Proofing the Collatz Conjecture by Mathematical Induction

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Abstract

In this article, the author applies the mathematical induction, classifies positive integers, and passes operations according to the operational rule, to achieve the goal that proves the Collatz conjecture finally.

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1. Introduction

The Collatz conjecture is also variously well-known 3n+1 conjecture, the Ulam conjecture, Kakutani's problem, the Thwaites conjecture, Hasse's algorithm, and the Syracuse problem etc. Yet it is still both unproved and un-negated a conjecture ever since named after Lothar Collatz in 1937.

2. Basic Concepts and Criteria

The Collatz conjecture states that take any positive integer n, if n is an even number, then divide n by 2; if n is an odd number, then multiply n by 3 and add 1. Repeat the above process indefinitely, then no matter which positive integer you start with, it will eventually reach a result of 1.

Let us regard above-mentioned operational stipulations as the operational rule of the conjecture or the operational rule for short.

Start with any positive integer to operate successive emerging positive integers by the operational rule, afterwards, regard consecutive integers plus synclastic arrowheads among them as an operational route.

If an integer (or an integer's expression) P_{ie} exists at an operational route, then may term the operational route "an operational route of P_{ie}" or "the operational route P_{ie}". Two operational routes of P_{ie} branch from an integer (or an integer's expression) after pass operations of P_{ie}.

Before make the proof, be necessary to prepare two theorems concerned, so as to reach certain of conclusions by them.

Theorem 1* If a positive integer or a positive integer's expression at operational route P_{ie} suits the conjecture, then P_{ie} suits the conjecture.

For examples: (1) If $27+2^3\eta$ suits the conjecture, and $P_{ie}=31+3^2\eta$, where $\eta\geq 0$, then from $27+2^3\eta\rightarrow 82+3\times 2^3\eta\rightarrow 41+3\times 2^2\eta\rightarrow 124+3^2\times 2^2\eta\rightarrow 62+3^2\times 2\eta\rightarrow 31+3^2\eta>27+2^3\eta$, conclude that $31+3^2\eta$ suits the conjecture. (2) If $4+3\mu$ suits the conjecture, and $P_{ie}=5+2^2\mu$, where $\mu\geq 0$, then from $5+2^2\mu\rightarrow 16+3\times 2^2\mu\rightarrow 8+3\times 2\mu\rightarrow 4+3\mu<5+2^2\mu$, conclude that $5+2^2\mu$ suits the conjecture.

Proof* Suppose C_{ie} suit the conjecture at an operational route of P_{ie}. If C_{ie} appears before P_{ie}, then the operations of C_{ie} via P_{ie} reached 1 already. If C_{ie} appears behind P_{ie}, then the operations of P_{ie} pass C_{ie}, afterwards continue along operational route of C_{ie} to reach 1.

Theorem 2* If a positive integer or a positive integer's expression at operational route Q_{ie} suits the conjecture, in addition the operational route

P_{ie} and the operational route Q_{ie} intersect, then P_{ie} suits the conjecture, where $P_{ie} \neq Q_{ie.}$ For example, the positive integer's expression $71+3^3\times2^5\varphi$ suits the conjecture, and $P_{ie}=95+3^2\times2^7\varphi$, where $\varphi\geq0$, so from $95+3^2\times2^7\varphi$ $\rightarrow286+3^3\times2^7\varphi\rightarrow143+3^3\times2^6\varphi\rightarrow430+3^4\times2^6\varphi\rightarrow215+3^4\times2^5\varphi\rightarrow646+3^5\times2^5\varphi$ $\rightarrow$ $323+3^5\times2^4\varphi\rightarrow970+3^6\times2^4\varphi\rightarrow485+3^6\times2^3\varphi\rightarrow1456+3^7\times2^3\varphi\rightarrow728+3^7\times2^2\varphi$ $\rightarrow364+3^7\times2\varphi\rightarrow182+3^7\varphi\rightarrow \ldots$

$\uparrow121+3^6\times2\varphi\leftarrow242+3^6\times2^2\varphi\leftarrow484+3^6\times2^3\varphi\leftarrow161+3^5\times2^3\varphi\leftarrow322+3^5\times2^4\varphi$ $\leftarrow107+3^4\times2^4\varphi\leftarrow214+3^4\times2^5\varphi\leftarrow71+3^3\times2^5\varphi$ $<$ $95+3^2\times2^7\varphi$, conclude that $95+3^2\times2^7\varphi$ suits the conjecture.

Proof* On the supposition that two operational routes intersect at A_{ie}, and D_{ie} suits the conjecture, then D_{ie} and A_{ie} exist at operational route Q_{ie}, so A_{ie} suits the conjecture according to the theorem 1. Like the reason, P_{ie} and A_{ie} exist at operational route P_{ie}, of course, P_{ie} suits the conjecture.

3. A Classified Proof by Mathematical Induction

Let us set to prove the Collatz conjecture by the mathematical induction:

1. From 2→1; 3→10→5→16→8→4→2→1; 4→2→1; 5→16→8→4→2→1; 6→3→10→5→16→8→4→2→1; 7→22→11→34→17→52→26→13→40→20→10 →5→16→8→4→2→1; 8→4→2→1; 9→28→14→7→22→11→34→17→52→26 →13→40→20→10→5→16→8→4→2→1; 10→5→16→8→4→2→1; 11→34→17 →52→26→13→40→20→10→5→16→8→4→2→1 ; 12→6→3→10→5→16→8→ 4→2→1 ; 13→40→20→10→5→16→8→4→2→1 ; 14→7→22→11→34→17→52 →26→13→40→20→10→5→16→8→4→2→1; 15→46→23→70→35→106→53→

160→80→40→20→10→5→16→8→4→2→1 ; 16→8→4→2→1; 17→52→26 →13 →40→20→10→5→16→8→4→2→1 ; 18→9→28→14→7→22→11→34→17→52 →26→13→40→20→10→5→16→8→4→2→1 and 19→58→29→88→44→22→ 11→34→17→52→26→13→40→20→10→5→16→8→4→2→1, reach the conclusion that each of positive integers≤ 19 suits the conjecture.

2. Suppose that positive integer n suits the conjecture, where n≥19.

3. Prove that positive integer n+1 suits the conjecture.

Proof * According to two theorems prepared in advance, to prove that n+1 suits the conjecture, so long as we can find out a direct relation between n+1 and a smaller integer ≤n with n≥19, or an indirect relation between n+1 and an integer's expression which suits the conjecture in accordance with the operational rule, then the problem is solved by us.

It is well known that all positive integers are divided into even numbers and odd numbers.

Firstly, for any even number 2k with k≥1, from 2k→k<2k, conclude that 2k suits the conjecture according to the theorem 1. So, if n+1 belongs within 2k, then n+1 suits the conjecture because 2k suits the conjecture.

Secondly, for all unproved odd numbers, first divide them into two genera, i.e. 5+4k and 7+4k, where k ≥4.

For 5+4k, from 5+4k→16+12k→8+6k→4+3k<5+4k, conclude that 5+4k suits the conjecture according to the theorem 1. So, if n+1 belongs within 5+4k, then n+1 suits the conjecture because 5+4k suits the conjecture.

For 7+4k, divide it into 3 sorts: 15+12c, 19+12c and 23+12c where c≥0.

For 23+12c, from 15+8c→46+24c→23+12c<15+8c, conclude that 23+12c suits the conjecture according to the theorem 1. So, if n+1 belongs within 23+12c, then n+1 suits the conjecture because 23+12c suits the conjecture.

For 15+12c and 19+12c, when c=0, they have been proved in advance, so merely need us to prove 15+12c and 19+12c where c≥1.

For 15+12c and 19+12c where c≥1, let us operate them right along, so that expound the relation that they act in accordance with either theorem.

Firstly, let us operate 15+12c by the operational rule, as listed below.

15+12c→46+36c→23+18c→70+54c→35+27c ♣

d=2e+1: 29+27e (**1**)   e=2f: 142+486f→71+243f ♥
♣35+27c↓→c=2d+1: 31+27d↑→d=2e: 94+162e→47+81e↑→e=2f+1:64+81f (**2**)
c=2d: 106+162d→53+81d↓→d=2e+1:67+81e↓→e=2f+1:74+81f (**3**)
d=2e:160+486e♦   e=2f: 202+486f→101+243f ♠

g=2h+1: 200+243h (**4**)   …
♥ 71+243f↓→f=2g+1:157+243g↑→g=2h: 472+1458h→236+729h↑→ …
f=2g: 214+1458g→107+729g↓→g=2h+1: 418+729h↓→…
g=2h: 322+4374h→… …

g=2h: 86+243h (**5**)
♠101+243f↓→f=2g+1:172+243g↑→g=2h+1:1246+1458h→…
f=2g: 304+1458g→152+729g ↓→…
…

…
♦160+486e→80+243e↓→e=2f+1: 970+1458f→485+729f↑→ …   …
e=2f:40+243f↓→f=2g+1:850+1458g→ 425+729g↑→ …
f=2g: 20+243g↓→g=2h: 10+243h (**6**)   …
g=2h+1:880+1458h→440+729h↑→ …

Annotation:

(1) Each of letters c, d, e, f, g, h …etc at listed above operational routes expresses each of natural numbers plus 0.

(2) Also, there are ♣↔♣, ♥↔♥, ♠↔♠, and ♦↔♦.

(3) Aforesaid two points are suitable to latter operational routes of 19+12c similarly.

In the course of operation for 15+12c/19+12c by the operational rule, if an operational result is smaller than a kind of 15+12c/19+12c, and that it first appears at an operational route of 15+12c/ 19+12c, then, let us term the operational result “№1 satisfactory operational result”.

Since each kind of 15+12c/19+12c derives from a №1 satisfactory operational result, and that the former is greater than the latter always, in addition, both of them coexist at an operational route of 15+12c/19+12c.

Thereupon, can conclude six kinds of 15+12c derived from six №1 satisfactory operational results at the bunch of operational routes of 15+12c monogamously to suit the conjecture, ut infra.

From c=2d+1 and d=2e+1, get c=2d+1=2(2e+1)+1=4e+3, then 15+12c= $51+48e=51+3\times2^4e\to154+3^2\times2^4e\to77+3^2\times2^3e\to232+3^3\times2^3e\to116+3^3\times2^2e$ $\to58+3^3\times2e\to29+27e$ where mark (1), and 29+27e < 51+48e, so we get that 51+48e suits the conjecture according to the theorem 1.

From c=2d+1, d=2e and e=2f+1, get c=2d+1=4e+1=4(2f+1)+1=8f+5, then $15+12c=75+96f=75+3\times2^5f\to 226+3^2\times2^5f\to113+3^2\times2^4f\to 340+3^3\times2^4f$ $\to170+3^3\times2^3f\to 85+3^3\times2^2f\to 256+3^4\times2^2f\to 128+3^4\times2^1f\to 64+81f$ where mark (2), and 64+81f < 75+96f, so we get that 75+96f suits the conjecture according to the theorem 1.

From c=2d, d=2e+1 and e=2f+1, get c=2d= 4e+2=4(2f+1)+2=8f+6, then $15+12c=87+96f=87+3\times2^5f\to262+3^2\times2^5f\to131+3^2\times2^4f\to394+3^3\times2^4f\to$ $197+3^3\times2^3f\to 592+3^4\times2^3f\to 296+3^4\times2^2f\to 148+3^4\times2^1f\to74+81f$ where mark (3) , and 74+81f < 87+96f, so we get that 87+96f suits the conjecture according to the theorem 1.

From c=2d+1, d=2e, e=2f, f=2g+1 and g=2h+1, get c=2d+1=4e+1= 8f+1=8(2g+1)+1=16g+9=16(2h+1)+9=32h+25, then 15+12c=315+384h= $315+3\times 2^7h\to 946+3^2\times 2^7h\to 473+3^2\times 2^6h\to 1420+3^3\times 2^6h\to 710+3^3\times 2^5h\to 355+3^3\times 2^4h\to 1066+3^4\times 2^4h\to 533+3^4\times 2^3h\to 1600+3^5\times 2^3h\to 800+3^5\times 2^2h \to 400+3^5\times 2^1h\to 200+243h$ where mark (4), and 200+243h < 315+384h, so we get that 315+384h suits the conjecture according to the theorem 1.

From c=2d, d=2e+1, e=2f, f=2g+1 and g=2h, get c=2d=2(2e+1)=4e+2= 8f+2=8(2g+1)+2=16g+10=32h+10, then 15+12c=135+384h= $135+3\times 2^7h \to 406+3^2\times 2^7h\to 203+3^2\times 2^6h\to 610+3^3\times 2^6h\to 305+3^3\times 2^5h\to 916+3^4\times 2^5h\to 458+3^4\times 2^4h\to 229+3^4\times 2^3h\to 688+3^5\times 2^3h\to 344+3^5\times 2^2h\to 86+243h$ where mark (5), and 86+243h<135+384h, so we get that 135+384h suits the conjecture according to the theorem 1.

From c=2d, d=2e, e=2f, f=2g and g=2h, get c=2d=32h, then 15+12c= $15+384h=15+3\times 2^7h\to 46+3^2\times 2^7h\to 23+3^2\times 2^6h\to 70+3^3\times 2^6h\to 35+3^3\times 2^5h \to 106+3^4\times 2^5h\to 53+3^4\times 2^4h\to 160+3^5\times 2^4h\to 80+3^5\times 2^3h\to 40+3^5\times 2^2h\to 10+243h$ where mark (6), and 10+243h<15+384h, so we get that 15+384h suits the conjecture according to the theorem 1.

Secondly, let us continues to operate 19+12c by the operational rule, as listed below.

19+12c→58+36c→ 29+18c→ 88+54c→ 44+27c ♣

d=2e: 11+27e (**α**)　　　　e=2f:37+81f (**β**)
♣ 44+27c↓→c=2d: 22+27d↑→d=2e+1:148+162e→74+81e↑→e=2f+1:466+486f ♥
c=2d+1: 214+162d→107+81d↓→d=2e:322+486e ♠
d=2e+1:94+81e↓→e=2f:47+81f (γ)
e=2f+1:175+162f ♦

g=2h: 119+243h (**δ**)　　　　…
f=2g+1:238+243g↑→g=2h+1:1504+1458h→752+729h↑→…
♥466+486f→233+243f↑→f=2g:700+1458g→350+729g↓→g=2h+1:3238+4374h↓
g=2h: 175+729h↓→…　…
…

g=2h+1:172+243h (**ε**)
f=2g: 101+243g↑→g=2h: 304+1458h→…
e=2f+1:202+243f↑→f=2g+1:1336+1458g→…
♠322+486e→161+243e↑→e=2f:484+1458f→…

♦175+162f→263+243f↓→f=2g:263+486g→…
f=2g+1: 253+243g↓→g=2h+1: 248+243h (ζ)
g=2h: 253+486h→…

Like that, we can conclude six kinds of 19+12c derived from six №1 satisfactory operational results at the bunch of operational routes of 19+12c monogamously to suit the conjecture, ut infra.

From c=2d and d=2e, get c=2d=4e, then $19+12c=19+48e=19+3\times 2^4e\to 58+3^2\times 2^4e\to 29+3^2\times 2^3e\to 88+3^3\times 2^3e\to 44+3^3\times 2^2e\to 22+3^3\times 2e\to 11+27e$ where mark (α), and 11+27e < 19+48e, so we get that 19+48e suits the conjecture according to the theorem 1.

From c=2d, d=2e+1 and e=2f, get c=2d = 2(2e+1) = 4e+2=8f+2, then $19+12c= 43+96f= 43+3\times 2^5f\to 130+3^2\times 2^5f\to 65+3^2\times 2^4f\to 196+3^3\times 2^4f\to 98+3^3\times 2^3f\to 49+3^3\times 2^2f\to 148+3^4\times 2^2f\to 74+3^4\times 2^1f\to 37+81f$ where mark (β), and 37+81f < 43+96f, so we get that 43+96f suits the conjecture according to the theorem 1.

From c=2d+1, d=2e+1 and e=2f, get c=2d+1=4e+3=8f+3, then 19+12c $=55+96f=55+3\times2^5f\rightarrow166+3^2\times2^5f\rightarrow83+3^2\times2^4f\rightarrow250+3^3\times2^4f\rightarrow125+3^3\times2^3f$ $\rightarrow376+3^4\times2^3f\rightarrow188+3^4\times2^2f\rightarrow94+3^4\times2^1f\rightarrow47+81f$ where mark (γ), and 47+81f < 55+96f, so we get that 55+96f suits the conjecture according to the theorem 1.

From c=2d, d=2e+1, e=2f+1, f=2g+1 and g=2h, get c=2d=2(2e+1)= 4e+2 =4(2f+1)+2=8f+6=8(2g+1)+6=16g+14=32h+14, then 19+12c=187+384h $=187+3\times2^7h\rightarrow562+3^2\times2^7h\rightarrow281+3^2\times2^6h\rightarrow844+3^3\times2^6h\rightarrow422+3^3\times2^5h$ $\rightarrow211+3^3\times2^4h\rightarrow634+3^4\times2^4h\rightarrow317+3^4\times2^3h\rightarrow952+3^5\times2^3h\rightarrow476+3^5\times2^2h$ $\rightarrow238+3^5\times2^1h\rightarrow119+243h$ where mark (δ), and 119+243h < 187+384h, so we get that 187+384h suits the conjecture according to the theorem 1.

From c=2d+1, d=2e, e=2f+1, f=2g and g=2h+1, get c=2d+1=4e+1= 4(2f+1)+1=8f+5=16g+5=16(2h+1)+5=32h+21, then 19+12c = 271+384h= $271+3\times2^7h\rightarrow814+3^2\times2^7h\rightarrow407+3^2\times2^6h\rightarrow1222+3^3\times2^6h\rightarrow611+3^3\times2^5h\rightarrow$ $1834+3^4\times2^5h\rightarrow917+3^4\times2^4h\rightarrow2752+3^5\times2^4h\rightarrow1376+3^5\times2^3h\rightarrow688+3^5\times2^2h$ $\rightarrow344+3^5\times2^1h\rightarrow172+243h$ where mark (ε), and 172+243h < 271+384h, so we get that 271+384h suits the conjecture according to the theorem 1.

From c=2d+1, d=2e+1, e=2f+1, f=2g+1 and g=2h+1, get c=2d+1= 2(2e+1)+1= 4e+3= 4(2f+1)+3= 8f+7= 8(2g+1)+7= 16(2h+1)+15= 32h+31, then 19+12c = 391+384h = $391+3\times2^7h\rightarrow1174+3^2\times2^7h\rightarrow587+3^2\times2^6h\rightarrow$ $1762+3^3\times2^6h\rightarrow881+3^3\times2^5h\rightarrow2644+3^4\times2^5h\rightarrow1322+3^4\times2^4h\rightarrow661+3^4\times2^3h\rightarrow$ $1984+3^5\times2^3h\rightarrow992+3^5\times2^2h\rightarrow496+3^5\times2^1h\rightarrow248+243h$ where mark (ζ), and

248+243h < 391+384h, so we get that 391+384h suits the conjecture according to the theorem 1.

It is obvious that if n+1 belongs within any kind of 15+12c/19+12c derived from a №1 satisfactory operational result, such as preceding 51+48e, 75+96f, 87+96f, 315+384h, 135+384h, 15+384h, 19+48e, 43+96f, 55+96f, 187+384h, 271+384h or 391+384h, then n+1 suits the conjecture according to the theorem 1.

Thus it can seen, that following two laws accord with objective reality.

Firstly, each kind of 15+12c/19+12c derived from a №1 satisfactory operational result and the №1 satisfactory operational result coexist at an operational route of 15+12c/19+12c, and that both of them share a variable.

Secondly, the greatest exponent of factor 2 of coefficient of variable within each kind of 15+12c/19+12c is exactly the number of times that divided by 2 in the operational course from the kind of 15+12c/19+12c to №1 satisfactory operational result.

Thereinafter, let us further explain the relation of mutual dependence of №1 satisfactory operational results and kinds of 15+12c/ 19+12c at the bunch of operational routes of 15+12c/ 19+12c.

It is observed that variables d, e, f, g, h … etc. of integer's expressions appear at the bunch of operational routes of 15+12c/ 19+12c, in fact, the purpose which substitutes c by them is that in order to avoid the confusion and for convenience. Thus, let χ represent intensively variables d, e, f, g, h … etc., but χ can not represent the variable c directly.

Evidently, the kind of 15+12c/ 19+12c which contains n+1 can be written into $(n+1)+3\times2^{y}\chi$ in accordance with the above discussions, where y is a exponent of 2, and $y > 4$.

After substitute variables d, e, f, g, h … etc by variable χ, the odevity of part integer's expressions that contain the variable χ at operational routes of 15+12c/19+12c is still indeterminate. Or rather, for every such integer's expression, both regard it as an odd number to operate, and regard it as an even number to operate. Thus, we label such integer's expressions "odd-even expressions".

For any odd-even expression at operational routes of 15+12c/19+12c, two kinds of operations synchronize at itself. After regard an odd-even expression as an odd number to operate, get a greater operational result>itself. Yet, after regard it as an even number to operate, get a smaller operational result< itself. Moreover, after pass operations for each odd-even expression, would add to an operational route inevitably.

Begin with any odd-even expression to operate continuously by the operational rule, every such operational route via consecutive greater operational results will be getting longer and longer, and that in general, the sum of the constant term plus the coefficient of χ of integer's expression is getting greater and greater along continuation of operations.

On the other, for a smaller operational result in synchronism with a greater operational result, if it can be divided by 2^{μ} with $\mu \geq 2$ to get an even smaller integer's expression, once the even smaller integer's expression is first smaller than a kind of 15+12c/19+12c, then the even smaller integer's expression is exactly №1 satisfactory operational result. Accordingly, it can derive a kind of 15+12c/19+12c from the №1 satisfactory operational result, so operations at the operational route may stop at here.

If the even smaller integer's expression is greater than any kind of 15+12c/ 19+12c still, or the smaller operational result itself is an odd expression, then it needs us to operate continuously.

By this token, on the one hand, the number of operational routes at the bunch of operational routes of 15+12c/ 19+12c increases continually; on the other hand, the number of operational routes at the bunch of operational routes of 15+12c/ 19+12c reduces continually.

Since every №1 satisfactory operational result can only appear at an operational route of 15+12c/ 19+12c, and every operational route of 15+12c/ 19+12c is an embranchment of the bunch of operational routes of 15+12c/ 19+12c, this shows that every №1 satisfactory operational result must exist at the bunch of operational routes of 15+12c/ 19+12c.

Yet every kind of 15+12c/ 19+12c can only derive from a №1 satisfactory operational result, so every №1 satisfactory operational result determines a kind of 15+12c/ 19+12c.

It is observed that every operational route from a kind of 15+12c/ 19+12c to the №1 satisfactory operational result contains a part of an operational route therein, at the bunch of operational routes of 15+12c/19+12c.

After start with 15+12c/ 19+12c to operate continuously, every gotten two operational routes intersect from each other undoubtedly. Namely every two operational routes at the bunch of operational routes of 15+12c/ 19+12c intersect from each other.

Therefore, an operational route gotten after operate each of kinds of 15+12c/ 19+12c, it intersects with any operational route except for itself at the bunch of operational routes of 15+12c/ 19+12c surely, including each of operational routes which contain №1 satisfactory operational results monogamously. Or else, the operational route exists not at the bunch of operational routes of 15+12c/19+12c, of course can't get a №1 satisfactory operational result, also derive not a kind of 15+12c/ 19+12c.

Since c≥1, then there are infinitely many kinds of 15+12c/19+12c, accordingly there are infinitely many operational routes of 15+12c/ 19+12c monogamously. Well then, there should be a №1 satisfactory operational result at every such operational route of 15+12c/19+12c.

Start with any confirmable kind of 15+12c/19+12c to operate by the operational rule, in a manner of speaking, operations in regard to it are possessed of finite length surely, because either there are some odd-even expressions at the operational route in regard to it, to lead to come into being smaller and smaller integer's expressions, up to №1 satisfactory operational result; or it is operated to intersect another operational route which contains smaller operational result < the confirmable kind itself.

Now that n+1 belongs within a kind of 15+12c/19+12c, of course, not excepting the kind of 15+12c/19+12c which contains n+1, either it is operated into №1 satisfactory operational result like each of listed above twelve kinds of 15+12c/19+12c to use the theorem 1; or it is operated to intersect another operational route which has an operational result<n+1 to use the theorem 2, like as $19+12(5+2^3j) = 79+2^5\times3j \rightarrow 238+2^5\times3^2j \rightarrow 119+2^4\times3^2j\rightarrow 358+2^4\times3^3j\rightarrow 179+2^3\times3^3j\rightarrow 538+2^3\times3^4j\rightarrow 269+2^2\times3^4j\rightarrow 808+2^2\times3^5j\rightarrow 404+2\times3^5j \rightarrow 202+3^5j\leftarrow67+3^4j\leftarrow\ldots$

Therefore, if n+1 belongs within a kind of 15+12c/19+12c, then n+1 suits the conjecture, because the kind of 15+12c/19+12c has been proved to suit the conjecture.

Hereto, n+1 has been proved to suit the conjecture, no matter which genus, which sort or which kind of odd numbers that contains n+1, or n+1 is exactly an even number.

Likewise, we can prove positive integers n+2, n+3, n+4 and otherwise up to every positive integer to suit the conjecture in the light of the old way of doing things.

The proof was thus brought to a close. As a consequence, the Collatz conjecture is tenable.

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