain:

$$\frac{m_{(P)}}{M_{(\Delta)}} = \frac{\mathcal{C}_{(P)}.\mathcal{V}_{(P)}}{\mathcal{C}_{(\Delta)}.\mathcal{V}_{(\Delta)}} = \frac{\mathcal{C}_{(P)}}{\mathcal{C}_{(\Delta)}}.\frac{\frac{4}{3}.\pi.r_{(P)}{}^3}{\frac{4}{3}.\pi.R_{(\Delta)}{}^3} = \frac{\mathcal{C}_{(P)}}{\mathcal{C}_{(\Delta)}}.\frac{r_{(P)}{}^3}{R_{(\Delta)}{}^3} = \frac{\mathcal{C}_{(P)}}{\mathcal{C}_{(\Delta)}}.\left(\frac{r_{(P)}}{R_{(\Delta)}}\right)^3$$

$$\Rightarrow \frac{m_{(P)}}{M_{(\Delta)}} = \underbrace{\frac{\mathcal{C}_{(P)}}{\mathcal{C}_{(\Delta)}}}_{=K}.\left(\frac{r_{(P)}}{R_{(\Delta)}}\right)^3$$

$$\Rightarrow \frac{m_{(P)}}{M_{(\Delta)}} = \underbrace{K}_{constant} \cdot \left(\frac{r_{(P)}}{R_{(\Delta)}}\right)^3$$

However, as the case is considered **(3.11),** we concluded the following:

$$\frac{m_{(P)}}{M_{(\Delta)}} \cong 0 \quad \Rightarrow \quad K.\left(\frac{r_{(P)}}{R_{(\Delta)}}\right)^3 \cong 0 \quad \Rightarrow \quad \frac{r_{(P)}}{R_{(\Delta)}} \cong 0$$

$\Rightarrow \; r_{(P)} \cong 0$ **,** if considered $R_{(\Delta)}$ constant.

From this result, we can conclude that the sphere *(P)* is ***a point zero-dimensional*** by definition for the hard-sphere ***(Δ).*** Therefore, we conduct a theoretical simulation of the particular case mentioned. Thus, we put the following definition and the proposition with proof.

Definition 3.1.1:

*We call **the fixed elastic collision**, the collision of the hard-sphere **(P)** with the hard-sphere **(Δ)** respectively, and the mass of the hard- sphere **(P)** is **negligible** with respect to the mass of the sphere **(Δ)** i.e.* $\frac{m_{(P)}}{M_{(\Delta)}} \cong 0.$

*So we can consider the hard-sphere **(P)** as **a material point** for the hard-sphere **(Δ).***

The text of the following proposition is named ***the law of the fixed elastic collision*** **[F.E.C]**, which will be fundamental when discussing the assumptions on which the ***Michelson experiment*** was based and which will be discussed in the chapter (**4**) of this article.

Proposition 3.1.1:

*In the field of **the fixed elastic collision*** **[F.E.C]** *relative to the Galilean reference* $\mathcal{R}(O;\vec{\imath};\vec{\jmath};\vec{k})$*; the velocity vector after the collision of the material point **(P)** with the hard-sphere **(Δ)** is written as follows:*

$$\overrightarrow{u_p} = \overrightarrow{v_p} - 2 \cdot \langle \overrightarrow{v_p} - \overrightarrow{v_\Delta} | \vec{A} \rangle . \vec{A} \qquad (3.12)$$

*And the velocity vector after the collision of the hard-sphere **(Δ)** is* $\overrightarrow{v_\Delta}$ *remaining without change.*

Proof

According to ***the Theorem 2.1,*** we have the equality (**2.9**) that:

$$\overrightarrow{u_p} = \overrightarrow{v_p} - 2 \cdot \frac{M_\Delta}{M_\Delta + m_P} . \langle \overrightarrow{v_p} - \overrightarrow{v_\Delta} | \vec{A} \rangle . \vec{A}$$

$$\Rightarrow \overrightarrow{u_p} = \overrightarrow{v_p} - 2 \cdot \frac{\frac{M_\Delta}{M_\Delta}}{\frac{M_\Delta}{M_\Delta} + \frac{m_P}{M_\Delta}} . \langle \overrightarrow{v_p} - \overrightarrow{v_\Delta} | \vec{A} \rangle . \vec{A}$$

$$\Rightarrow \overrightarrow{u_p} = \overrightarrow{v_p} - 2 \cdot \frac{1}{1 + \frac{m_P}{M_\Delta}} . \langle \overrightarrow{v_p} - \overrightarrow{v_\Delta} | \vec{A} \rangle . \vec{A}$$

However, as the case is considered (**3.11**), we deduce:

$$\overrightarrow{u_p} = \overrightarrow{v_p} - 2 \cdot \frac{1}{1 + \underbrace{\frac{m_P}{M_\Delta}}_{=0}} . \langle \overrightarrow{v_p} - \overrightarrow{v_\Delta} | \vec{A} \rangle . \vec{A}$$

$$\Rightarrow \overrightarrow{u_p} = \overrightarrow{v_p} - 2 \cdot \frac{1}{1 + 0} . \langle \overrightarrow{v_p} - \overrightarrow{v_\Delta} | \vec{A} \rangle . \vec{A}$$

$$\Rightarrow \overrightarrow{u_p} = \overrightarrow{v_p} - 2 \cdot \langle \overrightarrow{v_p} - \overrightarrow{v_\Delta} | \vec{A} \rangle . \vec{A}$$

Again, we have the equality (**2.10**) that:

$$\overrightarrow{u_\Delta} = \overrightarrow{v_\Delta} - 2 \frac{m_P}{M_\Delta + m_P} \langle \overrightarrow{v_\Delta} - \overrightarrow{v_p} | \vec{A} \rangle . \vec{A}$$

$$\Rightarrow \overrightarrow{u_\Delta} = \overrightarrow{v_\Delta} - 2\frac{\frac{m_{(P)}}{M_{(\Delta)}}}{1+\frac{m_{(P)}}{M_{(\Delta)}}}\langle \overrightarrow{v_\Delta} - \overrightarrow{v_p} | \overrightarrow{A}\rangle . \overrightarrow{A}$$

Thus, as the case is considered **(3.11)**, we deduce:

$$\overrightarrow{u_\Delta} = \overrightarrow{v_\Delta} - \underbrace{2\frac{0}{1+0}\langle \overrightarrow{v_\Delta} - \overrightarrow{v_p} | \overrightarrow{A}\rangle . \overrightarrow{A}}_{=0}$$

$$\Rightarrow \overrightarrow{u_\Delta} = \overrightarrow{v_\Delta}$$

3.2 Second step: From a material point with the sphere to a collision between a material point with a solid flat plate.

Furthermore, Guttler *et al.* **[7]** studied the type of collision and called it the normal collision, but not in all inertial reference frames, without giving details of the velocity vectors of the collision in the inertial reference frame.

So in this step, following the first step, we convert the collision from a material point with the hard-sphere to a collision between a material point with ***a Solid flat plate*** $(\boldsymbol{S})$**.** We mentioned in Chapter (**3**), that there is a tangent plane $\boldsymbol{S_\Delta}$ between the two spheres at the moment of collision. We have seen that the laws remain intact, all of which have logical correlations with ***proposition 2.5.***

With regard to ***the proposition 2.5,*** we may see that equality is not at all associated with the value of mass or metric dimensions of objects, only has correlations between velocity vector values of objects. However, the result of the collision is entirely related to the tangent plane $\boldsymbol{S_\Delta}$. In other words, the results necessarily relate to the situation of the normal line (**N**), which is distinguished by the direction vector $\overrightarrow{\boldsymbol{A}}$.

Additionally, we simulate the ***proposition 3.1*** in order to be valid for optics. Moreover, in classical dynamics, light is considered as an electromagnetic wave, which is described by Maxwell's equations **[15].** As it is well-known, as Newton's particle theory of light which states that light is only a particle subject to the laws of mechanics **[13].**

Newton proposed that light consists of little masses. This means that a horizontal beam of light near the earth is undergoing projectile motion, and forms a parabola. The straight line we observe is due to the fact that the speed of the particles is so great.

Compared to what we have already said in second Chapter, we can say that light or, in other words***, a photon*** is a ***material point***, and the mirror is a ***solid flat plate.***

So, for a central Galilean reference $\boldsymbol{\mathcal{R}(C;\vec{\imath};\vec{\jmath};\vec{k})}$**,** if we assume that a material point ***(P)*** has collided with a solid flat plate $(\boldsymbol{S})$ at point (***C***), the mass of the material point ***(P)*** is ***negligible*** for the solid flat plate$(\boldsymbol{S})$. It is necessarily subject to the text of the ***proposition 2.5*** we mentioned, but the normal line (**N**) of this collision is the same as for the solid flat plate$(\boldsymbol{S})$, since the solid flat plate $(\boldsymbol{S})$ is part of the tangent plane $\boldsymbol{S_\Delta}$**,** during the moment of collision. To simulate this, we can put this following definition with two propositions, and we may omit to prove them, because they are clear.

Definition 3.2.1:

*We call **the fixed elastic collision**, the collision of the material point **(P)** with **the solid flat plate** $(\boldsymbol{S})$ which the axis vector $\overrightarrow{\boldsymbol{A}}$, and the mass of the material point **(P)** is **negligible** with respect to the mass of **the solid flat plate (S)** i.e.* $\frac{\boldsymbol{m_{(P)}}}{\boldsymbol{M_{(S)}}} \cong \boldsymbol{0}$**.**

Proposition 3.2.1:

*In the field of **the fixed elastic collision*** **[F.E.C]** *relative to the Galilean reference* $\boldsymbol{\mathcal{R}(O;\vec{\imath};\vec{\jmath};\vec{k})}$*; the velocity vector after the collision of the material point **(P)** with **the solid flat plate (S)** which the axis vector* $\overrightarrow{\boldsymbol{A}}$ *is written as follows:*

$$\overrightarrow{u_p} = \overrightarrow{v_p} - 2\cdot\langle \overrightarrow{v_p} - \overrightarrow{v_S} | \overrightarrow{A}\rangle . \overrightarrow{A} \qquad (3.13)$$

Proposition 3.2.2:

If we consider that $\vec{A}$ *is the axis vector of* ***the fixed elastic collision*** **[F.E.C]** *between a material point* ***(P)*** *with* ***the solid flat plate (S).*** $(-\vec{A})$ *Is also an axis vector of* ***the fixed elastic collision*** **[F.E.C]** *between a material point* ***(P)*** *with* ***the solid flat plate (S).***

Proof

If we consider that $\vec{A}$ is the axis vector of **the fixed elastic collision [F.E.C]** between the spheres$(\boldsymbol{P})$ and $(\boldsymbol{\Delta})$ respectively. That is to say:

$$\overrightarrow{u_p} = \overrightarrow{v_p} - 2 \cdot \langle \overrightarrow{v_p} - \overrightarrow{v_S} | \vec{A} \rangle . \vec{A}$$

$$\Rightarrow \overrightarrow{u_p} = \overrightarrow{v_p} - 2 \cdot \langle \overrightarrow{v_p} - \overrightarrow{v_S} | -(-\vec{A}) \rangle . [-(-\vec{A})]$$

$$\Rightarrow \overrightarrow{u_p} = \overrightarrow{v_p} + 2 \cdot \langle \overrightarrow{v_p} - \overrightarrow{v_S} | (-\vec{A}) \rangle . [-(-\vec{A})]$$

$$\Rightarrow \overrightarrow{u_p} = \overrightarrow{v_p} - 2 \cdot \langle \overrightarrow{v_p} - \overrightarrow{v_S} | (-\vec{A}) \rangle . (-\vec{A})$$

We obtain the result that $(-\vec{A})$ is also an axis vector of **the fixed elastic collision [F.E.C]** between a material point ***(P)*** and ***the solid flat plate (S).***

4. CRITICAL ANALYSIS OF THE THEORETICAL FRAMEWORK OF THE MICHELSON-MORLEY EXPERIMENT.

Until the end of the nineteenth century, light was supposed to propagate in the medium called ether.

Then the idea of the experiment realized by Michelson was suggested to him by a letter which Maxwell wrote in 1865 **[15],** he set forth a method for measuring the velocity of the earth with respect to the ether, supposed motionless fluid in the space.

According to this hypothesis, Earth and the ether are in relative motion, implying that a so-called "ether wind" should exist. Although it would be possible, in theory, for the Earth's motion to match that of the ether at one moment in time, it was not possible for the Earth to remain at rest with respect to the ether at all times, because of the variation in both the direction and the velocity of the motion. At any given point on the Earth's surface, the magnitude and direction of the wind would vary with time of day and season. By analyzing the return speed of light in different directions at various different times, it was thought to be possible to measure the motion of the Earth relative to the ether. The expected relative difference in the measured speed of light was quite small, given that the velocity of the Earth in its orbit around the Sun has a magnitude of about one hundredth of one percent of the speed of light **[16].** Therefore, his goal of the experiment was to prove two issues:

1 - To prove the existence of the medium called the ether so that the electromagnetic waves transmitted by it.

2 - Prove the possibility of setting the velocity of the Earth for the sun.

Michelson built the theoretical framework of the experiment on this example: so that two swimmers swim in one river; while one swims with the river back and forth, the other starts from the same point first and swims in the width of the river back and forth and cuts the same distance as the first cut it and the same Time and it became clear to him from the law of the collection of velocities that the two cannot return at the same time because the casual swim arrives first; that is also the light. Therefore, he believed that the time spent by the first swimmer is:

$$\boldsymbol{t_1} = \frac{\boldsymbol{d}}{\boldsymbol{C_\beta + v}} + \frac{\boldsymbol{d}}{\boldsymbol{C_\beta - v}} \qquad (\mathbf{4.14})$$

As for the period of time spent by the second swimmer so that it is perpendicular to the flow of the river ***(Fig №05)*** it will be:

$$\boldsymbol{t_2} = \frac{\mathbf{2.}\boldsymbol{d}}{\sqrt{\boldsymbol{C_\beta}^2 - \boldsymbol{v}^2}} \qquad (\mathbf{4.15})$$

Thus, the difference between the times for the longitudinal path and the transverse path, taking into account:

$$v \ll C_\beta \Rightarrow \triangle t = t_1 - t_2 \cong \frac{d}{C_\beta}.\left(\frac{v}{C_\beta}\right)^2 \qquad (4.16)$$

In the case of Maxwell, the highest velocity of the Earth in its orbit is:

$$v = 3.10^4 m.s^{-1} \; and \; C_{light} = 3.10^8 m.s^{-1} \qquad (4.17)$$

This gives, for a length of:

$$\triangle t = 3.10^{-17}.s \qquad (4.18)$$

Michelson designed an entirely new and precise instrument that would enable him to measure the duration: ***the Michelson Interferometer***.

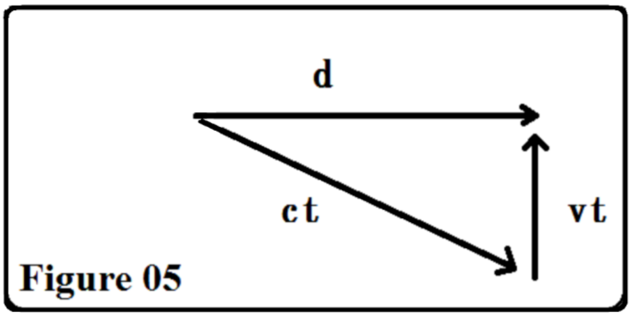


Figure 05

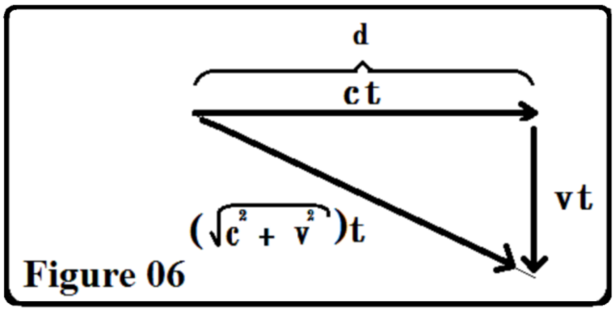


Figure 06

The difference of time between the return flow in the direction of the ether movement and in the orthogonal direction would therefore be:

$$\triangle t = t_1 - t_2 \cong \frac{d}{C_\beta}.\left(\frac{v}{C_\beta}\right)^2 \qquad (4.19)$$

One of the waves propagating a little faster than the other, they would be slightly out of phase, which would manifest itself by a shift of the fringes of interference compared to the case where the Earth would be motionless.

Unable to stop the Earth, Michelson's idea was to rotate his 90^0 interferometer while observing the fringes. The inversion of the role of the two arms of the interferometer therefore had to cease, at the level of the interference figure, a shift of the fringes due to the difference of the optical paths. How many fringes?

$$N = 2.C_\beta.\frac{\triangle t}{\lambda} \cong 2.\frac{L}{\lambda}.\left(\frac{v}{C_\beta}\right)^2 \qquad (4.20)$$

Either for $L = 1m \; and \; \lambda = 500nm$ a value of $N = 0.04$

But he did not observe any movement of fringe!!!**???**

Joined by Morley **[17],** they enlarged the apparatus to the point of foreseeing a displacement of 4 fringes. Once again, they saw no fringe movement. This time the result was unambiguous: ***there was no detectable ether wind.***

The speed of light is not influenced by the motion of the Earth, as it is based on its conclusions.

All experiments that have tried to measure the absolute motion of the Earth with respect to ether have ***failed***.

In principle, Michelson's idea is wonderful, but the conclusions related to this idea include several theoretical errors. If we try to examine previous conclusions, discuss the Michelson's idea, and look for errors in detail, we must discuss the assumptions of Michelson about what is true? What is wrong?

For the idea of ethers, swimmers and the river: Michelson assumed that the swimmer who is swimming orthogonally with the river bank in case of going has two different velocities respectively $[C_\beta]$ and $\left[\sqrt{{C_\beta}^2 - v^2}\right]$, while the current is silent and while moving respectively.

Let us ask: What if the absolute value of the flow velocity of the river is equal to the absolute value of the swimmer's velocity, that is:

$$\left\{V = \sqrt{{C_\beta}^2 - v^2} \quad and \quad C_\beta = v\right\} \Rightarrow V = 0 \tag{4.21}$$

That is, the velocity of the swimmer in the case of river flow is equal to zero, i.e. the swimmer will ***not move from its place***, and this is strange.

Let us ask again: What if the absolute value of the flow velocity of the river is greater than the absolute value of the swimmer's velocity, that is:

$$\left\{V = \sqrt{{C_\beta}^2 - v^2} \quad and \quad v > C_\beta\right\} \Rightarrow V = \underbrace{\sqrt{-1}}_{\notin \mathcal{R}} . \sqrt{v^2 - {C_\beta}^2} \tag{4.22}$$

That is, the swimmer's velocity value is not ***a real number***, and we live a realistic experience, that means Michelson's wrong assumption.

Also, for the swimmers to come and go, Michelson assumed in the case of the river's silent that the swimmers' departure was from ***a single point*** and their return was to ***a single point*** which is ***reasonable*** (see ***Fig №8, Fig №10***).

But if the current of the river is moving, Michelson assumed that the swimmers' departure was from a single point and their return was not to a single point, but rather their return to the river bank, which was ***unreasonable*** and ***unreliable.***

Again, What if we assume that in the case of the silence of the current of the river, where the second swimmer swims to the other bank with a deviation of the angle [θ]. And go back to the same point where he began, passing the same swimming distance and the return of the first swimmer who swim in parallel with the first bank, and then analyze the situation again if the river is in a state of running.

From this preliminary discussion, we see that the idea contains clear logical defects, so it must be free from logical defects and must be reformulated under logical conditions.

This assumption was not put forward once in all the experiments that relied on Michelson 's perceptions, and we can see the historical review of the experiment in this article **[18], [19]**, that his critics did not address the question of the validity of his assumptions or not. With such criticisms, it is clear that Michelson adopted the theoretical framework of the experiment quickly, and did not pay attention, not once, to the possibility that he was mistaken in his assumptions after repeating his experience in 1887**[17].** He repeated his assumptions in similar experiments, and the same with Morley and Miller **[25],[26],** Miller's work alone **[27],** the experiments of Piccard and Stahel **[31],[32],[33]**, the refinement of Kennedy **[28]**, and Illingworth **[30]**, the repetitions of Michelson et al. **[34]**, up to Joos **[35]**.

Once again with Kennedy **[29],** he started with Thorndike a new class of experiments: a null-result in the M-M experiment was assumed, thus implying a length-contraction in the context of special theory of relativity, the objective was then to test the ensuing time-dilation and/or the isotropy of the space. There were no radical changes to the theoretical framework of the experiment except with Sagnac [36] who tried to prove the opposite of what Michelson had adopted, and tried to prove the existence of the medium called the ether, and the validity of the law of addition of velocities, which depend on the old classical view of Newton, and its content that the speed of light increases and decreases By changing reference frames inertia.

From this analytical study, we will demonstrate that the results obtained by Michelson correspond exactly to the real reality, embodied by the results, which the waves that separated at the first point back to a single point at the same time. Whether it is the first point for the Galilean reference where the earth is centered, or another point for the Galilean reference where the Sun is centered.

And to conclude that there is no delay between them, and this is contrary to what Michelson assumed. This is what we will interpret theoretically by the fixed collision law **[F.E.C]** that we get from the third chapter.

Although we have discovered Michelson's errors in this preliminary discussion, how can we find the right solution to his assumptions?

4.1 Discussion and comment on the Michelson idea ($\mathbf{1}^{st}$part):

In order to correct the mistakes in the idea of the river and the swimmers, it should be replaced by the following idea, which satisfies the conditions of going to and from the other river bank, in both cases, without contradicting one another as it was with the first. The following idea specifications are almost identical to the function of an interferometer.

We should use this experiment using the ***Fig №07.*** Consider a square ship which he named **((T))** of length ***d***, situated on the river and installed in parallel with the banks of the river.

Assume that the four researchers **Copernicus, Galileo, Kepler** and **Newton** agree to make measurements and record the experimental data from the surface of the square ship installed in parallel with the banks of the river. **Copernicus** and **Galileo** agreed to walk back and forth on the edges of the square ship at the same velocity C_β, While **Kepler** remains constant at the head E of the square ship **((T)).**

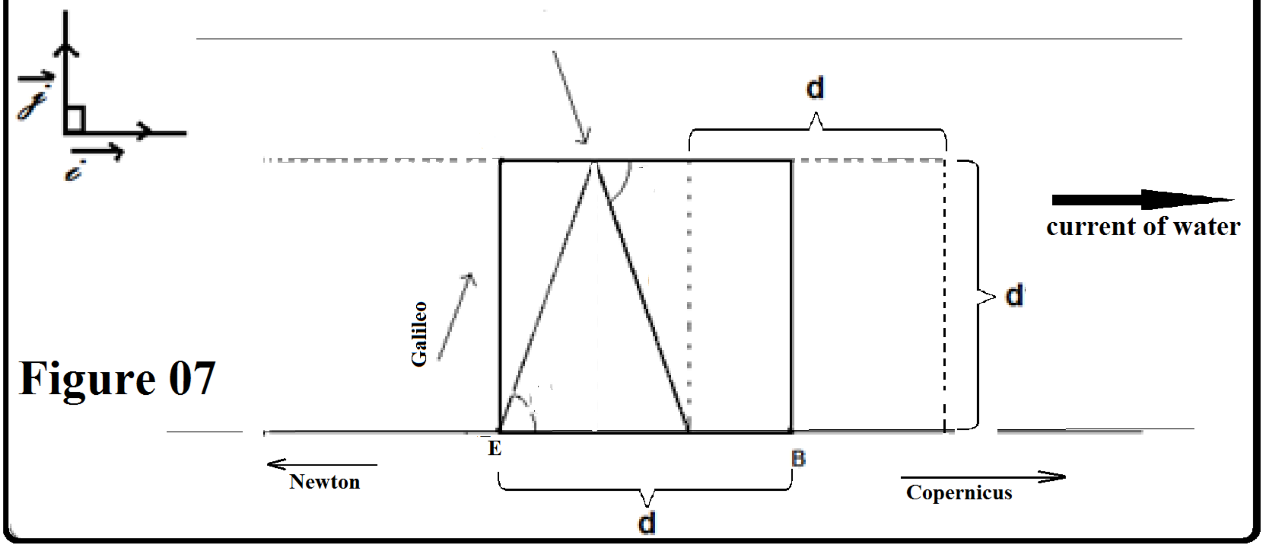


Figure 07

4.1.1 The 1^{st} Case that the river's water is still and does not move.

In the absence of the flow of the river water, this means that the value of the velocity of the square ship is equal to zero compared to **Newton** i.e. $\vec{V}_N = \vec{0}$, and therefore record the following information: **Copernicus** moves horizontally according to the equation that:

$$\overrightarrow{EM_C}(t) = \begin{cases} C_\beta . t . \vec{i} & if \;\; t \in \left[0, \dfrac{d}{C_\beta}\right] \\ (-C_\beta . t + 2.d).\vec{i} & if \;\; t \in \left[\dfrac{d}{C_\beta}, \dfrac{2.d}{C_\beta}\right] \end{cases} \qquad (4.23)$$

And **Galileo** moves vertically according to the equation that:

$$\overrightarrow{EM_G}(t) = \begin{cases} C_\beta . t . \vec{j} & if \; t \in \left[0, \dfrac{d}{C_\beta}\right] \\ (-C_\beta . t + 2.d).\vec{j} & if \;\; t \in \left[\dfrac{d}{C_\beta}, \dfrac{2.d}{C_\beta}\right] \end{cases} \qquad (4.24)$$

Thus, the plane of the paths of the movement is the ***Fig №08*****:**

The conclusions of this case are as follows: **Copernicus** and **Galileo** move at the same velocity C_β from the same point E, where the center of the reference frame $\Sigma_1(E, \vec{i}, \vec{j})$, and they return to the same velocity C_β at the same point ***E,*** and they move at the same distance $2.d$, and the dimensions of the square ship remain constant and equal to ***d.***

4.1.2 The 2^{nd} case that so the current of the river moves.

In this case, the current or river water moves forward at constant velocity, as the square ship moves by the current, and **Kepler** imagines that **Newton** is moving horizontally on the bank opposite the current. **Newton** - represented by point S which is the center of the reference frame $\Sigma_2(S, \vec{i}, \vec{j})$- moves horizontally relative to **Kepler** with the velocity vector $\vec{V}_N = -v.\vec{i}$, contrary to a movement of **Copernicus**. And therefore record the following information:

$$\overrightarrow{ES}(t) = -v.t.\vec{i} \quad if \;\; t \in \left[0, \frac{2.d}{C_\beta}\right] \qquad (4.25)$$

So:

$$\vec{V}_N = \frac{\partial}{\partial t}\overrightarrow{ES}(t) = -v.\vec{\imath} \quad (4.26)$$

But, with using ***Chasles relation***, we have that:

$$\begin{cases} \overrightarrow{SM_C} = \overrightarrow{SE} + \overrightarrow{EM_C} \\ \quad and \\ \overrightarrow{SM_G} = \overrightarrow{SE} + \overrightarrow{EM_G} \end{cases} \Rightarrow \begin{cases} \overrightarrow{SM_C} = \overrightarrow{EM_C} - \overrightarrow{ES} \\ \quad and \\ \overrightarrow{SM_G} = \overrightarrow{EM_G} - \overrightarrow{ES} \end{cases}$$

This means that:

$$\overrightarrow{SM_C}(t) = \begin{cases} (C_\beta + v).t.\vec{\imath} & if\ t \in \left[0, \frac{d}{C_\beta}\right] \\ [-(C_\beta - v).t + 2.d].\vec{\imath} & if\ t \in \left[\frac{d}{C_\beta}, \frac{2.d}{C_\beta}\right] \end{cases} \quad (4.27)$$

And:

$$\overrightarrow{SM_G}(t) = \begin{cases} C_\beta.t.\vec{\jmath} + v.t.\vec{\imath} & if\ t \in \left[0, \frac{d}{C_\beta}\right] \\ (-C_\beta.t + 2.d).\vec{\jmath} + v.t.\vec{\imath} & if\ t \in \left[\frac{d}{C_\beta}, \frac{2.d}{C_\beta}\right] \end{cases} \quad (4.28)$$

Therefore:

$$\overrightarrow{V_{NC}} = \frac{\partial}{\partial t}\overrightarrow{SM_C}(t) = \begin{cases} +(C_\beta + v).\vec{\imath} & if\ t \in \left[0, \frac{d}{C_\beta}\right] \\ -(C_\beta - v).\vec{\imath} & if\ t \in \left[\frac{d}{C_\beta}, \frac{2.d}{C_\beta}\right] \end{cases} \quad (4.29)$$

And:

$$\overrightarrow{V_{NG}} = \frac{\partial}{\partial t}\overrightarrow{SM_G}(t) = \begin{cases} C_\beta.\vec{\jmath} + v.\vec{\imath} & if\ t \in \left[0, \frac{d}{C_\beta}\right] \\ -C_\beta.\vec{\jmath} + v.\vec{\imath} & if\ t \in \left[\frac{d}{C_\beta}, \frac{2.d}{C_\beta}\right] \end{cases} \quad (4.30)$$

This means that the absolute values of the velocity vectors are written as follows:

$$\|\overrightarrow{V_{NC}}\| = \begin{cases} |C_\beta + v| & if\ t \in \left[0, \frac{d}{C_\beta}\right] \\ |C_\beta - v| & if\ t \in \left[\frac{d}{C_\beta}, \frac{2.d}{C_\beta}\right] \end{cases} \quad (4.31)$$

And:

$$\|\overrightarrow{V_{NG}}\| = \sqrt{{C_\beta}^2 + v^2} \quad with \quad t \in \left[0, \frac{2.d}{C_\beta}\right] \quad (4.32)$$

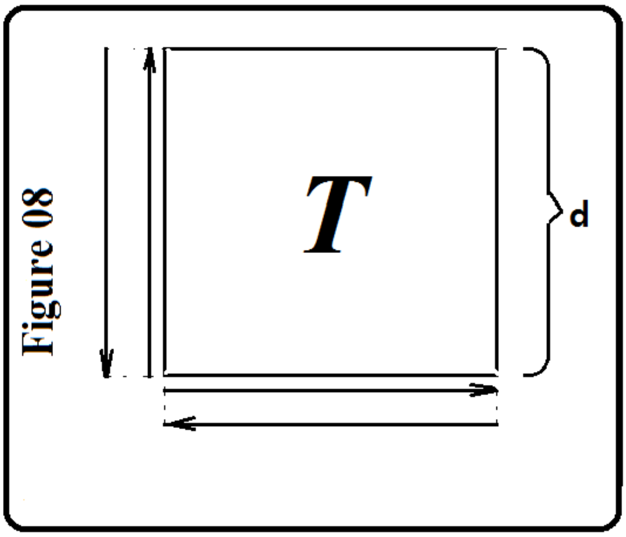
And the lengths of the paths of the movement are written as follows:

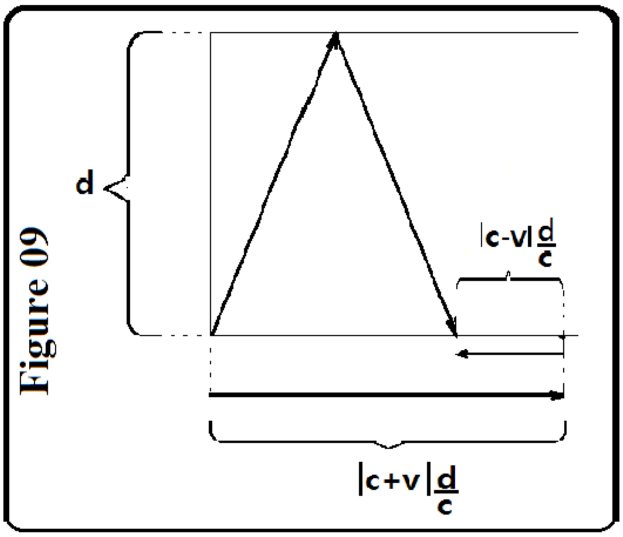
$$L(S, M_C) = \begin{cases} |C_\beta + v|.\frac{d}{C} & if\ t \in \left[0, \frac{d}{C_\beta}\right] \\ |C_\beta - v|.\ \frac{d}{C} & if\ t \in \left[\frac{d}{C_\beta}, \frac{2.d}{C_\beta}\right] \end{cases} \quad (4.33)$$

And:

$$L(S, M_G) = \frac{2.d}{C_\beta}.\sqrt{{C_\beta}^2 + v^2} \quad with \quad t \in \left[0, \frac{2.d}{C_\beta}\right] \quad (4.34)$$

Thus, the plane of the paths of the movement is the ***Fig №09:***





The conclusions of this case are as follows:

Copernicus and **Galileo** move from the same point, and they return to the other point (***a single point*)**, but they move with the different vectors of velocities.

By comparing the results obtained with the Michelson assumptions, we conclude the following:

1-Michelson neglected the details of the study by relying on a single reference frame, from which he took false data, but the data changes from one reference frame to another.

2- He also committed an error in assessing the velocity value, putting the value of $\left(\sqrt{C_\beta{}^2 - v^2}\right)$ (***Fig №05***) instead of $\left(\|\overrightarrow{V_{NG}}\| = \sqrt{C_\beta{}^2 + v^2}\right)$ **(see *Fig №06*)**, and the latter is correct **(4.32).** So it is ***an illusion.***

3-The dimensions of the solids objects (***square ship***) remain constant and independent of the system changes References, stable or mobile.

4-The paths of movement have changed each time the reference system has changed. This means that the measurement of the trajectory of the movement also changes **(4.33), (4.34).**

5-The durations of the times of the movements remain constant and independent of the change of references of the system, that is to say:

$$t_1 = \frac{|C_\beta + v|.\frac{d}{C_\beta}}{|C_\beta + v|} + \frac{|C_\beta - v|.\ \frac{d}{C_\beta}}{|C_\beta - v|} = \frac{d}{C_\beta} + \frac{d}{C_\beta} = 2.\frac{d}{C_\beta} \tag{4.35}$$

And:

$$t_2 = \frac{\frac{d}{C_\beta}\cdot\sqrt{C_\beta{}^2 + v^2}}{\sqrt{C_\beta{}^2 + v^2}} + \frac{\frac{d}{C_\beta}\cdot\sqrt{C_\beta{}^2 + v^2}}{\sqrt{C_\beta{}^2 + v^2}} = \frac{d}{C_\beta} + \frac{d}{C_\beta} = 2.\frac{d}{C_\beta} \tag{4.36}$$

On the other hand, that is to say:

$$t_1 = t_2 = 2.\frac{d}{C_\beta} \Rightarrow \triangle\, t = t_1 - t_2 = 0 \tag{4.37}$$

This contrasts with Michelson's assumptions **(4.14), (4.15), (4.16).**

4.2 Discussion and comment on the Michelson idea (2^{nd}part):

So now we want to discuss the second and last part of Michelson's work. What is exactly, the result obtained from the device; he named the ***Michelson's interferometer.*** But how do we apply these laws to this experiment? When Michelson put the theoretical framework in which he concluded the flow of river water and swimmers, he go to practically applied to a machine designed for a fundamental purpose.

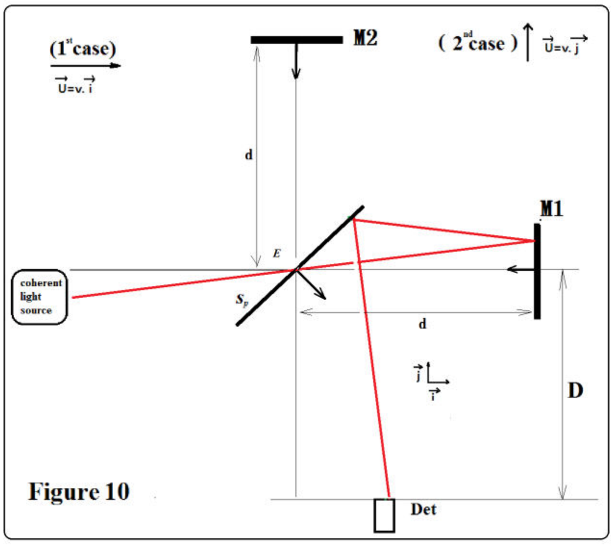


Figure 10

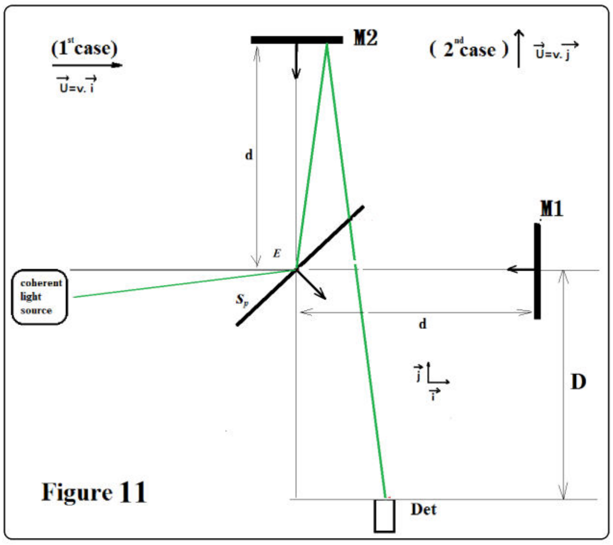


Figure 11

The device which has been called the ***Michelson's interferometer*** consists of a beam splitter $\mathbf{S_p}$ separating a light beam in two, and two mirrors **M1** and **M2** placed at Equal distance **d** of the blade. The two reflected beams are recombined by the semi-reflecting plate $\mathbf{S_p}$ and their interference pattern is observed on a screen.

From ***Fig №10*** and ***Fig №11***, we see that Michelson made another mistake, considering that the velocity vector for the motion of the earth is ***parallel*** to the sunlight beam, as it is in accordance with the first case. So it seems to the observer that the Earth is moving away from the sun, but the fact that the earth rotates around the velocity vector sun is also identical for the second case. But I want to study both cases using the laws already mentioned. In the analysis of the set of data in ***Fig №10*** and ***Fig №11***, we see that there are two movements of two distinct beams and after the separation of the two movements and plotted in the ***Fig №10*** and ***№11***, we note that each beam is exposed to a double reflection and I say to double collision.

Initially, we look at the first case where Michelson based his experiment. And to recall once more that they do not coincide with the rotation of the earth on the sun, but it is ***a linear withdrawal movement*** *(translation movement)* of the Earth away from the Sun.

To this point, we want to study the movement of the photon emitted by the sun in both cases that distinguish the movement of the Earth for the sun, and to begin first study for the Galilean reference where the sun center.

4.2.1 *The sun is the center of the reference system*

The sun is the center of our solar system, and its effect on the planets is clearly determined by the law of gravity. The influence of planets on the sun is negligible, and our assumption that it is the center of the reference frame remains intact in order to study the movement of any object within the solar system. And the photons resulting from the theory of the classical light version can be considered objects that are within the solar system, and focus on the movement of the photon, which emits from the sun towards the interferometer installed on the surface of earth. As the optical beam is divided into two parts as it reaches the separating mirror, we should assume that two photons are simultaneously launched and at the same speed from the sun, separated by the separator mirror.

So we assume that the photons represented by the points ($\boldsymbol{P_{h1}}$) and ($\boldsymbol{P_{h2}}$) move according to this equation

$$\overrightarrow{SP_{h1}} = \overrightarrow{SP_{h2}} = C.t.\vec{i} + R.\vec{i}$$

, before separation when they arrive at the beam splitter $\mathbf{S_p}$ and continue to search for the rest of the equations of motion for the two points until they reach the line of the detector $\mathbf{D_{et}}$.

Give the equations **(4.38)** and **(4.39)** respectively for the points ($\boldsymbol{P_{h1}}$) and ($\boldsymbol{P_{h2}}$) respectively, and we find them in both cases.

$$\overrightarrow{SP_{h1}} = \begin{cases} \vec{V}.t + R.\vec{\imath} & if\ t \in [0, t_1] \\ \overrightarrow{U_1}.t + \overrightarrow{r_1} & if\ t \in [t_1.t_2] \\ \overrightarrow{U_2}.t + \overrightarrow{r_2} & if\ t \in [t_2.t_3] \end{cases} \quad (4.38)$$

And:

$$\overrightarrow{SP_{h2}} = \begin{cases} \vec{V}.t + R.\vec{\imath} & if\ t \in \left[-\frac{R}{C}, 0\right] \\ \overrightarrow{U_1^0}.t + \overrightarrow{r_1^0} & if\ t \in [0, t_1^0] \\ \overrightarrow{U_2^0}.t + \overrightarrow{r_2^0} & if\ t \in [t_1^0.t_2^0] \end{cases} \quad (4.39)$$

4.2.1.1 Such that the two vectors $\vec{V}$ and $\overrightarrow{v_S}$ are parallel (1^{st} Case):

In this case where it corresponds to the motion of the Earth in relation to the sun as imagined by Michelson, the earth moves with ***a linear withdrawal movement***, moving away from the sun according to this equation **(4.40).**

$$\overrightarrow{SE} = v.t.\vec{\imath} + R.\vec{\imath} \quad (4.40)$$

And with an interferometer installed on the surface of the earth, the components of an interferometer, including mirrors, move with the same velocity vector **(4.41)** as the earth speed vector.

$$\overrightarrow{v_S} = v.\vec{\imath} \quad (4.41)$$

Since we are only studying the movement in a two-dimensional Galilean reference, the mirrors or plates are rendered by projection as straight lines, and therefore their equations must be written.

For the mirror **M1** represented by the straight line (**M1**) whose beam of direction vector ($\overrightarrow{A_1} = \vec{\imath}$), we also see that the point $\boldsymbol{p_1(v.t + d + R, 0)}$ belong to it, from which we conclude the following:

$$\overrightarrow{p_1p}\begin{pmatrix} x - v.t - d - R \\ y - 0 \end{pmatrix} \perp \overrightarrow{A_1}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow (x - v.t - d - R).1 + (y - 0).0 = 0$$

$$\Rightarrow x - v.t - d - R = 0$$

Thus, the equation of the straight line $(\boldsymbol{M1})$ is written as follows:

$$(M1) = \{x, y \in \mathbb{R} \quad / \quad x = d + R + v.t\} \quad (4.42)$$

For the mirror **M2** represented by the straight line (**M2**) whose beam of direction vector ($\overrightarrow{A_2} = \vec{\jmath}$), we also see that the point $\boldsymbol{p_2(0, d)}$ belong to it, from which we conclude the following:

$$\overrightarrow{p_2p}\begin{pmatrix} x - 0 \\ y - d \end{pmatrix} \perp \overrightarrow{A_2}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow (x - 0).0 + (y - d).1 = 0$$

$$\Rightarrow y - d = 0$$

Thus, the equation of the straight line (**M2**) is written as follows:

$$(M2) = \{x, y \in \mathbb{R} \quad / \quad y = d\} \quad (4.43)$$

For the beam splitter $\mathbf{S_p}$ represented by the straight line ($\mathbf{S_p}$) whose beam of direction vector $\overrightarrow{A_0} = \frac{\sqrt{2}}{2}(\vec{\imath} - \vec{\jmath})$, we also see that the point $\boldsymbol{E(v.t + R, 0)}$ belong to it, from which we conclude the following:

$$\overrightarrow{Ep}\begin{pmatrix} x-v.t-R \\ y-0 \end{pmatrix} \perp \overrightarrow{A_0}\begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{-\sqrt{2}}{2} \end{pmatrix} \Rightarrow (x-v.t-R).\frac{\sqrt{2}}{2}-(y-0).\frac{\sqrt{2}}{2}=0$$

$$\Rightarrow x-v.t-R=y$$

Thus, the equation of the straight line ($\mathbf{S_p}$) is written as follows:

$$(S_p)=\{x,y\in\mathbb{R}\quad/\quad x-v.t-R=y\} \tag{4.44}$$

For the detector $\mathbf{D_{et}}$ where the straight line ($\mathbf{D_{et}}$) whose beam of direction vector $\overrightarrow{A_2}=\vec{j}$, we also see that the point $p_3(-D,0)$ belong to it, from which we conclude the following:

$$\overrightarrow{p_3p}\begin{pmatrix} x-0 \\ y+D \end{pmatrix} \perp \overrightarrow{A_2}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow (x-0).0+(y+D).1=0$$

$$\Rightarrow y+D=0$$

Thus, the equation of the straight line ($\mathbf{S_p}$) is written as follows:

$$(Det)=\{x,y\in\mathbb{R}\quad/\quad y=-D\} \tag{4.45}$$

For the point P_{h1}**:**

From ***Fig №10***, the beam of light is collided with the mirror **M1** in the first collision, then it produces a light beam with a velocity vector $\overrightarrow{U_1}$.Thus, according to the ***proposition 4.1*** or law of a fixed collision **[F.E.C]**, the value of the vector is calculated as follows:

$$\overrightarrow{U_1}=\vec{V}-2\cdot\langle\vec{V}-\overrightarrow{v_S}|\overrightarrow{A_1}\rangle.\overrightarrow{A_1}\quad\Rightarrow\quad\vec{V}-\overrightarrow{U_1}=2\cdot\langle\vec{V}-\overrightarrow{v_S}|\overrightarrow{A_1}\rangle.\overrightarrow{A_1}$$

So, we have:

$$\vec{V}-\overrightarrow{U_1}=2\cdot\langle C.\vec{\imath}-v.\vec{\imath}|\vec{\imath}\rangle.\vec{\imath}$$
$$\Rightarrow\vec{V}-\overrightarrow{U_1}=2\cdot\langle(C-v).\vec{\imath}|\vec{\imath}\rangle.\vec{\imath}$$
$$\Rightarrow\vec{V}-\overrightarrow{U_1}=2\cdot(C-v).\underbrace{\langle\vec{\imath}|\vec{\imath}\rangle}_{=1}.\vec{\imath}$$
$$\Rightarrow\vec{V}-\overrightarrow{U_1}=2\cdot(C-v).\vec{\imath} \tag{4.46}$$

From this last equality **(4.46)** we conclude**:**

$$\overrightarrow{U_1}=\vec{V}-2\cdot\langle\vec{V}-\overrightarrow{v_S}|\overrightarrow{A_1}\rangle.\overrightarrow{A_1}$$
$$\Rightarrow\overrightarrow{U_1}=C.\vec{\imath}-2\cdot(C-v).\vec{\imath}$$
$$\Rightarrow\overrightarrow{U_1}=-(C-2.v).\vec{\imath} \tag{4.47}$$

After the first collision, the beam of light is last collision with the beam splitter $\mathbf{S_p}$, then it produces a light beam with a velocity vector $\overrightarrow{U_2}$.Thus, to the ***proposition 4.1*** or law of a fixed collision **[F.E.C]**, the value of the vector $\overrightarrow{U_2}$ is calculated as follows:

$$\overrightarrow{U_2}=\overrightarrow{U_1}-2.\langle\overrightarrow{U_1}-\overrightarrow{v_S}|\overrightarrow{A_2}\rangle\cdot\overrightarrow{A_2}\quad\Rightarrow\quad\overrightarrow{U_1}-\overrightarrow{U_2}=2.\langle\overrightarrow{U_1}-\overrightarrow{v_S}|\overrightarrow{A_2}\rangle\cdot\overrightarrow{A_2}$$

So, we have:

$$\overrightarrow{U_1}-\overrightarrow{U_2}=2.\left\langle-(C-2.v).\vec{\imath}-v.\vec{\imath}\,\middle|\,\frac{\sqrt{2}}{2}(\vec{\imath}-\vec{\jmath})\right\rangle\cdot\frac{\sqrt{2}}{2}(\vec{\imath}-\vec{\jmath})$$

$$\Rightarrow\overrightarrow{U_1}-\overrightarrow{U_2}=\underbrace{2.\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}}_{=1}\langle-(C-v).\vec{\imath}|(\vec{\imath}-\vec{\jmath})\rangle(\vec{\imath}-\vec{\jmath})$$

$$\Rightarrow \overrightarrow{U_1} - \overrightarrow{U_2} = -(C-v).\langle \vec{\imath} | (\vec{\imath} - \vec{\jmath}) \rangle (\vec{\imath} - \vec{\jmath})$$

$$\Rightarrow \overrightarrow{U_1} - \overrightarrow{U_2} = -(C-v).\left[\underbrace{\langle \vec{\imath} | \vec{\imath} \rangle}_{=1} - \underbrace{\langle \vec{\imath} | \vec{\jmath} \rangle}_{=0} \right] (\vec{\imath} - \vec{\jmath})$$

$$\Rightarrow \overrightarrow{U_1} - \overrightarrow{U_2} = -(C-v).(\vec{\imath} - \vec{\jmath}) \tag{4.48}$$

From this last equality **(4.48)** we conclude**:**

$$\overrightarrow{U_2} = \overrightarrow{U_1} + (C-v).(\vec{\imath} - \vec{\jmath})$$

$$\Rightarrow \overrightarrow{U_2} = -(C-2.v).\vec{\imath} + (C-v)(\vec{\imath} - \vec{\jmath})$$

$$\Rightarrow \overrightarrow{U_2} = [-(C-2.v) + (C-v)].\vec{\imath} - (C-v).\vec{\jmath}$$

$$\Rightarrow \overrightarrow{U_2} = v.\vec{\imath} - (C-v).\vec{\jmath} \tag{4.49}$$

In order to set the coordinates of point (P_{h1}) in the interval $[0, t_1]$, we have:

$$if\ t \in [0, t_1] \quad / \quad \overrightarrow{SP_{h1}} = \vec{V}.t + R.\vec{\imath}$$

$$\Rightarrow \overrightarrow{SP_{h1}} = C.\vec{\imath}.t + R.\vec{\imath}$$

$$\Rightarrow \overrightarrow{SP_{h1}} = (C.t + R).\vec{\imath}$$

So:

$$P_{h1}(C.t + R, 0)\ \ if\ t \in [0, t_1]$$

In order for point (P_{h1}) to belong to the straight line ($M1$) so that its equation **(4.42)** should be:

$$P_{h1} \in (M1) \Rightarrow \exists t_1 \in [0, t_1] \quad / \quad C.t_1 + R = d + R + v.t$$

$$\Rightarrow t_1 = \frac{d}{C-v} \tag{4.50}$$

Hence, if they:

$$t_1 \in [0, t_1] \cap [t_1.t_2] \Rightarrow \vec{V}.t_1 + R.\vec{\imath} = \overrightarrow{U_1}.t_1 + \overrightarrow{r_1}$$

$$\Rightarrow \overrightarrow{r_1} = \left(\vec{V} - \overrightarrow{U_1}\right).t_1 + R.\vec{\imath}$$

Using the result **(4.46),** we have:

$$\overrightarrow{r_1} = 2 \cdot (C-v).\frac{d}{C-v}.\vec{\imath} + R.\vec{\imath}$$

$$\Rightarrow \overrightarrow{r_1} = (2.d + R)\vec{\imath}$$

Otherwise, if:

$$t \in [t_1.t_2] \setminus \overrightarrow{SP_{h1}} = \overrightarrow{U_1}.t + \overrightarrow{r_1}$$

$$\Rightarrow \overrightarrow{SP_{h1}} = -(C-2.v).\vec{\imath}.t + (2.d + R).\vec{\imath}$$

$$\Rightarrow \overrightarrow{SP_{h1}} = [-(C-2.v).t + (2.d + R)].\vec{\imath}$$

So:

$$P_{h1}(-(C-2.v).t + (2.d + R), 0)\ \ if\ t \in [t_1.t_2]$$

Again, In order for point (P_{h1}) to belong to the straight line (S_P) so that its equation **(4.44)** should be:

$$P_{h1} \in (S_P) \Longrightarrow \exists t_2 \in [t_1.t_2] \quad / \quad \begin{cases} x = -(C-2.v).t_2 + (2.d+R) \\ y = 0 \\ x - v.t_2 - R = y \end{cases}$$

$$\Longrightarrow \begin{cases} x = -(C-2.v).t_2 + (2.d+R) \\ and \\ x = v.t_2 + R \end{cases}$$

$$\Longrightarrow (v.t_2 + R) - \big(-(C-2.v).t_2 + (2.d+R)\big) = 0$$

$$\Longrightarrow t_2.(v + C - 2.v) = 2.d \quad \Longrightarrow \quad t_2.(C-v) = 2.d$$

$$\Longrightarrow t_2 = \frac{2.d}{C-v} \qquad (4.51)$$

Hence, if they:

$$t_2 \in [t_1.t_2] \cap [t_2.t_3] \Longrightarrow \overrightarrow{U_1}.t_2 + \overrightarrow{r_1} = \overrightarrow{U_2}.t_2 + \overrightarrow{r_2}$$

$$\Longrightarrow \overrightarrow{r_2} = \big(\overrightarrow{U_1} - \overrightarrow{U_2}\big).t_2 + \overrightarrow{r_1}$$

Using the result **(4.48),** we have:

$$\overrightarrow{r_2} = -(C-v).(\vec{\imath} - \vec{\jmath}).2.\frac{d}{C-v} + (2.d+R)\vec{\imath}$$

$$\Longrightarrow \overrightarrow{r_2} = -2.d.(\vec{\imath} - \vec{\jmath}) + (2.d+R)\vec{\imath}$$

$$\Longrightarrow \overrightarrow{r_2} = R.\vec{\imath} + 2.d.\vec{\jmath}$$

Also, order to set the coordinates of point (P_{h1}) in the interval $[t_2.t_3]$, we have:

$$if\ t \in [t_2.t_3] \quad / \quad \overrightarrow{SP_{h1}} = \overrightarrow{U_2}.t + \overrightarrow{r_2}$$

$$\Longrightarrow \overrightarrow{SP_{h1}} = (v.\vec{\imath} - (C-v).\vec{\jmath}).t + R.\vec{\imath} + 2.d.\vec{\jmath}$$

$$\Longrightarrow \overrightarrow{SP_{h1}} = (v.t + R).\vec{\imath} + [2.d - (C-v).t].\vec{\jmath}$$

So:

$$P_{h1}(v.t + R, 2.d - (C-v).t) \ \ if\ t \in [t_2.t_3]$$

Finally, in order for point (P_{h1}) to belong to the straight line (Det) so that its equation **(4.45)** should be:

$$P_{h1} \in (Det) \Longrightarrow \exists t_2 \in [t_2.t_3] \quad / \quad \begin{cases} y = 2.d - (C-v).t \\ y = -D \end{cases}$$

$$\Longrightarrow 2.d - (C-v).t = -D \quad \Longrightarrow \quad (C-v).t_3 = 2.d + D$$

$$\Longrightarrow t_3 = \frac{2.d + D}{C-v} \qquad (4.52)$$

For the point P_{h2}**:**

In the same way, the beam of light is collided in the first collision with the beam splitter S_p, from the ***Fig №11***, then it produces a light beam with a velocity vector $\overrightarrow{U_1^0}$.Thus, to the ***proposition 4.1*** or law of a fixed collision **[F.E.C]**, the value of the vector is calculated as follows:

$$\overrightarrow{U_1^0} = \vec{V} - 2.\langle \vec{V} - \overrightarrow{v_S} | \overrightarrow{A_0} \rangle \cdot \overrightarrow{A_0} \quad \Longrightarrow \quad \vec{V} - \overrightarrow{U_1^0} = 2.\langle \vec{V} - \overrightarrow{v_S} | \overrightarrow{A_0} \rangle \cdot \overrightarrow{A_0}$$

So:

$$\vec{V}-\overrightarrow{U_1^0}=2.\left\langle C.\vec{\imath}-v.\vec{\imath}\middle|\frac{\sqrt{2}}{2}(\vec{\imath}-\vec{\jmath})\right\rangle\cdot\frac{\sqrt{2}}{2}(\vec{\imath}-\vec{\jmath})$$

$$\Rightarrow\vec{V}-\overrightarrow{U_1^0}=\underbrace{2.\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}}_{=1}\langle(C-v).\vec{\imath}|\vec{\imath}-\vec{\jmath}\rangle(\vec{\imath}-\vec{\jmath})$$

$$\Rightarrow\vec{V}-\overrightarrow{U_1^0}=(C-v).\left[\underbrace{\langle\vec{\imath}|\vec{\imath}\rangle}_{=1}-\underbrace{\langle\vec{\imath}|\vec{\jmath}\rangle}_{=0}\right](\vec{\imath}-\vec{\jmath})$$

$$\Rightarrow\vec{V}-\overrightarrow{U_1^0}=(C-v).(\vec{\imath}-\vec{\jmath})\qquad(4.53)$$

From this last equality **(4.53),** we conclude**:**

$$\overrightarrow{U_1^0}=\vec{V}-(C-v).(\vec{\imath}-\vec{\jmath})$$

$$\Rightarrow\overrightarrow{U_1^0}=C.\vec{\imath}-(C-v).(\vec{\imath}-\vec{\jmath})$$

$$\Rightarrow\overrightarrow{U_1^0}=[C-(C-v)].\vec{\imath}+(C-v).\vec{\jmath}$$

$$\Rightarrow\overrightarrow{U_1^0}=v.\vec{\imath}+(C-v).\vec{\jmath}\qquad(4.54)$$

After the first collision, the beam of light is last collision with the mirror **M1**, then it produces a light beam with a velocity vector $\overrightarrow{U_2^0}$. Thus, to the ***proposition 4.1*** or law of a fixed collision **[F.E.C]**, the value of the vector $\overrightarrow{U_2^0}$ is calculated as follows:

$$\overrightarrow{U_2^0}=\overrightarrow{U_1^0}-2.\left\langle\overrightarrow{U_1^0}-\overrightarrow{v_S}\middle|\overrightarrow{A_2}\right\rangle\cdot\overrightarrow{A_2}\quad\Rightarrow\quad\overrightarrow{U_1^0}-\overrightarrow{U_2^0}=2.\left\langle\overrightarrow{U_1^0}-\overrightarrow{v_S}\middle|\overrightarrow{A_2}\right\rangle\cdot\overrightarrow{A_2}$$

So:

$$\overrightarrow{U_1^0}-\overrightarrow{U_2^0}=2.\langle v.\vec{\imath}+(C-v).\vec{\jmath}-v.\vec{\imath}|\vec{\jmath}\rangle\cdot\vec{\jmath}$$

$$\Rightarrow\overrightarrow{U_1^0}-\overrightarrow{U_2^0}=2.\langle(C-v).\vec{\jmath}|\vec{\jmath}\rangle\cdot\vec{\jmath}$$

$$\Rightarrow\overrightarrow{U_1^0}-\overrightarrow{U_2^0}=2.(C-v).\underbrace{\langle\vec{\jmath}|\vec{\jmath}\rangle}_{=1}\cdot\vec{\jmath}$$

$$\Rightarrow\overrightarrow{U_1^0}-\overrightarrow{U_2^0}=2.(C-v)\cdot\vec{\jmath}\qquad(4.55)$$

From this last equality **(4.55),** we conclude**:**

$$\overrightarrow{U_2^0}=\overrightarrow{U_1^0}-2.(C-v)\cdot\vec{\jmath}$$

$$\Rightarrow\overrightarrow{U_2^0}=v.\vec{\imath}+(C-v).\vec{\jmath}-2.(C-v)\cdot\vec{\jmath}$$

$$\Rightarrow\overrightarrow{U_2^0}=v.\vec{\imath}-(C-v)\cdot\vec{\jmath}\qquad(4.56)$$

Hence, if they:

$$t\in\left[-\frac{\mathrm{R}}{\mathrm{C}},0\right]\cap\left[0,t_1^0\right]\setminus\overrightarrow{SP_{h2}}=C.t.\vec{\imath}+R.\vec{\imath}=\overrightarrow{U_1^0}.t+\overrightarrow{r_1^0}$$

$$\Rightarrow\overrightarrow{r_1^0}=R.\vec{\imath}$$

In order to set the coordinates of point (P_{h2}) in the interval $\left[0,t_1^0\right]$, we have:

$$if\ t\in\left[0,t_1^0\right]\quad/\quad\overrightarrow{SP_{h2}}=(v.\vec{\imath}+(C-v).\vec{\jmath}).t+R.\vec{\imath}$$

$$\Rightarrow\overrightarrow{SP_{h2}}=(v.t+R).\vec{\imath}+(C-v).t.\vec{\jmath}$$

So:

$$P_{h2}(v.t+R, 2.(C-v).t) \;\; if\ t \in \left[0, t_1^0\right]$$

In order for point (P_{h2}) to belong to the straight line **($M2$)** so that its equation **(4.43)** should be:

$$P_{h1} \in (M2) \Rightarrow \exists t_1^0 \in \left[0, t_1^0\right] \quad / \quad \begin{cases} y = (C-v).t_1^0 \\ \quad and \\ \quad y = d \end{cases}$$

$$\Rightarrow (C-v).t_1^0 = d$$

$$\Rightarrow t_1^0 = \frac{d}{C-v} \tag{4.57}$$

Hence, if:

$$t_1^0 \in \left[0, t_1^0\right] \cap \left[t_1^0 . t_2^0\right] \Rightarrow \overrightarrow{U_1^0}.t + \overrightarrow{r_1^0} = \overrightarrow{U_2^0}.t + \overrightarrow{r_2^0}$$

$$\Rightarrow \overrightarrow{r_2^0} = \left(\overrightarrow{U_1^0} - \overrightarrow{U_2^0}\right).t_1^0 + \overrightarrow{r_1^0}$$

Using the result **(4.55)**, we have:

$$\Rightarrow \overrightarrow{r_2^0} = 2.(C-v)\cdot\vec{j}.\frac{d}{C-v} + R.\vec{i}$$

$$\Rightarrow \overrightarrow{r_2^0} = 2.d\cdot\vec{j}.+R.\vec{i}$$

Otherwise, if:

$$t \in \left[t_1^0 . t_2^0\right] \quad / \quad \overrightarrow{SP_{h2}} = \overrightarrow{U_2^0}.t + \overrightarrow{r_2^0}$$

$$\Rightarrow \overrightarrow{SP_{h2}} = (v.\vec{i} - (C-v)\cdot\vec{j}).t + 2.d\cdot\vec{j}.+R.\vec{i}$$

$$\Rightarrow \overrightarrow{SP_{h2}} = (v.t+R).\vec{i} + (2.d-(C-v)\cdot t).\vec{j}$$

So:

$$P_{h2}(v.t+R, 2.d-(C-v)\cdot t) \;\; if\ t \in \left[t_1^0 . t_2^0\right]$$

Finally, in order for point (P_{h2}) to belong to the straight line **(Det)** so that its equation **(4.45)** should be:

$$P_{h2} \in (Det) \Rightarrow \exists t_2^0 \in \left[t_1^0 . t_2^0\right] \quad / \quad \begin{cases} y = 2.d-(C-v)\cdot t_2^0 \\ \quad and \\ \quad y = -D \end{cases}$$

$$\Rightarrow -D = 2.d-(C-v)\cdot t_2^0$$

$$\Rightarrow (C-v).t_2^0 = 2.d + D$$

$$\Rightarrow t_2^0 = \frac{2.d+D}{C-v} \tag{4.58}$$

Now that we have finished studying the movement of points (P_{h1}) and (P_{h2}), which represent the photons emitted from the sun, from their separation at the beam splitter $\mathbf{S_P}$ and their return to the detector $\mathbf{D_{et}}$ we conclude that the points (P_{h1}) and (P_{h2}), reach the detector $\mathbf{D_{et}}$ with the same velocity vector **(4.49), (4.56),** and the same period of time **(4.52), (4.56),** which means that they reach together at the same time. This is contrary to what Michelson assumed **(4.16)**.

4.2.1.2 Such that vectors $\vec{V}$ and $\overrightarrow{v_S}$ are orthogonal (2^{nd} Case):

In this case, unlike Michelson, although it is more correct, the rotation of the earth on the sun, through which the transverse velocity vector ($\overrightarrow{v_S}$) becomes perpendicular to the velocity vector $\vec{V} = C.\vec{\imath}$ of the photon, emitted from the sun. And considering that the motion of the Earth for the Sun is not straight linear, but for a very short period of time that the photon moves within the interferometer, we can consider the motion of the Earth for the Sun linear for that short period of time, and its velocity vector is orthogonal with the velocity vector of the photon. In this very short period, we can see that the earth moves within this equation **(4.59).**

$$\overrightarrow{SE} = v.t.\vec{\jmath} + R.\vec{\imath} \qquad (4.59)$$

So, all the elements of the interferometer installed on the earth's surface, including the mirrors and the detector move with the same velocity vector **(4.60),** for the sun-centered Galilean reference.

$$\overrightarrow{v_S} = v.\vec{\jmath} \qquad (4.60)$$

Again, since we are only studying the movement in a two-dimensional Galilean reference, the mirrors or plates are rendered by projection as straight lines, and therefore their equations must be written.

For the mirror **M1** represented by the straight line (**M1**) whose beam of direction vector ($\overrightarrow{A_1} = \vec{\imath}$), we also see that the point $p_1(d+R,0)$ belong to it, from which we conclude the following:

$$\overrightarrow{p_1p}\begin{pmatrix} x-d-R \\ y-0 \end{pmatrix} \perp \overrightarrow{A_1}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow (x-d-R).1 + (y-0).0 = 0$$

$$\Rightarrow x-d-R = 0$$

Thus, the equation of the straight line $(M1)$ is written as follows:

$$(M1) = \{x, y \in \mathbb{R} \quad / \quad x = d+R\} \qquad (4.61)$$

For the mirror **M2** represented by the straight line (**M2**) whose beam of direction vector ($\overrightarrow{A_2} = \vec{\jmath}$), we also see that the point $p_2(R, v.t+d)$ belong to it, from which we conclude the following:

$$\overrightarrow{p_2p}\begin{pmatrix} x-R \\ y-v.t-d \end{pmatrix} \perp \overrightarrow{A_2}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow (x-R).0 + (y-v.t-d).1 = 0$$

$$\Rightarrow y-v.t-d = 0$$

Thus, the equation of the straight line (**M2**) is written as follows:

$$(M2) = \{x, y \in \mathbb{R} \quad / \quad y = v.t+d\} \qquad (4.62)$$

For the beam splitter $\mathbf{S_p}$ represented by the straight line ($\mathbf{S_p}$) whose beam of direction vector $\overrightarrow{A_0} = \frac{\sqrt{2}}{2}(\vec{\imath} - \vec{\jmath})$, (we also see that the point $E(R, v.t)$ belong to it, from which we conclude the following:

$$\overrightarrow{Ep}\begin{pmatrix} x-R \\ y-v.t \end{pmatrix} \perp \overrightarrow{A_0}\begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{-\sqrt{2}}{2} \end{pmatrix} \Rightarrow (x-R).\frac{\sqrt{2}}{2} - (y-v.t).\frac{\sqrt{2}}{2} = 0$$

$$\Rightarrow x-R = y-v.t$$

Thus, the equation of the straight line ($\mathbf{S_p}$) is written as follows:

$$(\mathrm{S_p}) = \{x, y \in \mathbb{R} \quad / \quad x-R = y-v.t\} \qquad (4.63)$$

For the detector $\mathbf{D_{et}}$ where the straight line ($\mathbf{D_{et}}$) whose beam of direction vector $\overrightarrow{A_2} = \vec{j}$, we also see that the point $p_3(R, v.t - D)$ belong to it, from which we conclude the following:

$$\overrightarrow{p_3p}\begin{pmatrix} x - R \\ y - v.t + D \end{pmatrix} \perp \overrightarrow{A_2}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow (x - R).0 + (y - v.t + D).1 = 0$$

$$\Rightarrow y - v.t + D = 0$$

Thus, the equation of the straight line ($\mathbf{S_p}$) is written as follows:

$$(Det) = \{x, y \in \mathbb{R} \quad / \quad y = v.t - D\} \tag{4.64}$$

For the point P_{h1}:

From ***Fig №10***, the beam of light is collided with the mirror **M1** in the first collision, then it produces a light beam with a velocity vector $\overrightarrow{U_1}$.Thus, according to the ***proposition 4.1*** or law of a fixed collision **[F.E.C]**, the value of the vector is calculated as follows:

$$\overrightarrow{U_1} = \vec{V} - 2 \cdot \langle \vec{V} - \overrightarrow{v_S} | \overrightarrow{A_1} \rangle . \overrightarrow{A_1} \quad \Rightarrow \quad \vec{V} - \overrightarrow{U_1} = 2 \cdot \langle \vec{V} - \overrightarrow{v_S} | \overrightarrow{A_1} \rangle . \overrightarrow{A_1}$$

So:

$$\vec{V} - \overrightarrow{U_1} = 2 \cdot \langle C.\vec{i} - v.\vec{j} | \vec{i} \rangle . \vec{i}$$

$$\Rightarrow \vec{V} - \overrightarrow{U_1} = 2 \cdot \left[C.\underbrace{\langle \vec{i} | \vec{i} \rangle}_{=1} - v.\underbrace{\langle \vec{j} | \vec{i} \rangle}_{=0} \right] . \vec{i}$$

$$\Rightarrow \vec{V} - \overrightarrow{U_1} = 2 \cdot C.\vec{i} \tag{4.65}$$

From this last equality **(4.65),** we conclude**:**

$$\overrightarrow{U_1} = \vec{V} - 2 \cdot C.\vec{i}$$

$$\Rightarrow \overrightarrow{U_1} = C.\vec{i} - 2 \cdot C.\vec{i}$$

$$\Rightarrow \overrightarrow{U_1} = -C.\vec{i} \tag{4.66}$$

After the first collision, the beam of light is last collision with the beam splitter $\mathbf{S_p}$, then it produces a light beam with a velocity vector $\overrightarrow{U_2}$.Thus, to the ***proposition 4.1*** or law of a fixed collision **[F.E.C]**, the value of the vector $\overrightarrow{U_2}$ is calculated as follows:

$$\overrightarrow{U_2} = \overrightarrow{U_1} - 2.\langle \overrightarrow{U_1} - \overrightarrow{v_S} | \overrightarrow{A_0} \rangle \cdot \overrightarrow{A_0} \quad \Rightarrow \quad \overrightarrow{U_1} - \overrightarrow{U_2} = 2.\langle \overrightarrow{U_1} - \overrightarrow{v_S} | \overrightarrow{A_0} \rangle \cdot \overrightarrow{A_0}$$

So:

$$\overrightarrow{U_1} - \overrightarrow{U_2} = 2.\left\langle -C.\vec{i} - v.\vec{j} \middle| \frac{\sqrt{2}}{2}(\vec{i} - \vec{j}) \right\rangle \cdot \frac{\sqrt{2}}{2}(\vec{i} - \vec{j})$$

$$\Rightarrow \overrightarrow{U_1} - \overrightarrow{U_2} = \underbrace{2.\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2}}_{=1} \langle -C.\vec{i} - v.\vec{j} | \vec{i} - \vec{j} \rangle (\vec{i} - \vec{j})$$

$$\Rightarrow \overrightarrow{U_1} - \overrightarrow{U_2} = \left[-C.\underbrace{\langle \vec{i} | \vec{i} \rangle}_{=1} + v.\underbrace{\langle \vec{j} | \vec{j} \rangle}_{=1} \right] (\vec{i} - \vec{j})$$

$$\Rightarrow \overrightarrow{U_1} - \overrightarrow{U_2} = -(C - v)(\vec{i} - \vec{j}) \tag{4.67}$$

From this last equality **(4.67),** we conclude**:**

$$\overrightarrow{U_2} = \overrightarrow{U_1} + (C - v)(\vec{i} - \vec{j})$$

$$\Rightarrow \overrightarrow{U_2} = -C.\vec{i} + (C - v)(\vec{i} - \vec{j})$$

$$\Rightarrow \overrightarrow{U_2} = [-C + (C - v)].\vec{i} - (C - v).\vec{j}$$

$$\Rightarrow \overrightarrow{U_2} = -v.\vec{i} - (C - v).\vec{j} \tag{4.68}$$

Again, if:

$$t \in [0, t_1] \quad / \quad \overrightarrow{SP_{h1}} = \vec{V}.t + R.\vec{\imath}$$

$$\Rightarrow \overrightarrow{SP_{h1}} = C.\vec{\imath}.t + R.\vec{\imath} \quad \Rightarrow \quad \overrightarrow{SP_{h1}} = (C.t + R).\vec{\imath}$$

So:

$$P_{h1}(C.t + R, 0) \;\; if\; t \in [0, t_1]$$

In order for point (P_{h1}) to belong to the straight line $(M1)$ so that its equation **(4.61)** should be:

$$P_{h1} \in (M1) \Rightarrow \exists t_1 \in [0, t_1] \quad / \quad C.t_1 + R = d + R$$

$$\Rightarrow t_1 = \frac{d}{C} \tag{4.69}$$

Hence, if:

$$t_1 \in [0, t_1] \cap [t_1.t_2] \Rightarrow \vec{V}.t_1 + R.\vec{\imath} = \overrightarrow{U_1}.t_1 + \overrightarrow{r_1}$$

$$\Rightarrow \overrightarrow{r_1} = \left(\vec{V} - \overrightarrow{U_1}\right).t_1 + R.\vec{\imath}$$

Using the result **(4.65),** we have:

$$\overrightarrow{r_1} = 2 \cdot C.\vec{\imath}.\frac{d}{C} + R.\vec{\imath} \quad \Rightarrow \quad \overrightarrow{r_1} = (2.d + R)\vec{\imath}$$

Otherwise, if:

$$t \in [t_1.t_2] \quad / \quad \overrightarrow{SP_{h1}} = \overrightarrow{U_1}.t + \overrightarrow{r_1}$$

$$\Rightarrow \overrightarrow{SP_{h1}} = -C.t.\vec{\imath} + (2.d + R).\vec{\imath} \quad \Rightarrow \quad \overrightarrow{SP_{h1}} = \big(-C.t + (2.d + R)\big).\vec{\imath}$$

So:

$$P_{h1}(-C.t + (2.d + R), 0) \;\; if\; t \in [t_1.t_2]$$

Again, in order for point (P_{h1}) to belong to the straight line (S_P) so that its equation **(4.63)** should be:

$$P_{h1} \in (S_P) \Rightarrow \exists t_2 \in [t_1.t_2] \quad / \quad \begin{cases} x = -C.t_2 + (2.d + R) \\ y = 0 \\ x - R = y - v.t_2 \end{cases}$$

$$\Rightarrow \begin{cases} x = -C.t_2 + (2.d + R) \\ and \\ x = -v.t_2 + R \end{cases}$$

$$\Rightarrow (-v.t_2 + R) - [-C.t_2 + (2.d + R)] = 0 \quad \Rightarrow \quad t_2.(C - v) = 2.d$$

$$\Rightarrow t_2 = \frac{2.d}{C - v} \tag{4.70}$$

Hence, if:

$$t_2 \in [t_1.t_2] \cap [t_2.t_3] \Rightarrow \overrightarrow{U_1}.t_2 + \overrightarrow{r_1} = \overrightarrow{U_2}.t_2 + \overrightarrow{r_2}$$

$$\Rightarrow \overrightarrow{r_2} = \left(\overrightarrow{U_1} - \overrightarrow{U_2}\right).t_2 + \overrightarrow{r_1}$$

Using the result **(4.67),** we have:

$$\Rightarrow \overrightarrow{r_2} = -(C - v)(\vec{\imath} - \vec{\jmath}).\frac{2.d}{C - v} + (2.d + R)\vec{\imath}$$

$$\Rightarrow \overrightarrow{r_2} = -2.d.(\vec{\imath} - \vec{\jmath}) + (2.d + R)\vec{\imath}$$

$$\Rightarrow \overrightarrow{r_2} = R.\vec{\imath} + 2.d.\vec{\jmath}$$

Otherwise, if:

$$t \in [t_2.t_3] \quad / \quad \overrightarrow{SP_{h1}} = \overrightarrow{U_2}.t + \overrightarrow{r_2}$$

$$\Rightarrow \overrightarrow{SP_{h1}} = [-v.\vec{\imath} - (C-v).\vec{\jmath}].t + R.\vec{\imath} + 2.d.\vec{\jmath}$$

$$\Rightarrow \overrightarrow{SP_{h1}} = (-v.t + R).\vec{\imath} + [2.d - (C-v).t].\vec{\jmath}$$

So:

$$P_{h1}(-v.t + R, 2.d - (C-v).t) \;\; if\; t \in [t_2.t_3]$$

Finally, in order for point (P_{h1}) to belong to the straight line (Det) so that its equation **(4.64)** should be::

$$P_{h1} \in (Det) \Rightarrow \exists t_3 \in [t_2.t_3] \quad / \quad \begin{cases} y = 2.d - (C-v).t_3 \\ y = -D + v.t_3 \end{cases}$$

$$\Rightarrow 2.d - (C-v).t_3 = -D + v.t_3$$

$$\Rightarrow 2.d + D = (C-v).t_3 + v.t_3 \quad \Rightarrow \quad 2.d + D = C.t_3$$

$$\Rightarrow t_3 = \frac{2.d + D}{C} \tag{4.71}$$

For the point P_{h2}**:**

In the same way, the beam of light is collided in the first collision with the beam splitter $\mathbf{S_p}$, from the ***Fig №11***, then it produces a light beam with a velocity vector $\overrightarrow{U_1^0}$. Thus, to the ***proposition 4.1*** or law of a fixed collision **[F.E.C]**, the value of the vector is calculated as follows:

$$\overrightarrow{U_1^0} = \vec{V} - 2.\langle \vec{V} - \overrightarrow{v_S} | \overrightarrow{A_0} \rangle \cdot \overrightarrow{A_0} \quad \Rightarrow \quad \vec{V} - \overrightarrow{U_1^0} = 2.\langle \vec{V} - \overrightarrow{v_S} | \overrightarrow{A_0} \rangle \cdot \overrightarrow{A_0}$$

So:

$$\vec{V} - \overrightarrow{U_1^0} = 2.\left\langle C.\vec{\imath} - v.\vec{\jmath} \middle| \frac{\sqrt{2}}{2}(\vec{\imath} - \vec{\jmath}) \right\rangle \cdot \frac{\sqrt{2}}{2}(\vec{\imath} - \vec{\jmath})$$

$$\Rightarrow \vec{V} - \overrightarrow{U_1^0} = \underbrace{2.\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2}}_{=1} \langle C.\vec{\imath} - v.\vec{\jmath} | \vec{\imath} - \vec{\jmath} \rangle (\vec{\imath} - \vec{\jmath})$$

$$\Rightarrow \vec{V} - \overrightarrow{U_1^0} = \left[C.\underbrace{\langle \vec{\imath} | \vec{\imath} \rangle}_{=1} + v.\underbrace{\langle \vec{\jmath} | \vec{\jmath} \rangle}_{=1} \right] (\vec{\imath} - \vec{\jmath})$$

$$\Rightarrow \vec{V} - \overrightarrow{U_1^0} = (C+v)(\vec{\imath} - \vec{\jmath}) \tag{4.72}$$

From this last equality **(4.72)** we conclude**:**

$$\overrightarrow{U_1^0} = \vec{V} - (C+v)(\vec{\imath} - \vec{\jmath}) \quad \Rightarrow \quad \overrightarrow{U_1^0} = C.\vec{\imath} - (C+v)(\vec{\imath} - \vec{\jmath})$$

$$\Rightarrow \overrightarrow{U_1^0} = -v.\vec{\imath} + (C+v)\vec{\jmath} \tag{4.73}$$

After the first collision, the beam of light is last collision with the mirror **M1**, then it produces a light beam with a velocity vector $\overrightarrow{U_2^0}$. Thus, to the ***proposition 4.1*** or law of a fixed collision **[F.E.C]**, the value of the vector $\overrightarrow{U_2^0}$ is calculated as follows:

$$\overrightarrow{U_2^0} = \overrightarrow{U_1^0} - 2.\left\langle \overrightarrow{U_1^0} - \overrightarrow{v_S} \middle| \overrightarrow{A_2} \right\rangle \cdot \overrightarrow{A_2} \quad \Rightarrow \quad \overrightarrow{U_1^0} - \overrightarrow{U_2^0} = 2.\left\langle \overrightarrow{U_1^0} - \overrightarrow{v_S} \middle| \overrightarrow{A_2} \right\rangle \cdot \overrightarrow{A_2}$$

So:

$$\overrightarrow{U_1^0} - \overrightarrow{U_2^0} = 2.\langle -v.\vec{\imath} + (C+v)\vec{\jmath} - v.\vec{\jmath} | \vec{\jmath} \rangle \cdot \vec{\jmath}$$

$$\Rightarrow \overrightarrow{U_2^0} = \overrightarrow{U_1^0} - 2.\langle -v.\vec{\imath} + (C+v)\vec{\jmath} - v.\vec{\jmath} | \vec{\jmath} \rangle \cdot \vec{\jmath}$$

$$\Rightarrow \overrightarrow{U_1^0} - \overrightarrow{U_2^0} = 2.\langle -v.\vec{\imath} + C.\vec{\jmath}|\vec{\jmath}\rangle \cdot \vec{\jmath}$$

$$\Rightarrow \overrightarrow{U_1^0} - \overrightarrow{U_2^0} = 2.\left[-v\,\underbrace{\langle \vec{\imath}|\vec{\jmath}\rangle}_{=0} + C.\underbrace{\langle \vec{\jmath}|\vec{\jmath}\rangle}_{=1}\right] \cdot \vec{\jmath}$$

$$\Rightarrow \overrightarrow{U_1^0} - \overrightarrow{U_2^0} = 2.C.\vec{\jmath} \qquad (4.74)$$

From this last equality **(4.74),** we conclude**:**

$$\overrightarrow{U_2^0} = \overrightarrow{U_1^0} - 2.C.\vec{\jmath} \quad \Rightarrow \quad \overrightarrow{U_2^0} = -v.\vec{\imath} + (C+v)\vec{\jmath} - 2.C.\vec{\jmath}$$

$$\Rightarrow \overrightarrow{U_2^0} = -v.\vec{\imath} - (C-v).\vec{\jmath} \qquad (4.75)$$

Thus, if:

$$t \in \left[-\frac{R}{C}, 0\right] \cap \left[0, t_1^0\right] \quad / \quad \overrightarrow{SP_{h2}} = C.t.\vec{\imath} + R.\vec{\imath} = \overrightarrow{U_1^0}.t + \overrightarrow{r_1^0}$$

$$\Rightarrow \overrightarrow{SP_{h2}} = C.0.\vec{\imath} + R.\vec{\imath} = \overrightarrow{U_1^0}.0 + \overrightarrow{r_1^0}$$

$$\Rightarrow \overrightarrow{r_1^0} = R.\vec{\imath}$$

On the other hand, if:

$$t \in \left[0, t_1^0\right] \quad / \quad \overrightarrow{SP_{h2}} = \overrightarrow{U_1^0}.t + \overrightarrow{r_1^0}$$

$$\Rightarrow \overrightarrow{SP_{h2}}(-v.\vec{\imath} + (C+v)\vec{\jmath}).t + R.\vec{\imath}$$

$$\Rightarrow \overrightarrow{SP_{h2}} = (-v.t + R).\vec{\imath} + (C+v).t.\vec{\jmath}$$

So:

$$P_{h2}(-v.t + R, (C+v).t) \;\; if\; t \in \left[0, t_1^0\right]$$

In order for point (P_{h2}) to belong to the straight line $(M2)$ so that its equation **(4.62)** should be:

$$P_{h2} \in (M2) \Rightarrow \exists t_1^0 \in \left[0, t_1^0\right] \setminus \begin{cases} y = (C+v).t_1^0 \\ \quad and \\ y = d + v.t_1^0 \end{cases}$$

$$\Rightarrow (C+v).t_1^0 = d + v.t_1^0 \quad \Rightarrow \quad C.t_1^0 = d$$

$$\Rightarrow t_1^0 = \frac{d}{C} \qquad (4.76)$$

Hence, if:

$$t_1^0 \in \left[0, t_1^0\right] \cap \left[t_1^0.t_2^0\right] \Rightarrow \overrightarrow{U_1^0}.t_1^0 + \overrightarrow{r_1^0} = \overrightarrow{U_2^0}.t_1^0 + \overrightarrow{r_2^0}$$

$$\Rightarrow \overrightarrow{r_2^0} = \left(\overrightarrow{U_1^0} - \overrightarrow{U_2^0}\right).t_1^0 + \overrightarrow{r_1^0}$$

From this last equality **(4.74),** we conclude**:**

$$\overrightarrow{r_2^0} = 2.C.\frac{d}{C}.\vec{\jmath} + R.\vec{\imath} \quad \Rightarrow \quad \overrightarrow{r_2^0} = 2.d \cdot \vec{\jmath} + R.\vec{\imath}$$

On the other hand, if:

$$t \in \left[t_1^0.t_2^0\right] \quad / \quad \overrightarrow{SP_{h2}} = \overrightarrow{U_2^0}.t + \overrightarrow{r_2^0}$$

$$\Rightarrow \overrightarrow{SP_{h2}} = (-v.\vec{\imath} - (C-v).\vec{\jmath}).t + 2.d \cdot \vec{\jmath} + R.\vec{\imath}$$

$$\Rightarrow \overrightarrow{SP_{h2}} = (-v.t + R).\vec{\imath} + (2.d - (C - v) \cdot t).\vec{\jmath}$$

So:

$$P_{h2}(-v.t + R, 2.d - (C - v) \cdot t) \;\; if\; t \in [t_1^0 . t_2^0]$$

Finally, in order for point (P_{h2}) to belong to the straight line **(Det)** so that its equation **(4.64)** should be:

$$P_{h2} \in (Det) \Rightarrow \exists t_2^0 \in [t_1^0 . t_2^0] \quad / \quad \begin{cases} y = 2.d - (C - v) \cdot t_2^0 \\ \quad and \\ y = -D + v.t_1^0 \end{cases}$$

$$\Rightarrow -D + v.t_1^0 = 2.d - (C - v) \cdot t_2^0$$

$$\Rightarrow (C - v) \cdot t_2^0 + v.t_1^0 = 2.d + D$$

$$\Rightarrow C.t_2^0 = 2.d + D$$

$$\Rightarrow t_2^0 = \frac{2.d + D}{C} \tag{4.77}$$

Again, in this case, which exemplifies exactly the Michelson experiment, after studying the movement of points (P_{h1}) and (P_{h2}), which represent the photons emitted from the sun, from their separation at the beam splitter $\mathbf{S_P}$ and their return to meet at the detector $\mathbf{D_{et}}$. Data obtained that points (P_{h1}) and (P_{h2}) reach the detector $\mathbf{D_{et}}$ with the same velocity vector **(4.68), (4.75),** and the same time period **(4.71), (4.77),** that means that they reach together at the same moment Time, and there is no delay between them. This is contrary to what Michelson once again assumed **(4.16).**

4.2.2 The earth is the center of the reference system

We have studied the case **4.2.1** and in each of the two states in which the earth moves, where the Galilean reference is centered on the sun, and we follow how the points (P_{h2}) and (P_{h2}) representing the photons emitted from the sun reached to the detector at the same time and the same vector velocity, and we found that there was no delay between them, contrary to what Michelson assumed.

Now in this case where the Earth is the center of the Galilean reference, we will study the movement of the points (P_{h2}) and (P_{h2}) that represent the photons emitted from the sun or any other source. In my opinion, this subsequent study clearly reflects the results obtained by Michelson.

Thus we assume that the movement of the two points (P_{h2}) and (P_{h2}) are reflected in the equations **(4.78)** and **(4.79),** and then we assign the unknown values.

$$\overrightarrow{EP_{h1}} = \begin{cases} \overrightarrow{C_\infty}.t \;\; if\; t \in [0, t_1] \\ \overrightarrow{C_{1,1}}.t + \overrightarrow{r_1} \;\; if\; t \in [t_1 . t_2] \\ \overrightarrow{C_{2,1}}.t + \overrightarrow{r_2} \;\; if\; t \in [t_2 . t_3] \end{cases} \tag{4.78}$$

And

$$\overrightarrow{EP_{h2}} = \begin{cases} \overrightarrow{C_\infty}.t \;\; if\; t \in [-\mathrm{T}, 0] \\ \overrightarrow{C_{1,2}}.t + \overrightarrow{r_1^0} \;\; if\; t \in [0, t_1^0] \\ \overrightarrow{C_{2,2}}.t + \overrightarrow{r_2^0} \;\; if\; t \in [t_1^0 . t_2^0] \end{cases} \tag{4.79}$$

Again, since we are only studying the movement in a two-dimensional Galilean reference, the mirrors or plates are rendered by projection as straight lines, and therefore their equations must be written.

For the mirror **M1** represented by the straight line (**M1**) whose beam of direction vector ($\overrightarrow{A_1} = \vec{\imath}$), we also see that the point $p_1(d, 0)$ belong to it, from which we conclude the following:

$$\overrightarrow{p_1p}\begin{pmatrix} x-d \\ y-0 \end{pmatrix} \perp \overrightarrow{A_1}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow (x-d).1+(y-0).0=0$$

$$\Rightarrow x-d=0$$

Thus, the equation of the straight line $(M1)$ is written as follows:

$$(M1)=\{x,y\in\mathbb{R} \quad / \quad x=d\} \tag{4.80}$$

For the mirror **M2** represented by the straight line (**M2**) whose beam of direction vector ($\overrightarrow{A_2}=\vec{j}$), we also see that the point $p_2(0,d)$ belong to it, from which we conclude the following:

$$\overrightarrow{p_2p}\begin{pmatrix} x-0 \\ y-d \end{pmatrix} \perp \overrightarrow{A_2}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow (x-0).0+(y-d).1=0$$

$$\Rightarrow y-d=0$$

Thus, the equation of the straight line (**M2**) is written as follows:

$$(M2)=\{x,y\in\mathbb{R} \quad / \quad y=d\} \tag{4.81}$$

For the beam splitter $\mathbf{S_p}$ represented by the straight line ($\mathbf{S_p}$) whose beam of direction vector $\overrightarrow{A_0}=\frac{\sqrt{2}}{2}(\vec{i}-\vec{j})$, we also see that the point $E(0,0)$ belong to it, from which we conclude the following:

$$\overrightarrow{Ep}\begin{pmatrix} x-0 \\ y-0 \end{pmatrix} \perp \overrightarrow{A_0}\begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{-\sqrt{2}}{2} \end{pmatrix} \Rightarrow (x-0).\frac{\sqrt{2}}{2}-(y-0).\frac{\sqrt{2}}{2}=0$$

$$\Rightarrow x=y$$

Thus, the equation of the straight line ($\mathbf{S_p}$) is written as follows:

$$(S_P)=\{x,y\in\mathbb{R} \quad / \quad x=y\} \tag{4.82}$$

For the detector $\mathbf{D_{et}}$ where the straight line ($\mathbf{D_{et}}$) whose beam of direction vector $\overrightarrow{A_2}=\vec{j}$, we also see that the point $p_3(0,-D)$ belong to it, from which we conclude the following:

$$\overrightarrow{p_3p}\begin{pmatrix} x-0 \\ y+D \end{pmatrix} \perp \overrightarrow{A_2}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow (x-0).0+(y+D).1=0$$

$$\Rightarrow y+D=0$$

Thus, the equation of the straight line ($\mathbf{D_{et}}$) is written as follows:

$$(Det)=\{x,y\in\mathbb{R} \quad / \quad y=-D\} \tag{4.83}$$

By assuming that the earth is the center of the reference system, this means that the velocity vector of the interferometer installed on the surface of the earth is a zero-vector ($\overrightarrow{v_S}=\vec{0}$), and that all components of the device remain constant including mirrors and detectors. And assume that the photons emitted from the sun moving according to the velocity vector:

$$\overrightarrow{C_\infty}=C_\infty(\cos\theta.\vec{i}+\sin\theta.\vec{j}) \tag{4.84}$$

For the point P_{h1}**:**

From ***Fig №10,*** the beam of light is collided with the mirror **M1** in the first collision, then it produces a light beam with a velocity vector $\overrightarrow{C_{1,1}}$. Thus, according to the ***proposition 4.1*** or law of a fixed collision **[F.E.C]**, the value of the vector is calculated as follows:

$$\overrightarrow{C_{1,1}} = \overrightarrow{C_\infty} - 2\langle \overrightarrow{C_\infty} - \overrightarrow{v_S} | \overrightarrow{A_1} \rangle . \overrightarrow{A_1} \quad \Rightarrow \quad \overrightarrow{C_\infty} - \overrightarrow{C_{1,1}} = 2\langle \overrightarrow{C_\infty} - \overrightarrow{v_S} | \overrightarrow{A_1} \rangle . \overrightarrow{A_1}$$

So:

$$\overrightarrow{C_\infty} - \overrightarrow{C_{1,1}} = 2\langle C_\infty(\cos\theta . \vec{i} + \sin\theta . \vec{j}) | \vec{i} \rangle . \vec{i}$$

$$\Rightarrow \overrightarrow{C_\infty} - \overrightarrow{C_{1,1}} = 2. C_\infty . \langle \cos\theta . \vec{i} + \sin\theta . \vec{j} | \vec{i} \rangle . \vec{i}$$

$$\Rightarrow \overrightarrow{C_\infty} - \overrightarrow{C_{1,1}} = 2. C_\infty . \left[\cos\theta . \underbrace{\langle \vec{i} | \vec{i} \rangle}_{=1} + \sin\theta . \underbrace{\langle \vec{i} | \vec{j} \rangle}_{=0} \right] . \vec{i}$$

$$\Rightarrow \overrightarrow{C_\infty} - \overrightarrow{C_{1,1}} = 2. C_\infty . \cos\theta . \vec{i} \qquad (4.85)$$

From this last equality **(4.85),** we conclude**:**

$$\overrightarrow{C_{1,1}} = \overrightarrow{C_\infty} - 2\langle \overrightarrow{C_\infty} - \overrightarrow{v_S} | \overrightarrow{A_1} \rangle . \overrightarrow{A_1} \quad \Rightarrow \quad \overrightarrow{C_{1,1}} = \overrightarrow{C_\infty} - 2. C_\infty . \cos\theta . \vec{i}$$

$$\Rightarrow \overrightarrow{C_{1,1}} = C_\infty(\cos\theta . \vec{i} + \sin\theta . \vec{j}) - 2. C_\infty . \cos\theta . \vec{i}$$

$$\Rightarrow \overrightarrow{C_{1,1}} = C_\infty(-\cos\theta . \vec{i} + \sin\theta . \vec{j}) \qquad (4.86)$$

After the first collision, the beam of light is last collision with the beam splitter $\mathbf{S_p}$, then it produces a light beam with a velocity vector (($\overrightarrow{U_2}$)).Thus, to the ***proposition 4.1*** or law of a fixed collision **[F.E.C]**, the value of the vector $\overrightarrow{U_2}$ is calculated as follows:

$$\overrightarrow{C_{2,1}} = \overrightarrow{C_{1,1}} - 2\langle \overrightarrow{C_{1,1}} - \overrightarrow{v_S} | \overrightarrow{A_0} \rangle . \overrightarrow{A_0} \quad \Rightarrow \quad \overrightarrow{C_{1,1}} - \overrightarrow{C_{2,1}} = 2\langle \overrightarrow{C_{1,1}} - \overrightarrow{v_S} | \overrightarrow{A_0} \rangle . \overrightarrow{A_0}$$

So:

$$\overrightarrow{C_{1,1}} - \overrightarrow{C_{2,1}} = 2. \left\langle C_\infty(-\cos\theta . \vec{i} + \sin\theta . \vec{j}) - \vec{0} \middle| \frac{\sqrt{2}}{2}(\vec{i} - \vec{j}) \right\rangle . \frac{\sqrt{2}}{2}(\vec{i} - \vec{j})$$

$$\Rightarrow \overrightarrow{C_{1,1}} - \overrightarrow{C_{2,1}} = \underbrace{2. \frac{\sqrt{2}}{2} . \frac{\sqrt{2}}{2}}_{=1} . C_\infty \langle -\cos\theta . \vec{i} + \sin\theta . \vec{j} | \vec{i} - \vec{j} \rangle . (\vec{i} - \vec{j})$$

$$\Rightarrow \overrightarrow{C_{1,1}} - \overrightarrow{C_{2,1}} = C_\infty . \left[-\cos\theta . \underbrace{\langle \vec{i} | \vec{i} \rangle}_{=1} - \sin\theta . \underbrace{\langle \vec{j} | \vec{j} \rangle}_{=1} \right] . (\vec{i} - \vec{j})$$

$$\Rightarrow \overrightarrow{C_{1,1}} - \overrightarrow{C_{2,1}} = -C_\infty . (\cos\theta + \sin\theta) . (\vec{i} - \vec{j}) \qquad (4.87)$$

From this last equality **(4.87),** we conclude**:**

$$\overrightarrow{C_{2,1}} = \overrightarrow{C_{1,1}} + C_\infty . (\cos\theta + \sin\theta) . (\vec{i} - \vec{j})$$

$$\Rightarrow \overrightarrow{C_{2,1}} = C_\infty(-\cos\theta . \vec{i} + \sin\theta . \vec{j}) + C_\infty . (\cos\theta + \sin\theta) . (\vec{i} - \vec{j})$$

$$\Rightarrow \overrightarrow{C_{2,1}} = C_\infty[-\cos\theta . \vec{i} + (\cos\theta + \sin\theta) . \vec{i} + \sin\theta . \vec{j} - (\cos\theta + \sin\theta) . \vec{j}]$$

$$\Rightarrow \overrightarrow{C_{2,1}} = C_\infty . (\sin\theta . \vec{i} - \cos\theta . \vec{j}) \qquad (4.88)$$

Furthermore, point (P_{h1}) is characterized by coordinates in the interval $[0, t_1]$**:**

$$P_{h1}(C_\infty . t . \cos\theta , C_\infty . t . \sin\theta) \ \ if\ t \in [0, t_1]$$

In order for point (P_{h1}) to belong to the straight line ($M1$) so that its equation **(4.80)** should be:

$$P_{h1} \in (M1) \Rightarrow \exists t_1 \in [0, t_1] \quad / \quad C_\infty . t_1 . \cos\theta = d$$

$$\Rightarrow t_1 = \frac{d}{C_\infty . \cos\theta} \qquad (4.89)$$

Otherwise, if:

$$t_1 \in [0, t_1] \cap [t_1 . t_2] \quad \Rightarrow \quad \overrightarrow{C_\infty}. t_1 = \overrightarrow{C_{1,1}}. t_1 + \overrightarrow{r_1}$$

$$\Rightarrow \overrightarrow{r_1} = t_1 \left(\overrightarrow{C_\infty} - \overrightarrow{C_{1,1}} \right)$$

Using the result **(4.85),** we have:

$$\Rightarrow \overrightarrow{r_1} = \frac{d}{C_\infty . \cos\theta} (2. C_\infty . \cos\theta . \vec{\imath})$$

$$\Rightarrow \overrightarrow{r_1} = 2. d. \vec{\imath}$$

Otherwise, if:

$$t \in [t_1 . t_2] \quad / \quad \overrightarrow{EP_{h1}} = \overrightarrow{C_{1,1}}. t + \overrightarrow{r_1}$$

$$\Rightarrow \overrightarrow{EP_{h1}} = C_\infty(-\cos\theta . \vec{\imath} + \sin\theta . \vec{\jmath}) + 2. d. \vec{\imath}$$

$$\Rightarrow \overrightarrow{EP_{h1}} = (-\cos\theta . C_\infty . t + 2. d). \vec{\imath} + C_\infty . \sin\theta . \vec{\jmath}$$

So:

$$P_{h1}(-\cos\theta . C_\infty . t + 2. d, \sin\theta . C_\infty . t) \quad if\ t \in [t_1, t_2]$$

Again, in order for point (P_{h1}) to belong to the straight line (S_p) so that its equation **(4.82)** should be:

$$P_{h1} \in (S_p) \Rightarrow \exists t_2 \in [t_1, t_2] \quad / \quad -\cos\theta . C_\infty . t_2 + 2. d = \sin\theta . C_\infty . t_2$$

$$\Rightarrow 2. d = \sin\theta . C_\infty . t_2 + \cos\theta . C_\infty . t_2 \quad \Rightarrow \quad 2. d = t_2 . C_\infty . (\sin\theta + \cos\theta)$$

$$\Rightarrow t_2 = \frac{2. d}{C_\infty . (\sin\theta + \cos\theta)} \tag{4.90}$$

Well, on the other hand, if:

$$t_2 \in [t_1 . t_2] \cap [t_2 . t_3] \quad \Rightarrow \quad \overrightarrow{C_{1,1}}. t_2 + \overrightarrow{r_1} = \overrightarrow{C_{2,1}}. t_2 + \overrightarrow{r_2}$$

$$\Rightarrow \overrightarrow{r_2} = t_2 . \left(\overrightarrow{C_{1,1}} - \overrightarrow{C_{2,1}} \right) + \overrightarrow{r_1}$$

Using the result **(4.87),** we have:

$$\overrightarrow{r_2} = -2 \frac{d}{C_\infty . (\sin\theta + \cos\theta)} . C_\infty . (\cos\theta + \sin\theta). (\vec{\imath} - \vec{\jmath}) + 2. d. \vec{\imath}$$

$$\Rightarrow \overrightarrow{r_2} = -2. d. (\vec{\imath} - \vec{\jmath}). +2. d. \vec{\imath}$$

$$\Rightarrow \overrightarrow{r_2} = 2. d. \vec{\jmath}$$

Otherwise, if:

$$t \in [t_2 . t_3] \quad / \quad \overrightarrow{EP_{h1}} = \overrightarrow{C_{2,1}}. t + \overrightarrow{r_2}$$

$$\Rightarrow \overrightarrow{EP_{h1}} = C_\infty . (\sin\theta . \vec{\imath} - \cos\theta . \vec{\jmath}). t + \ 2. d. \vec{\jmath}$$

$$\Rightarrow \overrightarrow{EP_{h1}} = C_\infty . \sin\theta . t. \vec{\imath} + (2. d - C_\infty . \cos\theta . t). \vec{\jmath}$$

So:

$$P_{h1}(C_\infty . \sin\theta . t, 2. d - C_\infty . \cos\theta . t) \quad if\ t \in [t_2, t_3]$$

Finally, in order for point (P_{h1}) to belong to the straight line (***Det***) so that its equation **(4.83)** should be:

$$P_{h1} \in (Det) \Longrightarrow \exists t \in [t_2, t_3] \quad / \quad 2.d - C_\infty.\cos\theta.t = -D$$

$$\Longrightarrow C_\infty.\cos\theta.t_3 = 2.d + D$$

Lastly, we find:

$$\Longrightarrow t_3 = \frac{2.d + D}{C_\infty.\cos\theta} \tag{4.91}$$

For the point P_{h2}**:**

In the same way, the beam of light is collided in the first collision with the beam splitter $\mathbf{S_p}$, from the ***Fig №12***, then it produces a light beam with a velocity vector $\overrightarrow{U_1^0}$.

Thus, to the ***proposition 4.1*** or law of a fixed collision **[F.E.C]**, the value of the vector is calculated as follows:

$$\overrightarrow{C_{1,2}} = \overrightarrow{C_\infty} - 2\langle \overrightarrow{C_\infty} - \overrightarrow{v_S} | \overrightarrow{A_0} \rangle.\overrightarrow{A_0} \quad \Rightarrow \quad \overrightarrow{C_\infty} - \overrightarrow{C_{1,2}} = 2\langle \overrightarrow{C_\infty} - \overrightarrow{v_S} | \overrightarrow{A_0} \rangle.\overrightarrow{A_0}$$

So:

$$\overrightarrow{C_\infty} - \overrightarrow{C_{1,2}} = 2\langle \overrightarrow{C_\infty} - \overrightarrow{v_S} | \overrightarrow{A_0} \rangle.\overrightarrow{A_0}$$

$$\Rightarrow \overrightarrow{C_\infty} - \overrightarrow{C_{1,2}} = 2\left\langle C_\infty(\cos\theta.\vec{\imath} + \sin\theta.\vec{\jmath}) - \vec{0} \middle| \frac{\sqrt{2}}{2}(\vec{\imath} - \vec{\jmath}) \right\rangle.\frac{\sqrt{2}}{2}(\vec{\imath} - \vec{\jmath})$$

$$\Rightarrow \overrightarrow{C_\infty} - \overrightarrow{C_{1,2}} = \underbrace{2.\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}}_{=1}.C_\infty\langle \cos\theta.\vec{\imath} + \sin\theta.\vec{\jmath} | \vec{\imath} - \vec{\jmath} \rangle.(\vec{\imath} - \vec{\jmath})$$

$$\Rightarrow \overrightarrow{C_\infty} - \overrightarrow{C_{1,2}} = \underbrace{2.\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}}_{=1}.C_\infty.\left[\cos\theta.\underbrace{\langle\vec{\imath}|\vec{\imath}\rangle}_{=1} - \sin\theta.\underbrace{\langle\vec{\jmath}|\vec{\jmath}\rangle}_{=1}\right].(\vec{\imath} - \vec{\jmath})$$

$$\Rightarrow \overrightarrow{C_\infty} - \overrightarrow{C_{1,2}} = C_\infty.(\cos\theta - \sin\theta).(\vec{\imath} - \vec{\jmath}) \tag{4.92}$$

From this last equality **(4.92),** we conclude**:**

$$\overrightarrow{C_{1,2}} = \overrightarrow{C_\infty} - C_\infty.(\cos\theta - \sin\theta).(\vec{\imath} - \vec{\jmath})$$

$$\Rightarrow \overrightarrow{C_{1,2}} = C_\infty(\cos\theta.\vec{\imath} + \sin\theta.\vec{\jmath}) - C_\infty.(\cos\theta - \sin\theta).(\vec{\imath} - \vec{\jmath})$$

$$\Rightarrow \overrightarrow{C_{1,2}} = C_\infty[\cos\theta.\vec{\imath} + \sin\theta.\vec{\jmath} - \cos\theta.\vec{\imath} + \sin\theta.\vec{\imath} + \cos\theta.\vec{\jmath} - \sin\theta.\vec{\jmath}]$$

$$\Rightarrow \overrightarrow{C_{1,2}} = C_\infty.(\sin\theta.\vec{\imath} + \cos\theta.\vec{\jmath}) \tag{4.93}$$

After the first collision, the beam of light is last collision with the beam splitter $\mathbf{S_p}$, then it produces a light beam with a velocity vector (($\overrightarrow{U_2}$)).Thus, to the ***proposition 4.1*** or law of a fixed collision **[F.E.C]**, the value of the vector $\overrightarrow{U_2}$ is calculated as follows:

$$\overrightarrow{C_{2,2}} = \overrightarrow{C_{1,2}} - 2\langle \overrightarrow{C_{1,2}} - \overrightarrow{v_S} | \overrightarrow{A_2} \rangle.\overrightarrow{A_2} \quad \Rightarrow \quad \overrightarrow{C_{1,2}} - \overrightarrow{C_{2,2}} = 2\langle \overrightarrow{C_{1,2}} - \overrightarrow{v_S} | \overrightarrow{A_2} \rangle.\overrightarrow{A_2}$$

So:

$$\Rightarrow \overrightarrow{C_{1,2}} - \overrightarrow{C_{2,2}} = 2.\langle C_\infty.(\sin\theta.\vec{\imath} + \cos\theta.\vec{\jmath}) - \vec{0} | \vec{\jmath} \rangle.\vec{\jmath}$$

$$\Rightarrow \overrightarrow{C_{1,2}} - \overrightarrow{C_{2,2}} = 2.C_\infty.\left[\sin\theta.\underbrace{\langle\vec{\imath}|\vec{\jmath}\rangle}_{=0} + \cos\theta.\underbrace{\langle\vec{\jmath}|\vec{\jmath}\rangle}_{=1}\right].\vec{\jmath}$$

$$\Rightarrow \overrightarrow{C_{1,2}} - \overrightarrow{C_{2,2}} = 2.C_\infty.\cos\theta.\vec{\jmath} \tag{4.94}$$

From this last equality **(4.94)**, we conclude**:**

$$\overrightarrow{C_{2,2}} = \overrightarrow{C_{1,2}} - 2.C_\infty.\cos\theta.\vec{\jmath}$$

$$\Rightarrow \overrightarrow{C_{2,2}} = C_\infty.(\sin\theta.\vec{\imath} + \cos\theta.\vec{\jmath}) - 2.C_\infty.\cos\theta.\vec{\jmath}$$

$$\Rightarrow \overrightarrow{C_{2,2}} = C_\infty.(\sin\theta.\vec{\imath} - \cos\theta.\vec{\jmath}) \quad (4.95)$$

Hence, if:

$$t \in [0, t_1^0] \quad / \quad \overrightarrow{EP_{h2}} = \overrightarrow{C_{1,2}}.t$$

$$\Rightarrow \overrightarrow{EP_{h2}} = C_\infty.\sin\theta.t.\vec{\imath} + C_\infty.\cos\theta.t.\vec{\jmath}$$

So:

$$P_{h2}(C_\infty.\sin\theta.t, C_\infty.\cos\theta.t) \ \ if\ t \in [0, t_1^0]$$

In order for point (P_{h2}) to belong to the straight line ($M2$) so that its equation **(4.81)** should be:

$$P_{h2} \in (M2) \Rightarrow \exists t_1^0 \in [0, t_1^0] \quad / \quad C_\infty.\cos\theta.t_1^0 = d$$

$$t_1^0 = \frac{d}{C_\infty.\cos\theta} \quad (4.96)$$

Hence, if:

$$t_1^0 \in [0, t_1^0] \cap [t_1^0.t_2^0] \Rightarrow \overrightarrow{C_{1,2}}.t_1^0 = \overrightarrow{C_{2,2}}.t_1^0 + \overrightarrow{r_1^0}$$

$$\Rightarrow \overrightarrow{r_1^0} = (\overrightarrow{C_{1,2}} - \overrightarrow{C_{2,2}}).t_1^0$$

Using the result **(4.94),** we have:

$$\overrightarrow{r_1^0} = 2.C_\infty.\cos\theta.\vec{\jmath}.\frac{d}{C_\infty.\cos\theta} \quad \Rightarrow \quad \overrightarrow{r_1^0} = 2.d.\vec{\jmath}$$

On the other hand, if:

$$t \in [t_1^0.t_2^0] \quad \Rightarrow \quad \overrightarrow{EP_{h2}} = \overrightarrow{C_{2,2}}.t + \overrightarrow{r_1^0}$$

$$\Rightarrow \overrightarrow{EP_{h2}} = C_\infty.(\sin\theta.\vec{\imath} - \cos\theta.\vec{\jmath}).t + 2.d.\vec{\jmath}$$

$$\Rightarrow \overrightarrow{EP_{h2}} = C_\infty.\sin\theta.t.\vec{\imath} + (2.d - C_\infty.\cos\theta.t).\vec{\jmath}$$

So:

$$P_{h2}(C_\infty.\sin\theta.t, 2.d - C_\infty.\cos\theta.t) \ \ if\ t \in [t_1^0.t_2^0]$$

Finally, in order for point (P_{h2}) to belong to the straight line **(Det)** so that its equation **(4.83)** should be:

$$P_{h2} \in (Det) \Rightarrow \exists t_2^0 \in [t_1^0.t_2^0] \quad / \quad 2.d - C_\infty.\cos\theta.t_2^0 = -D$$

$$\Rightarrow C_\infty.\cos\theta.t_2^0 = 2.d + D$$

Lastly, we find:

$$t_2^0 = \frac{2.d + D}{C_\infty.\cos\theta} \quad (4.97)$$

In this case, the earth is the center of the Galilean reference, after studying the movement of points (P_{h2}) and (P_{h2}), which represent the photons emitted from any source, including the sun, from their separation at the beam splitter $\mathbf{S_P}$ and their return to the detector $\mathbf{D_{et}}$. We conclude from the data obtained that the two points (P_{h2}) and (P_{h2}) reach the detector $\mathbf{D_{et}}$ with the same vector speed **(4.88), (4.95)** and the same time period **(4.91), (4.97).** This means that they reach together at the same time, and there is no delay between them.

4.3 Re-formatting the data obtained above:

By analyzing the movement of the points ($\boldsymbol{P_{h2}}$) and ($\boldsymbol{P_{h2}}$), which represent the photons emitted from any source, including the sun, within the interferometer, in all cases where the earth moves for any source, we find that they reach together to the detector line at the same time. There is no delay between them. This leads us to the fact that the results obtained by Michelson were already positive and identical to reality, contrary to what he had expected.

Now we must correlate the computational results to determine the relationships between the branches with the last branch of the study and to determine the absolute values of the speed of light in both references where the Earth and the Sun are central. We start with the first case.

The first case:

With using ***Chasles relations***, we have that:

$$\overrightarrow{SP_{h1}} = \overrightarrow{SE} + \overrightarrow{EP_{h1}} \Rightarrow \overrightarrow{EP_{h1}} = \overrightarrow{SP_{h1}} - \overrightarrow{SE}$$

$$\Rightarrow \frac{\partial}{\partial t}\overrightarrow{EP_{h1}}(t) = \frac{\partial}{\partial t}\overrightarrow{SP_{h1}}(t) - \frac{\partial}{\partial t}\overrightarrow{SE}(t) \tag{4.98}$$

$$\Rightarrow \frac{\partial}{\partial t}\overrightarrow{EP_{h1}}(t) = \begin{cases} \vec{V} - v.\vec{\imath} & if\ t \in [0, t_1] \\ \overrightarrow{U_1} - v.\vec{\imath} & if\ t \in [t_1.t_2] \\ \overrightarrow{U_2} - v.\vec{\imath} & if\ t \in [t_2.t_3] \end{cases} \Rightarrow \frac{\partial}{\partial t}\overrightarrow{EP_{h1}}(t) = \begin{cases} C.\vec{\imath} - v.\vec{\imath} & if\ t \in [0, t_1] \\ -(C - 2.v).\vec{\imath} - v.\vec{\imath} & if\ t \in [t_1.t_2] \\ v.\vec{\imath} - (C - v).\vec{\jmath} - v.\vec{\imath} & if\ t \in [t_2.t_3] \end{cases}$$

$$\Rightarrow \frac{\partial}{\partial t}\overrightarrow{EP_{h1}}(t) = \begin{cases} +(C - v).\vec{\imath} & if\ t \in [0, t_1] \\ -(C - v).\vec{\imath} & if\ t \in [t_1.t_2] \\ -(C - v).\vec{\jmath} & if\ t \in [t_2.t_3] \end{cases} \tag{4.99}$$

Again, we've got:

$$\frac{\partial}{\partial t}\overrightarrow{EP_{h1}}(t) = \begin{cases} \overrightarrow{C_\infty} & if\ t \in [0, t_1] \\ \overrightarrow{C_{1,1}} & if\ t \in [t_1.t_2] \\ \overrightarrow{C_{2,1}} & if\ t \in [t_2.t_3] \end{cases}$$

$$\Rightarrow \frac{\partial}{\partial t}\overrightarrow{EP_{h1}}(t) = \begin{cases} C_\infty(\cos\theta.\vec{\imath} + \sin\theta.\vec{\jmath}) & if\ t \in [0, t_1] \\ C_\infty(-\cos\theta.\vec{\imath} + \sin\theta.\vec{\jmath}) & if\ t \in [t_1.t_2] \\ C_\infty.(\sin\theta.\vec{\imath} - \cos\theta.\vec{\jmath}) & if\ t \in [t_2.t_3] \end{cases} \tag{4.100}$$

In comparing between **(4.99)** and **(4.100),** we conclude the following:

$$\Rightarrow \begin{cases} C_\infty(+\cos\theta.\vec{\imath} + \sin\theta.\vec{\jmath}) = +(C - v).\vec{\imath} \\ C_\infty(-\cos\theta.\vec{\imath} + \sin\theta.\vec{\jmath}) = -(C - v).\vec{\imath} \\ C_\infty.(+\sin\theta.\vec{\imath} - \cos\theta.\vec{\jmath}) = -(C - v).\vec{\jmath} \end{cases} \Rightarrow \begin{cases} C_\infty = (C - v) \\ and \\ \theta = 0 \end{cases} \tag{4.101}$$

$$\Rightarrow \left[t_1 = \frac{d}{C_\infty} \quad \wedge \quad t_2 = \frac{2.d}{C_\infty} \quad \wedge \quad t_3 = \frac{2.d + D}{C_\infty}\right]$$

Again, we've got:

$$\overrightarrow{SP_{h1}} = \begin{cases} \vec{V}.t + R.\vec{\imath} & if\ t \in [0, t_1] \\ \overrightarrow{U_1}.t + \overrightarrow{r_1} & if\ t \in [t_1.t_2] \\ \overrightarrow{U_2}.t + \overrightarrow{r_2} & if\ t \in [t_2.t_3] \end{cases} \Rightarrow \frac{\partial}{\partial t}\overrightarrow{SP_{h1}}(t) = \begin{cases} \vec{V} & if\ t \in [0, t_1] \\ \overrightarrow{U_1} & if\ t \in [t_1.t_2] \\ \overrightarrow{U_2} & if\ t \in [t_2.t_3] \end{cases}$$

$$\Rightarrow \left\|\frac{\partial}{\partial t}\overrightarrow{SP_{h1}}(t)\right\| = \begin{cases} \|\vec{V}\| = \|C.\vec{\imath}\| & if\ t \in [0, t_1] \\ \|\overrightarrow{U_1}\| = \|-(C - 2.v).\vec{\imath}\| & if\ t \in [t_1.t_2] \\ \|\overrightarrow{U_2}\| = \|v.\vec{\imath} - (C - v).\vec{\jmath}\| & if\ t \in [t_2.t_3] \end{cases}$$

$$\Rightarrow \left\|\frac{\partial}{\partial t}\overrightarrow{SP_{h1}}(t)\right\| = \begin{cases} C & if\ t \in [0, t_1] \\ C - 2.v & if\ t \in [t_1.t_2] \\ \sqrt{(C - v)^2 + v^2} & if\ t \in [t_2.t_3] \end{cases} \tag{4.102}$$

When we compensate **(4.101)** at **(4.102),** we get the result following:

$$\Rightarrow \left\|\frac{\partial}{\partial t}\overrightarrow{SP_{h1}}(t)\right\| = \begin{cases} C_\infty + v & if\ t \in [0, t_1] \\ C_\infty - v & if\ t \in [t_1.t_2] \\ \sqrt{C_\infty{}^2 + v^2} & if\ t \in [t_2.t_3] \end{cases}$$

Again, we have:

$$\overrightarrow{SP_{h2}} = \begin{cases} \vec{V}.t + R.\vec{\imath} & if\ t \in \left[-\frac{R}{C}, 0\right] \\ \overrightarrow{U_1^0}.t + \overrightarrow{r_1^0} & if\ t \in [0, t_1^0] \\ \overrightarrow{U_2^0}.t + \overrightarrow{r_2^0} & if\ t \in [t_1^0.t_2^0] \end{cases} \Rightarrow \frac{\partial}{\partial t}\overrightarrow{SP_{h2}} = \begin{cases} \vec{V} & if\ t \in \left[-\frac{R}{C}, 0\right] \\ \overrightarrow{U_1^0} & if\ t \in [0, t_1^0] \\ \overrightarrow{U_2^0} & if\ t \in [t_1^0.t_2^0] \end{cases}$$

$$\Rightarrow \left\|\frac{\partial}{\partial t}\overrightarrow{SP_{h2}}\right\| = \begin{cases} \|\vec{V}\| = \|C.\vec{\imath}\| & if\ t \in \left[-\frac{R}{C}, 0\right] \\ \left\|\overrightarrow{U_1^0}\right\| = \|v.\vec{\imath} + (C - v).\vec{\jmath}\| & if\ t \in [0, t_1^0] \\ \left\|\overrightarrow{U_2^0}\right\| = \|v.\vec{\imath} - (C - v).\vec{\jmath}\| & if\ t \in [t_1^0.t_2^0] \end{cases}$$

$$\Rightarrow \left\|\frac{\partial}{\partial t}\overrightarrow{SP_{h2}}\right\| = \begin{cases} C & if\ t \in \left[-\frac{R}{C}, 0\right] \\ \sqrt{(C - v)^2 + v^2} & if\ t \in [0.t_2^0] \end{cases} \tag{4.103}$$

When we compensate **(4.101)** at **(4.103),** we get the following:

$$\Rightarrow \left\|\frac{\partial}{\partial t}\overrightarrow{SP_{h2}}\right\| = \begin{cases} C_\infty + v & if\ t \in \left[-\frac{R}{C}, 0\right] \\ \sqrt{C_\infty{}^2 + v^2} & if\ t \in [0.t_2^0] \end{cases} \tag{4.104}$$

From the initial observation of these results, after comparing them with the data of branch **4.1**, we obtain the match of the results **(4.31)** and **(4.32),** with **(4.103)** and **(4.104),** if we consider (P_{h1}) and (P_{h2}) represent points (M_C) and (M_G), we derive the following result:

$$C_\infty + v = C_\beta + v \Rightarrow C_\infty = C_\beta$$

This confirms once again the validity of our assumptions, compared to Michelson's assumptions.

Second case:

Now we must correlate between the computational results to determine the relationships between sections with the last section of study. Let us begin with the first case.

Using the result **(4.98),** we have:

$$\frac{\partial}{\partial t}\overrightarrow{EP_{h1}}(t) = \frac{\partial}{\partial t}\overrightarrow{SP_{h1}}(t) - \frac{\partial}{\partial t}\overrightarrow{SE}(t)$$

$$\Rightarrow \frac{\partial}{\partial t}\overrightarrow{EP_{h1}}(t) = \begin{cases} \vec{V} - v.\vec{\jmath} & if\ t \in [0, t_1] \\ \overrightarrow{U_1} - v.\vec{\jmath} & if\ t \in [t_1.t_2] \\ \overrightarrow{U_2} - v.\vec{\jmath} & if\ t \in [t_2.t_3] \end{cases} \Rightarrow \frac{\partial}{\partial t}\overrightarrow{EP_{h1}}(t) = \begin{cases} C.\vec{\imath} - v.\vec{\jmath} & if\ t \in [0, t_1] \\ -C.\vec{\imath} - v.\vec{\jmath} & if\ t \in [t_1.t_2] \\ -v.\vec{\imath} - (C - v).\vec{\jmath} - v.\vec{\jmath} & if\ t \in [t_2.t_3] \end{cases}$$

$$\Rightarrow \frac{\partial}{\partial t}\overrightarrow{EP_{h1}}(t) = \begin{cases} C.\vec{\imath} - v.\vec{\jmath} & if\ t \in [0, t_1] \\ -C.\vec{\imath} - v.\vec{\jmath} & if\ t \in [t_1.t_2] \\ -v.\vec{\imath} - C.\vec{\jmath} & if\ t \in [t_2.t_3] \end{cases} \tag{4.105}$$

Again, we have:

$$\begin{cases} C_\infty(\cos\theta.\vec{\imath} + \sin\theta.\vec{\jmath}) = +C.\vec{\imath} - v.\vec{\jmath} & if\ t \in [0, t_1] \\ C_\infty(-\cos\theta.\vec{\imath} + \sin\theta.\vec{\jmath}) = -C.\vec{\imath} - v.\vec{\jmath} & if\ t \in [t_1.t_2] \\ C_\infty.(\sin\theta.\vec{\imath} - \cos\theta.\vec{\jmath}) = -v.\vec{\imath} - C.\vec{\jmath} & if\ t \in [t_2.t_3] \end{cases} \Rightarrow \begin{cases} C_\infty.\cos\theta = C \\ and \\ C_\infty.\sin\theta = -v \end{cases}$$

$$\Rightarrow C_\infty = \sqrt{C^2 + v^2}\ and \begin{cases} \cos\theta = \dfrac{C}{\sqrt{C^2 + v^2}} \\ \sin\theta = \dfrac{-v}{\sqrt{C^2 + v^2}} \end{cases} \tag{4.106}$$

$$\Rightarrow \left[t_1 = t_1^0 = \frac{d}{C} \quad \wedge \quad t_2 = \frac{2.d}{C - v} \quad \wedge \quad t_3 = t_2^0 = \frac{2.d + D}{C} \quad \right]$$

From this case, which actually represents the Earth's motion for the Sun, which is a rotation, we conclude that the absolute value of the speed of light varies from any frame of reference to another. Again, we do not see any trace of value ($\sqrt{C^2 - v^2}$) in this analysis by the fixed elastic collision law **[F.E.C],** as Michelson hypothesized.

So, can we set the earth's velocity for the sun by the interferometer? This question is very accurate, since we have confirmed that there is no delay between points (P_{h1}) and (P_{h2}) when they reach the detector $\mathbf{D_{et}}$. The answer is that the speed value can be set, but in practice I doubt it will succeed, and the detail as follows:

Any light source - whatever its velocity - produces the interference fringes by the interferometer.

To determine the Earth's speed for the sun, we should carry out two independent and consecutive experiments on a single interferometer.

The first experiment is done with a constant source of light for the interferometer on a monochromatic light, and the second experiment is carried out by a moving source - the sun - for the interferometer on the same monochromatic light. Assuming in the first experiment that the light comes from a constant source, we may realize that calculating the path difference as follows:

We know that wavelength is the multiplication between the speed of light with the period of the wave T, assuming that the velocity of monochrome light varies from two sources, one fixed and the other moving. Also, let's say that the monochrome velocity of the constant source is C and C_∞ from the moving source.

So, the wavelength in each of the source situations is:

$$\lambda = C.T \quad and \quad \lambda_\infty = C_\infty.T$$

When the wave is subjected to wave interference by the interferometer, it gives the fringe after passing through two slits, separated by a distance d and strike a screen a distance D, from the slits, where the lengths are as follows:

$$y \cong \frac{n.\lambda.D}{d} \quad and \quad y_\infty \cong \frac{n.\lambda_\infty.D}{d} \quad / \quad n = 1, 2, 3, \dots\dots$$

By dividing the length of the fringe over the other, we conclude this relationship:

$$\frac{y_\infty}{y} \cong \left(\frac{n.\lambda_\infty.D}{d}\right) \div \left(\frac{n.\lambda.D}{d}\right) = \frac{\lambda_\infty}{\lambda} = \frac{C_\infty.T}{C.T} = \frac{C_\infty}{C} = \frac{\sqrt{C^2 + v^2}}{C}$$

$$\Rightarrow \sqrt{C^2 + v^2} = C.\frac{y_\infty}{y} \Rightarrow v^2 = C^2\left(\left(\frac{y_\infty}{y}\right)^2 - 1\right) \Rightarrow v = C.\sqrt{\left(\frac{y_\infty}{y}\right)^2 - 1} \qquad (4.107)$$

As I mentioned, its designation depends on the quality of the technology, knowing that the length of the fringes is in the order of millimeters. We calculate arithmetically that the value of the difference between the fringes is difficult to observe in any work well, and using the relationship **(4.17)** with the values **(4.107),** we have:

$$\frac{y_\infty}{y} \cong \frac{\sqrt{C^2 + v^2}}{C} \Rightarrow y_\infty \cong y.\frac{\sqrt{C^2 + v^2}}{C} = y.\sqrt{1 + \frac{v^2}{C^2}} \Rightarrow \Delta y = y_\infty - y = y\left(\sqrt{1 + \frac{v^2}{C^2}} - 1\right)$$

$$\Rightarrow \Delta y \cong y\left(1 + \frac{v^2}{2.C^2} - 1\right) \cong y\frac{v^2}{2.C^2} \cong y\frac{(30.10^3)^2}{2.(300.10^6)^2} \cong y.\frac{10^{-8}}{2}$$

For the ether, is there medium called ether? It is known that the physicists agreed to reject the idea of ether and ruled out its existence after being confused by the null results of the *Michelson-Morley* experiment. And not necessarily any object characterized by the nature wave, there is a medium transmitted by it, for example: the iron spring is vibrated by the powers of mechanical pressure, and if moving in the absolute space, it does not mean there is a medium where it moves by shaking.

5. CONCLUSION

In order to strengthen his interpretation and to reinforce his hypothesis about the experiment mathematically, Lorentz presented the theory of Lorentz transformations, and like other physicists he believed that Michelson's assumptions were correct, although we demonstrated the contrast that there was no delay between the motion of the two light beams when they were separated and returned to the detector in any Galilean reference. Hence, no matter how much we try to reform or update his work - Lorentz's transformations- this reform will not work, because it is based on corrupt assumptions. Also, experiments that support their validity are no lesson to them, and we can refute them by more credible counter-experiments.

We have concluded that with the change of the Galilean references, the paths change and their length changes, but the metric dimensions of the moving object remain constant, independent of the change in the Galilean references, which is contrary to Lorentz's hypothesis.

As for Einstein's postulate, it is very clear from this study that the speed of light changes from one reference to another, and we have never had to deal with it as constant in all references, and it is contrary to the principle of relativity, although it is the basis of modern physics. And in the future, we can reform a great deal of theories that have been associated with relativity and Lorentz transformations.

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