Title: Formula to find prime numbers and composite numbers with termination 9
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Abstract: The prime numbers greater than 5 have 4 terminations in their unit to infinity (1,3,7,9) and the composite numbers divisible by numbers greater than 3 have 5 terminations in their unit to infinity, these are (1,3,5,7,9). This paper develops an expression to calculate the prime numbers and composite numbers with ending 9.

Keywords: Prime numbers, composite numbers.

Introduction

The study of the prime numbers is wonderful, But to understand them, first study the composite numbers, in the absence of an expression that involves all of them I have investigated and I have discovered a brilliant expression that contains all the prime numbers greater than 3 and all composite number that are not divisible by 2 and by 3. This expression comes from investigating first how they are distributed the composite numbers with termination 9, this allowed me to explore its order and understand its mechanism. The expression of the prime numbers with termination 9 is its result.
This paper has 8 demonstrations.

Theorem 1

Numbers with termination 9. These numbers are interleaved between prime numbers greater than 3 and composite numbers divisible by numbers greater than 3. These are distributed in two well-known sequences.

Formula to obtain numbers with termination 9 within the sequence β

$$\beta = (6 * n \pm 1)$$

β_a=$(6 * n + 1) =$ 7,13,19,25,31,37,43,49,55,61,67,73,79,85,……..

β_b= $(6 * n - 1) =$ 5,11,17,23,29,35,41,47,53,59,65,71,77,83,89 …….

Within the beta sequence we find composite numbers and prime numbers. To be able to locate only the numbers that end with 9 we will go to the next point (**Theorem 2).**

Theorem 2

At point A we will look for numbers with ending 9 within the sequence $\beta_b = (6 * n - 1)$
At point B we will look for composite numbers with ending 9 within the sequence $\beta_b = (6 * n - 1)$

$\beta_b = (6 * n - 1) =$ 5,11,17,23,29,35,41,47,53,59,65,71,77,83,89, ...
n> 0

Reference A007528 (The On-line Enciclopedia of integers sequences)

A) Formula for numbers with termination 9 within the sequence β_b

$$N_{(b)t9} = (30 * n + 29)$$

$N_{(b)t9}$= numbers with termination 9 within the sequence β_b
$N_{(b)t9}$= 29,59,89,119,149,179,209,239,269,299,329,359,389,419,.........
n≥ 0
z≥ 0

Reference A128469 (The On-line Enciclopedia of integers sequences)

Demonstration 1

B) Formula for composite numbers with termination 9 within the sequence β_b

Composite numbers congruent to 29 (mod 30) within the sequence $\beta_b = (6 * n - 1)$

$$Nc_{(\mathbf{b})\ \mathbf{t9}} = (30 * n + 29)_{=\beta*(\delta+30*z)}$$

n≥ 0
z≥ 0

β has infinite values

Formed by the sequence $\beta = (6 * n \pm 1)$= 5,7,11,13,17,19,23,25,29,31, ...

β_1= 5, β_2= 7, β_3= 11, β_4= 13......

δ has 10 variants

These 10 variables are always ordered in the same way and are repeated simultaneously until infinity as we add more values of β.

5 Nothing	17	19	23	7	11	13	25 Nothing	31	29

The multiples of 5 are green and carry the word (nothing), these are not calculated since no multiples of 5 generate numbers with termination 9.

$Nc_{(\mathbf{b})\ \mathbf{t9}}$= **Composite numbers termination 9**

$$Nc_{(\mathbf{b})\mathbf{t9}} = (30 * n + 29)$$

$=\beta_1\ Nothing$
$=\beta_2*(17+30*z)$
$=\beta_3*(19+30*z)$
$=\beta_4*(23+30*z)$
$=\beta_5*(7+30*z)$
$=\beta_6*(11+30*z)$
$=\beta_{b7}*(13+30*z)$
$=\beta_8\ Nothing$
$=\beta_9*(31+30*z)$
$=\beta_{10}*(29+30*z)$
$-------$
$=\beta_{11}\ Nothing$
$=\beta_{12}*(17+30*z)$
$=\beta_{13}*(19+30*z)$
$=\beta_{14}*(23+30*z)$
$=\beta_{15}*(7+30*z)$
$=\beta_{16}*(11+30*z)$
$=\beta_{17}*(13+30*z)$
$=\beta_{18}\ Nothing$
$=\beta_{19}*(31+30*z)$
$=\beta_{20}*(29+30*z)$
$-------$
$continue\quad infinitely$

$$= (30 * n + 29)$$

$=5\ Nothing$
$=7*(17+30*z)$
$=11*(19+30*z)$
$=13*(23+30*z)$
$=17*(7+30*z)$
$=19*(11+30*z)$
$=23*(13+30*z)$
$=25\ Nothing$
$=29*(31+30*z)$
$=31*(29+30*z)$
$--------$
$=35\ Nothing$
$=37*(17+30*z)$
$=41*(19+30*z)$
$=43*(23+30*z)$
$=47*(7+30*z)$
$49*(11+30*z)$
$53*(13+30*z)$
$=55\ Nothing$
$=59*(31+30*z)$
$=61*(29+30*z)$
$--------$
$continue\quad infinitely$

The series is repeated every 10 blocks, (nothing,17,19,23,7,11,13,nothing,31,29) .
We can add more β numbers and expand the formula infinitely.

Demonstration 2

We solve when z = 0, z=1, z=2,…….

$$Nc_{(\mathbf{b})\mathbf{t9}} = (30 * n + 29)$$

$=5\ Nothing$
$=7*(17+30*z)$
$=11*(19+30*z)$
$=13*(23+30*z)$
$=17*(7+30*z)$
$=19*(11+30*z)$
$=23*(13+30*z)$
$=25\ Nothing$
$=29*(31+30*z)$
$=31*(29+30*z)$
$continue\quad infinitely$

$$=(30 * n + 29)$$

$= -$
=119,329,539,.....
=209,539,869,......
=299,689,1079,.....
=119,629,1139,.....
=209,7791349 ,.....
=299,989,1679,....
$= -$
=899,1769,2639,....
=899,1829,2759,.......
$continue\quad infinitely$

Demonstration 3

C) Distances between composite numbers with termination 9 in column b.

The distance between composite numbers with termination 9 when we use the same value for β is equal to:

Distance between composite number $\boldsymbol{D_9}$**= 30*** $\boldsymbol{\beta}$

$\boldsymbol{D_9}$**=** Distance between composite number (Termination 9).

Example

$\beta = 7; \quad D_9 = 30 * 7 = 210$
$\beta = 11; \quad D_9 = 30 * 11 = 330$
$\beta = 13; \quad D_9 = 30 * 13 = 390$

Theorem 3

At point A we will look for numbers with ending 9 within the sequence $\beta_a = (6 * n + 1)$
At point B we will look for composite numbers with ending 9 within the sequence $\beta_a = (6 * n + 1)$

β_a=$(6 * n + 1)$ = 7,13,19,25,31,37,43,49,55,61,67,73,79,85,........
n> 0

Reference A016921 (The On-line Enciclopedia of integers sequences)

A) Formula for numbers with termination 9 within the sequence β_a

$$N_{(a)t9} = (30 * n + 19)$$

$N_{(a)t9}$= 19,49,79,109,139,169,199,229,259,289,319,349,379,409,.........
n≥ 0
z≥ 0

Reference A156376 (The On-line Enciclopedia of integers sequences)

Demonstration 4

B) Formula for composite numbers with termination 9 within the sequence β_a

Composite numbers congruent to 19 (mod 30) within the sequence $\beta_a = (6 * n + 1)$

$$Nc_{(\mathbf{a})\ \mathbf{t9}} = (30 * n + 19)_{=\beta *(\delta + 30 * z)}$$

n≥ 0
z≥ 0

β has infinite values

Formed by the sequence $\beta = (6 * n \pm 1)$= 5,7,11,13,17,19,23,25,29,31, ...

β_1= 5, β_2= 7, β_3= 11, β_4= 13......

δ has 10 variants

These 10 variables are always ordered in the same way and are repeated simultaneously until infinity as we add more values of β.

5 Nothing	7	29	13	17	31	23	25 Nothing	11	19

The multiples of 5 are green and carry the word (nothing), these are not calculated since no multiples of 5 generate numbers with termination 9.

$Nc_{(a)\ t9}$= Composite numbers termination 9

$Nc_{(a)t9} = (30*n+19)$
$=\beta_1\ Nothing$
$=\beta_2*(7+30*z)$
$=\beta_3*(29+30*z)$
$=\beta_4*(13+30*z)$
$=\beta_5*(17+30*z)$
$=\beta_6*(31+30*z)$
$=\beta_{b7}*(23+30*z)$
$=\beta_8\ Nothing$
$=\beta_9*(11+30*z)$
$=\beta_{10}*(19+30*z)$
$-------$
$=\beta_{11}\ Nothing$
$=\beta_{12}*(7+30*z)$
$=\beta_{13}*(29+30*z)$
$=\beta_{14}*(13+30*z)$
$=\beta_{15}*(17+30*z)$
$=\beta_{16}*(31+30*z)$
$=\beta_{17}*(23+30*z)$
$=\beta_{18}\ Nothing$
$=\beta_{19}*(11+30*z)$
$=\beta_{20}*(19+30*z)$
$-------$
$continue\ \ infinitely$

$= (30*n+19)$
$=\cancel{5}\ Nothing$
$=7*(7+30*z)$
$=11*(29+30*z)$
$=13*(13+30*z)$
$=17*(17+30*z)$
$=19*(31+30*z)$
$=23*(23+30*z)$
$=\cancel{25}\ Nothing$
$=29*(11+30*z)$
$=31*(19+30*z)$
$-------$
$=\cancel{35}\ Nothing$
$=37*(7+30*z)$
$=41*(29+30*z)$
$=43*(13+30*z)$
$=47*(17+30*z)$
$=49*(31+30*z)$
$=53*(23+30*z)$
$=\cancel{55}\ Nothing$
$=59*(11+30*z)$
$=61*(19+30*z)$
$-------$
$continue\ infinitely$

The series is repeated every 5 blocks, on the left (nothing 7,29,13,17,31,23, nothing,11,19) and also on the right (nothing ,29,17,23,11) to infinity. we can add more β numbers and expand the formula infinitely.

Demonstration 5

We solve when z = 0, z=1, z=2,…….

$Nc_{(a)t9} = (30*n+19)$
$=5\ Nothing$
$=7*(7+30*z)$
$=11*(29+30*z)$
$=13*(13+30*z)$
$=17*(17+30*z)$
$=19*(31+30*z)$
$=23*(23+30*z)$
$=25\ Nothing$
$=29*(11+30*z)$
$=31*(19+30*z)$
$continue\ \ infinitely$

$=(30*n+19)$
$= -$
=49,259,469,.....
=319,649,979,......
=169,559,949,.....
=289,799,1309,.....
=589,1159,1729,.....
=529,1219,1909,....
$= -$
=319,1189,2059,....
=589,1519,2449 ....
$continue\ \ infinitely$

Demonstration 6

A) Distances between composite numbers with termination 9 in column a.

The distance between composite numbers with termination 9 when we use the same value for β is equal to:

Distance between composite number $\boldsymbol{D_9}$**= 30*** $\boldsymbol{\beta}$

$\boldsymbol{D_9}$**=** Distance between composite number (Termination 9).

Example

A. $\beta = 7;\quad D_9 = 30*7 = 210$
B. $\beta = 11;\ D_9 = 30*11 = 330$
C. $\beta = 13;\ D_9 = 30*13 = 390$

<u>Theorem 4</u> We will use the same information that we obtained to calculate the numbers composed in the theorem 2, but in this occasion we will use the inequality to obtain only results of prime numbers.

At point A we will look for prime numbers with ending 9 within the sequence $\beta_b = (6*n-1)$

$\beta_b = (6*n-1) =$ 5,11,17,23,29,35,41,47,53,59,65,71,77,83,89, …….
n> 0

Reference A007528 (The On-line Enciclopedia of integers sequences)

Demonstration 7

A) Formula for Prime numbers with termination 9 within the sequence $\beta_b = (6*n-1)$

$$P_{(\mathbf{b})\,\mathbf{t9}} = (30*n+29)_{\neq \beta*(\delta+30*z)}$$

n≥ 0
z≥ 0

β has infinite values

Formed by the sequence $\beta = (6*n \pm 1)$= 5,7,11,13,17,19,23,25,29,31, ….

β_1= 5, β_2= 7, β_3= 11, β_4= 13……

δ has 10 variants

These 10 variables are always ordered in the same way and are repeated simultaneously until infinity as we add more values of β.

5 Nothing	17	19	23	7	11	13	25 Nothing	31	29

The multiples of 5 are green and carry the word (nothing), these are not calculated since no multiples of 5 generate numbers with termination 9.

Primes congruent to 29 (mod 30) within the sequence $\beta_b = (6*n-1)$

$$P_{(b)t9} = \textbf{Prime numbers } \boldsymbol{termination}\ \mathbf{9}$$

$P_{(\mathbf{b})\mathbf{t9}} = (30*n+29)$	$\neq 5\ Nothing$	$=(30*n+29)$	$\neq -$
	$\neq 7*(17+30*z)$		$\neq 119,329,539,\ldots..$
	$\neq 11*(19+30*z)$		$\neq 209,539,869,\ldots...$
	$\neq 13*(23+30*z)$		$\neq 299,689,1079,\ldots..$
	$\neq 17*(7+30*z)$		$\neq 119,629,1139,\ldots..$
	$\neq 19*(11+30*z)$		$\neq 209,7791349\ ,\ldots..$
	$\neq 23*(13+30*z)$		$\neq 299,989,1679,\ldots.$
	$\neq 25\ Nothing$		$\neq -$
	$\neq 29*(31+30*z)$		$\neq 899,1769,2639,\ldots.$
	$\neq 31*(29+30*z)$		$\neq 899,1829,2759,\ldots....$
	$continue\ \ infinitely$		$continue\ \ infinitely$

$P_{(\mathbf{b})\mathbf{t9}}$= 29, 59, 89, 149, 179, 239, 269, 359, 389, 419, 449, 479, 509, 569, 599, 659, 719, 809, 839, 929, 1019, 1049, 1109, 1229, 1259, 1289, 1319, 1409, 1439, 1499, 1559, 1619, 1709, 1889, 1949, 1979, 2039, 2069, 2099, 2129, 2309, 2339, 2399, 2459, 2549, 2579,……

Reference A132236 (The On-line Enciclopedia of integers sequences)

Theorem 5 We will use the same information that we obtained to calculate the composite numbers in the theorem 3, but in this occasion we will use the inequality to obtain only results of prime numbers.

At point A we will look for prime numbers with ending 9 within the sequence $\beta_a = (6 * n + 1)$

β_a=$(6 * n + 1)$ = 7,13,19,25,31,37,43,49,55,61,67,73,79,85,……..
n> 0

Reference A016921 (The On-line Enciclopedia of integers sequences)

Demonstration 8

A) Formula for Prime numbers with termination 9 within the sequence $\beta_a = (6 * n + 1)$

$$P_{(\mathbf{a})\ \mathbf{t9}} = (30 * n + 19)_{\neq \beta *(\delta + 30 * z)}$$

n≥ 0
z≥ 0

β has infinite values

Formed by the sequence $\beta = (6 * n \pm 1)$= 5,7,11,13,17,19,23,25,29,31, ….

β_1= 5, β_2= 7, β_3= 11, β_4= 13……

δ has 10 variants

These 10 variables are always ordered in the same way and are repeated simultaneously until infinity as we add more values of β.

5 Nothing	7	29	13	17	31	23	25 Nothing	11	19

The multiples of 5 are green and carry the word (nothing), these are not calculated since no multiples of 5 generate numbers with termination 9.

Primes congruent to 19 (mod 30) within the sequence $\beta_a = (6 * n + 1)$

$$P_{(a)t9} = \textbf{Prime numbers } \boldsymbol{termination}\ \mathbf{9}$$

$P_{(\mathbf{a})\mathbf{t9}} = (30 * n + 19)$

$\neq 5\ Nothing$
$\neq 7*(7+30*z)$
$\neq 11*(29+30*z)$
$\neq 13*(13+30*z)$
$\neq 17*(17+30*z)$
$\neq 19*(31+30*z)$
$\neq 23*(23+30*z)$
$\neq 25\ Nothing$
$\neq 29*(11+30*z)$
$\neq 31*(19+30*z)$
$continue \quad infinitely$

$=(30 * n + 19)$

$\neq -$
$\neq 49,259,469,.....$
$\neq 319,649,979,......$
$\neq 169,559,949,.....$
$\neq 289,799,1309,.....$
$\neq 589,1159,1729,.....$
$\neq 529,1219,1909,....$
$\neq -$
$\neq 319,1189,2059,....$
$\neq 589,1519,2449\ ....$
$continue \quad infinitely$

$\boldsymbol{P}_{(\mathbf{a})\mathbf{t9}}=$ 19, 79, 109, 139, 199, 229, 349, 379, 409, 439, 499, 619, 709, 739, 769, 829, 859, 919, 1009, 1039, 1069, 1129, 1249, 1279, 1399, 1429, 1459, 1489, 1549, 1579, 1609, 1669, 1699, 1759, 1789, 1879, 1999, 2029, 2089, 2179, 2239, 2269, 2389, 2539, 2659, 2689,........

Reference A132234 (The On-line Enciclopedia of integers sequences)

Theorem 6 Graphic table 1, with termination 9.

In yellow the prime numbers

Composite number in red Prime numbers with termination 9 in green					
A β_a				B β_b	
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
97	98	99	100	101	102
103	104	105	106	107	108
109	110	111	112	113	114
115	116	117	118	119	120
121	122	123	124	125	126
127	128	129	130	131	132
133	134	135	136	137	138
139	140	141	142	143	144

In red the composite numbers

Prime number in Yellow Composite numbers with termination 9 in light blue					
A β_a				B β_b	
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
97	98	99	100	101	102
103	104	105	106	107	108
109	110	111	112	113	114
115	116	117	118	119	120
121	122	123	124	125	126
127	128	129	130	131	132
133	134	135	136	137	138
139	140	141	142	143	144

Conclusion

The numbers with ending 9 are ordered every 30 numbers interspersed between composite numbers and prime numbers. These have two variables.
The first variable shows that the numbers of the formula $N_{(b)t9} = (30 * n + 29)$ are located in the column (B.) The sum of their digits always generates the sequence 2,5,8.
The second variable shows that the numbers of the formula $N_{(a)t9} = (30 * n + 19)$ are located in another column (A). the sum of its digits always generates the sequence 1,4,7.
By means of equalities and inequalities we can condition these formulas to obtain all prime numbers greater than 3 and all composite numbers divisible by numbers greater than 3 by means of a simple, unique and infinite expression.
By equalities we obtain composite numbers divisible by numbers greater than 3 whit termination 9.
By inequalities we obtain the prime numbers greater than 3 whit termination 9.
The formula developed with the 10 variables of the delta letter allows you to obtain them infinitely.
These 10 variables are the key for the formula to work.
Thanks to this expression we can understand how the prime numbers and the composite numbers with ending 9 are distributed in the unit.
The multiples of 5 in beta are excluded since they do not generate numbers with ending 9.

This formula demonstrates that it is possible to calculate and obtain the sequence of prime numbers with ending 9 and also that of the composite numbers.
This model is applied to the other three terminations (1,3,7) although the locations of the delta numbers vary, since these are the same but they are located differently.

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