Some topological paradoxes of relativity (EPR) -II[1)]

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To turn again to the article of A. Aspect "BELL's THEOREM: the naive view of the experimenter" we were forced by some publications, for example, [2]. We were convinced once again of the *conceptual correctness* of the problem of EPR in the Aspect's article.

Conceptually, in the "*naive presentation of EPR*" from A. Aspect no "gluing" of probability measures in different spaces is not required. The presentation of the A. Aspect is logically closed and complete. By simple examples, the existence of a problem related to the violation of Bell's inequality is shown.

The real possibility of solving this problem today is, in our opinion, only the relational interpretation of quantum mechanics [3], [4], since the relational interpretation of quantum mechanics by Rowelli "puts out of brackets" the local causality in the EPR paradox, replacing it with the concept of the integrity of the relations of the observed systems and, thereby, abandon the dubious from the point of view of view of quantum mechanics the concept of speed as a derivative in the *TMK-topology* of space-time relations. And that's apparently what physics has been "pregnant" with for a long time! But it is the difficulties of resolving the dilemma of completeness and local causality that is associated with the lack of the concept of velocity in the form of a space-time derivative in a scalar form. And this is a General problem of quantum mechanics, which the relational concept intends to solve .

The proposed article has all the logical "moments" , each of which can be said - it is not so! It should be emphasized that none of the points [2] was "glued" to any of our "moments".

A. Probability space

The probability space is defined by Kolmogorov [1] as a triple of objects $(\boldsymbol{\Omega}, \boldsymbol{\mathcal{F}}, \boldsymbol{P})$, where

$\boldsymbol{\Omega}$ − space of elementary outcomes;

$\boldsymbol{\mathcal{F}}$ −σ-algebra of events;

$\boldsymbol{P}$ − a probability measure on an event algebra.

For finite algebras, the conditions must be satisfied:

If $A \in \boldsymbol{\mathcal{F}}, B \in \boldsymbol{\mathcal{F}}$, then $A \cup B, A \cap B, \bar{A} \in \boldsymbol{\mathcal{F}}$.

In this regard, let us consider the example given in section 2 of [2].

1. The space of elementary outcomes for three dichotomous variables a_1, a_2, a_3in this example contains $2^3 = 8$ elements. Three-dimensional discrete space is required for their representation (see Fig.).

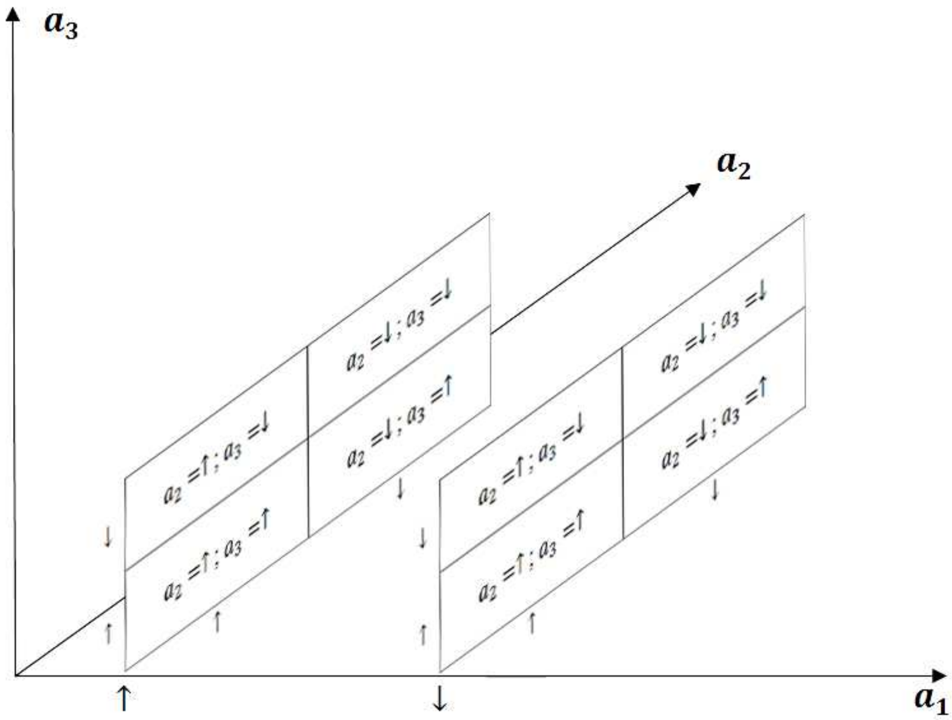
2. The space of elementary outcomes for the two dichotomous variables, chosen from the three $a_1, a_2, a_3,$ will contain the number of elements, depending on the distinction or non-distinction of the order of entry to the selected pair, according to the formula for the number of placements A_n^m, or the number of combinations C_n^m

$$A_n^m = \frac{n!}{(n-m)!}, \qquad C_n^m = \frac{n!}{(n-m)!\,m!}:$$

$$a) \quad A_3^2 = \frac{3 \cdot 2}{1} = 6,$$

$$b) \quad C_3^2 = \frac{3 \cdot 2}{1 \cdot 2} = 3.$$

1) **I beg your pardon for my not very good English!** The original text in Russian: http://vixra.org/pdf/1804.0312v1.pdf



Let us associate the dichotomous variables a_1, a_2, a_3 with spin parameters ($\sigma = 1/2$) for quantum particles, which can take two values, traditionally denoted by the arrows - ↑ and ↓.

The situations considered (1 and 2) allow us to see the following.

Case a) can be represented as a set of elementary outcomes

$$\{(a_1, a_2), (a_2, a_1), (a_1, a_3), (a_3, a_1), (a_2, a_3), (a_3, a_2)\} \qquad \text{(a.1)}$$

or in spin-mnemonics

$$\uparrow\uparrow, \uparrow\uparrow, \uparrow\downarrow, \downarrow\downarrow, \downarrow\downarrow, \downarrow\uparrow, \qquad \text{(a.2)}$$

$$\uparrow\uparrow, \uparrow\downarrow, \downarrow\downarrow, \downarrow\uparrow. \qquad \text{(a.3)}$$

Comparing (a.1) and (a.2), it is possible to see the emergence of the same configurations, for example - the first two: ↑↑, ↑↑ . However, there is nothing surprising here. The expression $\{(a_1, a_2), (a_2, a_1)\}$ in (a.1) reflects the result of the procedure of replacement of the first particles to the second and vice versa; the same fragment in the expression (a.2) reflects the fact of equality of projections of spins of the rearranging particles. This is a typical situation of spin's degeneration, which is removed by the introduction of additional distinguishing characteristics of the particles. With preservation of individuality of particles (a.2) can take the form of (a.3), which is understandable.

Case b) can also be represented as a set of elementary outcomes

$$\{(a_1, a_2), (a_1, a_3), (a_2, a_1)\} \qquad \text{(b.1)}$$

or in spin-mnemonics

$$\uparrow\uparrow, \uparrow\downarrow, \downarrow\downarrow. \qquad \text{(b.2)}$$

This situation can be considered as a case of identical particles. Therefore, there are no terms that differ in the order of occurrence in the set of elementary outcomes.

Thus, in the presented example of section 2 of work [2], it is possible to consider at least three different problems on different sets of elementary outcomes $\boldsymbol{\Omega}$, and hence in different probability spaces.

B. Marginal distributions

Marginal distribution is the result of convolution of the general distribution upon several variables represented by random variables.

For example, let $f(x_{1,}x_2,x_3)$ represent the distribution density of a random vector of continuous quantities (X_1,X_2,X_3), then

$$g(x_{1,}x_2) = \int_{-\infty}^{+\infty} f(x_{1,}x_2,x_3)dx_3; \quad h(x_1) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x_{1,}x_2,x_3)dx_2dx_3; \tag{1}$$

$$1 = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x_{1,}x_2,x_3)dx_1dx_2dx_3 \tag{2}$$

Here: : $f(x_{1,}x_2,x_3) -$ PB [2] of 3-vector (X_1,X_2,X_3) RBs X_1,X_2,X_3; $g(x_{1,}x_2) -$ PB of 2-vector (X_1,X_2) of RBs X_1,X_2. The latter ratio represents a convolution of all variables and takes the form of normalization of PB.

Similar definitions exist in the discrete case. Denote by $P_{a_i,a_j,a_k}$the probabilities of the events (a_i,a_j,a_k), presented above. Then

$$P_{a_i,a_j} = \sum_{a_k} P_{a_i,a_j,a_k}; \quad P_{a_i,a_j} = \sum_{a_k}\sum_{a_j} P_{a_i,a_j,a_k}; \tag{3}$$

$$1 = \sum_{a_k}\sum_{a_j}\sum_{a_i} P_{a_i,a_j,a_k}; \tag{4}$$

It is quite natural to consider marginal distributions on the same probabilistic space as the initial one, but with the correct convolution by "excess" variables. However, the correctness of convolutions in summation in (3) and (4) is determined by the choice of the space of elementary outcomes, for example, presented in the previous section.

It is easy to notice that marginal distributions "hide" information about additional "degrees of freedom" of the considered system. As a rule, this information is not subject to correct recovery, except, perhaps, in purely private and special cases.

However, it is clear that this has nothing to do with the problem of EPR except that only from the possibility of artificial simulation of the situation with hidden parameters.

An illustration of the impossibility of recovery of the general distribution from the marginal in its extreme form (convolution of all variables in the form of normalization) can be an attempt to restore the PDB from the ratio of normalization: $\int \mathrm{PD} = 1 \ \rightarrow \mathrm{PDF}$. Such problems are known in mathematics (see, for example, [8]) as the recovery of the PDB by moments of all orders of RB and like the recovery of the analytical function by all derivatives at a given point using the Taylor series. However, with the help of such methods it is impossible to solve the problem of restoring the full distribution from marginal one.

Thus, it is possible to obtain various marginal distributions from the full probability distribution, but the inverse problem generally does not have a correct solution.

As a results of the discussions presented in the last two sections, should be said about example 2 [2] the following: there is cannot single probability measure that is compatible with the family of measures of randomized events in example 2 [2]. This is due to the fact that these events belong to different spaces of elementary outcomes ***Ω*** . Because of this, it becomes impossible to construct a General probability measure ***P*** on an ambiguous event algebra $\mathcal{F}$ However, this fact has nothing to do with the solution of the "hidden parameters - nonlocality" dilemma in the EPR. And all of this has nothing to do

[2] PD - probability density; PDF - probability distribution function; RV - random variable

with Bell's inequalities mentioned in Aspect's article. Below, for generality, we will repeat this conclusion.

C. Bell's Inequality

Let us return once again to the conclusion of Bell's inequality, following the Aspect's article , but in one place and entirely.

Consider the dichotomous variables A and B, wich taking the values of ± 1. We introduce a parameter λ with symmetric[3)] density of probability distribution $\rho(\lambda)$ obey the standard conditions[4)]

$$\rho(\lambda) \geq 0, \qquad \int d\lambda \rho(\lambda) = 1. \tag{9, 5}$$

Here λ plays the role of an external correlating factor (hidden) between variables A and B. Let parametrize this connection as a functional dependence of A and B on λ and additional parameters of $-$ vectors $\mathbf{a}$ and $\mathbf{b}$:

$$A = A(\lambda, \mathbf{a}) = \pm 1, \qquad B = B(\lambda, \mathbf{b}) = \pm 1. \tag{10, 6}$$

Let introduce the notation

$$P_{+}(\mathbf{a}) = \int d\lambda \rho(\lambda) \frac{[A(\lambda, \mathbf{a}) + 1]}{2}, \tag{11, 7}$$

$$P_{++}(\mathbf{a}) = \int d\lambda \rho(\lambda) \frac{[A(\lambda, \mathbf{a}) + 1]}{2} \frac{[B(\lambda, \mathbf{b}) + 1]}{2}, \tag{11, 8}$$

$$P_{+-}(\mathbf{a}, \mathbf{b}) = \int d\lambda \rho(\lambda) \frac{[A(\lambda, \mathbf{a}) + 1]}{2} \frac{[1 - B(\lambda, \mathbf{b})]}{2}, \tag{11, 9}$$

$$P_{-+}(\mathbf{a}, \mathbf{b}) = \int d\lambda \rho(\lambda) \frac{[1 - A(\lambda, \mathbf{a})]}{2} \frac{[B(\lambda, \mathbf{b}) + 1]}{2}. \tag{11, 10}$$

The expression (7) "collects " the probabilities $d\lambda\rho(\lambda)$ for which the value $\frac{[A(\lambda,\mathbf{a})+1]}{2}$ is not 0,that is, by virtue of dichotomicity A is 1. Similarly, (8), (9), (10) "collect" probabilities for which $A = 1,\ B = 1$; $A = 1,\ B = -1$; $A = -1, B = 1$.

The correlation coefficient $E(\mathbf{a}, \mathbf{b})$is determined by the ratio

$$E(\mathbf{a}, \mathbf{b}) = P_{++}(\mathbf{a}, \mathbf{b}) + P_{--}(\mathbf{a}, \mathbf{b}) - P_{+-}(\mathbf{a}, \mathbf{b}) - P_{-+}(\mathbf{a}, \mathbf{b}), \tag{5,11}$$

Substituting (8), (9), (10) into (11),
given $P_{++} = P_{--}$, we get the correlation coefficient $E(\mathbf{a}, \mathbf{b})$:

$$E(\mathbf{a}, \mathbf{b}) = \int d\lambda\ \rho(\lambda) A(\lambda, \mathbf{a}) \cdot B(\lambda, \mathbf{b}). \tag{12,12}$$

[3)] Concerning the symmetry of PD $\rho\ (\lambda)$ of the hidden parameter λ, the following should be said. The distribution of the normalized RV when taking into account two moments – the average (position measure) and the variance (spread measure), according to the principle of maximum entropy (maximum uncertainty and, therefore, information capacity) is the normal Gaussian distribution. For an indefinite random parameter, this distribution provides the most reliable prediction of the result.

[4)] The color will display the formula numbers of the original text of the Aspect's article and necessary explanations.

because

$$P_{++}(\mathbf{a},\mathbf{b}) + P_{--}(\mathbf{a},\mathbf{b}) = 2\cdot\frac{[A(\lambda,\mathbf{a})+1]}{2}\frac{[B(\lambda,\mathbf{b})+1]}{2} = \frac{1}{2}\{A(\lambda,\mathbf{a})\cdot B(\lambda,\mathbf{b}) + A(\lambda,\mathbf{a}) + B(\lambda,\mathbf{b}) + 1\}.$$

$$P_{+-}(\mathbf{a},\mathbf{b}) + P_{-+}(\mathbf{a},\mathbf{b}) = \frac{[A(\lambda,\mathbf{a})+1]}{2}\frac{[1-B(\lambda,\mathbf{b})]}{2} + \frac{[1-A(\lambda,\mathbf{a})]}{2}\frac{[B(\lambda,\mathbf{b})+1]}{2} =$$

$$= \frac{1}{4}\{A(\lambda,\mathbf{a}) - A(\lambda,\mathbf{a})\cdot B(\lambda,\mathbf{b}) + 1 - B(\lambda,\mathbf{b}) + B(\lambda,\mathbf{b}) + 1 - A(\lambda,\mathbf{a})\cdot B(\lambda,\mathbf{b}) - A(\lambda,\mathbf{a})\} =$$

$$\frac{1}{4}\{A(\lambda,\mathbf{a}) - A(\lambda,\mathbf{a})\cdot B(\lambda,\mathbf{b}) + 1 - B(\lambda,\mathbf{b}) + B(\lambda,\mathbf{b}) + 1 - A(\lambda,\mathbf{a})\cdot B(\lambda,\mathbf{b}) - A(\lambda,\mathbf{a})\} =$$

$$= \frac{1}{2}\{1 - A(\lambda,\mathbf{a})\cdot B(\lambda,\mathbf{b})\}.$$

$$\boldsymbol{E}(\mathbf{a},\mathbf{b}) = \frac{1}{2}\cdot\int d\lambda\ \rho(\lambda)\{\{A(\lambda,\mathbf{a})\cdot B(\lambda,\mathbf{b}) + A(\lambda,\mathbf{a}) + B(\lambda,\mathbf{b}) + 1\} - \{1 - A(\lambda,\mathbf{a})\cdot B(\lambda,\mathbf{b})\}\} =$$

$$= \frac{1}{2}\cdot\int d\lambda\ \rho(\lambda)\{2\cdot A(\lambda,\mathbf{a})\cdot B(\lambda,\mathbf{b}) + A(\lambda,\mathbf{a}) + B(\lambda,\mathbf{b})\} =$$

$$\int d\lambda\ \rho(\lambda)A(\lambda,\mathbf{a})\cdot B(\lambda,\mathbf{b}) + \int d\lambda\ \rho(\lambda)\{A(\lambda,\mathbf{a}) + B(\lambda,\mathbf{b})\} = \int d\lambda\ \rho(\lambda)A(\lambda,\mathbf{a})\cdot B(\lambda,\mathbf{b}).$$

It is obvious that due to the symmetry of PD $\rho(\lambda)$

$$\int d\lambda\ \rho(\lambda)\{A(\lambda,\mathbf{a}) + B(\lambda,\mathbf{b})\} = 0.$$

Let this

$$s = A(\lambda,\mathbf{a})B(\lambda,\mathbf{b}) - A(\lambda,\mathbf{a})B(\lambda,\mathbf{b}') + A(\lambda,\mathbf{a}')B(\lambda,\mathbf{b}) + A(\lambda,\mathbf{a}')B(\lambda,\mathbf{b}') =$$
$$= A(\lambda,\mathbf{a})[B(\lambda,\mathbf{b}) - B(\lambda,\mathbf{b}')] + A(\lambda,\mathbf{a}')[B(\lambda,\mathbf{b}) + B(\lambda,\mathbf{b}')]. \quad (17,13)$$

Given that the four numbers A and B only take ± 1, a simple analysis of the second line of the expression (13) shows that

$$s(\lambda,\mathbf{a},\mathbf{a}',\mathbf{b},\mathbf{b}') = \pm 2. \quad (18,14)$$

Averaging s over λ, we find that the value of this parameter is between +2 and -2:

$$-2 \le \int d\,\lambda\rho(\lambda)s(\lambda,\mathbf{a},\mathbf{a}',\mathbf{b},\mathbf{b}') \le 2. \quad (18,15)$$

According to (12), we can rewrite these inequalities as

$$-2 \le S(\lambda,\mathbf{a},\mathbf{a}',\mathbf{b},\mathbf{b}') \le 2, \quad (20,16)$$

where

$$S(\lambda,\mathbf{a},\mathbf{a}',\mathbf{b},\mathbf{b}') = E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{b}') + E(\mathbf{a}',\mathbf{b}) + E(\mathbf{a}',\mathbf{b}'). \quad (21,17)$$

D. Violation of Bell's inequality

The substitution of quantum-mechanical probability values in (17) demonstrates a clear violation of inequality (16). This is discussed in detail and demonstrated in [5]: the Appendix contains an elementary conclusion of the probabilities, and in table C and figure B - the results of numerical calculations.

E. Philosophical generalization problems

The proposed statement of tasks at the level of philosophical generalization is given in works [6], [7].

Links

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V.A. Kasimov. *Some topological paradoxes of relativity (EPR)-II*

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