Time-Independent Solution of Navier-Stokes Equations

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ABSTRACT

In Navier-Stokes equations, we discover time-independent solutions by Newton potential.

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1. Introduction

We discover the time-independent solutions in Navier-Stokes equation by Newton potential.

According NASA's Navier-Stokes Equations(3-dimensional-unsteady),

Coordinate: (x, y, z), Time: t, Pressure: p, Heat Flux: q

Density: ρ, Stress: τ, Reynolds Number: R_e,

Velocity Components: (u, v, w) Total Energy: E_t Plandtl Number: P_r

Continuity: $$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \qquad (1)$$

X-Momentum: $$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z}$$

$$= -\frac{\partial p}{\partial x} + \frac{1}{R_e}[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}] \qquad (2)$$

Y-Momentum: $$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z}$$

$$= -\frac{\partial p}{\partial y} + \frac{1}{R_e}[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}] \qquad (3)$$

Z-Momentum: $$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho wv)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z}$$

$$= -\frac{\partial p}{\partial z} + \frac{1}{R_e}[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}] \qquad (4)$$

Energy: $$\frac{\partial E_t}{\partial t} + \frac{\partial(E_t u)}{\partial x} + \frac{\partial(E_t v)}{\partial y} + \frac{\partial(E_t w)}{\partial z} = -\frac{\partial(\rho u)}{\partial x} - \frac{\partial(\rho v)}{\partial y} - \frac{\partial(\rho w)}{\partial z}$$

$$-\frac{1}{R_e P_r}[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}]$$

$$+\frac{1}{R_e}[\frac{\partial}{\partial x}(u\tau_{xx} + v\tau_{xy} + w\tau_{xz}) + \frac{\partial}{\partial y}(u\tau_{xy} + v\tau_{yy} + w\tau_{yz}) + \frac{\partial}{\partial z}(u\tau_{xz} + v\tau_{yz} + w\tau_{zz})]$$

(5)

2. Time-Independent solutions

For we solve equations, we use Newton potential. If we think the solution of Laplace equation,

$$u = \frac{C_1}{r^3} x \ , \ v = \frac{C_1}{r^3} y, w = \frac{C_1}{r^3} z, \ r = \sqrt{x^2 + y^2 + z^2}, \ \rho = \rho_0 \qquad (6)$$

In this case, we solve Eq(1).

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=\rho_0[-\frac{3C_1}{r^5}(x^2+y^2+z^2)+\frac{3C_1}{r^3}]=0 \quad (7)$$

Second point, in Eq(2)

$$\tau_{xx}=\frac{C_2}{r^6}x^2, \quad \tau_{xy}=\frac{C_2}{r^6}xy, \qquad \tau_{xz}=\frac{C_2}{r^6}xz \quad ,\dots,\tau_{ij}=\frac{C_2}{r^6}x^i x^j \quad (8)$$

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho u^2)}{\partial x}+\frac{\partial(\rho uv)}{\partial y}+\frac{\partial(\rho uw)}{\partial z}$$

$$=\rho_0[-\frac{6C_1^{\ 2}}{r^8}(x^3+xy^2+xz^2)+\frac{C_1^{\ 2}}{r^6}(2x+x+x)]=-\rho_0\frac{2C_1^{\ 2}}{r^6}x$$

$$=-\frac{\partial p}{\partial x}+\frac{1}{R_e}[\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial z}]$$

$$=-\frac{\partial p}{\partial x}+\frac{1}{R_e}[-\frac{6C_2}{r^8}(x^3+xy^2+xz^2)+\frac{C_2}{r^6}(2x+x+x)]$$

$$=-\frac{\partial p}{\partial x}-\frac{1}{R_e}\frac{2C_2}{r^6}x \quad (9)$$

Hence, in Eq(2),in Eq(3) and in Eq(4),

$$\frac{\partial p}{\partial x}=\frac{2x}{r^6}(\rho_0 C_1^{\ 2}-\frac{C_2}{R_e})=\frac{2C_3}{r^6}x \quad , \quad C_3=\rho_0 C_1^{\ 2}-\frac{C_2}{R_e}$$

$$\frac{\partial p}{\partial y}=\frac{2y}{r^6}(\rho_0 C_1^{\ 2}-\frac{C_2}{R_e})=\frac{2C_3}{r^6}y, \frac{\partial p}{\partial z}=\frac{2z}{r^6}(\rho_0 C_1^{\ 2}-\frac{C_2}{R_e})=\frac{2C_3}{r^6}z \quad (10)$$

Therefore,

$$p=-\frac{1}{2}\frac{C_3}{r^4} \quad , \quad C_3=\rho_0 C_1^{\ 2}-\frac{C_2}{R_e} \quad (11)$$

In Eq(5), if

$$E_t=E_0, \ q_x=\frac{C_4}{r^3}x, q_y=\frac{C_4}{r^3}y, q_z=\frac{C_4}{r^3}z, \quad (12)$$

$$\frac{\partial E_t}{\partial t}+\frac{\partial(E_t u)}{\partial x}+\frac{\partial(E_t v)}{\partial y}+\frac{\partial(E_t w)}{\partial z}=0$$

$$-\frac{\partial(pu)}{\partial x}-\frac{\partial(pv)}{\partial y}-\frac{\partial(pw)}{\partial z} \qquad , \ (pv,pu,pw)=-\frac{1}{2}\frac{C_1C_3}{r^7}(x,y,z)$$

$$=-7\frac{C_3C_1}{2r^9}(x^2+y^2+z^2)+3\frac{C_3C_1}{2r^7}=-2\frac{C_1C_3}{r^7} \quad (13)$$

$$\frac{1}{R_e P_r}[\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}+\frac{\partial q_z}{\partial z}]=0 \qquad (14)$$

$$\tau_{ij}=\frac{C_2}{r^6}x^i x^j, \quad (\tau_{xj}v,\tau_{yj}u,\tau_{zj}w)=\frac{C_1C_2}{r^9}(x^2,y^2,z^2)x^j$$

$$\frac{1}{R_e}[\frac{\partial}{\partial x}(u\tau_{xx}+v\tau_{xy}+w\tau_{xz})+\frac{\partial}{\partial y}(u\tau_{xy}+v\tau_{yy}+w\tau_{yz})+\frac{\partial}{\partial z}(u\tau_{xz}+v\tau_{yz}+w\tau_{zz})]$$

$$=\frac{1}{R_e}[-9\frac{C_1C_2}{r^{11}}x^2(x^2+y^2+z^2)+\frac{C_1C_2}{r^9}(3x^2+y^2+z^2)$$

$$-9\frac{C_1C_2}{r^{11}}y^2(x^2+y^2+z^2)+\frac{C_1C_2}{r^9}(x^2+3y^2+z^2)$$

$$-9\frac{C_1C_2}{r^{11}}z^2(x^2+y^2+z^2)+\frac{C_1C_2}{r^9}(x^2+y^2+3z^2)]$$

$$=\frac{1}{R_e}[-9\frac{C_1C_2}{r^7}+5\frac{C_1C_2}{r^7}]=-\frac{1}{R_e}\frac{4C_1C_2}{r^7} \qquad (15)$$

Hence, Eq(5) is

$$0=-2\frac{C_1C_3}{r^7}-\frac{1}{R_2}\frac{4C_1C_2}{r^7} \quad , \quad C_3=-\frac{2C_2}{R_e}$$

$$C_3=\rho_0 C_1^2-\frac{C_2}{R_e}=-\frac{2C_2}{R_e} \quad , \quad C_2=-\rho_0 R_e C_1^2$$

$$p=-\frac{1}{2}\frac{C_3}{r^4}=\frac{C_2}{R_e}\frac{1}{r^4}=-\rho_0\frac{C_1^2}{r^4} \qquad (16)$$

3. Conclusion

Therefore, solutions of Navier-Stokes equations(3-dimensional-unsteady),

Pressure: $p=-\rho_0\frac{C_1^2}{r^4}$, Heat Flux: $q_x=\frac{C_4}{r^3}x, q_y=\frac{C_4}{r^3}y, q_z=\frac{C_4}{r^3}z$

Density: $\rho=\rho_0$,

Stress: $\tau_{ij}=\frac{C_2}{r^6}x^i x^j=-\rho_0\frac{R_e}{r^6}C_1^2 x^i x^j$

Reynolds Number: R_e,

Velocity Components: $(u,v,w)=\frac{C_1}{r^3}(x,y,z)$

Total Energy: $E_t=E_0$

Reference

[1]Three-dimensional unsteady form of the Navier-Stokes equations: Glenn Research Center,NASA