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# The 'constant Lagrangian' fit of galaxy rotation curves as caused by Hubble space expansion under baryonic energy conservation conditions.

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## ABSTRACT

In my opinion, the problem of the galaxy rotation curves can be solved on the basis of the combined and competing principles of space expansion and energy conservation on the one hand and gravitational contraction and the virial theorem on the other hand. Thus far it has been assumed that the existence of galaxies is proof of the dominance of gravitational contraction in galaxies. In this paper I present arguments in favor of my conviction that this assumption is wrong and that space expansion is one of the two dominating and competing principles active in galaxies. I propose to reset, rethink and rescale the presupposed boarder between Newtonian gravitational contraction and Hubble space expansion. This effectively identifies Dark Matter attributed effects in galaxies as Dark Energy manifestations.

*Keywords:* Galaxies: kinematics and dynamics, Galactic rotation curves, Dark Matter, MOND, Dark Energy

## 1. INTRODUCTION

In a recent paper submitted to this Journal, I introduced a 'constant Lagrangian' model for galactic dynamics (de Haas 2018). In the paper I presented the analysis of the full set of 175 galaxies at the SPARC database, as provided by (Lelli et al. 2016) in the file Rotmod-LTG.zip. After the rotation curve fitting I subsequently categorizes the fitting curves. I realized that it allowed me to go from a rather weak deductive to a more robust inductive justification of the 'constant Lagrangian' model.

Of the 175 galaxies of the database, 73 percent allowed for perfect to moderate analysis on the basis of the constant Lagrangian model. In this paper I will briefly give a possible reason, or explanation, for the appearance of the constant Lagrangian galactic rotation curves. In my opinion, the explanation of the galactic rotation curves can be presented in terms of the cosmic competition between Hubble space expansion on the one hand and Newtonian gravitational contraction on the other hand. Newtonian gravitational contraction follows the virial curve, Hubble space expansion the Lagrangian curve. Where the two meet face to face with equal strengths, complex galactic rotation curves result.

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## 2. A 'CONSTANT LAGRANGIAN' MODEL GALAXY WITH FLATTENING ROTATION CURVE

In the submitted paper I presented a model galaxy consisting of a model bulge and a model outside of the bulge. I presented this in the context of both a geodetic Lagrangian approach and a GNSS atomic clock satellite syntonization application as a completely syntonized model galaxy. Fundamental my approach of galactic dynamics was to analyze gravity using relative frequency shifts, and thus $\frac{d\tau}{dt}$, as one of the basic experimental inputs. The key to my approach was to extend this clock frequency perspective towards gravity from geodesy to galaxies. When I connected the GNSS result for clock-rate syntonization

$$\frac{d\tau^2}{dt^2} = 1 + \frac{2\Phi}{c^2} - \frac{v_{orbit}^2}{c^2} = 1 - \frac{2L}{U_0} \tag{1}$$

to the problem of the galactic rotation curve, I realized that the flat rotation curve implies atomic clock syntonization in those areas. In order to study the relativistic clock-rate behavior in the inner regions of galaxies, I had construct a model galaxy that was completely syntonized of the entire rotation curve. This model galaxy was build of a model bulge with mass $M$ and radius $R$ and a Schwardschild metric emptiness around it. The model bulge was a quasi solid sphere.

The gravitational potential in such a case is well known. Inside the sphere the potential is

$$\Phi = -\frac{GM}{2R}\left(3 - \frac{r^2}{R^2}\right), \tag{2}$$

and outside the sphere the potential is

$$\Phi = -\frac{GM}{r}. \tag{3}$$

Inside this bulge the classical virial theorem would hold side by side of the constant Lagrangian condition. So $2K = -V$, with on the boundary where $r = R$ we would have $\frac{K}{m} = \frac{GM}{2R}$ and $\frac{L}{m} = \frac{K-V}{m} = \frac{3GM}{2R}$. At the center of the rotating sphere, $K = 0$ and we also have $\frac{L}{m} = \frac{3GM}{2R}$. One can further calculate that $L = K - V$ is a constant in between $r = 0$ and $r = R$, so everywhere' inside this quasi-solid sphere. We can write for this region:

$$\frac{L}{m} = \frac{v_{orbit}^2}{2} + \frac{GM}{2R}\left(3 - \frac{r^2}{R^2}\right) = \frac{3GM}{2R} = constant. \tag{4}$$

In the model galaxy that I thus constructed, $L = constant$ inside the model bulge. I then added the condition that $L = constant$ too in the whole of space outside the bulge. In that region however, the virial theorem was assumed invalid, without changing the Newtonian potential.

This leads to $K = L + V$ and $L = V(r = 0)$, so for the region $0 \le r \le R$ we get

$$v_{orbit}^2 = \frac{GM}{R} \cdot \frac{r^2}{R^2} \tag{5}$$

and outside the model bulge, where $R \le r \le \infty$, we have

$$v_{orbit}^2 = \frac{3GM}{R} - \frac{2GM}{r}. \tag{6}$$

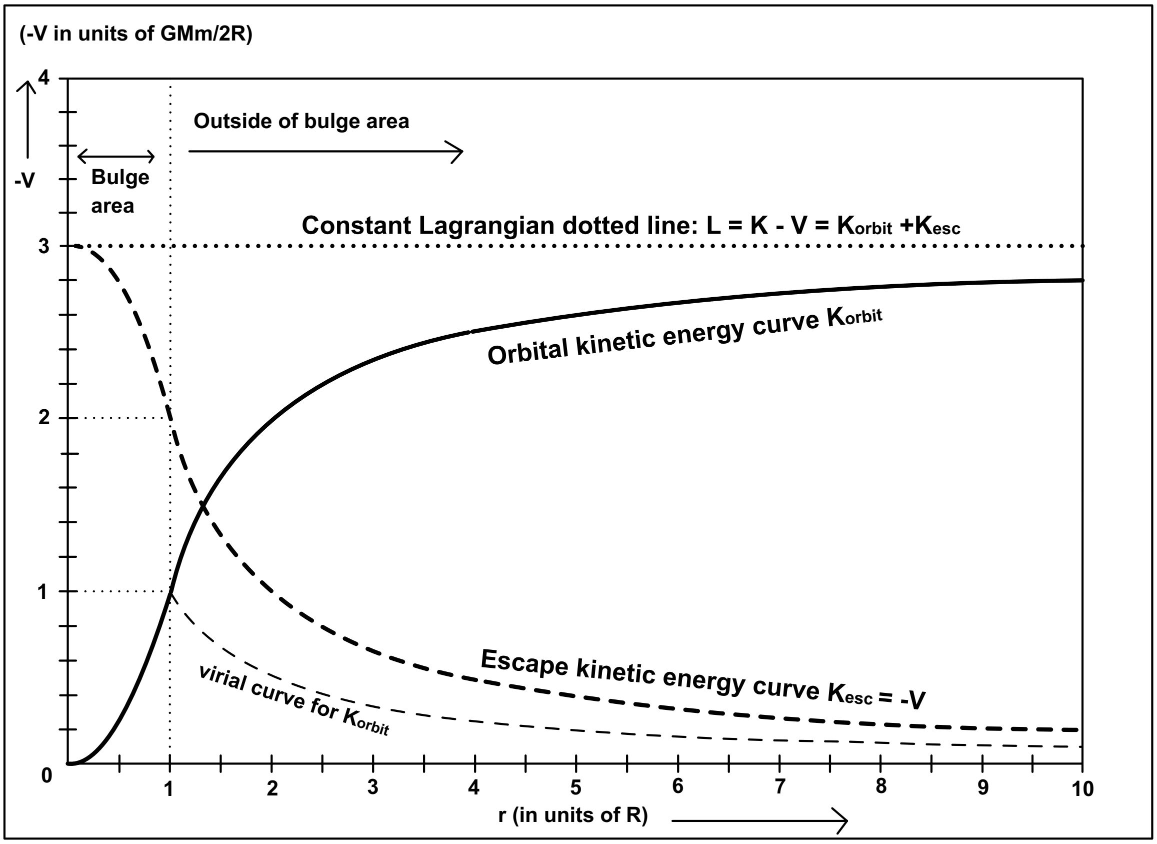



**Figure 1.** The square of the orbital velocity profile in the model galaxy with $L = constant$.

In Fig.(1) I sketched the result, with $-V = +K_{escape}$.

From the perspective of a free fall Einstein elevator observer, the free fall on a radial geodetic from infinity towards the center of the bulge, the other free fall tangential geodetics seem to abide the law of conservation of energy, because the escape kinetic energy plus the orbital kinetic energy is a constant on my model galaxy with galactic constant $L$. An Einstein elevator system with test mass $m$ that would be put in an orbital collapse situation, magically descending from orbit to orbit in a process in thermodynamic equilibrium, would have constant total kinetic energy, from the radial free fall perspective. This can be expressed as $L = K_{orbit} - V = K_{orbit} + K_{escape} = K_{final}$.

## 3. SPACE EXPANSION VERSUS GRAVITATIONAL CONTRACTION

During the analysis of real galactic rotation curves, I began to realize that the constant Lagrangian rotation curve had to be understood from the inside out, not from the outside in as with virial rotation curves. The latter one's are under the Newtonian regime of gravitational contraction. From an energy perspective it makes sense to analyze Newtonian gravitation as a system that is in a natural state of contraction. Contracting mass looses gravitational energy, of which, according to the virial theorem, one half is converted into orbital energy and one half has to be dissipated into any other form of energy. Dissipation examples are heat, internal rotation and emission of radiation.

For the constant Lagrangian curves, it makes more sense to look at them as situations where orbital kinetic energy is being transformed into gravitational energy under extremely stable conditions of total Lagrangian energy conservation. This is a process that implies some form of escape from the central mass. On a galactic scale, only cosmic expansion of space seems to provide such an escape mechanism. So my hypothesis is that the constant Lagrangian curve is a product of the combination of Hubble space expansion and baryonic energy conservation.

In the model galactic bulge, the virial theorem and the constant Lagrangian are both valid, implying that gravitational contraction and space expansion balance each other out with a perfectly stable bulge as a result. But outside the bulge, the extreme low density presumed in the model makes that gravitational contraction is no match for space expansion. If space expansion was to happen without affecting the orbital velocity and kinetic energy, then gravitational energy would increase automatically due to space expansion. Then the cosmic expansion of space would create energy out of its own process, so out of space expansion. The appearance of the constant Lagrangian rotation curve of galaxies show that, at least as far as the involved mass is concerned, baryonic conservation of energy is maintained in the process of metric driven space expansion: baryonic orbital kinetic energy is the source of the increasing baryonic gravitational energy. This baryonic mass related conservation of energy during Hubble space expansion has to be viewed as separate from the energy analysis of the metric itself.

Perceived from the inside out, the mass realizing the flattening rotation curve in the outer regions of galaxies started out as mass being close to the bulge. This mass then slowly expanded away from the center, not on its own initiative but by being dragged along the expansion of space itself as a purely metric process. But during the metric driven process this mass had to maintain its own energy balance. The only available free source for balancing the increasing gravitational energy was its orbital energy. The result is a 'constant Lagrangian' rotation curve for my model galaxy.

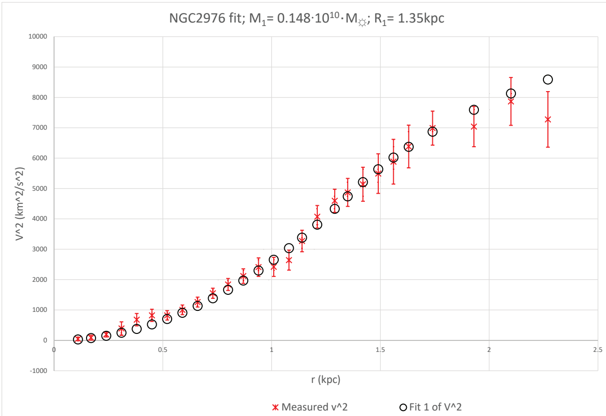
But in regions of galaxies where the matter density is high, gravitational contraction might win the battle with space expansion and the virial theorem partly reasserts itself. In regions where space expansion is negligible relative to mass density, the virial gravitational contraction rules alone.

## 4. IDENTIFYING SPACE EXPANSION ZONES VERSUS GRAVITATIONAL CONTRACTION ZONES IN REAL GALAXY ROTATION CURVES

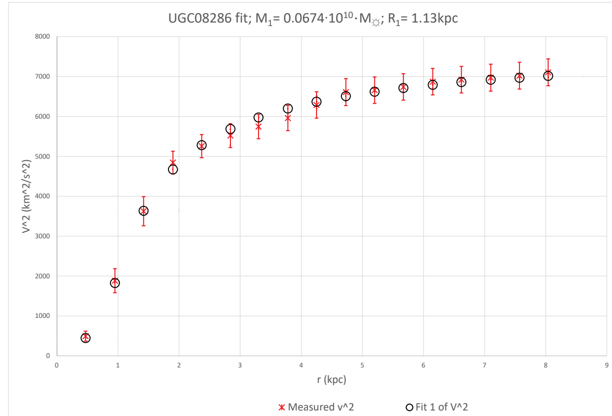
In the results of Fig.(2, UGC01281) and Fig.(2, NGC2976) the three aspects of the model curve are clearly present. First the model bulge patter is clearly present in the ascending parabolic part of the curve. This part of the model is classical because it combines the virial theorem and the constant Lagrangian. Then secondly the shift from bulge to free space as a continuous increasing function instead of the abrupt decrease as would be expected classically with the virial theorem. Thirdly is the type of ascending towards a maximum. This part of the graph is more clearly visible in Fig.(2, UGC08286). Using the analysis of this paper I can conclude that these two galaxy rotation curves are the product of space expansion under energy conservation as the dominating process outside their bulge.

The other two galaxies could only be fitted on two curves, with an intermediate zone of rotation curve descent. In the analysis of this paper, in these last zones gravitational contraction has managed to assert itself relative to space expansion. But the result is a mixture of both because the descent is not by far on a pure virial curve. The pure virial curve would have resulted in a much steeper descent. These zones should be recognizable as having a relatively high mass density, allowing gravity and gravitational contraction to partially reassert itself.

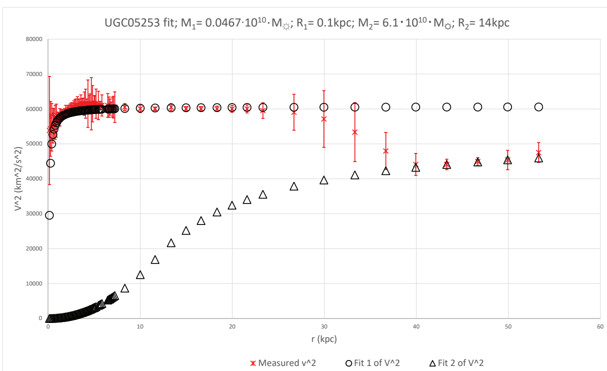
But as we go further out, mass densities inevitably reduces to very low values and space expansion, which should be a function of the amount of space available, inevitably has to come out as the




NGC2976




UGC08286

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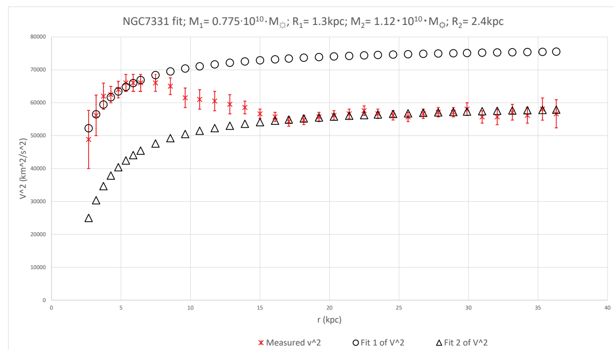


NGC7331

**Figure 2.** Two single fit constant Lagrangian galaxies and two dual fit constant Lagrangian galaxies. Where the actual measurements descend from the first fit to the second, gravitational contraction reasserted itself relative to space expansion.

dominating factor in the far out regions of all galaxies. This also leads to the conclusion that only mass that originates from a galaxy can be part of its constant Lagrangian rotation curve. In many galaxies here will be mass that didn't originate from the inside of the galaxy concerned and that mass should have a different energy situation relative to that galaxy. Mass that has been gravitationally captured by a galaxy from the outside can originally have any rotation curve relative to the host galaxy. Only as far as and for the time it has participated in the space expansion process of the galaxy that has trapped it can it be on a constant Lagrangian curve of its own inside that galaxy.

## 5. MOND, VERLINDE AND THE CONNECTION BETWEEN BTF AND HUBBLE EXPANSION

One of the few non-particle approaches to the problem of Dark Matter is MOND or MOdified Newtonian Dynamics. MOND started in 1983 with two seminal paper of Milgrom. Instead of assuming the Newtonian theory to remain valid in and around galaxies, Milgrom modified Newtons second law by making inertia a function of acceleration (Milgrom 1983b). Milgrom replaced $m_g\mathbf{a} = \mathbf{F}$ by

$$m_g\mu\left(\frac{a}{a_0}\right)\mathbf{a} = \mathbf{F}. \tag{7}$$

With such a deviation only reveals itself for accelerations with $a \approx a_0$. When $a \gg a_0$, $\mu \approx 1$ and the Newtonian regime reasserts itself. This resulted in the capacity to reasonably fit most of the galaxy rotation curves and it lead to an intrinsic connection to the baryonic Tully-Fisher relation as $V_\infty^4 = a_0 GM$ (Milgrom 1983a).

The original Tully-Fisher relation is a relation between the luminosity of a spiral galaxy and its, maximum, rotation velocity (Tully & Fisher 1977). The physical basis of the Tully-Fisher relation is the relation between a galaxy's total baryonic mass and the velocity at the flat end of the rotation curve, the final velocity. According to McGaugh both stellar and gas mass of galaxies have to be taken into account in the relation that is referred to as the Baryonic Tully-Fisher (BTF) relation. In 2005 McGaugh determined the baryonic version of the LT relation as $M_d = 50 v_f^4$, see (McGaugh 2005) and Fig(3). In this form, $M_d$ is expressed in solar mass $M_\odot = 1,99 \cdot 10^{30}\ kg$ units and the final velocity of the galactic rotation velocity curve $v_f$ is expressed in $km/s$. If we express the galactic mass in $kg$ and the velocity in $m/s$ we get the total baryonic mass, final velocity relations in SI unit values as $M_b = 1,0 \cdot 10^{20} v_f^4$.

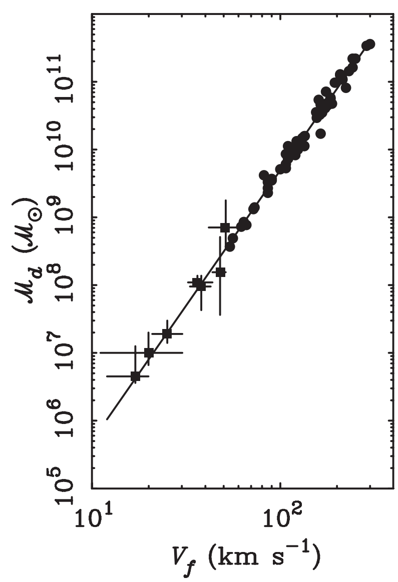


**Figure 3.** The Baryonic Tully-Fisher relation. Reprint from McGaug 2005 (McGaugh 2005).

In 1983, Milgrom interpreted the BTF relation as indicative of his proposed deviation from Newtonian gravity, justifying his modification of Newtonian dynamics or MOND (Milgrom 1983b). Using McGaug's 2005 values in SI units, Milgrom's presentation of the BTF relation can be cast in the form $v_f^4 = 1,0 \cdot 10^{-20} M_b = G a_0 M_b$, resulting in an acceleration $a_0 = 1,5 \cdot 10^{-10}\ m/s^2$ in McGaug's values. Milgrom hypothesized that this relation should hold exactly, thus interpreting it as an inductively found law of nature, instead of looking at it as just a coincidental empirical relation (Milgrom 1983a). The resulting acceleration can be written as $5 \cdot a_0 \approx c H_0$, with the velocity of light $c$ and the Hubble constant $H_0$. According to Milgrom, the deeper significance of this relation between this special galactic acceleration and the Hubble acceleration should be revealed by future cosmological insights (Milgrom 1983b). At the moment, (Verlinde 2016, p. 38) calls it the Hubble acceleration scale and assumes $6 \cdot a_0 \simeq cH_0$. Then we get the approximate BTF-MOND-Hubble relation

$$v_f^4 \simeq \frac{GcH_0M_b}{6}. \tag{8}$$

Thus, in galactic research, the connection between galaxy rotation curves and the Hubble expansion parameter is a recognized empirical peculiarity.

The expansion of the cosmos is determined by the Hubble constant. It is a universal constant, because the same in all directions where we can observe receding galaxies. The deviations from the virial rotation curve always appears below a certain acceleration, when

$$\frac{GM}{r^2} < \frac{cH_0}{2}, \tag{9}$$

see (Verlinde 2016, 2017). In my perspective, this clearly indicates that Newtonian gravitational contraction is limited in its capacity to counterbalance Hubble space expansion. The MOND acceleration $a_0$ or Verlinde's acceleration scale $a_M$ present the practical limit in this sense. In today's Standard Cosmology Dark Matter effects kick in beyond this limit. In my perspective, beyond this limit Hubble space expansion starts to measurably dominate over Newtonian gravitational contraction. This Hubble space expansion is a process within the principle of conservation of energy and the mass dragged along with this expanding space therefore follows the constant Lagrangian rotation curves, implicating that orbital kinetic energy is being transformed into gravitational energy during this expansion.

The Hubble acceleration defined as $g_H \simeq cH_0$ indicates the order of magnitude below which value expansion starts to dominate contraction. The notion that this already happens deep inside galaxies and not only well outside galaxies and even outside galaxy clusters is the 'revolutionary' new of my interpretation of the'constant Lagrangian' approach towards the galaxy rotation curve issue. What I propose to reset, rethink and rescale is the presupposed boarder between Newtonian contraction and Hubble expansion.

## 6. CONCLUSION

In my opinion, the problem of the galaxy rotation curves can be solved on the basis of the combined and competing principles of space expansion and energy conservation on the one hand and gravitational contraction and the virial theorem on the other hand. Thus far it has been assumed that the existence of galaxies is proof of the dominance of gravitational contraction in galaxies. It is my conviction that this assumption is wrong and that space expansion is one of the two dominating principles active in galaxies. The other being gravitational contraction. Rotation curves of Galaxy clusters should allow analogous analysis, with perhaps a somewhat different balance between space expansion and gravitational contraction. My analysis effectively identifies Dark Matter attributed effects in galaxies as Dark Energy manifestations.

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